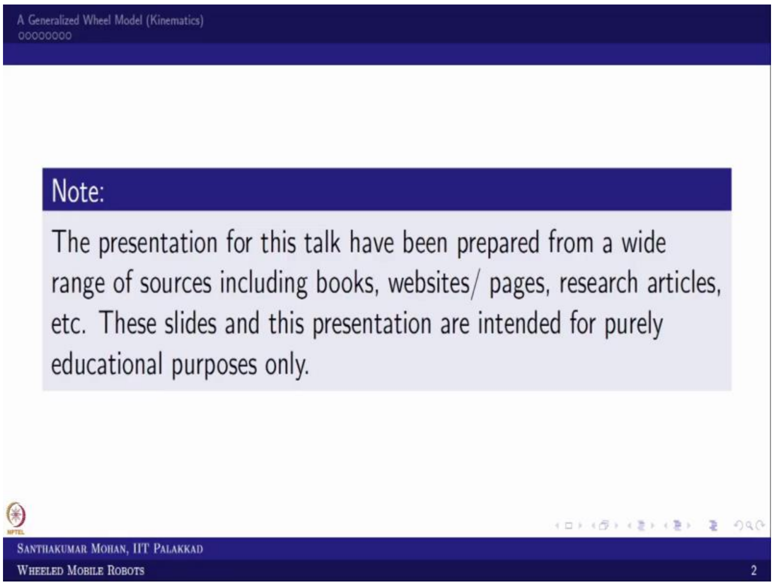
Wheeled Mobile Robots
Prof. Santhakumar Mohan
Department of Mechanical Engineering
Indian Institute of Technology, Palakkad

Lecture - 07
A Generalized Wheel (Kinematic) Model

Yeah. Welcome back to the course on Wheeled Mobile Robots. So, this particular lecture called lecture 7, A Generalized Wheel Kinematic Model. We are going to derive in this particular lecture. So, last class itself I gave a introduction. So, in the next class we would be talking about wheel that too like generalized wheel model. So, this particular lecture is going to talk about all.

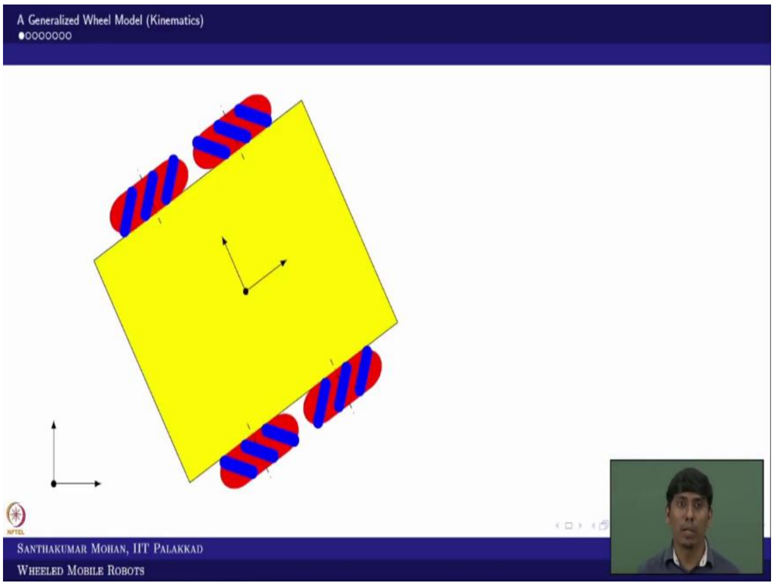
(Refer Slide Time: 00:33)



(Refer Slide Time: 00:35)

So, in that sense what we are trying to cover here is very simple segment as generalized wheel model that too like what we are covering here is a kinematic model. So, in that sense what one can see? So, you know there are four types of wheel we have seen in the lecture 4. So, where we have talk about the solid wheel; in the solid wheel where we are talk about conventional and unconventional or non conventional wheel.

