Wheeled Mobile Robots
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Lecture - 40
Cascaded or Back-stepping Control of Mobile Robots

Welcome back to Wheeled Mobile Robot course. So, today what we are going to see like more you can say combination of what you have seen in the last two lectures. So, the previous lecture to previous what we have seen? We have seen like what is kinematic control of a mobile robot and in the previous lecture we have seen actually like how to do a dynamic control with you can say second order error dynamics.

So, the first order error dynamics we have seen as a kinematic control, the second order error dynamics we consider as a you can say dynamic control. So, like that kinematic and dynamic we combined and give as a called cascaded loop. So, that is what we call actually like double loop or cascaded control for a mobile robot that is what we are going to see in this particular lecture.

So, before going to see that this particular cascaded has another game called back-stepping. So, why it is called back-stepping that is also like we are going to see in dηil in this particular lecture.

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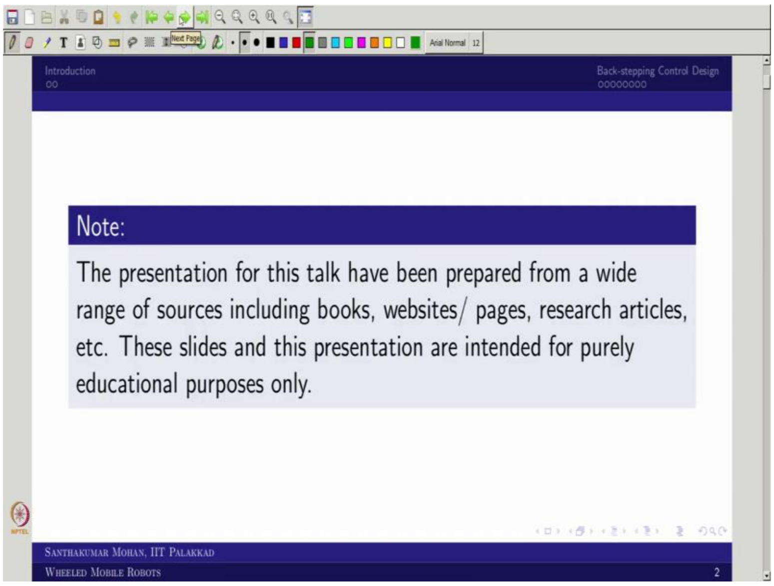

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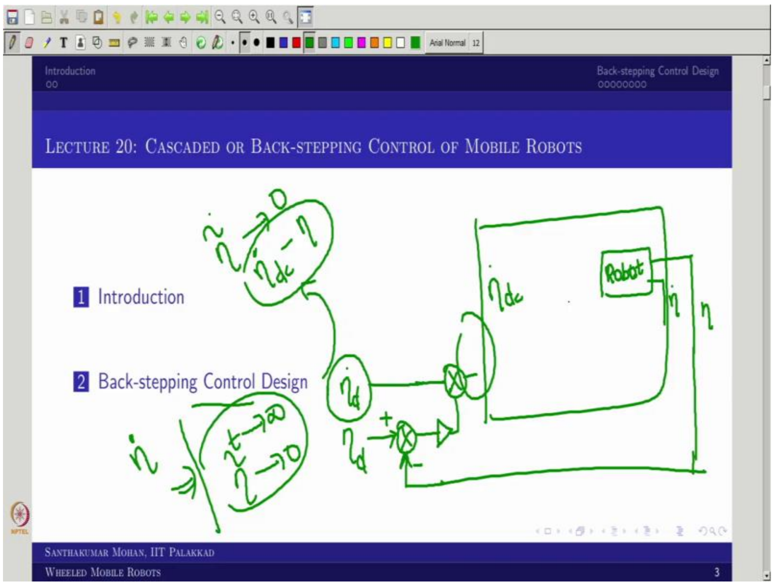

So, let us actually like move to the slide where we were actually like seeing about. So, what is back-stepping control design and how that is going to see? So, before going to see that let me brief introduce so, what we are going to see. So, for example, you assume that this is a robot ok the robot would be giving two things.

So, the robot would give actually like I assume that the velocity of this and position of this. So, I am taking this position as the outer loop ok. So, I assume that the η desired is known and I am trying to control that. So, this I am actually like trying to take as the prime most ok.

So, then what I am trying to see that the η desired dot also available to me. So, this also like I am actually like combining it. So, then what I can do it? So, I am trying to do actually like two stage control. So, where each stage I am controlling as single you can say state control.

So, in the sense what I am trying to do it? So, I will be writing this second order differential equation of the mobile robot, I will be writing in a two first order you can say state equation. So, what then I will do? So, first I will try to control this as 0 when t tends to infinity, then I will see that this would be achievable only with the second state we call η desired dot then the second case what I will take?

This $\tilde{\eta}_d \to 0$ where here I will not put this η desired you can say dot as it is like coming from this derivative ok. So, that is the change which we are trying to incorporate.

In the sense you can see like this is going to do one inner loop based on what you obtain that you call as $\dot{\eta}_d$, but this $\dot{\eta}_d$ combine with another one we are actually trying to do. So, this I will call as a control ok. So, then what you can see? This is outer and this is inner that is what we are trying to do in this particular lecture.

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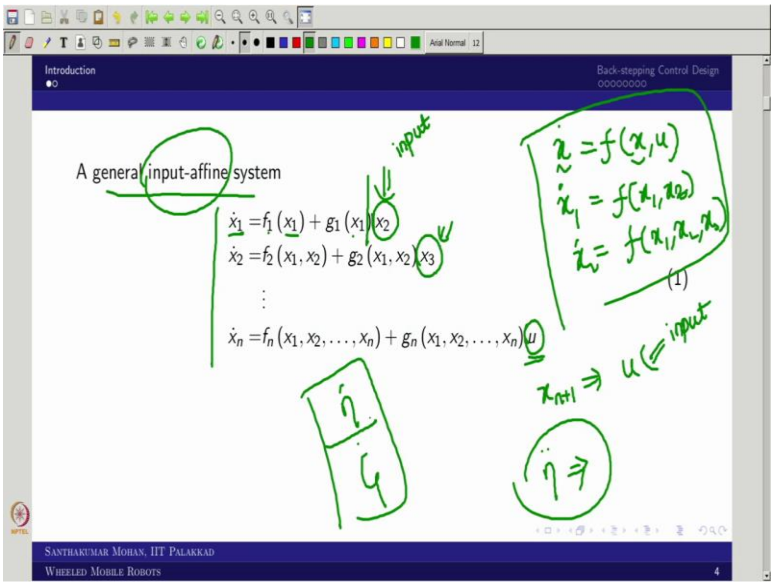

We will go in dηil along with the lecture. So, first for understanding this you know nth order system can be rewritten as n first order system right. So, now, if we write that is what you call state space equation, but generally what we usually write? So, $\dot{x}$ we will write as function of you can say x , u , t. I am just skipping that.

