Wheeled Mobile Robots
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Lecture - 37
Simulation of Land-based Mobile Robots along with Kinematic Control Part 2

In the last lecture, what we have seen is actually like we have done the Kinematic Control Simulation, for you call omnidirectional and magnum wheel. But what we have done? We have done with line diagram and I said that we will do the what you call animation along with the simulation. So, that is what we are going to do in this particular lecture.

So, let us move to the MATLAB window and then, see how to incorporate this particular simulation along with the animation.

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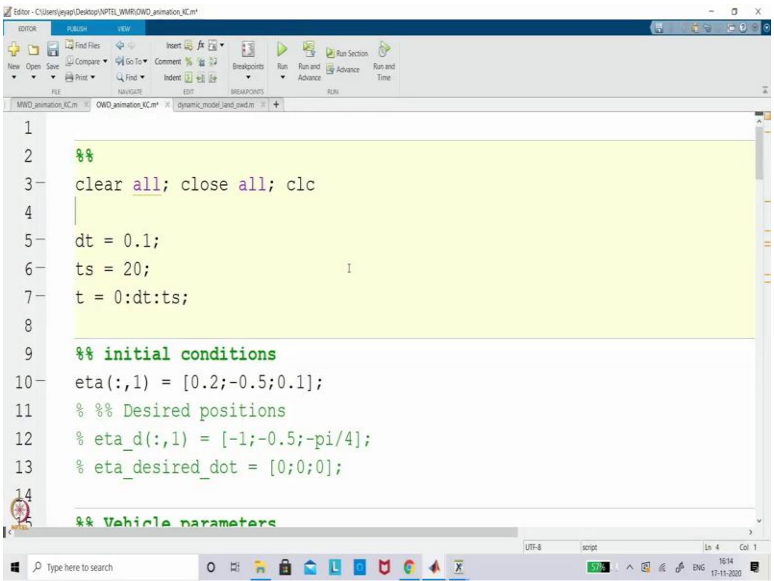

I already told right, we will bring the animation part.

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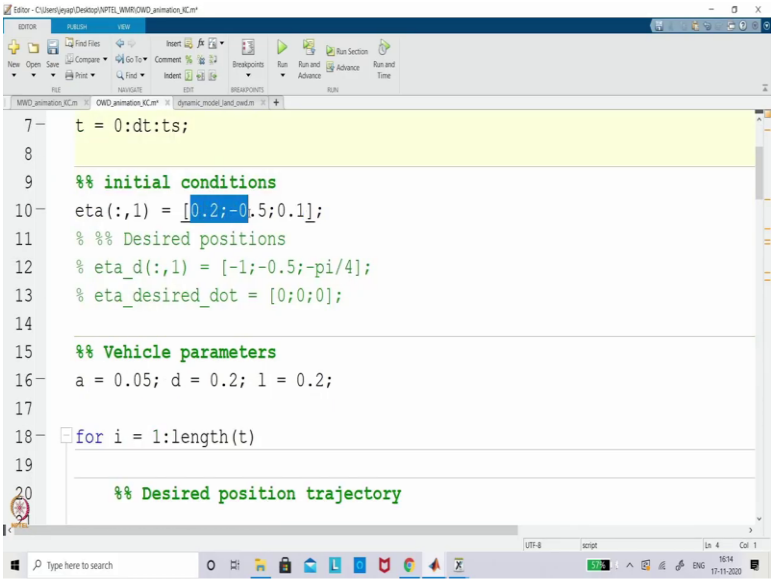

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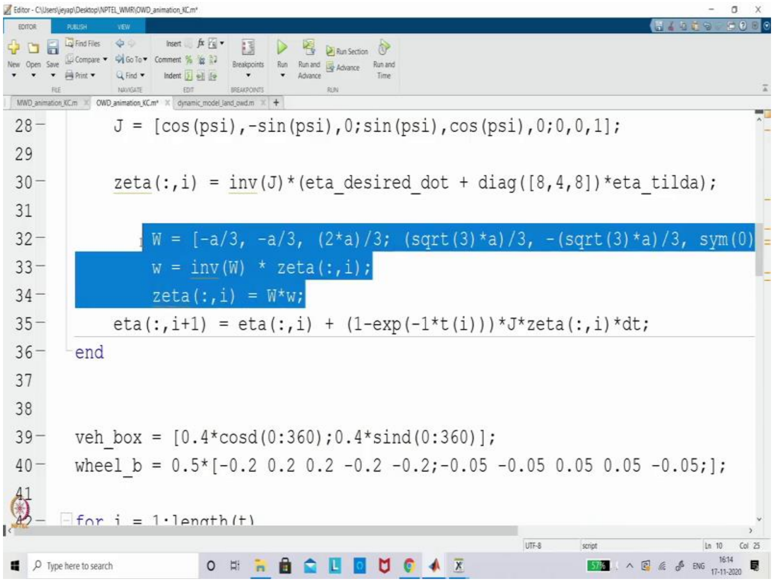

So, now, I will change the initial condition also for just for your understanding. So, what I did in the animation part? I incorporate the what you call omni wheel directional you can say in mobile base, where the first wheel would be 30° with respect to x axis, then 150° the second wheel and the third wheel would be 270°. So, if you look at the model, you will actually like get it understand.

So, right now what I am trying to see that the vehicle would be a circular base ok, so not a rectangular box. In the circular there would be three wheels would be associated; one in 30, one in 150, one in 270.

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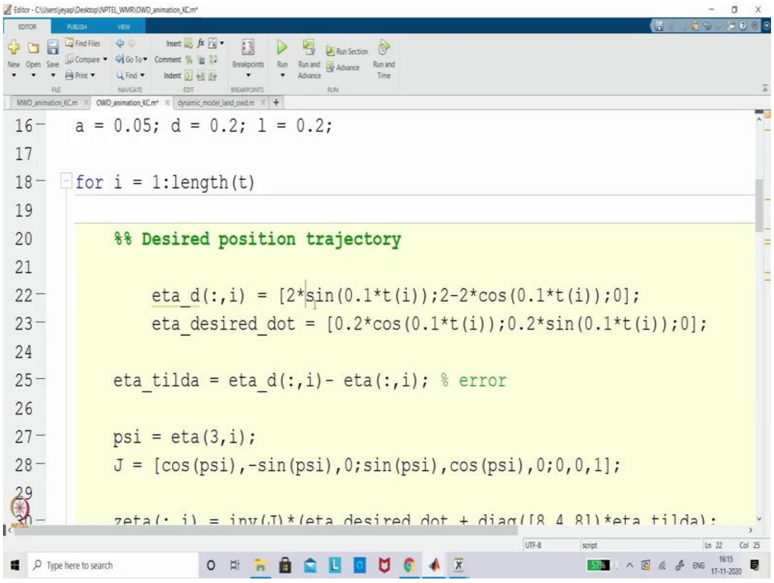

So, now what I did? I have actually like taken the desired point as a you can say x and y as a circular basis it would be running, where the Ψ angle would be 0; but the initial condition I change just for your understand. So, the initially the Ψ would be 0.1 radiance and your y axis position is - 0.5 and your x position is 0.2 meter.

So, now, if I actually like run what one can expect. So, this will try to follow a circle. But it may not complete because it is 0.1 radian per second. So, if you see that the 20 second, so it may actually like take a half a circle or probably almost like close to a quarter circle ok. So, we will see how it is actually like running.

