Wheeled Mobile Robots
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Lecture - 35
Kinematic control of Land-based Mobile Robots

Welcome back to Wheeled Mobile Robot course. So, you know like last class what we have seen, we have seen actually like what is robot motion control and what are the types. So, in that itself I said one of the type we would be discussing here. So, if you look at in the you can see previous versions how we started kinematics and dynamics, so similarly here also the Kinematic control we will do it for a general land based mobile robot.

What that mean? We would not be considering the wheeled configuration, we write the equation for a general case. So, after that, we will come back with if it is a holonomic how would be, non-holonomic how would be, that would be covered in the next lecture. But this lecture what we are trying to cover, so what is actually like kinematic control for a given land based mobile robot.

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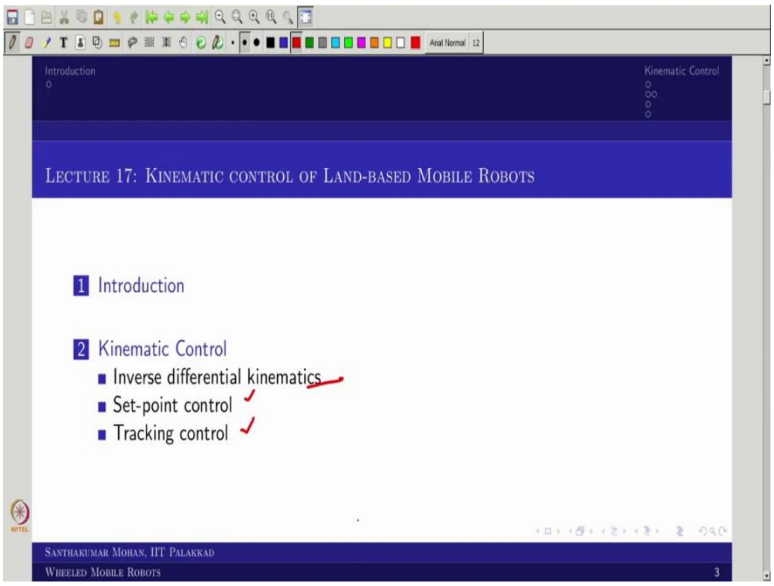

So, if that is the case, so what we can expect from here? So, we will see like what is inverse differential kinematics, and what is set point control, and how we can go for a tracking control with respect to kinematics. So, that is what we are trying to cover in this particular lecture.

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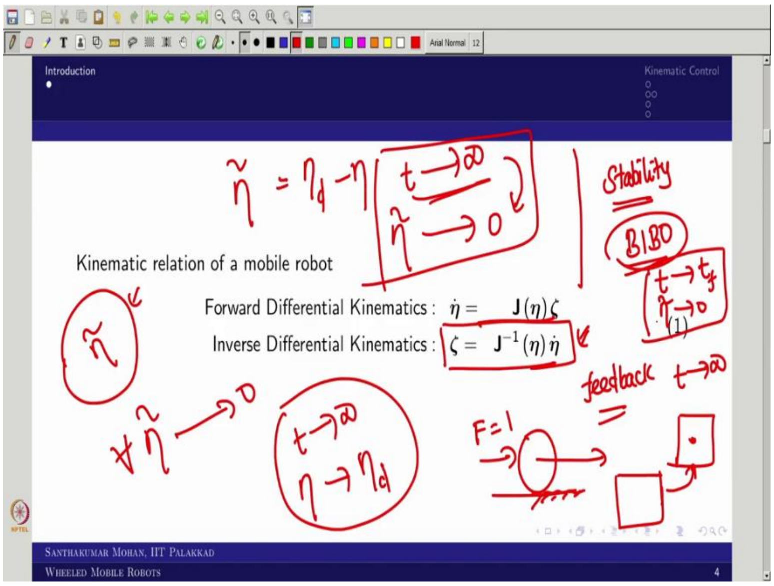

So, let us actually like move what we know we know one of the relation, so that is what we usually write $\dot{\eta} = J(\Psi) \times \xi$ that is I am writing it ok. So, instead of Ψ, I am writing in a general. So, here inverse differential kinematics, I can write in this form. So, now, in that sense, so what we are trying to see is this is one equation. This equation we are actually like modifying based on the feedback. So, then you can actually like make it as a kinematic control.

Then what would be the necessary thing? So, you know like any control that would be the prime most is actually like stability. The stability how you can actually like recall, so we can recall in so BIBO in the sense Bounded Input and Bounded Output stable. So, what that mean? If you are giving an input within certain bound for example delta, the output is actually like bounded within certain parameter called η or probably ε, then you call it is bounded input and bounded output.