So, now, this is actually like vector, but similarly you write rewrite this nth order system into n first order system, then most of the people will write like this ok. So, $\dot{x}_2$ would be like function of x_1, x_2 and $x_3$like that, but this will not give any back stepping control design.

So, then what supposed to be for back stepping control design? The system supposed to be input affine. So, you know affine right. So, it is suppose to be input affine. So, in the sense every single state equation you write as a sub system, that subsystem has supposed to be having a independent control input you can see right.

So, this x dot is actually like function of x_1, but that is independent with x_2. So, now, the x_2 is actually like considered as a input for the overall $\dot{x}_1$ dot equation. So, similarly $\dot{x}_1$ you can see that x_3 as the independent input. So, like that you go at the end where $x_n + 1$ I am calling as a_u, this is what the original input ok this original input I am actually like rewriting.

Now, you see in that sense the nth order system I have rewritten as n you can say first order differential equation along with in such a way that it is input affine. Some people call it is a strictly feedback form, but you can take it either way, but it is a input affine system. So, now then you can actually like recall what we have derived 2 sub systems in the robotics. One is what you call kinematic model the other one is dynamic model right.

So, now, you can rewrite this into like this you can say second order differential equation in the form of η or what you can write $\dot{\eta}$ and $\dot{\xi}$. Now, what you can do? You can see that the second order differential equation I can write as a 2 you can say first order differential equations right.

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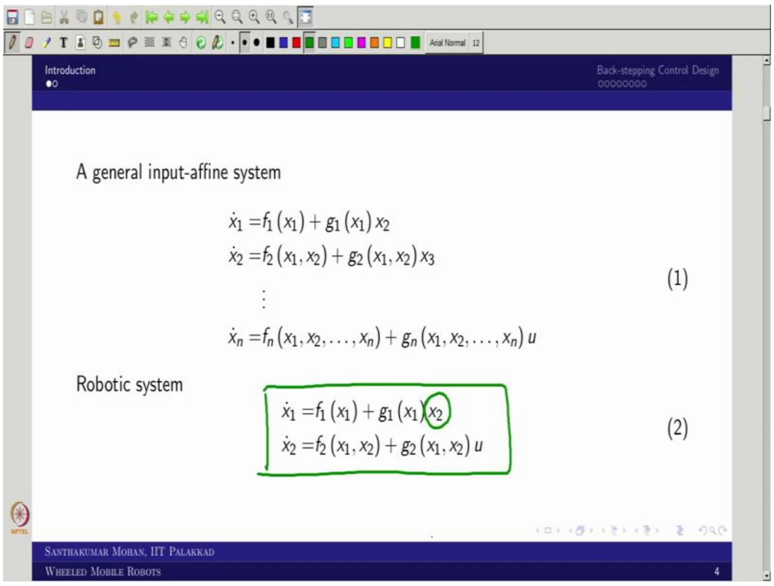

So, that is what we are rewriting you can see right. So, this is what I wanted the robotic system I am you know I know like it is a second order differential equation I am rewriting in the form of $\dot{x}_1$ and $\dot{x}_2$where x_2 is actually like the second system you can say state variable where u is the overall system input.

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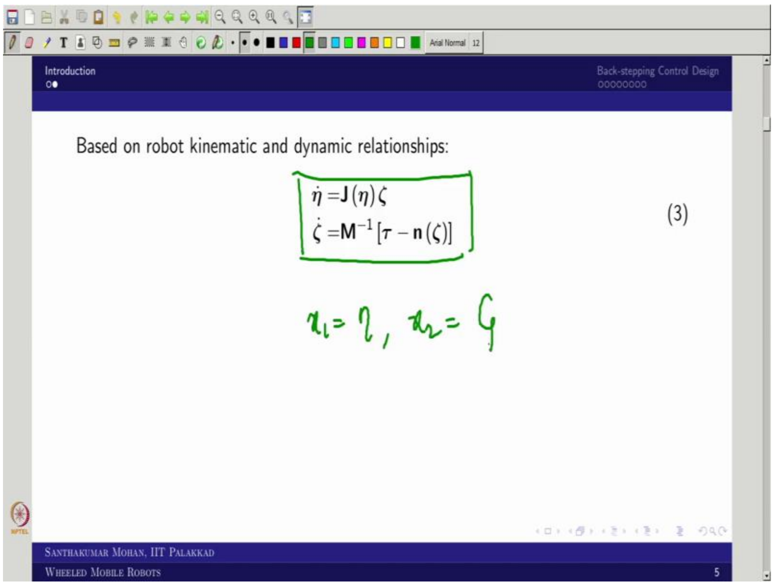

So, that is what we are trying to make it. So, for that we will recall this is what the equation we obtain in the kinematic and dynamic equation or derivation. So, now, we recall this into what we have seen in this way. So, then what I can write? x_1 I can write as η and x_2 I can write as a ξ right. So, then I can rewrite the remaining everything.