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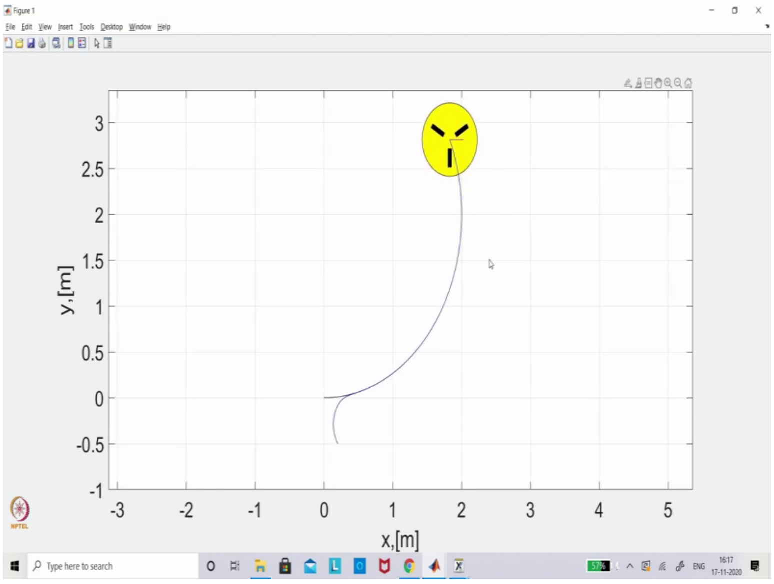

So, now, you can see that this is the vehicle configuration. You can see it actually like it is starting from non-zero and it is trying to follow. So, this is the actual. So, this is the blue one is actual; the black one is actually desired. It is trying to follow after a certain time. So, that you can actually like see from the you can say the time trajectory profile.

But now, you can see this is the 30° wheel, and this is 150° wheel and this is 270° wheel. So, this wheel would be taking directly lateral motion and these two will be combinedly taking you call horizontal motion that is what you can expect. So, now, this black line which is actually like given as the desired and the blue line which is following as a actual path.

So, initially what we said? The actual exposition would be somewhere and y position is actually like - 0.5 and x 1 x is 0.2 and the angle also like a point you call one or radian or something. So, now, you can see like it is trying to follow and this red line what indicates? It is a orientation. Right now, you can see like it is a horizontal or you can say parallel to x axis. In the sense, the Ψ is 0 all the cases.

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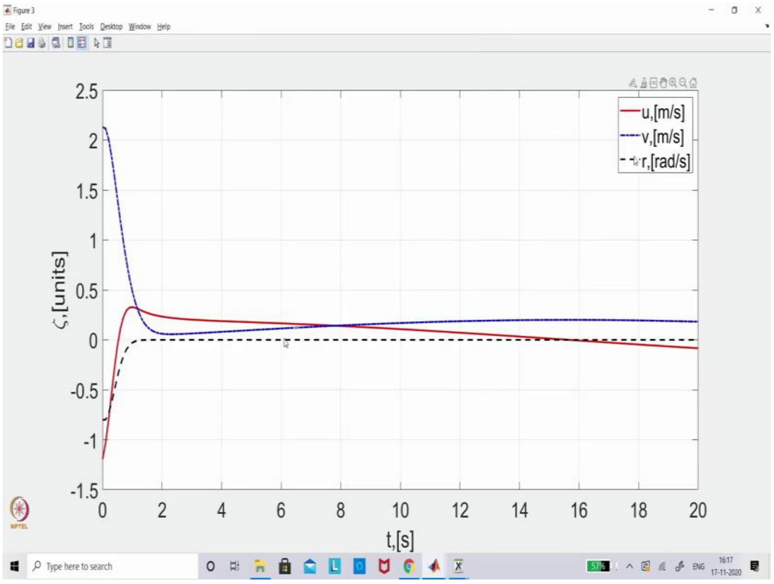

So, now this is what your u, v, r which is what you call the you can say vector of control input, that is actually like varying based on your given what you call the computer velocity control.

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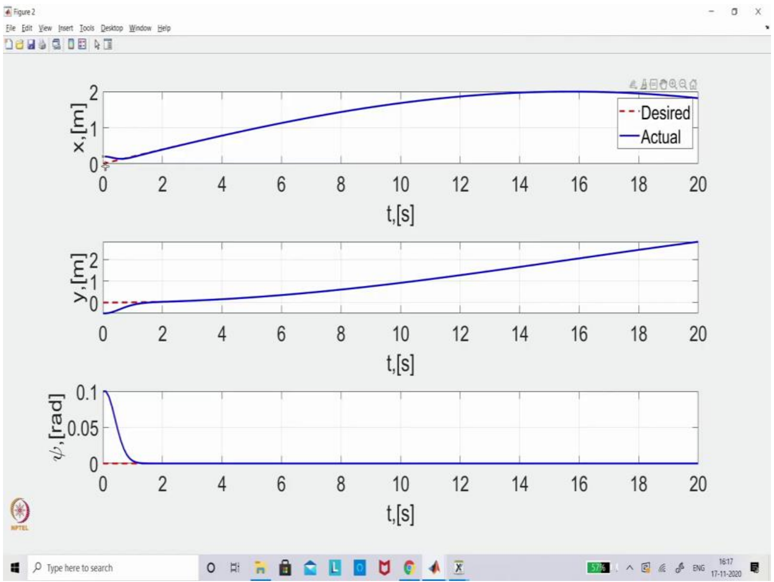

But the case which you are interested to see this. So, you see that the initially the exposition is somewhere what you call 0.2 meter, but you have actually given the desired is actually like 0 that is trying to follow. But you can see like within less than probably 1 second, it reached and it is trying to converge. Similarly, you can see that the second one which is y axis, it started 0.5; but it is supposed to start from 0.0 like a 0, but it is taking some time to converge and similarly, you can see the Ψ.

So, now in order to give that little more clarity, I will just run probably only you can say 20 second; but the speed I am going to increase because here, we do not have any restriction. So, in the sense, what this would be? This would be 1 and this is also 0.5. Just I am trying to increase the what you call the frequency of that ok. So, now, what you can actually like expect?

This would go faster; it is 5 times or you can say 5 times faster than the earlier curve. So, that is what you can expect. So, in the sense, I will just put it only 10 second that is sufficient.

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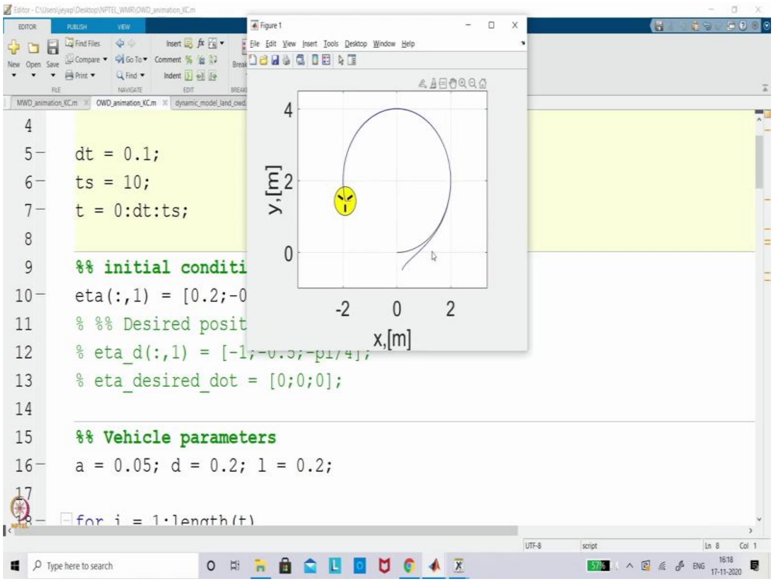

So, now, if I actually like a run. So, you will see the same thing, it would be starting from non-zero initial condition; then, it actually like try to go and actually like you can see it is trying to follow that circular profile, whatever given. And now, you brought the magnum sorry omni wheel directional configuration right. So, right now, I have given 30°, 150 and 270. Even you can change that and further, what I have given? I have given the radius of the wheel also like given as 0.05.