For example, if I am giving a mobile robot for 10 Newton as a force as bounded, the output also supposed to be bounded in certain way. It should not keep on rolling ok. But in mobile robot, it is going to be keep on roll.

For example, you take this is a mobile robot and you apply a constant force F which is 1 Newton, and you assume it is actually like slippery in the sense frictionless surface. So, what will happen? This would be having actually like keep on rolling right. So, in the sense, what one can see this system is actually like not BIBO stable.

Then what one supposed to know, so if I make a closed loop whether the system is stable or not. So, in the other way round people actually tried to see the system in a asymptotically stable, in the sense I have decide, I have actual.

So, if this error $\tilde{\eta}$ I am writing, this $\tilde{\eta}$ is actually like tends to 0 when t → ∝. So, what that mean? So, when t tends to infinity means this is asymptotic. So, it is actually like $\tilde{\eta} \rightarrow 0$, in other sense $\eta \rightarrow \eta_d$ when t → ∝.

If this is the system, then we call it is asymptotically stable. So, whereas, this $\tilde{\eta}$ can be anything ok. You choose any $\tilde{\eta}$ and that $\tilde{\eta} \rightarrow 0$, then you call globally asymptotically stable, or you choose this $\tilde{\eta}$ in one some range, so then it is locally asymptotically stable ok. So, these things we will see in detail little later. Right now you imagine this is what one aspect we are interested.

What that mean? t $\rightarrow \propto$ your $\tilde{\eta} \rightarrow 0$. So, for example, you take a car parking problem where you have this is what the initial car. This car supposed to be stopped like this. So, you imagine. So, what you will do?

You will take a turn and stop. But what I am seeing that at t $\rightarrow \propto$ if this car is parked here or not. So, now the other way round also t $\rightarrow \propto$, so then if your output $\tilde{\eta} \rightarrow 0$ then you call finite time stable, but right now we are not discussing.

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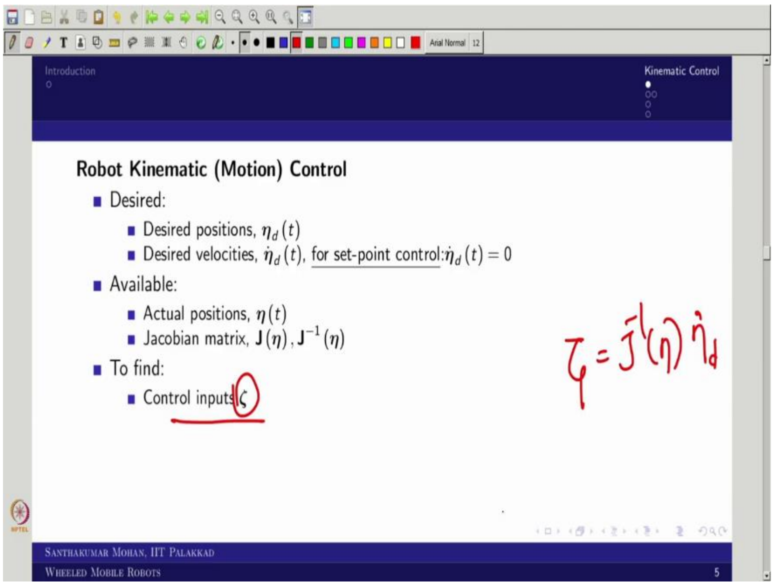

Why that is important here? So, there is a usual way. So, for that, I am taking a very gentle case. I am talking about the motion control in kinematics. What would be given to us? The desired thing right. So, what is the desired here? So, η_d, and $\dot{\eta}_d$ would be given to us right.

If it is a set point control, the $\dot{\eta}_d$ = 0 right. So, this is given to you as a desired. So, what would be available I already told in the control beginning itself, so we assume that all these sensors are available to measure all the desired state.

So, in the sense, what you can see that the available is η is available which is what you call actual positions. In the sense, you are x y and Ψ would be available to you. Then what you are interested, you are interested to find out? So, what would be your η? From η what you can actually like find your interested is ξ.

So, for that what one supposed to know? You supposed to know the relationship. So, here the relationship is coming from Jacobian matrix. So, we assume that the Jacobian matrix is available. So, in the sense, you know J sorry you know η, so you can find J(η) right. So, that is what we are actually like obtained.