(Refer Slide Time: 00:59)

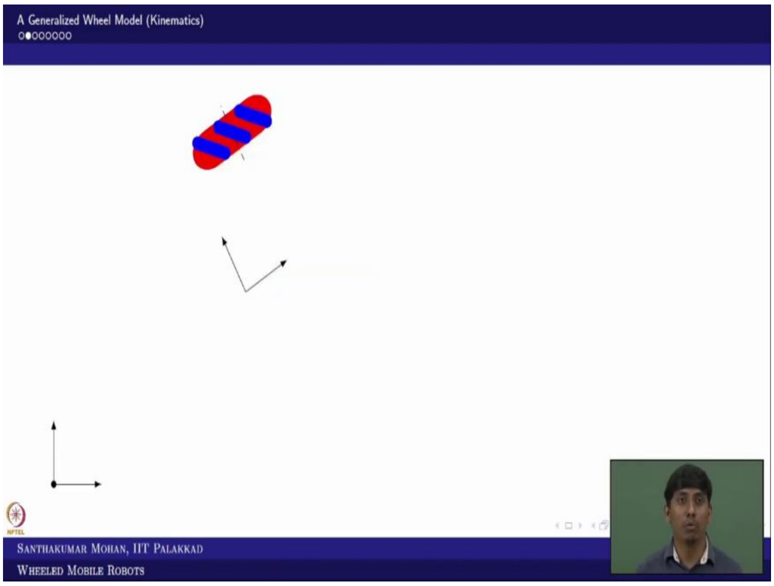


So, I am taking a generalized wheel model; I am taking a very complex one which is nothing but the mecanum wheel; where the mecanum wheel there is a passive roller which would be in inclined manner. So, this is the complex one where the generalized wheel model is mainly required for to derive this particular wheel system.

So, in that sense I am taking one of the mobile robot which has four mecanum wheel. So, in the sense the degree of maneuverability is actually like four, but what we are trying to do?

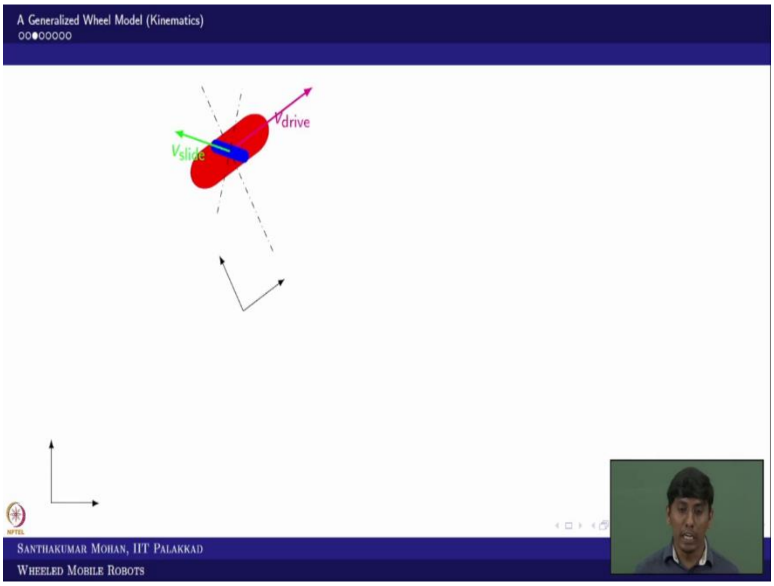
We are trying to derive one particular wheel where the angular velocity which is going to be written in the form of; so, your body fixed velocity in the sense at instantaneously what would be the u v r; would give the relation to the omega i. So, that is what we are trying to derive.

(Refer Slide Time: 01:44)



So, for that what we are trying to take? We are trying to take only one wheel. So, in the sense this is actually like what you call the body frame and this is the wheel which I am going to take here ok. So, this particular course we are taking one wheel at a time.

(Refer Slide Time: 01:56)



So, now if you take this particular wheel, I am taking away the roller. So, now you assume that only wheel is there, there is no passive roller then what one can see based on the pure rolling? So, what one can see? So, the wheel if it is having a angular velocity of ω, so, then that would be having a tangential velocity of rω.