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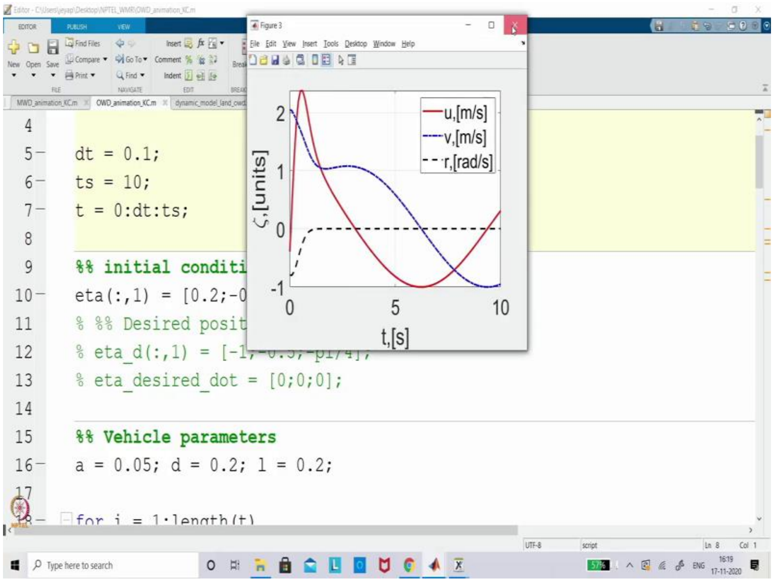

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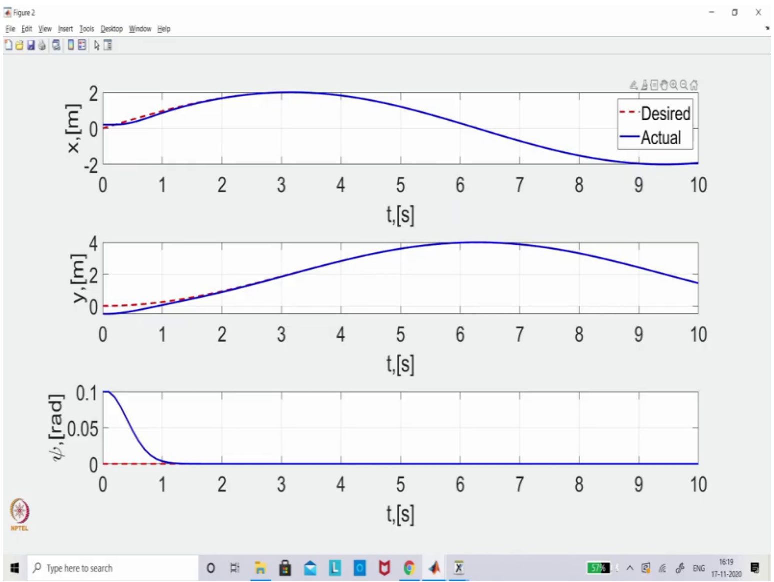

Now, you can actually like even change that. So, in that sense, you can see like these all actually like getting change right. So, for example, this is I am taking that the way radius is actually like double.

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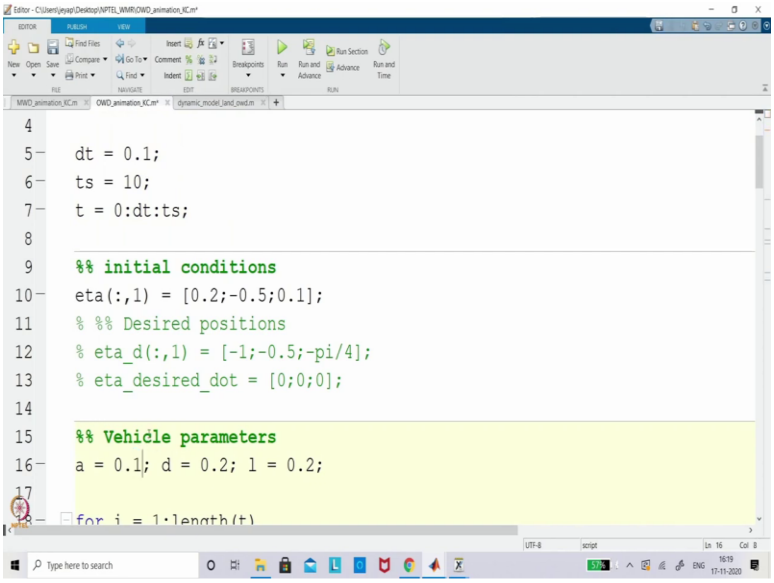

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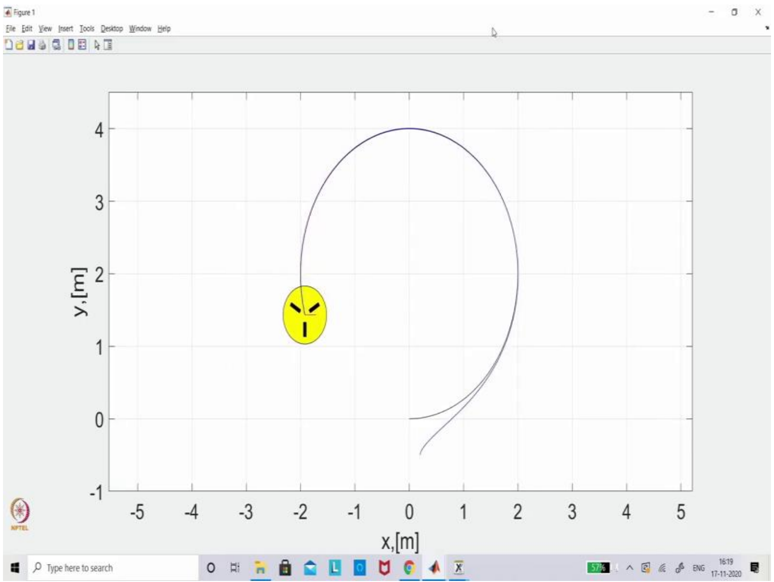

So, now it may be actually little faster than the earlier so that variations and all you can actually like see it. So, that is what we say that this is a simulation. So, now, you can see that the variation also like you can find it. So, where you can vary length, you can vary you call the radius of the wheel and you can vary the control game and even you can vary the profile.