So, in the sense what one can actually like see, so what we are interested? We are interested to find the you call vector of input commands which is we simply call control inputs hereafter, the ξ we need to find out. So, then do not write that the $\xi = J^{-1}(\eta)\dot{\eta}_d$ ok this may not work all the time, so that is what I wanted to explain.

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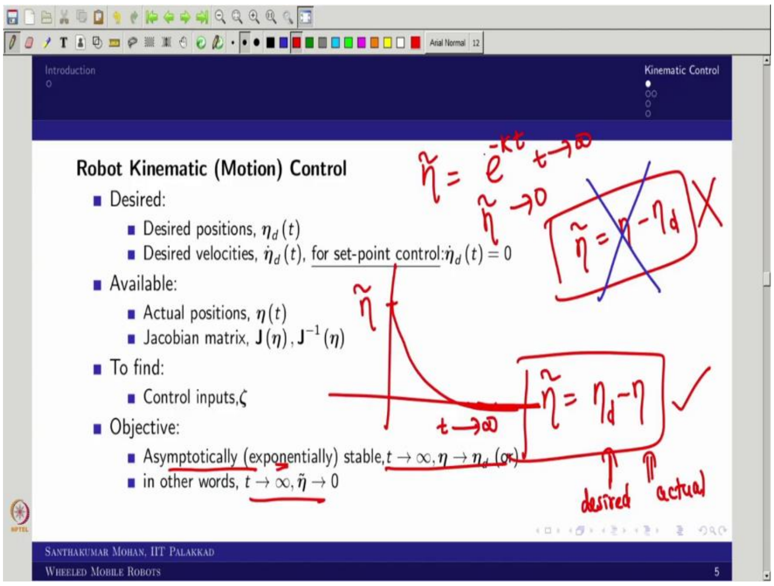

So, now what would be the desired objective? So, one of the objective is actually like you are thinking about the stability aspect, where so you see t tends to infinity, my $\eta \to \eta_d$. In other way, so I write t → ∝, my $\tilde{\eta} \to 0$. Then what I will write $\tilde{\eta} = \eta_d - \eta$.

So, this is what I am assuming. So, this is desired and this is the actual ok. So, that is what I am actually like taking it because most of the control theory people they call like this ok. So, we are actually like a different. So, that is what I am actually like using this. So, in the sense, what we are saying that this is we are not going to use ok. We are going to use this relationship.

So, in the sense, what would be the objective? Objective is asymptotically stable. So, here I am giving a key word exponentially stable. So, what that mean? You take your $\tilde{\eta}$

as something as here. So, when t $\longrightarrow \propto$, this supposed to goes to 0. What is the easiest choice? I take a exponential curve, and then actually like you can see that it would tends to 0.

In the sense, what I can take $\tilde{\eta}$ I can take as an exponential form ok. The exponential form I take some of the negative value of a positive constant with respect to time I assume that it is a scalar. What happened to this? So, $\tilde{\eta} \rightarrow 0$ when t $\longrightarrow \propto$. Why? This become $\frac{1}{\propto} = 0$ right. So, that is what we are trying to do.

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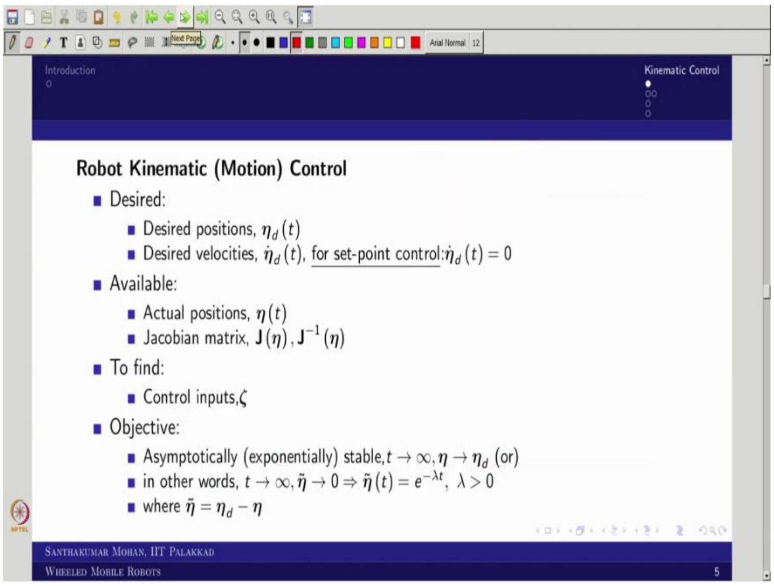