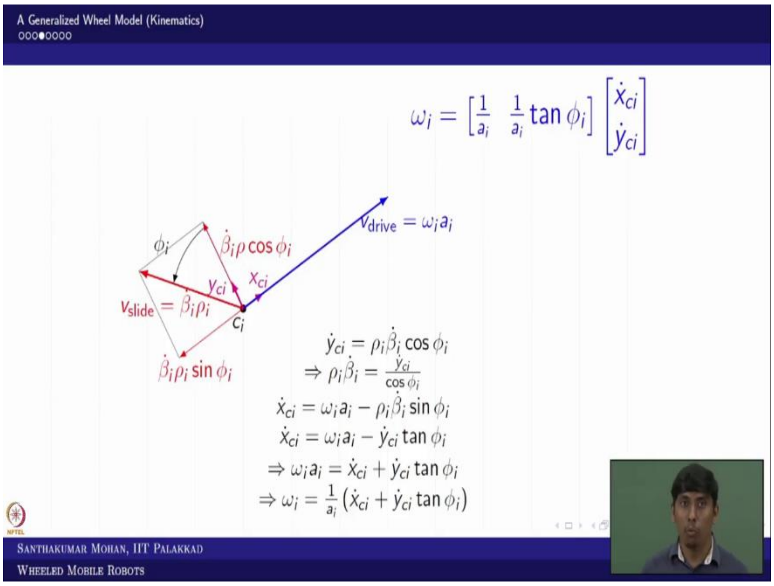
Since r we have used as the angular velocity of the vehicle, so, I am taking w is the radius of the wheel. So, then a ω would be the tangential velocity. Since it is a pure rolling, so, there is no lateral slip. In the sense there is no tangential, you can say there is no normal velocity in the sense there is no lateral slip. So, only tangential velocity would be there, then simply a ω would be directly called as V_{drive}.

So, the V_{drive} what I can write as a ω_i, but before that let me actually like brought. So, this is what V_{drive}. Now I brought the roller. So, now, this passive roller axis is actually like look like this. In the sense the blue is actually like rotating in this way, you can say across the wheel. So, the wheel is red colour patch where the blue is actually like passive roller; the roller is actually like rolling on the across the wheel.

So, now in that sense, so, what one can see? This wheel would be actually like giving a V_{drive}, but this V_{drive} would be taken away by the what you call the passive roller. Then what it allows? There is a slip it would be allowed. The slip can go either upside or you can say downward direction. I am taking it is actually like happening in upward direction, there is a reason ok.

Now, that particular velocity, I am calling V_{slide} because it is sliding on the lateral plane. So, now, I need to bring some kind of coordinate right. Otherwise what happen? V_{drive} in one axis and V_{slide} is in the inclined axis.

(Refer Slide Time: 03:46)



So, for understanding that, what I am bringing it? I am bringing it as actually like a small point which is I call center of this wheel. So, what I call this point? I am calling as a c. So, since it is i^{th} wheel I am calling that is actually like c_i.

Now, the c_i is actually like going to have a coordinate frame. So, for that what I am trying to see? So, this is what the V_{drive} axis. So, that V_{drive} axis I am going to call as x_{ci} ok. So, now, this x_{ci} is actually like this as per the right hand rule, the Y_{ci} is actually like upper direction right.

So, now this is what we brought it. So, now, I am bringing the V_{drive} and V_{slide}. So, what I can actually like know? This V_{drive} is actually like I already told. So, ω_i is the angular velocity of the wheel and ai is the radius of the wheel.

So, since it is a pure you can say rolling, the V_{drive} I can written as ωiai. But what happened to the V_{slide}? The V_{slide} also like pure rolling as per our understanding; because we have taken as an independent aspect so, where we assume that the wheel is independently taken and as well as roller also independently taken.

In that sense the $\dot{\beta}_i$ is my rotational velocity of the passive roller and rho i is the radius of the passive roller then $\dot{\beta}_i \rho_i$ is the V_{slide}. Now, you got V_{slide} and V_{drive}, but the V_{slide} is actually like not you can say actuated velocity. It is actually like passively obtained due to the V_{drive}.