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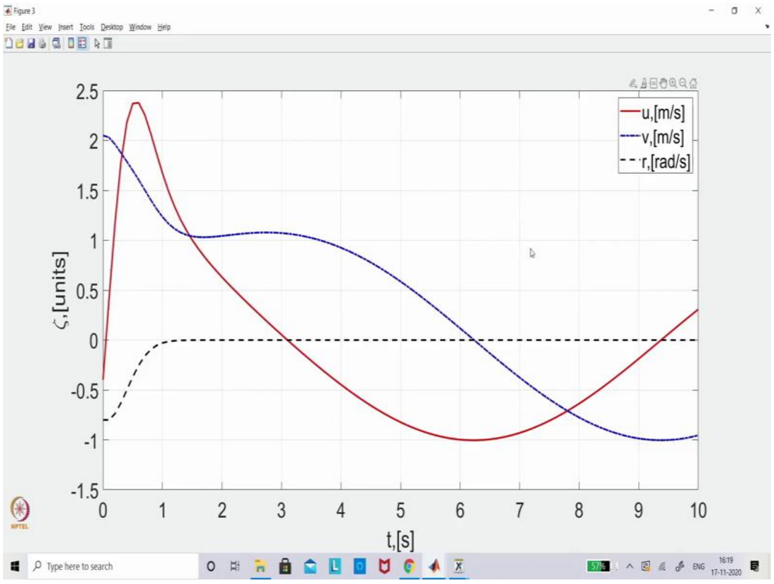

So, the profile varying is actually like one scenario, but your system parameter you can actually like vary ok.

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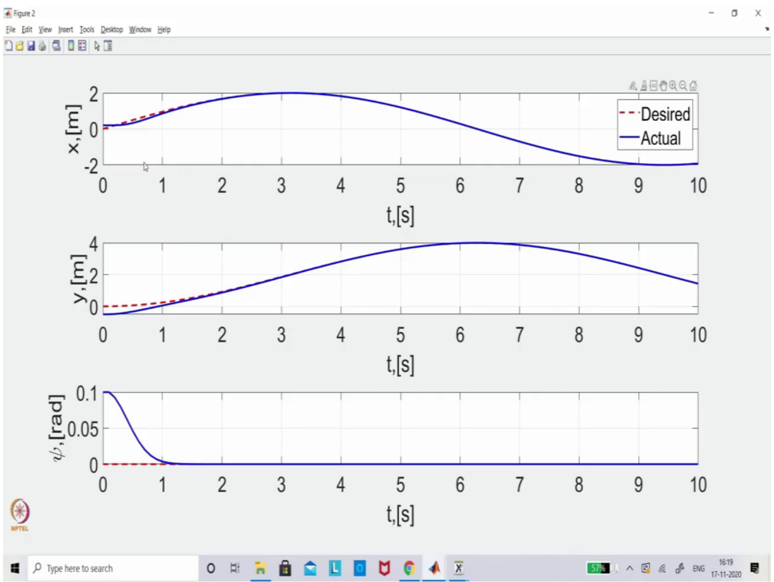

So, now this is what your u, v, r and even I can also plot the similar sense x, y, Ψ. But right now, I am actually showing along with the desired you can see right. So, I increase the profile faster and it is trying to do it. In the same thing, you can bring it to a magnum wheel. So, I am trying to show that in a magnum wheel case also. So, now, you can see that the magnum wheel, I am taking a simple what you call set point control.

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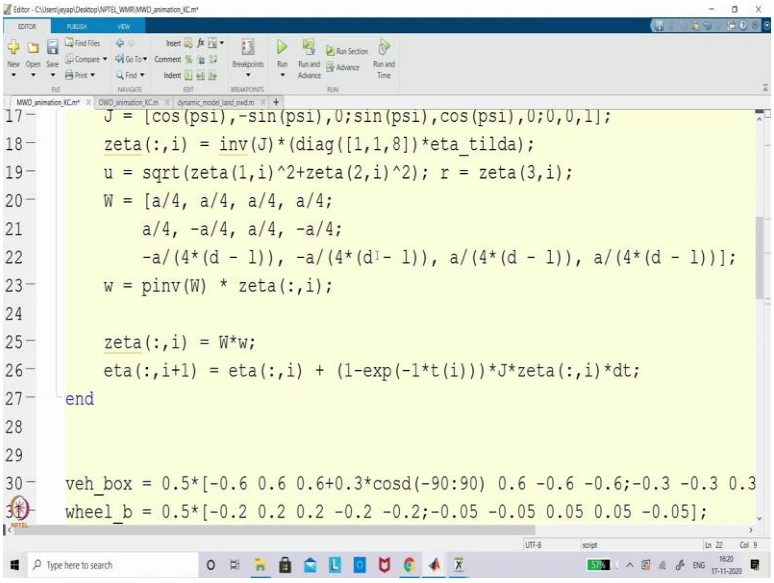

So, where you can see the magnum wheel, the W matrix brought in. So, I will just show you in your benefit. So, these are the you can say 3 rows and 4 columns are there. So, since it is actually like 4, you can say columns are there. So, obviously, what you have to do? You have to do the pseudo inverse. I am using pseudo inverse straight away as a p^{-1}; but you can actually like use even the same thing, in this case it is the right inverse.

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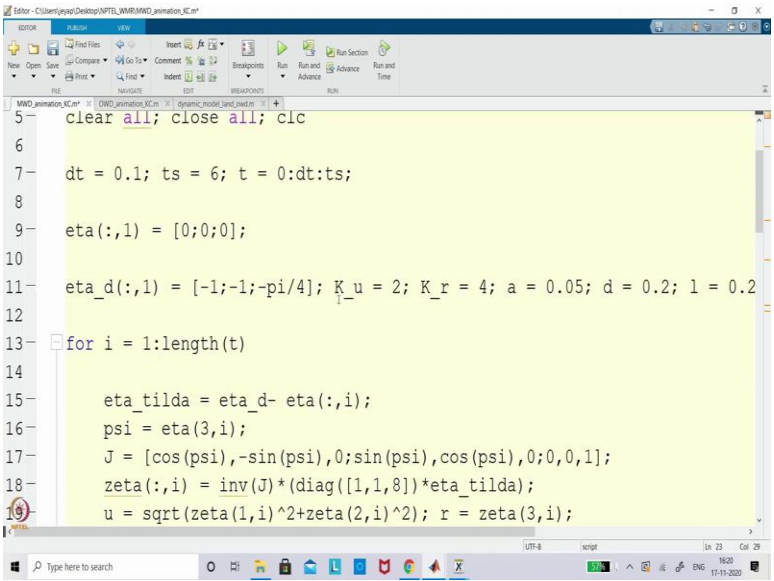

But now, if I actually like give the condition as starting from 0 and it supposed to go - 1, - 1 and - 45 ° and you can see how that is going. So, these two are not required, this is for the other model.