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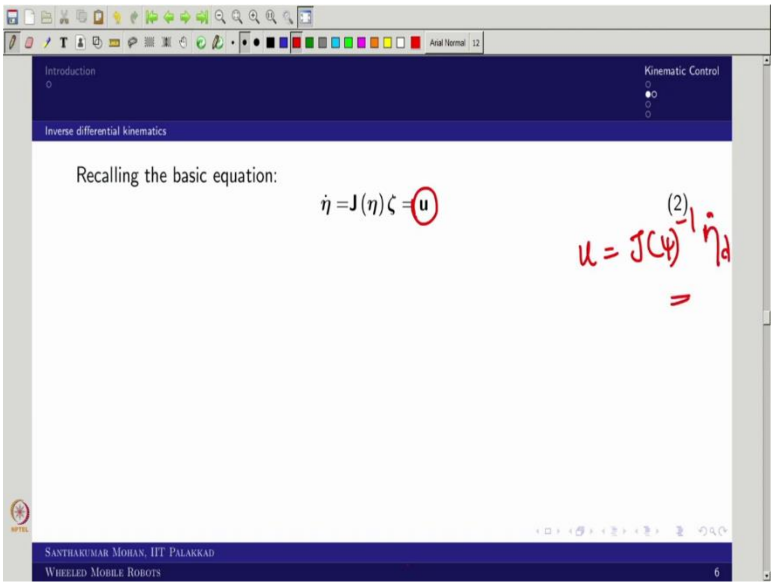

So, you can see that is what I am actually like going to write. So, you can see like this is the basic equation. So, what we are actually like writing that equation I am writing as a u. I am trying to do all combination whatever in your mind arises all the combination I am trying to do. So, first combination, so I already wrote. So, this, so if I actually like write this, so can I achieve? Yes. But what situation? That is what we are first attempting.

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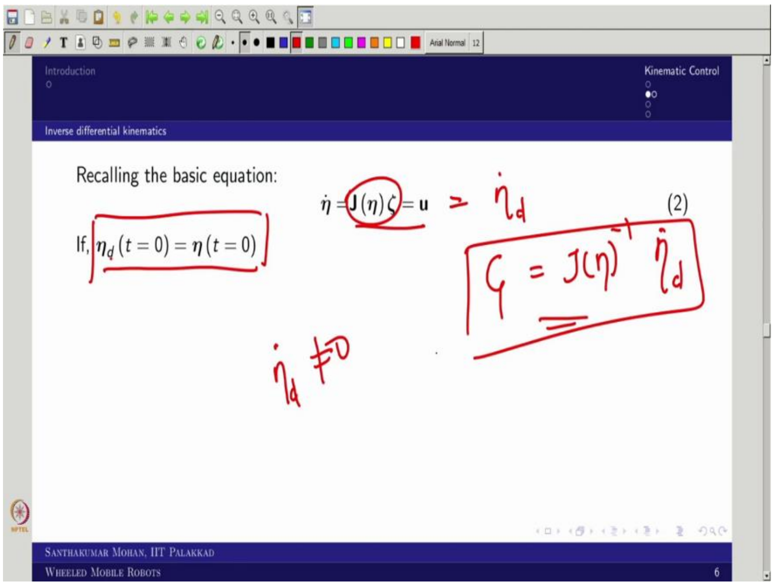

So, in the sense, I am assuming that this entire thing as u. So, then what I can do, u as actually like $\dot{\eta}_d$ then that is fine right. So, then what would be ξ? $\xi = J^{-1}(\eta)\dot{\eta}_d$. So, this is nothing but your inverse differential kinematics, but this would be applicable only certain situation.

What are the situation? η_d(t = 0) and η = η(t = 0), both are same; in addition to that $\dot{\eta}_d \neq 0$ ok. If η_d $=$ 0, then you can see this is actually like non you can say differentiable form right because the ξ = 0 right, so that is what we are actually like writing it ok.