So, then we can actually bring the relation between V_{drive} and V_{slide}. For that I am bringing one of the angle with respect to Y_{ci}. So, the Y_{ci} to V_{slide} what I am going to call as ϕ_i.

So, now, based on the ϕ_i what i can bring it? I can actually like decompose this V_{slide} into two component. One would be in Y_{ci} axis. So, one would be in X_{ci} axis right. So, this is what would be along with the X_{ci} direction which is $\dot{\beta}_i \rho_i \sin \phi_i$. So, the other one is what I can see? It is $\dot{\beta}_i \rho_i \cos \phi_i$.

So, now in that sense what one can brought? So, still you are actually like giving it $\dot{\beta}_i \rho_i$ right. So, I hope there is a small typo it is written as ρ. So, you can actually like see it. So, now, if you rewrite this equation, so, what happened the c_i would be having a you can say longitudinal velocity and lateral velocity. The lateral velocity I am calling as Y_{ci} $\dot{\alpha}$; that would be definitely equal to $\dot{\beta}_i \rho_i \cos \phi_i$ right.

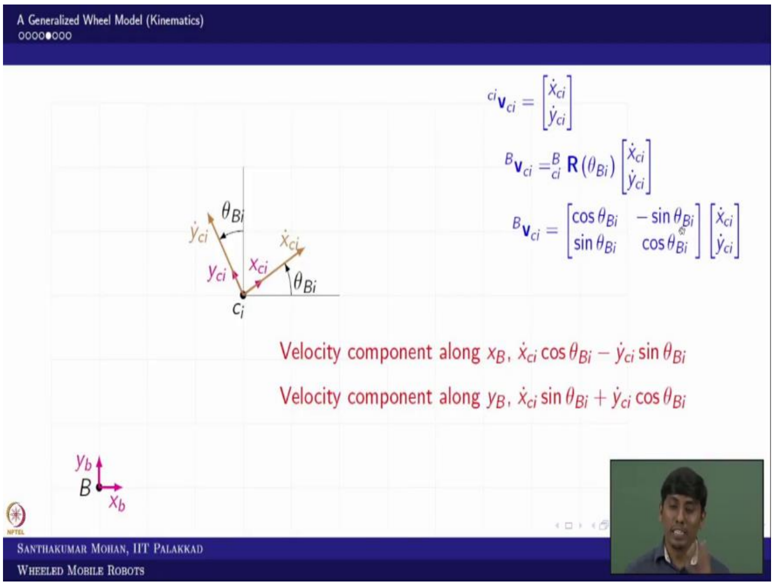
So, now similarly if I take it as a $\dot{x}_{ci}$, what would be that? That would be the vector addition of vector addition of $\omega_i a_i$ - $\dot{\beta}_i \rho_i \sin \phi_i$. But from the previous equation, what one can understand that $\dot{\beta}_i \rho_i$; I can rewrite as $\rho_i \dot{\beta}_i = \frac{\dot{y}_{ci}}{\cos \phi_i}$. So now why that is brought in?

Now you can see that the $\dot{\beta}_i \rho_i$, I can take away from the equation. So, what I can bring it? So, $\dot{x}_{ci}$ I can write as $\omega_i a_i$ - $\dot{y}_{ci} \tan\phi_i$. So, why it is required? Because we are thinking about writing the angular velocity of the wheel which is what you call V_{drive}; so, that is actually like bringing it in one sense.

So, what I can write? So, the V_{drive}, I can write as $\dot{x}_{ci} + \dot{y}_{ci} \tan \phi$. So, this is one equation. So, based on that what I can bring it? So, the ω_i, I can rewrite and this form. So, this form is actually like very much beneficial why because this I can even rewrite as you can say a column vector and a row vector I can make it.

In the sense you can see the ω_i, I can write as $\omega_i = \frac{1}{a_i}(\dot{x}_{ci} + \dot{y}_{ci} \tan\phi_i)$ as one of the you call row vector. And there is another one is actually a column vector, I can rewrite. Now this go into helpful in the further slides, why?