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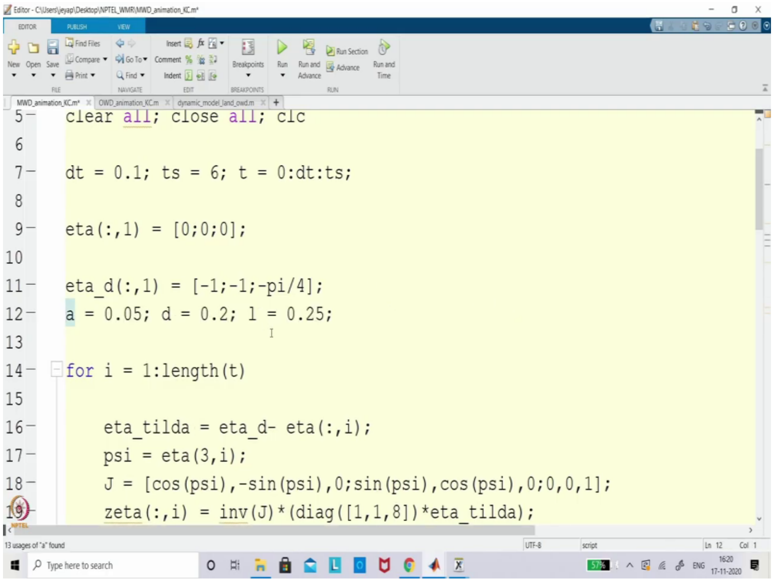

So, now the wheel radius and there you can say distance all given. So, now, what I am trying to plot? I am trying to plot the initial point and the final point and trying to show how the vehicle will move. I will just run this and you can actually like get it out.

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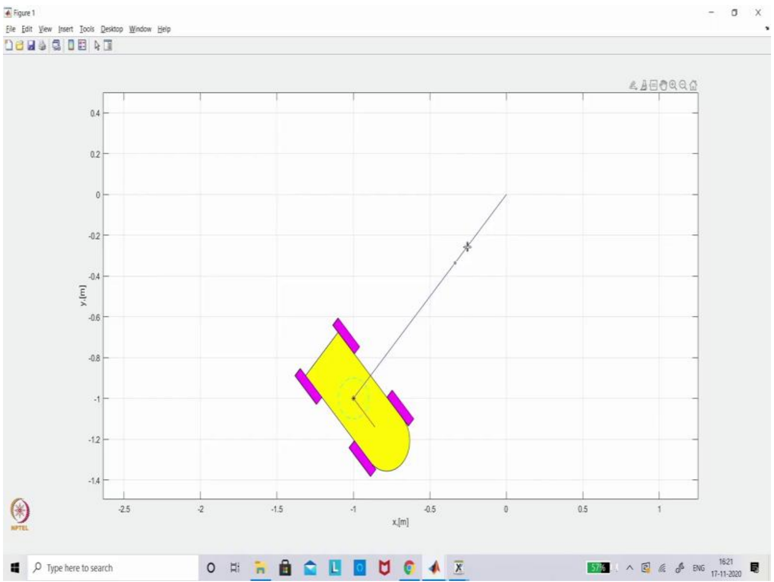

So, this is the magnum wheel. So, these are the 4 magnum wheel, I will just to show as a purple or you can say pink colour and this is the final target. It start from 0, 0 point and it is trying to reach here right. Since, it is actually like you call holonomic vehicle, it can go in the what you call a lateral direction ok, that is what it is doing. So, initially it is trying to adjust along with this line. So, after that, it is just to following the line as a minimum case.

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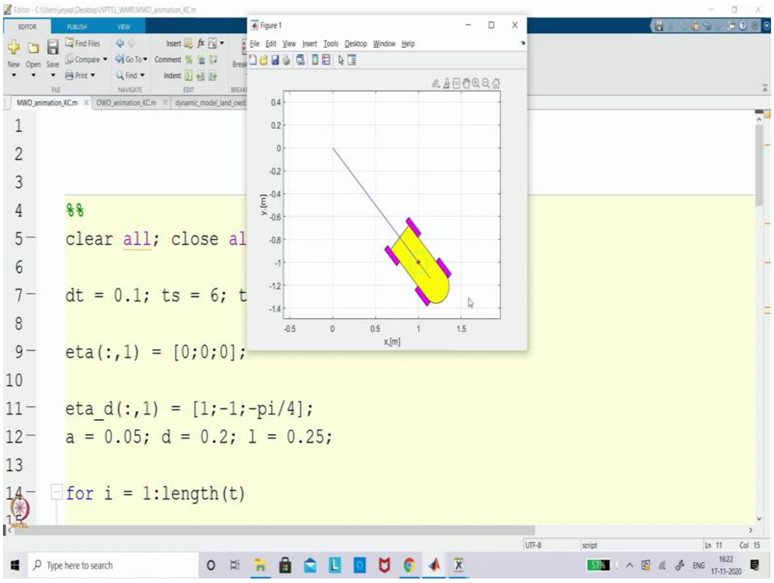

So, now you can see even you change the condition. For example, I am saying that this is the other way round, still this particular vehicle will go. Now, you can see the initial condition and the final condition has changed. Now, you can see right. So, the vehicle is actually like making a alignment as a line of sight and it is trying to follow. But the same situation may not be applicable to certain vehicles right.

So, very simple example you take a unicycle or differential wheel drive or tricycle, these things will not be eligible. Why? Because these vehicles are or these mobile robots are all called nonholonomic; in the sense, not all the three states can be achieved, only certain states would be achievable. So, that is what we are going to see in the slide.

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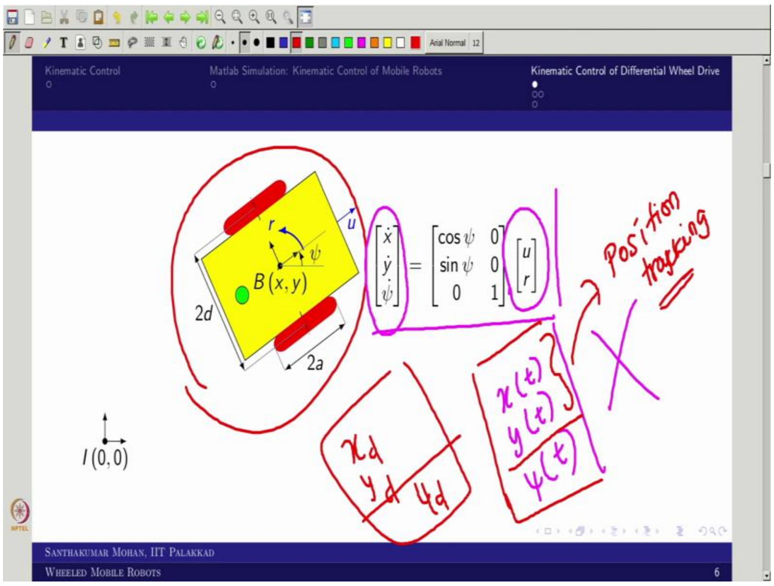

Let us actually like move to the slide and then, we can actually like a try to see. So, for the nonholonomic case, what we have taken? So, we have taken a differential wheel drive. So, you know this is actually like one of the differential wheel drive which we usually consider for all analysis. So, I am taking this is what $\dot{x}$, $\dot{y}$ and $\dot{\Psi}$. If you substitute.

So, the v also then you will get something; but right now, what you can see the final equation, since it is actually like a lateral resistant is infinite, you can see that the v is no longer exist. So, in the sense, what one can see? It can be controllable only two states. So, now what you are actually looking? So, you need to follow this right. So, in the sense, this is not at all possible straight away.

So, then what can be possible? You can actually like give some constant x_d, y_d and Ψ_d as a step input, then that is follow; you can say followable. But if you are giving this way either of you can say three, only two of the states can be controllable, usually we will go for a position tracking.

So, usually like a differential wheel will go for position tracking ok. So, position tracking is achievable; whereas, the all three states are not achievable. But this is achievable with two stage; so, where first you reach the position and then, you orient. So, that is what we are trying to see in this kinematic control loop.