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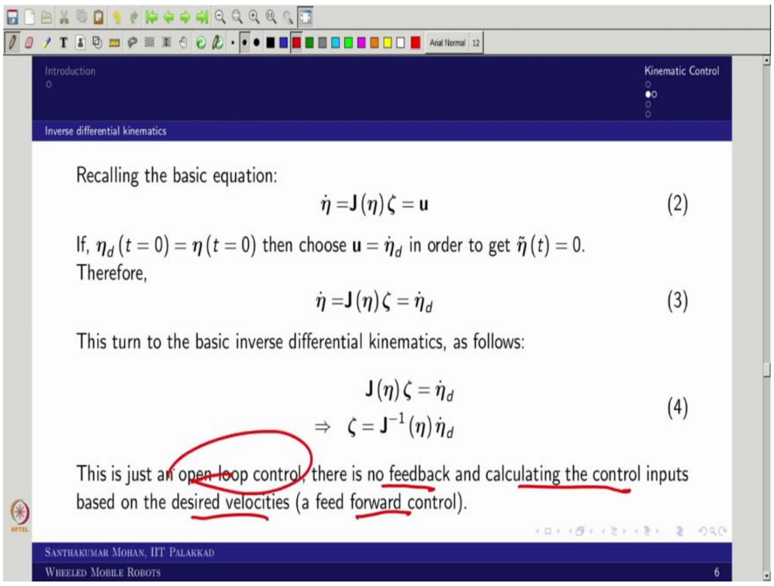

So, in order to get this, so this I can choose. So, now, what you can actually like see it. So, this is what the equation. I already said right the ξ I can write it straight away. So, that is what we are actually like doing it. So, now what we have done? We have done no feedback. And what we are doing?

We are calculating the control input based on the desired thing. What that mean? This is a simple feed forward control. This is what you call inverse differential kinematic that what we call open loop right.

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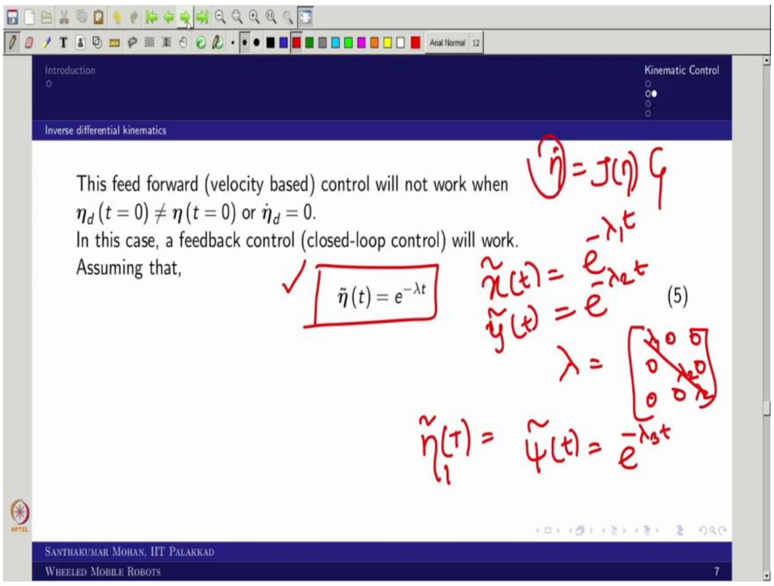

So, these all we have discussed. Now, what would be the special here? Now, imagine this feed forward control if this is the case and this is like situation, so this definitely would not work. Then what one can actually like see can I actually like think about a feedback control? So, definitely yes that will work. So, how that will work?

So, for that what I am taking? I am taking a $\tilde{\eta}$ in this form, where here the λ is actually like a matrix ok which would be diagonally arranged, in the sense it is a diagonal matrix. For simplicity, I am taking this way. So, then what you can see that $\tilde{\eta}$ of you can say 1, I will write 1 T, in the sense I can say $\tilde{x}(t) = e^{-\lambda_1 t}$. So, $\tilde{x}(t)$, I am just simplifying ok. So, then $\tilde{\Psi}(t)$ is actually like e.

So, what that mean? So, I can rewrite everything in a first order equation right. So, that is what we are trying to do. So, now, if I rewrite this equation in this form, what I can do? I can actually like relate what I know. What I know? This. Can I rewrite in this form? Yes. What I can do for that? This differentiable required.

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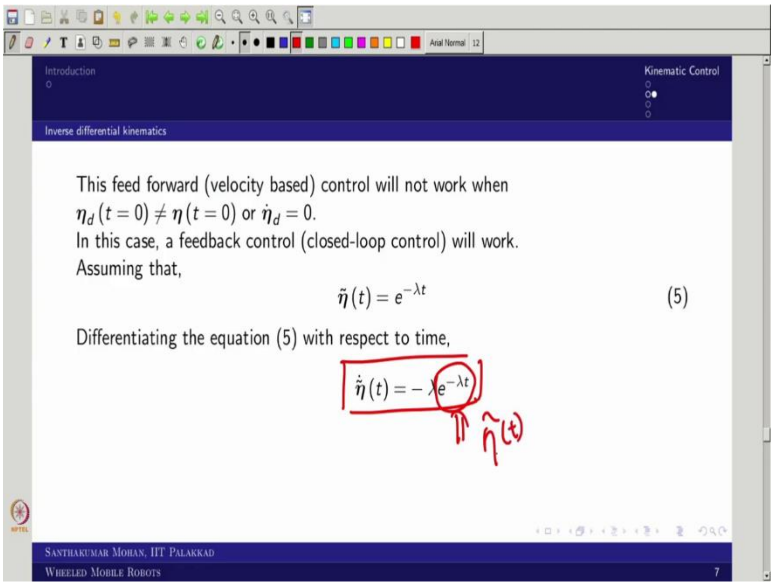