(Refer Slide Time: 07:57)



You can see now this is what you call the c_i frame. This ci frame is having a velocity longitudinal which is I call a $\dot{X}_{ci}$, lateral I call $\dot{Y}_{ci}$. This I can actually like bring it with a vehicle frame. What that mean? So, I am bringing the vehicle frame as B. For convenient I am assuming that the V like B frame is actually like a parallel to xy plane of the board. So, now, X_b and Y_b is given.

So, now what one can see? If I know the angle between X_b and X_{ci}, I call that as a θ_{Bi}. So, what when can I find it? I can resolve this $\dot{X}_{ci}$ and $\dot{Y}_{ci}$ along with X_b and Y_b right. So, this is what I am doing it. So, for that before doing this, I am just writing as.

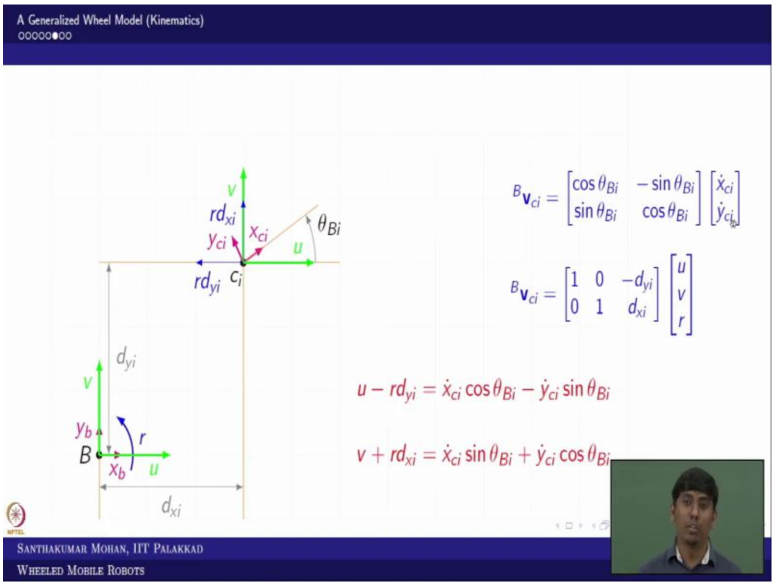
So, the V is a vector which is actually like a call linear velocity vector of ci with a frame of ci that easily written as $\dot{X}_{ci}$ and $\dot{Y}_{ci}$. But what I wanted here the V_{ci} with respect to B frame ok. So, for that you know already we did rotational you can say matrix which is we call transformation matrix in general. So, this matrix we got it, but just you do the transformation ok.

So, before that I just resolve this $\dot{X}_{ci}$ and $\dot{Y}_{ci}$ projected on the B frame. So, what I will get? Velocity component along X_B, what would be coming? $\dot{X}_{ci}\cos\theta_{Bi}$ - $\dot{Y}_{ci}\sin\theta_{Bi}$. Similarly, if

I project that $\dot{Y}_{ci}$ in the B frame so, then what you will get? The velocity component along with Y_B would be $\dot{X}_{ci}\sin\theta_{Bi} + \dot{Y}_{ci}\cos\theta_{Bi}$ right.

So, if you brought this you bring it back what would be the V_{ci} with respect to B frame? So, this is what we are going to get. So, now what one can see? Already you have given a relation which is ω_i as one relation of $\dot{X}_{ci}$ and $\dot{Y}_{ci}$, but now you are actually like making much more you can say simplicity right.

(Refer Slide Time: 10:04)



So, now we go little further. I assume that the B frame and c_i frame are actually like part of a single body because the vehicle is actually like connecting the wheel. So, wherever the vehicle goes, the wheel also go. In the sense if B rotated c_i also like parallely rotated along with the B frame.

In the sense what one can see? If the B frame is having instantaneous velocity of $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$, what would be that equivalent velocity at point c? Here actually like ith point. So, what would be the equivalent velocity which happened due to the $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$ at point c_i? So, for that what we can see? We can just directly brought. For understanding much more, so, I am bringing out the coordinates.

So, the c_i is actually like away from B in x axis d_{xi} and y axis d_{yi}, already I know that θ_{Bi} is the angle between x_B and x_{ci}. So, now why this two other coordinates brought? Because the B frame is having an angular velocity of r so, which gives the tangential velocity at point c_i so that is what we are bringing.