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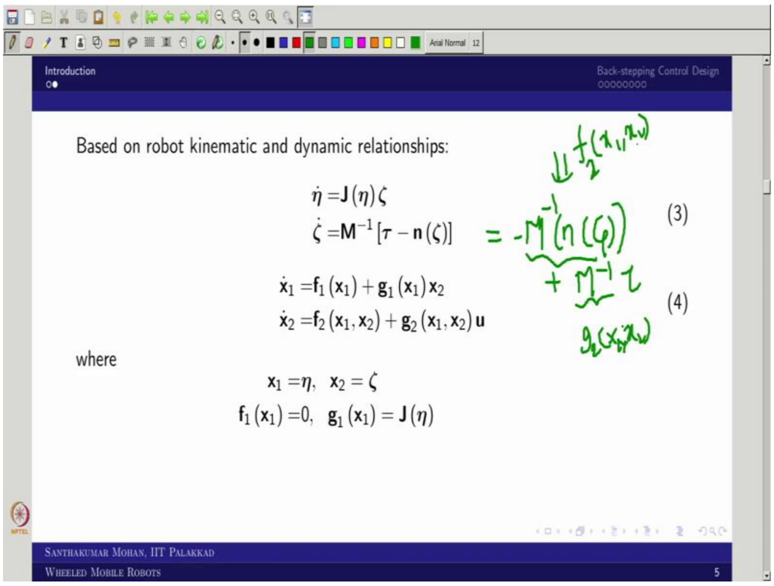

So, that is what I am writing in this form. So, now, I can see that I am recalling this equation recall x_1 as η and x_2 as ξ. So, now, I can actually like go again with what is f_1, x_1 in this case there is no function of η. So, then it is 0 right and what is you can look at is $g_1(x_1)$. So, this is actually like x_1 right η is x_1.

So, this is look like $g_1(x_1)$. So, in the sense what you can write $g_1(x_1)$ as what you call J(η). So, similarly you come to the second equation. So, the second equation I am rewriting as like this. So, then you can rewrite as Ʈ. So, you can see like this is $g_2(x_2)$, x_1 , x_2 and this would be you can say rewrite as. So, function of $f_2 x_1$, $x_2$right.

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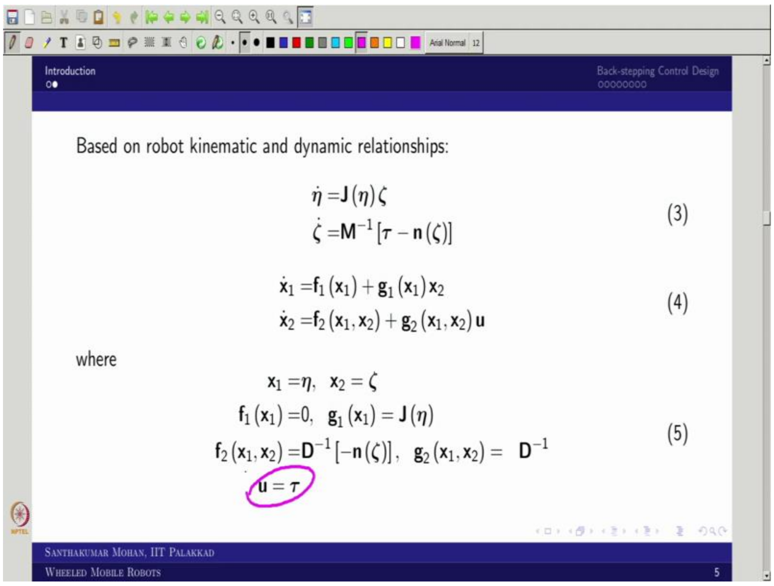

So, that is what I am actually like rewriting you can see that way right. So, then f_2, x_1 , x_2 is actually like this equation right. So, whereas, actually like this whole product would be your g(x_2) right $g_2(x_1, x_2)$. So, now, these all obtained. So, what you can see?

So, already you know this particular equation in the input affine form. So, now, you are actually like ready to use this particular model for the back stepping control design ok. So, now, you know the u is nothing but τ.

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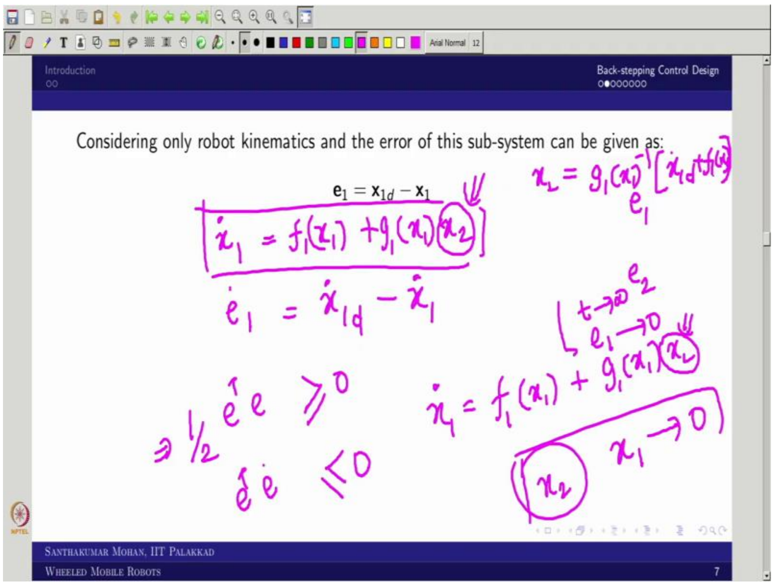

So, what is back stepping that we will see in the flow ok. So, now, you know like this is what we have derived.

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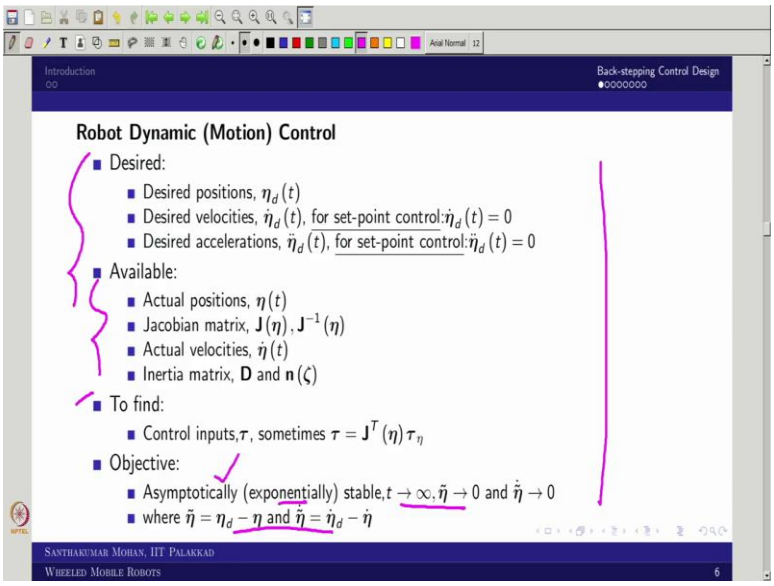

So, before that I will actually like recall what we have done in the motion control. So, this we have actually like assumed that these are actually like given and these are available and this is what we are finding with assumption that the t → ∝ where $\tilde{\eta}$ and $\dot{\tilde{\eta}}$ → 0 which is nothing but asymptotically or exponentially stable.