So, I am differentiating this $\tilde{\eta}(t)$ with respect to time. What I will get? So, this right. So, if I get this, what this is? This is nothing but $\tilde{\eta}(t)$ right.

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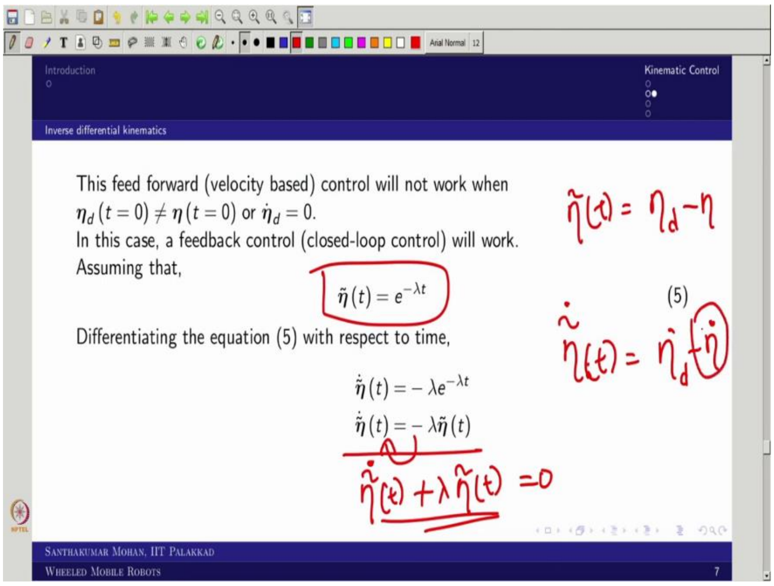

So, I can substitute. So, then what it becomes. So, now, you rewrite this in this form in this side. So, what you will get? $\dot{\tilde{\eta}}(t) + \lambda\tilde{\eta}(t) = 0$, this is actually like simple first order system that to like homogeneous system. What that mean? So, there is no you can say input. So, if this is the case, what would be the output? This is what the output right.

So, now you further go this. So, what this? So, $\dot{\eta}_d - \dot{\eta}$ right because $\tilde{\eta}(t)$ what you have written, $\eta_d - \eta$ right. So, you take differentiation this is what going to come.

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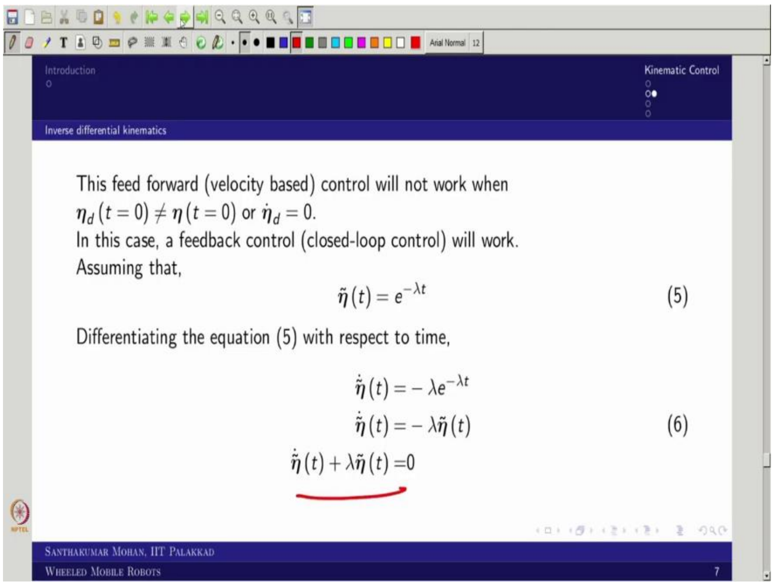