So, if you see the longitudinal velocity and lateral velocity, whenever the vehicle is moving so, the same velocity would be across all the point. In addition to that the angular velocity will give tangential component. So, these are the two tangential component will come.

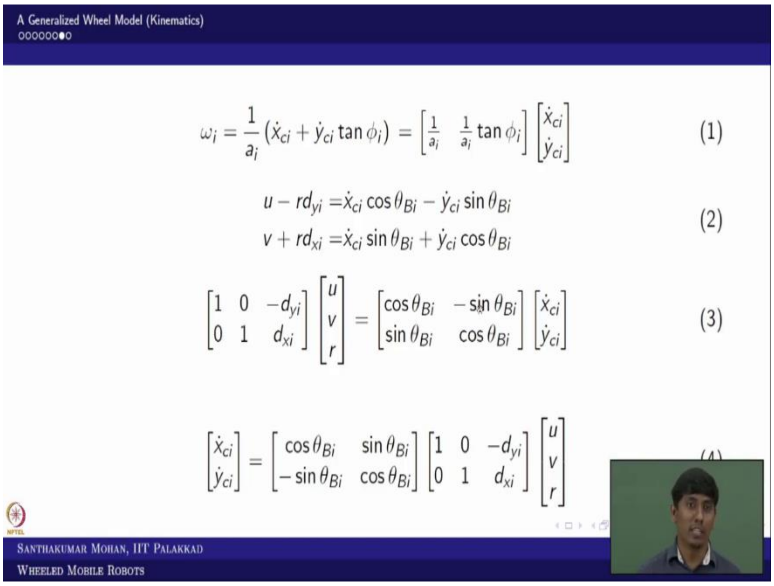
So, now rd_{yi} will give in x axis, but it is opposite direction because the rotation of r is actually like counterclockwise. So, in that sense the distance you call you can see the d_{yi} and r would give you the tangential velocity towards you can say opposite direction of x_B whereas, the $d_{xi} \times r$ would be giving the same direction of you can say y_B; you can see right.

So, now in that case what one can see? So, $u - rd_{yi}$ would be equivalent to what you derived already; so, the velocity along x_B direction. So, what it is actually like? $V + rd_{xi}$ would be giving the velocity component along y_B direction, right. This is what we obtained in the previous slide, but what we obtained right now?

$u - rd_{yi}$ supposed to be equal to the $\dot{X}_{ci} \cos \theta_{Bi} - \dot{Y}_{ci} \sin \theta_{Bi}$. Similarly so, the $V + rd_{xi}$ supposed to be equal to $\dot{X}_{ci} \sin \theta_{Bi} + \dot{Y}_{ci} \cos \theta_{Bi}$. So, why this particular relation is required here? You can actually see we have already achieved what we wanted. What we wanted?

We wanted omega i in the form of $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$. So, now, you rewrite this in a matrix form, you can see that the V_{ci} with respect to B, I can write in terms of $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$. In the same way, I can write in terms of $\dot{X}_{ci}$ and $\dot{Y}_{ci}$. Already you know like ω_i; we have written in this form.

(Refer Slide Time: 13:04)