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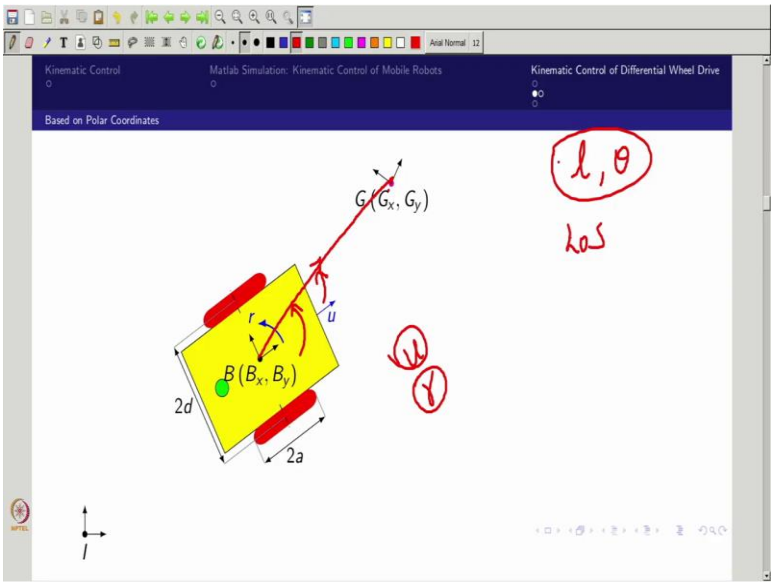

So, now you know this I, B and as well as u and r are the control input. So, now, we will actually like move to the polar coordinate. So, I assume that there is a goal point somewhere here. So, I am just showing that the goal point has given; this is the start point and this is the goal point. So, I am trying to follow this particular profile or you can say in this case, it is a set point.

So, what I can do? I can draw a line which is joining. So, this line if I actually like align this robot, what one can see? This robot will actually like go along the line. So, in the sense, u and r is sufficient. So, initially I take r, where I actually like orient this vehicle along this line. So, after that only u is sufficient right.

This can be achieved in two way. I can write this overall equation into only a polar coordinate; where, l are in θ form or I can write what I did earlier. So, I can do it in a line of sight method. So, we will see one by one. First we will do the polar coordinate base.

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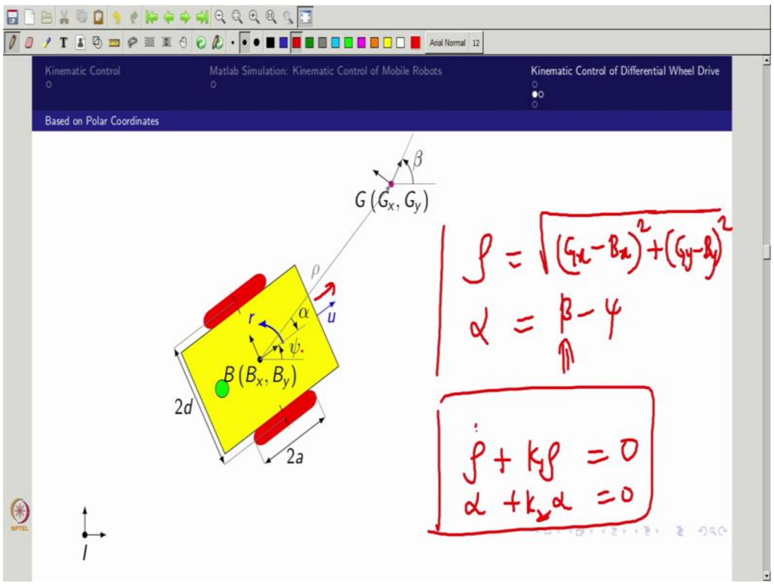

So, what I can do? So, now, I can actually like draw a line between B and G. So, which I can write that distance, which I call radius ρ. So, now, the angle between what you call the you call x, you can say x_B to x_G that angle I can call as actually like one of the angle called line of sight angle ok.

So, this α is what I call line of sight angle. But you know the vehicle already actually like orient with respect to I frame as a Ψ; but what we are actually looking at? We are looking at to reach this point and then, actually like try to follow this β angel. So, definitely the β directly cannot be achievable. So, we will do one by one. So, what we can do? We can take ρ and α.

So, α I can write as actually like so β - Ψ because this is given and ρ, I can actually like take it. So, the G_x - B_x whole squared as a distance relation. So, G_y - B_y relation right. So, now, these two I can obtain. So, what you can see? Further, I can write rewrite this equation as $\dot{\rho} + k\,\rho = 0$ ok.

So, then what I can write similarly $\alpha + k_2\alpha = 0$. So, then what it look like? It is a first order error dynamics what we did in a kinematic loop right. It is very close to that. So, similar direction, we can do it. So, in the sense, what one can actually like do it? So, that is what we are trying to do.

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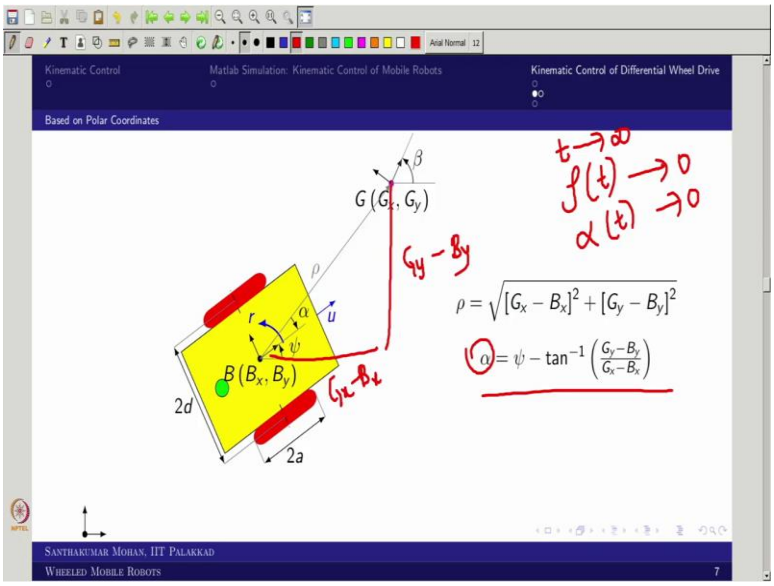

So, we will first write the ρ equation and then, the α equation. So, the α equation, we can even rewrite. So, if we make this triangle. So, what this? So, this is G_y - B_y and what this? This is actually like G_x - B_x right. So, I can get it that and then, I can actually like get this particular what you call α. Once I know ρ and α, what is my ultimate objective?

At every instant ok, this supposed to reduce and $t \to \propto$; $\rho(t) = 0$. So, this is what I was actually like interested. So, that is why I called the first order error dynamics base. So, similarly, $t \to \propto$, my $\alpha = 0$. Further, I will do a second stage from 0 to β; I will do as a second loop.