So, for that what we are actually like trying to write? So, now, there are two errors will come one is e_1, the other one is e_2 2 where e_1 would be the first system equation where $\dot{x}_1$ I am writing as. So, $f_1 x_1 +$ you can write $g_1 x_1 \times x_2$. If I choose x_2 properly where $x_1 \to 0$, right.

So, now here I am actually like rewriting that in the other way around if I choose this x_2, I can make this $e_1 \to 0$ when $t \to \propto$. So, this is a general, but I am bringing with a close loop. So, what then supposed to be? I have to choose x_2 in such a way that this equation actually like make it.

So, I am rewriting that equation. So, where $\dot{x}_1$ as $f_1 x_1 + g_1 x_1 \times x_2$ where I am differentiating this $\dot{e}_1$. So, why? I can actually like I can recall the Lyapunov stability; I can actually take the Lyapunov function as like this ok. So, this would be always positive when it would be 0 when e = 0. If I take the derivative of this then the derivative is actually like negative and negative definite.

So, then what I can say that would be what you call a stable system. So, now, for that I am taking the differentiation this. So, then what I will get? So, this. So, for that I am actually like trying to find out what is $\dot{e}_1$. So, $\dot{e}_1$ is actually like $\dot{x}_{d1} - \dot{x}_1$; already I know $\dot{x}_1$ is in this form right I substitute that.

So, what you can see this x_2 is the control input that will come into a picture. So, now, what would be the x_2? x_2 would be the choice where the $(g_1 x_1)^{-1}$ inverse ok multiply with all the other things, what all other things? So, you can see that $\dot{x}_{d1}$ ok. So, then plus $f_1 x_1$. So, these all actually like going to come right.

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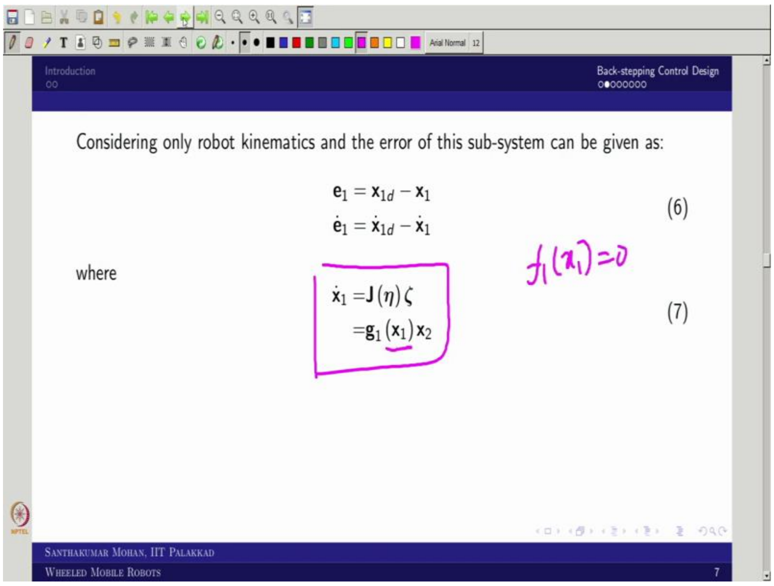

So, that is what we are also like trying to see. So, for that I am actually like taking this e 1 dot and I am actually like substituting this equation since it is actually like $f_1(x_1)$ is 0 in this case. So, straight away it is coming.

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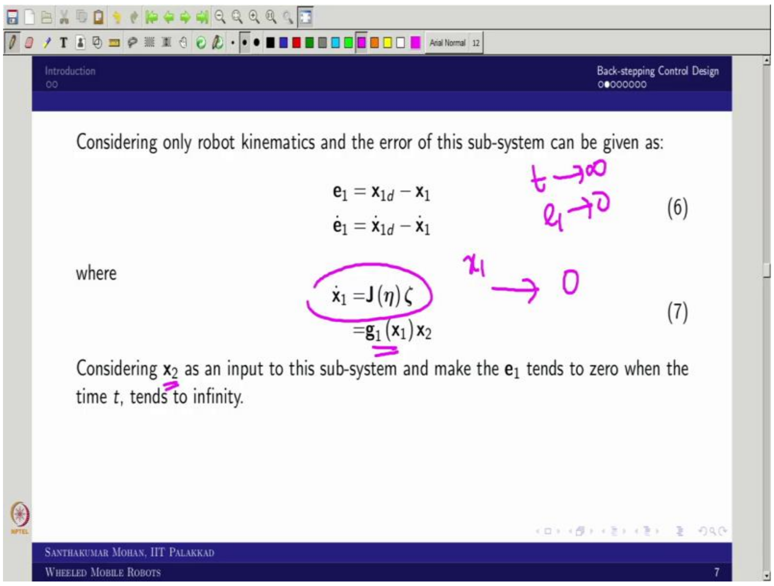

So, now I am actually like taking x_2 as the input and I am actually substituting as a subsystem then what I will get? So, this $x_1 \to 0$ right. So, in the sense $e_1 \to 0$ in our case.