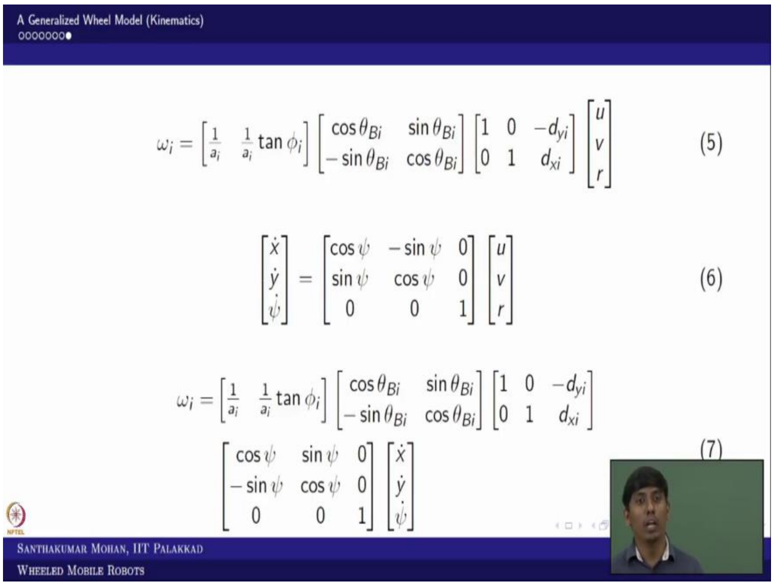


So, I brought that equation. So, this is the equation which we derived in the very beginning. So, now, based on that you have written in a matrix and vector form right. So, now, this particular omega i in the form of $\begin{bmatrix} \dot{X}\text{ci} \\ \dot{Y}\text{ci} \end{bmatrix}$, but what we know the relation. So, this relation we know. So, based on that what one can see?

The matrix and vector form, you can bring it. So, you can see that $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$ in one side and $\begin{bmatrix} \dot{X}\text{ci} \\ \dot{Y}\text{ci} \end{bmatrix}$ is in another side. So, now, this is a matrix. So, now, you take this matrix and invert this, what you will get? You will get $\begin{bmatrix} \dot{X}\text{ci} \\ \dot{Y}\text{ci} \end{bmatrix}$ would be getting this, vector you will be getting. So, that is what we are actually getting it.

So, you know this is actually like orthogonal matrix that to like orthonormal vectors in the sense this is just a transpose. So, you have done. So, now what you got it? $\begin{bmatrix} \dot{X}\text{ci} \\ \dot{Y}\text{ci} \end{bmatrix}$ is you obtained. So, now, you substitute these all in this first equation, what do you will get? ω i in the form of $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$.

(Refer Slide Time: 14:06)



This is what you want it. So, you got it now the generalized wheel model already obtained, where this is purely function of your wheel radius and your roller, you can say axis inclination with respect to y_B frame or y_{ci} frame and you can see that how much angle deviated from your c with respect to B that is here and how much distance it is deviated from c to B and your body fixed velocities right.

So, now you go further, what further? You know like $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$ can be written in the derivative of generalized coordinate in the form of $j(\Psi)\times\begin{bmatrix} u \\ v \\ r \end{bmatrix}$. So, now, you can actually like recall this equation into $\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\Psi} \end{bmatrix}$. So, that everything you can write in a single frame, no need to bring the body fix which is instantaneous frame. You can just keep it everything as simple with respect to the initial fixed frame.

So, in that sense what one can know? So, you take the inverse of this. So, here also it is orthonormal vectors, this matrix is orthogonal matrix. So, the inverse is just a transpose, you just do that and then you substitute instead of $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$; you substitute this inverse of this. So, this is what the matrix. So, now, what you obtain? You obtain the generalized wheel model which is nothing but a kinematic model you obtain.

So, you recall the lecture 4, where I have written as actual like η, which is the you call vector of you call input commands or you can say velocity of input commands which is nothing, but $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$ can be written as w multiply with you call the wheel angular velocities which is nothing, but vector of angular velocity which we written as ω.

So, in the sense the $\eta =$ w $\times$ ω that you can obtain when you know number of wheels where you can actually get ω_i, where the i vary from you can say 1 : n wheel; then you can actually like merge and you bring the w matrix which is what you call the wheel input matrix or input configuration matrix, whatever way you can call. With that the generalized wheel model is done.

So, the next class, we will do a few example with a generalized model; how we can derive the w matrix, how the w matrix can be categorized. So, based on the nature of or you can say based on nature of this particular w matrix, you can actually like define the robot whether the robot is actually like one kind or the other kind, you can actually like define.

So, in the sense the next lecture is more about examples in the sense you take different wheel configurations which we discussed in the lecture 4; where the degree of maneuverability, we have shown several examples right. So, those example we will brought in and we will derive the generalized wheel model and then we will actually like simulate in the end of this week.

So, with that so, I am saying thank you and see you in the next lecture. Bye.