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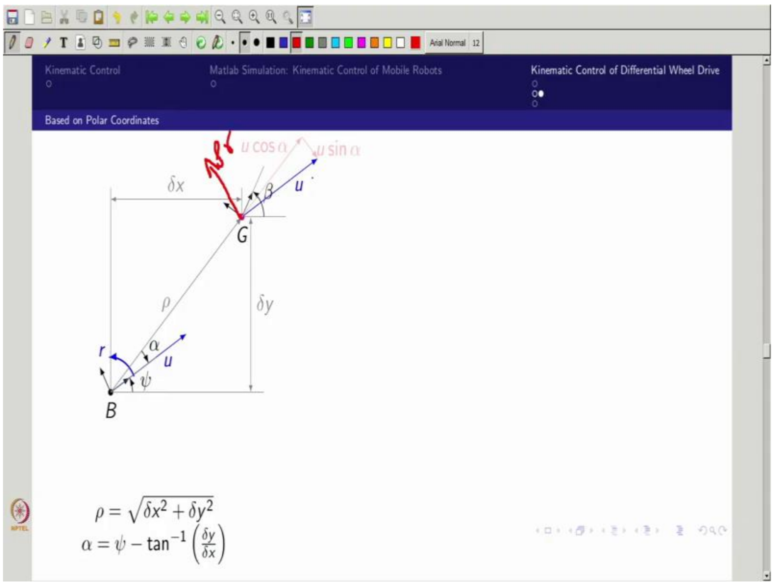

So, that is what we are trying to do. So, then what we are trying to see? What is your control activity? That is u and what is the other control activity? Which is r. So, these two, we will take and try to see how we can achieve ρ and α → 0, when t → ∝. So, for that, what one can rewrite this equation? So, I assume this is delta y and delta x. So, I rewritten.

So, now what I am trying to do? So, I take this u straight away up and if I take this r, what are the components will come? There are two components will come due to the tangential you call velocity; otherwise ρ × r will be perpendicular to this right. So, either you can write you can say ρ r and u. So, these two velocity component, I can take it to directly g and then, I can resolve it.

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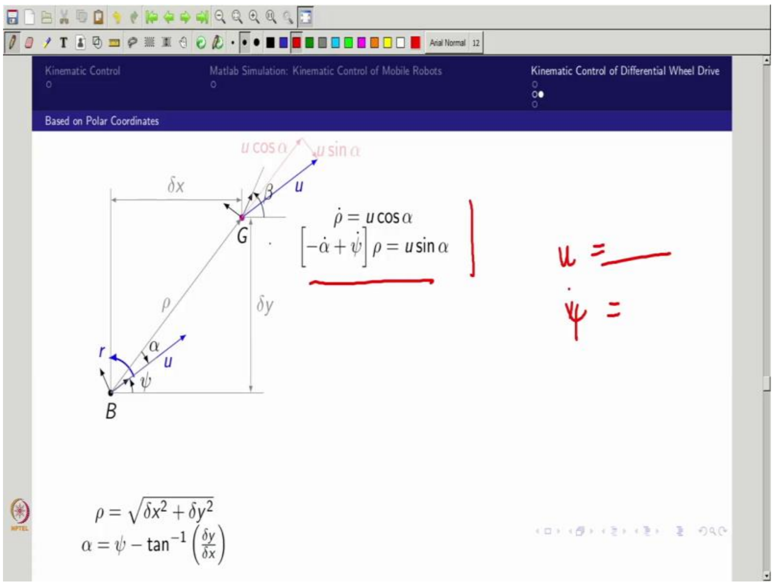

So, that is what I am trying to do. The $\dot{\rho}$ dot, I can equate on this line direction. I can equate as ucosα. So, then the other one is I can write $\dot{\alpha} + \dot{\Psi}$, where α we have written in the clockwise and the Ψ, we have written in the counterclockwise. So, that is why this - sign is coming; multiply with ρ supposed to be equal to what this usinα.

Now, these two equations are there and we rewrite this. So, now, I choose u as a control activity and you can write, you can say the rewrite. So, $\dot{\Psi}$ and u, I can write as a control activity, I can rewrite in a simple first order error dynamics.

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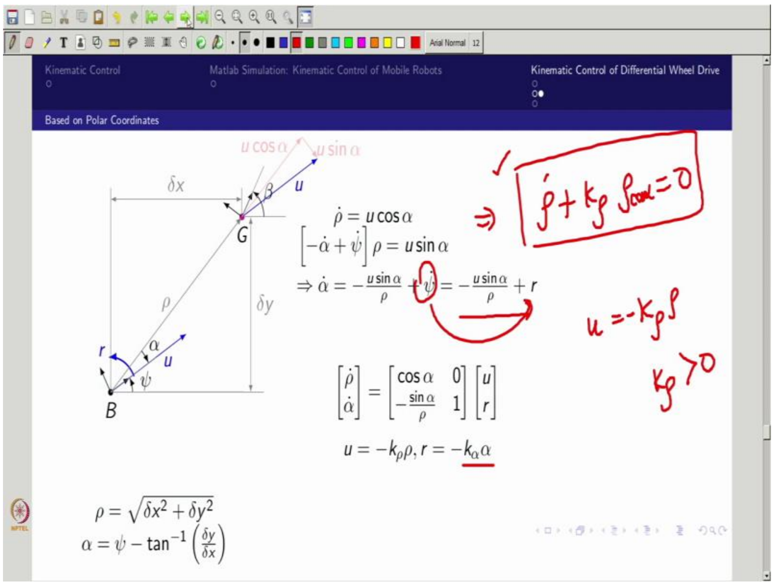

That is what we are trying to do. I am first rewriting this equation in a bigger form, where the sinα and cosα I can actually like take it consideration. So, now, what I can actually like see, I can write in a you can say state space form. So, now, the state space form this supposed to be clarified. So, for that, what we are choosing? If I choose u as so $k_\rho \times \rho$ that to like opposite direction, what it looks like?

So, this k ρ is actually like as long as positive value. So, if I substitute that here, what it look like? So, $\dot{\rho} + k_\rho \times \rho = 0$ right. So, only thing here actually like cosα would be residue will come, but that is not a problem right because cosα I can actually take it as a positive or negative according to the situation. So, now, what one can see?

This error dynamics goes to 0 right. Similar way, if I take r as - $k_\alpha \alpha$. So, then you can see this, you can actually like substitute this equation, where usinα I can substitute in the form of r. Then, you can actually like rewrite this all the equation, the $\dot{\Psi}$ I can rewrite in r; then, what you will get? So, this r, I will substitute as $k_\alpha \alpha$. Then, I can rewrite this whole equation in this form ok.

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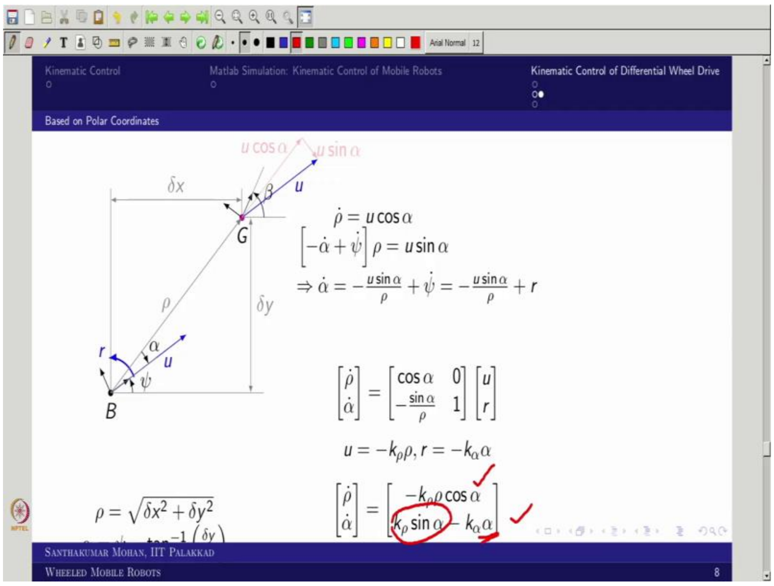

So, now, I already know that this error dynamics goes 0 and based on that, this also like will converge; in the sense, the second system also like would be what you can say go as a simple first order system. So, this is one way you can do it as a polar coordinate. So, usually people will do this for unicycle and differential wheel; but I prefer personally the other one, which is what you call the general function as a line of sight.