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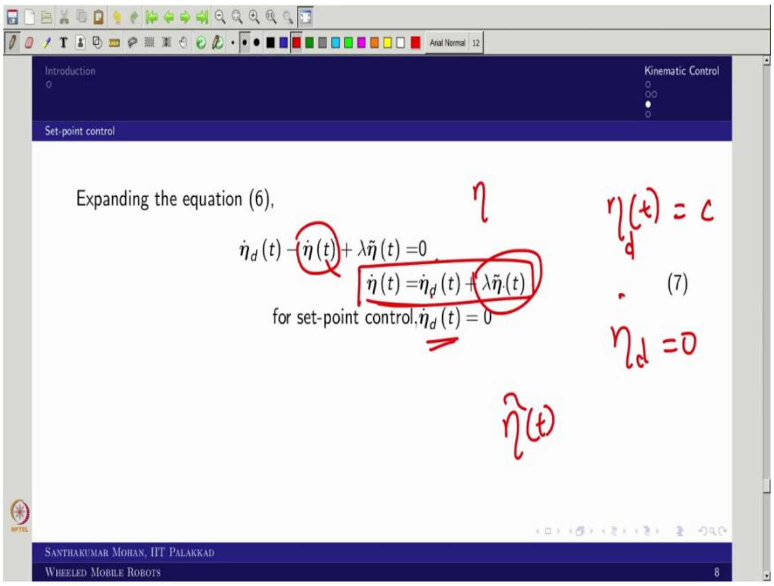

So, now you know this relation, you substitute that. So, then you will get it right. So, I am rewriting this. And I am substituting the relationship. First I m making it bigger, and then I am substituting this ok. So, if it is the set point, so this would be 0; otherwise I can actually like take it this way. So, now, what I have written, this I am keeping one side, and this all other terms I am keeping in the other side. So, now, this has come ok. So, now, you can see this has come.

So, now, what one can do? So, you can actually like use if it is a simple set point control where $\tilde{\eta}(t)$ is constant, in the sense $\dot{\tilde{\eta}}(t)$ sorry η(t) is constant and $\dot{\eta}$ is actually like desired is 0. So, that way it is actually like set point. But what you have done you have taken $\tilde{\eta}(t)$ in the sense what you have done you have done a feedback ok.

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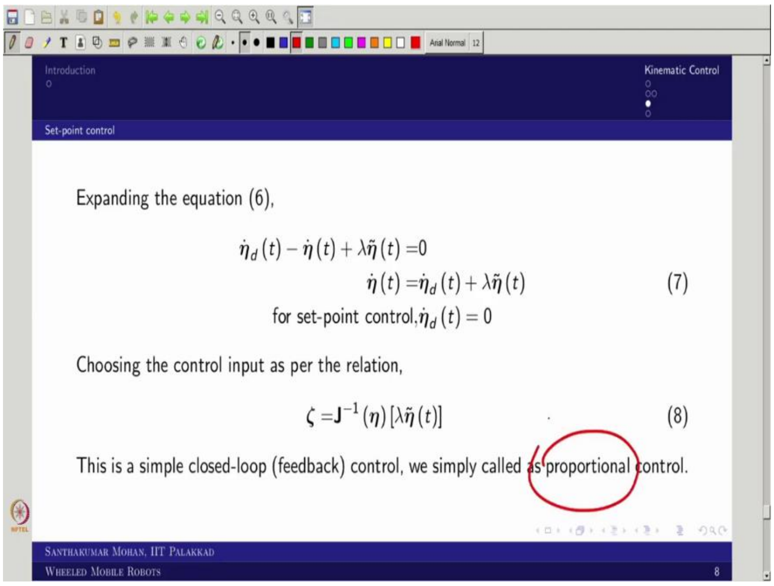

So, that is what we call a simple feedback control ok. So, now, we will move further. If ok there is a simple closed loop control that to like simple proportional control, this will work in a kinematic level.