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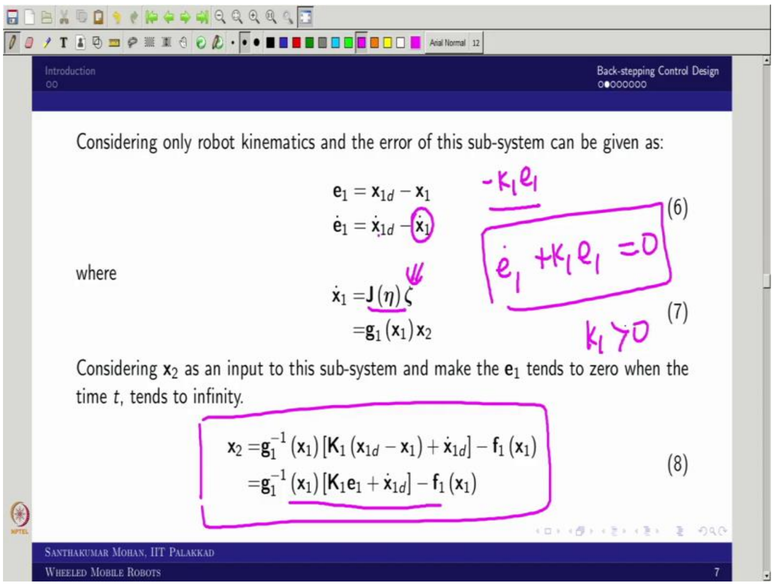

So, that is what we are actually like achieving it. So, for that I am choosing this x_2 in this form ok. So, now, for that what I am trying to bring? So, I am trying to bring the error dynamic since this way ok in order to find this error dynamics what I know this equation right.

So, here I choose x 1 dot in such a way that. So, this goes and it will give only $K_1 e_1$ as the residue that to negative sign then I am done, but what is $\dot{x}_1$ is in this form? So, now, this ξ I can choose in such a way that this is actually like happening. So, that is what we have done here where the simple first order error dynamics is actually like become 0. So, then you can see that as long this K_1 is positive. So, this particular case is done.

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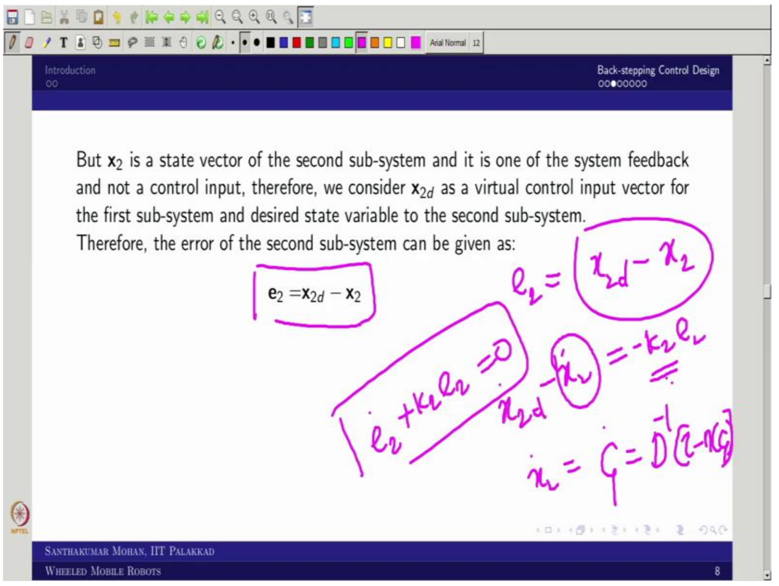

So, that is what we are actually like taken care here, but what the condition here is this x_2 is not a control input that you should be known. So, then what x_2? x_2 is the state vector right then what you have to do it? The x_2 you cannot actually like play anything right, but instead of that what you can do a play you can take x_{d2} desired as a virtual control input for that particular subsystem and then you can do it, then you can see that the x 2 desired and x 2 there would be a difference right.

This difference if it is 0, then you are actually like achieved x_{2d} as x_2 or x_2 is x_{2d} both are same then what you have seen in the previous? This is achieved right. So, in the sense what you are doing is actually like you are coming backward right. So, that is why it is called back stepping.

In the sense the x_2 you assume as a virtual input and you have done all the control strategy and then you are going the next system and once you are going the next system, then you are actually like seeing that no this is a state variable I have to actually like go back and change it.

So, that is what you are actually like going in the backward direction that is why it is called back stepping design. The design is based on you can say your previous state and then decide and then going further. Since it is actually like only second order system you have done only 2 step input right 2 stages where e_1 and e_2 you are calculating.

Whereas, if it is actually like nth order then you would be doing every stage n times right you are going in a backward direction. So, that is what we are actually like saying a back stepping. Now, this e_2 supposed to be 0 then what you can see? The e_2 can be 0 when this $\dot{x}_2$ if you take the $\dot{e}_2 + k_2 e_2 = 0$ right in the sense what you can see? This $\dot{x}_{2d} - \dot{x}_2$ you can say supposed to be equal to something like $k_2 e_2$ that is what you have to write right. So, that to like minus.

So, then what you have to see? You have to choose x_2 in such a way that this would be fulfilling what is x_2? x_2 is actually like you know this is $\dot{\xi}$ right where $D^{-1} \times (\tau - n(\xi))$ right. So, now, you can actually like substitute the τ in such a way that this would be fulfilling.

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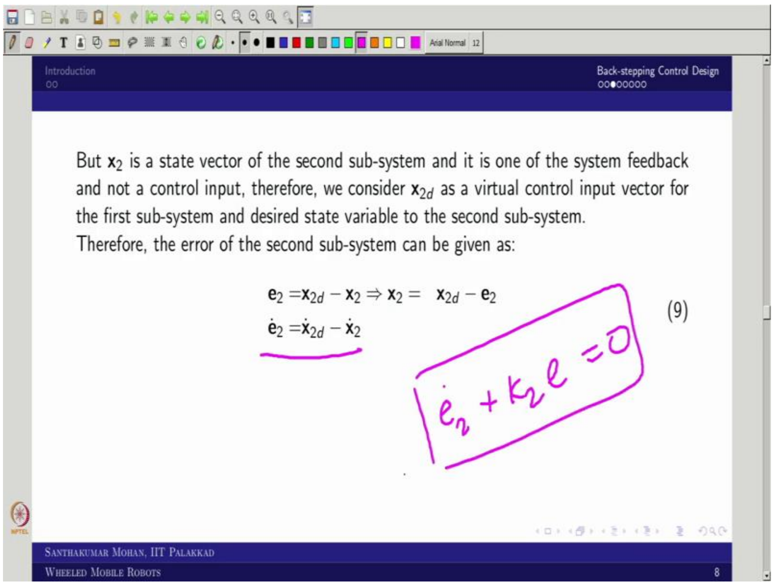

So, that is what the second loop all about, but what one supposed to know? You have taken x_2 as the input, but the x_2 is actually like difference of you can say x_2 - error this error need to go 0, for that we are actually like playing the further thing where the error dynamics $\dot{e}_2$ plus something into this form we are trying to find out.