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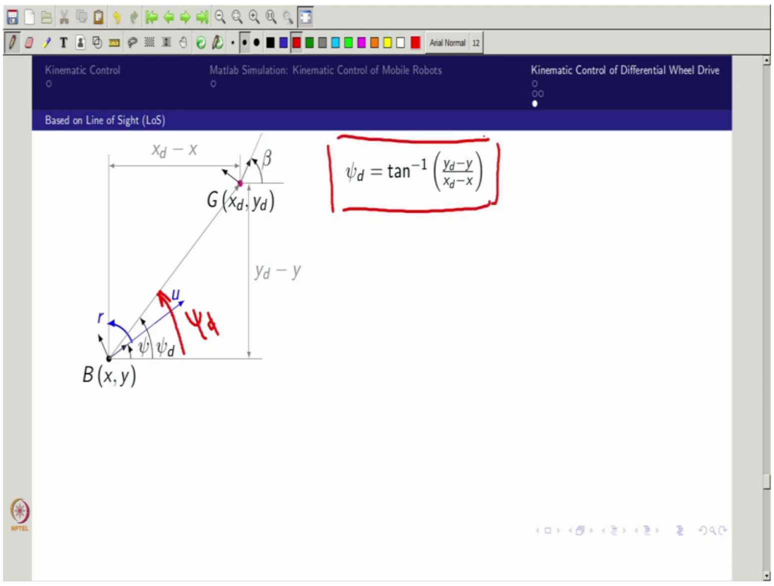

You see that the same scenario we are trying to do; but only thing what we are trying to do, we are taking this as a line of sight angle and then, we are trying to follow it. So, what we are trying to do? So, this whole I am trying to take as a Ψ_d ok. The Ψ_d, I can obtain in this form and then, I can actually like do it as it is.

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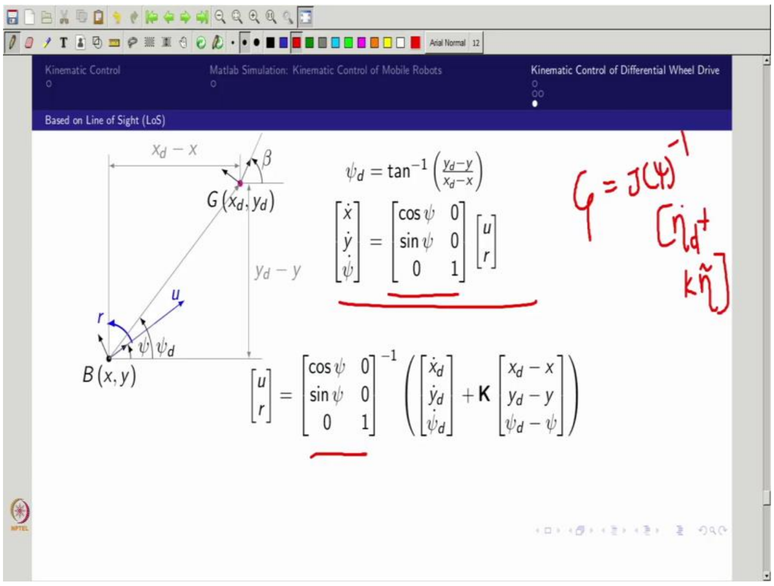

So, what that mean. So, I am taking a simple kinematic controller, what you have taken as so the ξ I can rewrite as so J you can say $\Psi^{-1} \times \dot{\eta}_d + k \times \tilde{\eta}$ right. The same thing, I can actually like do it without doing any issue; only thing here J(Ψ) would be reduced matrix because the v would not be there. So, that is what this is the J(Ψ) and the J(Ψ^{-1}) is look like this.

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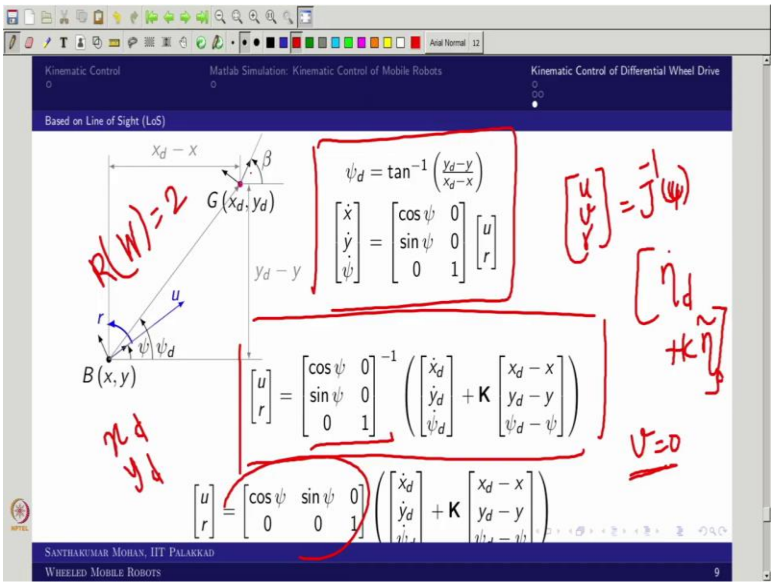

If you take even the pseudo inverse, the final form would be look like this. So, right now you can actually like see this we can do it right. So, this is actually like same as the earlier one; only thing the J^{-1} as modified. So, in the sense, it is actually like easy right.

So, you can now apply this straight forward, even you no need to bother; you can simply you take u, v, r and you do it as a $J^{-1}(\Psi)$. And you can actually like substitute this as $\dot{\eta}_d + k \times \tilde{\eta}$.

So, then you can actually like take v = 0 in all the cases, that is all. So, then also it will work. So, that is why I said the second method which is what you call line of sight method, I personally preferred because it will not actually like get any confusion. So, for example, now we take the earlier model, what we have used for a magnum wheel or omni wheel directional you can say robot and all, the same model you can take it.

Only thing this change you can do it or you can do this change, either way you can do it and then you can make it. Only one thing you need to get understand, you will be bringing this as a line of sight. So, that you need to keep it in your mind because this is a nonholonomic.

In this case, the rank of the W is actually like is two; in the sense, only two states can be controllable at a time. So, here we are trying to control x_d and y_d. So, once we achieved this point, then we will do what you call the orientation control. So, that is what we are trying to see in the MATLAB window now. We will move to the MATLAB and see how we can actually like do it yeah.

Now you can actually like see that what we have done. We have done the nonholonomic in specific one particular case called the differential wheel drive a mobile robot. We have taken for a kinematic control. We have taken two stage; so, one is polar coordinate or the other one is line of sight. So, these two equations we have seen.

So, now in the next lecture, we will see how these equation can be incorporate in a MATLAB environment and see how the vehicle will perform based on this particular control scheme. So, in that sense, so the next lecture, we will see the animation of this particular control schemes for a differential wheel drive along with a few other specific non-holonomic mobile robots. Until then, see you, bye.