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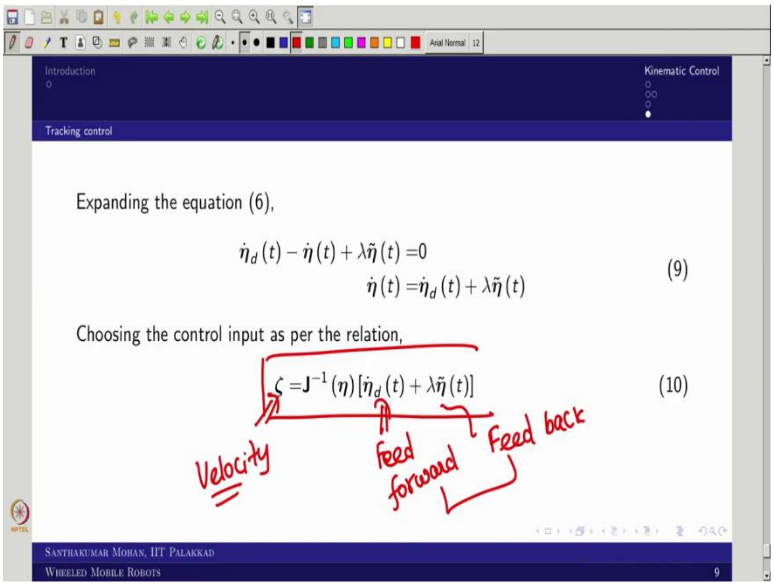

So, now we will actually like go the same equation. But you actually like substitute this and take it as a further. So, what you are doing it? So, this is a feed forward term, because you are actually like calculating based on the desired, and this is what this is a feedback term ok. So, in the sense, it is a combination of feed forward and feedback.

So, what we simply seeing? So, we are computing the velocity. So, this is a velocity command right. So, we are actually like computing velocity. So, in robotics we usually have a control called computed tar control. The similar sense here we are computing the velocity we simply call it is a computing or computed velocity control.

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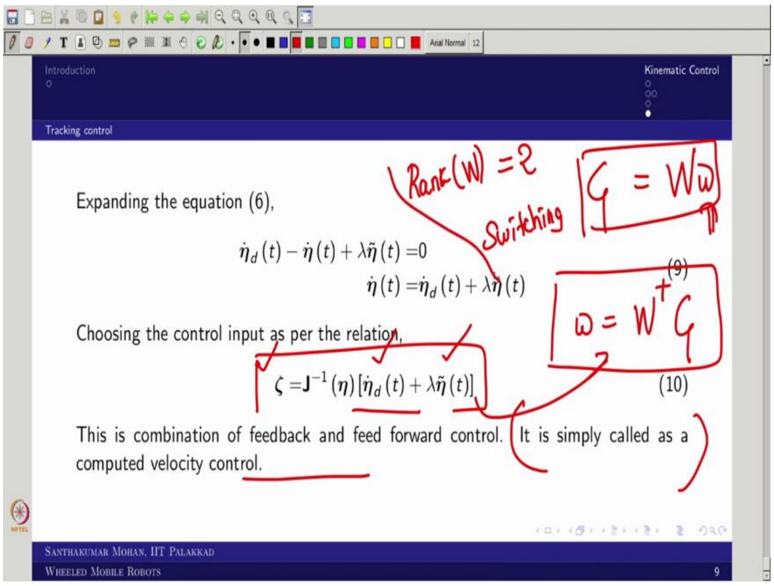

So, that is what we are actually like done here. So, now, what you can see like we have actually like make it this as a computed velocity control where the feed forward term and feedback term is there. So, now, this control you apply in a kinematic level, the two general land base it would be you can say workable.

Now, this called η you can further bring it down to this. And based on the wheel configuration, you can actually like see whether the ξ is achievable or not. Based on that, you can find the you call omega. In the sense what you have to see? So, this is what your final control activity. So, I am putting plus because sometime the matrix would be a rectangular. So, this is what? So, first you calculate this, and then you substitute this.

So, why I am doing this two stage because this is actually like generally you can do after that based on your constraint you can do it. So, that is what we are trying to do in the next lecture. So, in the next lecture, what we are going to bring? We are going to bring the wheel configuration, and then trying to do the same kinematic control but with this.

So, in that sense you can see like some of the mobile robot which we have seen mobile bases, for example, differential wheel, or you can see a tricycle, or unicycle, these things are not achievable all the three states. So, then you have to actually like compromise some of the performance. In the sense the rank the rank of what you call W is actually like less than 3, so in this case 2. So, then you can see that only two states are controllable, then you have to actually like move as a switching control.

So, what that mean? So, you will control first position, then orientation or something like a polar, and then actually like polar coordinate control, then you will bring to the Cartesian control. Like that you will actually like make the switching between one to another and then trying to do. So, in the sense what we are trying to see in the next lecture, we will see kinematic control of you call non-holonomic mobile robot in the beginning, then we will see the holonomic robot in the second ok.

With that I am actually like closing this particular lecture. See you then in the next lecture where we can see the mobile robot inside, and then understand how the kinematic control for certain special cases of vehicle, for example, differential wheel or tricycle. Until then see you bye.