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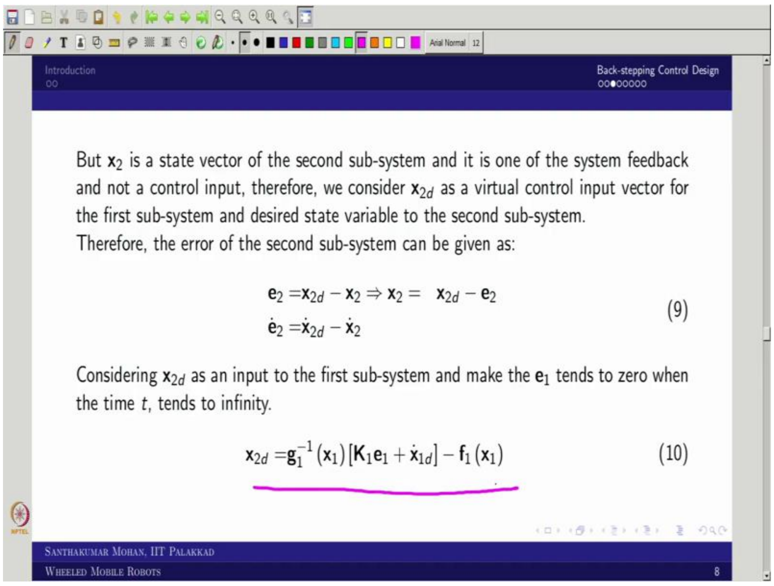

So, far that what we are trying to do? We are actually like going back with your known. So, this is actually like already known from the previous you call what you call previous slide. So, this you know this is now become x_{2d}. So, that is what we are actually like used.

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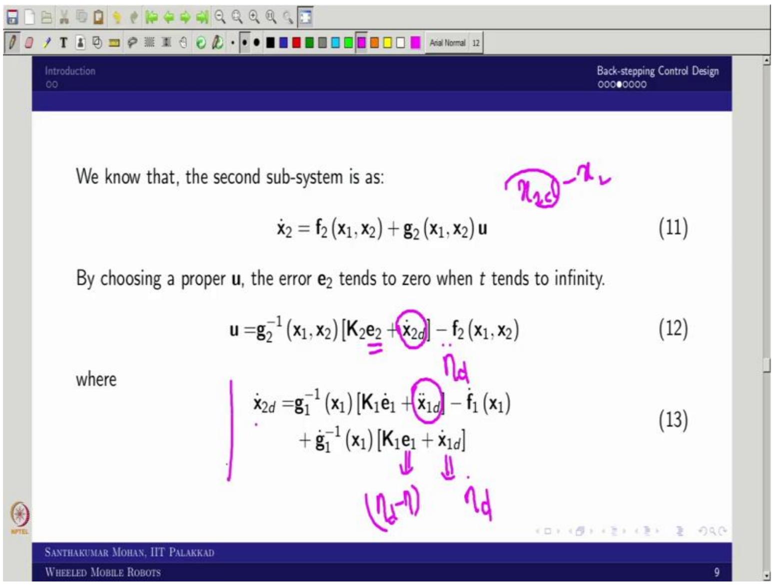

So, now based on that you are going further. So, what further? You are taking $\dot{x}_2$ equation and you know this u equation. So, now, you can actually like choose in such a way that this particular case coming into a picture you substitute that $\dot{x}_2$ and then you replace and then find the Ʈ. So, the Ʈ will come in this form ok. So, nothing but that is u.

So, now you rewrite that into a generalized way where this e_2 is actually like you know $x_{2d} - x_2$ where the x_{2d} you can write in the other form, then $\dot{x}_{2d}$ is actually like in this form.

So, here you can see that this is nothing, but $\ddot{\eta}_d$ and this is actually like $\dot{\eta}_d$ and you can actually like see that here it is $\eta_d - \eta$ right. So, whatever you have given that all actually like resolved and got it as a simple input that is what we are actually like used.

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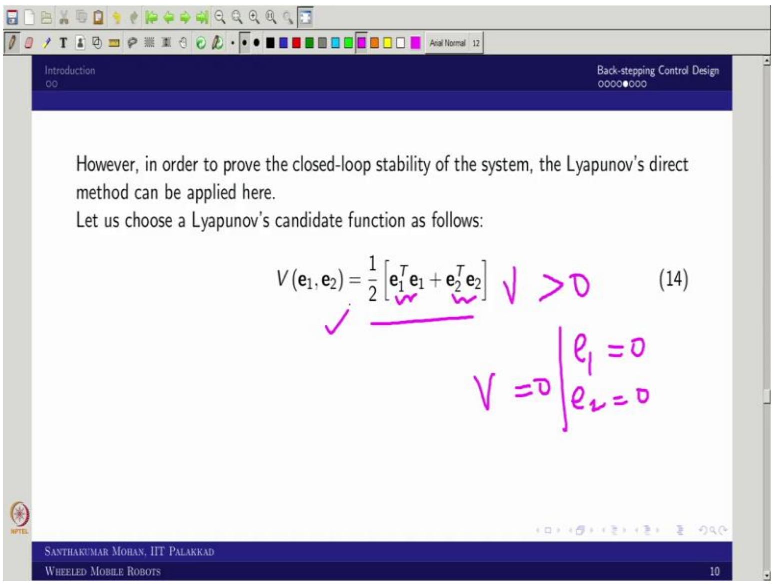

So, in that sense what you can actually like see that. So, you can actually like ensure the closed loop stability, but we will get it that with the Lyapunov direct method where we actually like bring the Lyapunov candidate function as.

So, $V(\mathbf{e}_1, \mathbf{e}_2) = \frac{1}{2}\left[\mathbf{e}_1^T\mathbf{e}_1 + \mathbf{e}_2^T\mathbf{e}_2\right]$. So, now, this is actually like positive all the cases because it is something like e_1^2 e_2^2 right. So, now, when this would be 0? When both $e_1 = 0$ and $e_2 = 0$, then only the V = 0 right. So, then you can see this is always positive definite that you can ensure.

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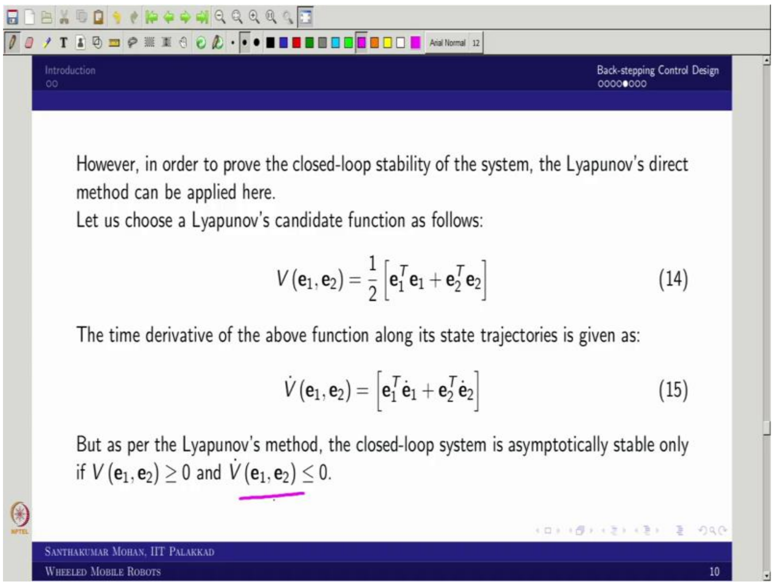

So, now you differentiate this. So, where $\dot{e}_1$ and $\dot{e}_2$ already we have calculated and you substitute. So, what you can actually like see that? So, this condition is actually like fulfilling or not.

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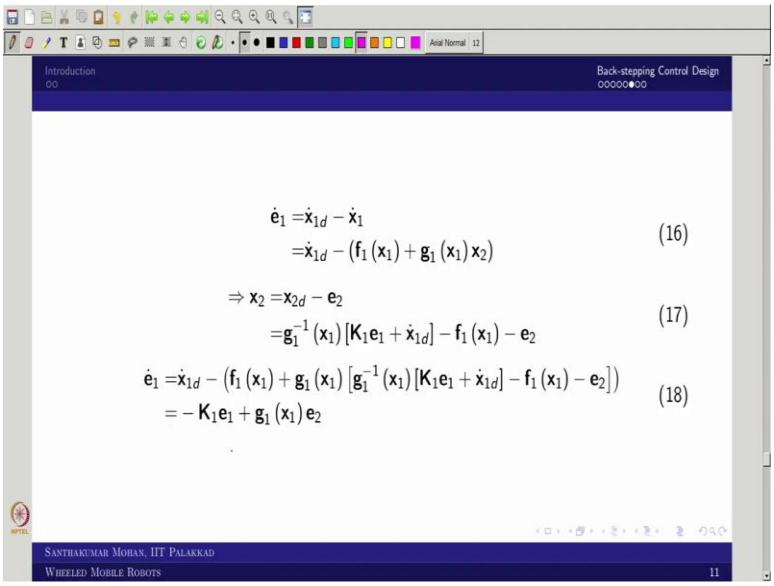

You substitute that $\dot{e}_1$ you have already obtained and $\dot{e}_2$ also like you have obtained ok $\dot{e}_1$ you have obtained and $\dot{e}_2$ also like you can obtain in this way.

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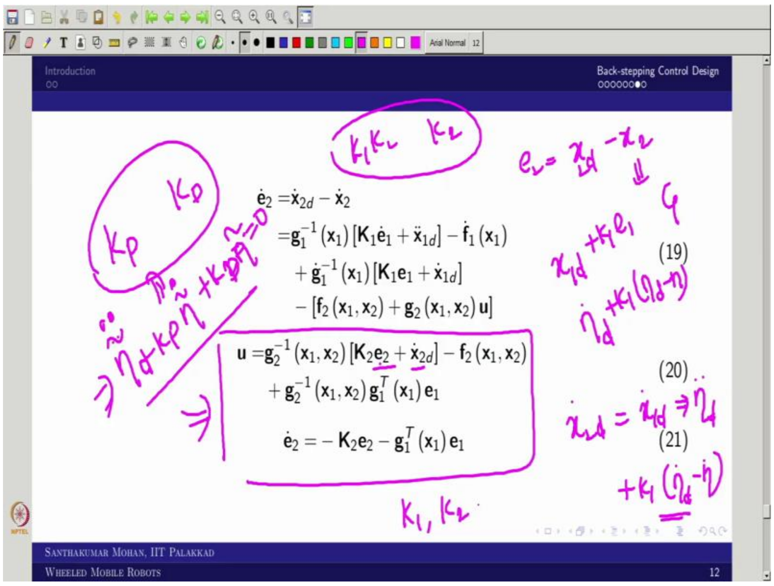

And then what you can actually like see?

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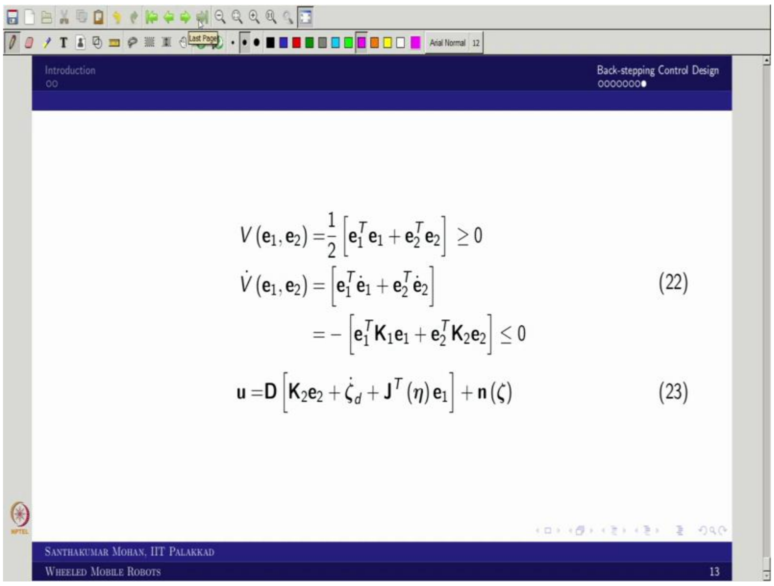

So, these two relations you substitute in this equation and that will give you can say the negative definite ok. So, in the sense what one can see the closed loop stability also ensured. So, now, what you can recall this equation? So, where you have substituted this right. So, this you recall. So, what is e_2 2? So, e_2 is actually like $x_{2d} - x_2$. What is x_2 is this case? It is ξ, right. What is x_{2d}?

So, that is actually like $\dot{x}_{1d} + K_1 e_1$ right. So, what is this? So, this is actually like what you can actually like feel it. So, this is you can say this is not η this $\dot{\eta}_d$. So, this is K_1. So, $\eta_d - \eta$ right this is for the e_2. What is $\dot{x}_{2d}$? So, this is actually like you can differentiate this.

So, it is $\dot{x}_1$ and already you have actually like one dot which is actually like in the sense. So, what you can see? This would be equal to $\ddot{\eta}$ and this would be $K_1 \dot{\eta}_d - \dot{\eta}$ right. So, in the sense you can see like these two are coming as it is what you have seen in the what you call the computed input control where only thing here K_1 and K_2 and K_2 are actually like playing a role.

So, that you have to actually see. So, earlier we have taken simply K_p and K_d, but this would be combination of $K_1 K_2$ as a combination. So, that is the major difference, but this would be ensure that this is completely asymptotically stable whereas, here you have to properly choose, but here what we have taken? The second order error dynamics of the overall system we have taken as you can say tends to 0 in the sense we have written as like this.

So, I will put it. So, this equation we have actually like taken as 0. So, where $\ddot{\tilde{\eta}}$ and $\dot{\tilde{\eta}}$. So, that actually like equation we have taken where K_1 and K_2 was actually like positive there we are put K_p and K_d these two are as long positive this would be ensured; right now here what we have modified as K_1 and K_2 are actually like positive, right.

But what the difference between these two? So, here everything we have written in the one single form called $\tilde{\eta}$ or η form in a very simple sense. So, overall the position trajectory we have done in a inertial frame and we have understood everything in the initial frame itself.

Whereas, in the cascaded you done you are actually like doing everything the kinematic level in inner you call inertial space and the inner loop you are doing in the body frame respect, in the sense you are doing with respect to actual body configuration. So, the dynamics you are trying to compensate in a body frame where your kinematics you are correcting in the what you call inertial frame.

So, that is why it is called double loop or some people call it is actually like what you call cascaded or composite control. So, I will just show it only one additional aspect.

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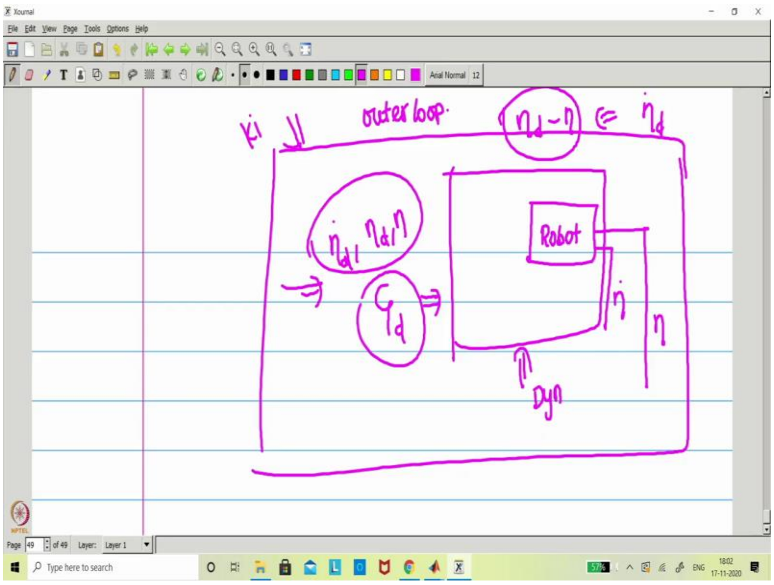

So, what that means? So, you can see like. So, you are actually getting the robot I will finish this diagram. So, what you are actually getting it? So, you are getting $\dot{\eta}$ and η. So, what we are actually like trying to do as a outer loop. So, in the outer loop. So, I am actually like seeing that this is the outer loop. In the outer loop, so, you are actually like comparing η_d and η. So, that you are actually like making it along with what you are bringing the $\dot{\eta}_d$.

So, once that is done then what you are giving? So, $\dot{\eta}_d$, the η_d and η. So, this you are actually like combining and what you are giving? The ξ_d you are actually like giving; once this ξ_d is given then what you are trying to do? You are doing everything in the inner loop.

So, that is why it is called actually like a dual loop or double loop where the outer loop would be in a kinematic level and the inner loop in the dynamic level. So, that is what the beauty of this particular control. This is more common in the robotics community whether it is a manipulator or mobile robot this is one of the common phenomena used in you can say robotics.

So, now this outer loop can be combined with other control strategy. Here we have done with simple proportional control whereas, you can even you can use something like LQR or learning base control or robust adaptive similarly the inner loop even you can combine with you can say something like a disturbance, compensated control all those things you can actually like incorporate.

In the sense you can say that these are two different segment only thing you have to achieve like give proper desired vector for the inner loop that is the only important thing which is what you call the virtual control input that you need to give it to the inner loop. So, with that, so the control part is over.

So, now we will actually like move to the you can say modern robotics, where we will see like what is the difference between what you call automated guided vehicle and autonomous mobile robot and the other aspect what are the modern robot getting into a challenge why it is actually like so difficult. So, all those things we will address in the next lectures. So, till then we are actually like saying bye see you then.