Wheeled Mobile Robots

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Lecture – 03
Introduction to Mobile Robot Kinematics

Welcome back to the lecture on you can see Introduction to Mobile Robot Kinematics. So, the course on Wheeled Mobile Robots. As I already mentioned in the last lecture we were actually like talking about locomotion and types of locomotion, at end of the lecture I told that we would be talking more about kinematics in the 3rd lecture. So, that is what we are going to focus here.

So, in this particular 3rd lecture, we would be more focused on what is land based mobile robot, so what would be the kinematic relationship? So, how we can obtain that kinematic relationship? So, based on the kinematic relationship how we can actually like go forward in the further robot kinematic aspects; for example, forward and inverse differential kinematics.

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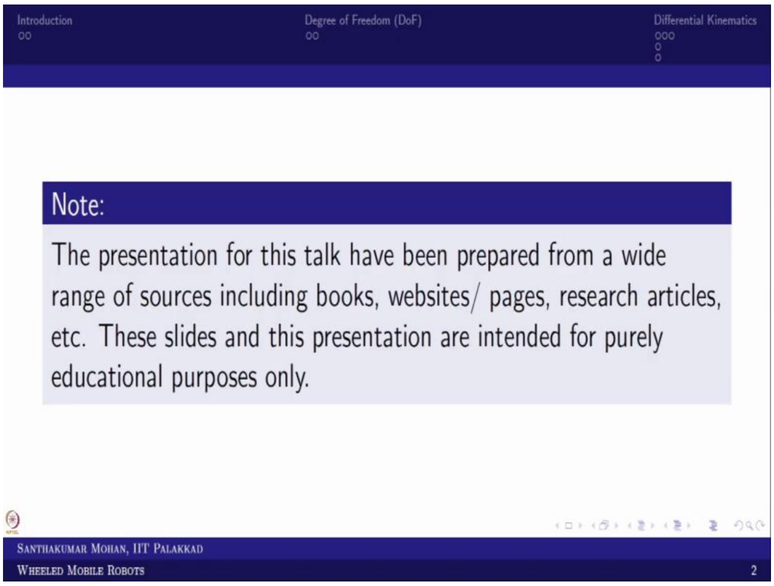

So, in that sense as we did in the last two lectures a similar way. So, this is the note.

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So, let us move to the particular topic called lecture 3 in this way. So, this particular topic or lecture would be focusing as I already mentioned, it would be focusing mainly on the mobile robot kinematics. As I already told mobile robot means in general it is a land base. So, land based means it is actually like having you call planner movement.

So, we will not be seeing the off planner movement. So, that is what the overall idea. So, let us start with the basic introduction about a mobile robot kinematics and then we move forward to what is degree of freedom and what is differential kinematics. So, this is what the overall flow which we planned for this lecture 3.

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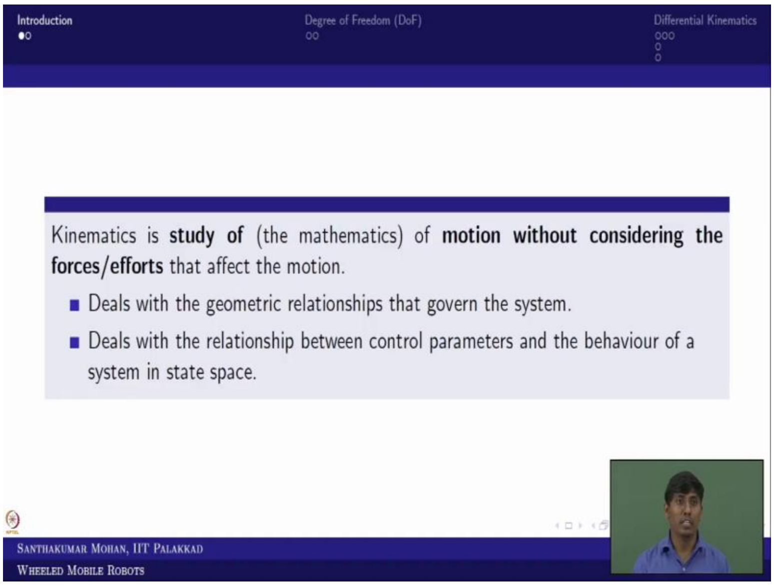

So, let us start with the kinematics. So, kinematics you already know it is one of the branch of you call physics, so where you talk about statics and dynamics. So, inside dynamics you know one of the you can say subsection called kinematics. But what this is all about; kinematics means study of motion without considering the forces or effects that affect the motion. So, this is what we have seen.

So, now, in that sense what we are actually like trying to bring here is the mathematical relation which is bringing the motion or you can say the geometrical relationship that govern the motion of the system. This is what we are trying to correlate. But robot kinematics means it is little more than this. So, what that mean? We are actually trying to map.

So, we are trying to map actually like two spaces or we are trying to map between the input and output. Although here the input is not force or you can say movement, but the input in the sense the control parameter and the system parameter what you call motion parameter we are trying to make a mapping. So, this mapping what we call kinematics in robotics. So, that is what we are trying to cover.

As I already mentioned you can see that kinematics what it deals with the geometrical relationship that govern the system. So, the other one is it deals with the relationship between control parameters and the behavior of the system in state space. This is what actually like one important thing.

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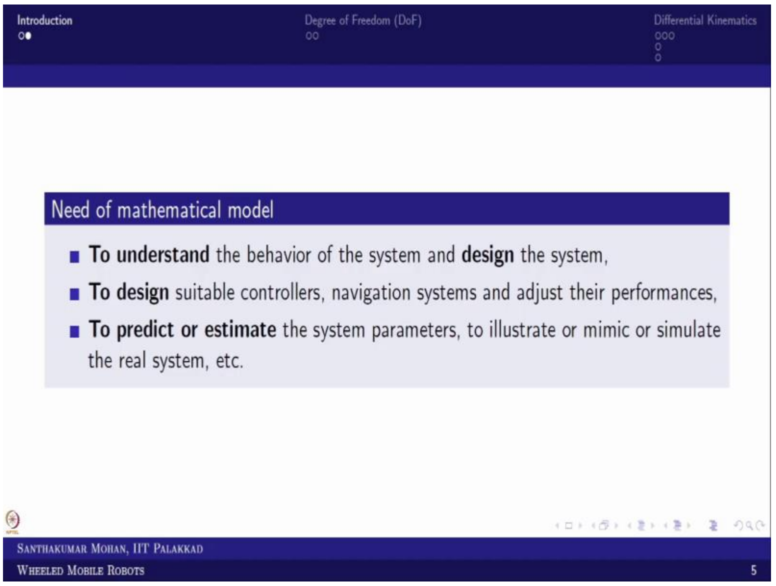

So, let us actually like move forward in that case. So, why it is required? So, the kinematical model or mathematical model, why it is required? There are 3 purposes which we are putting forward. So, one is actually like to understand the system. So, definitely, so what kinematics means? It is study of motion. So, in the sense we are trying to understand the system motion. So, that is one thing.

Second thing is what happens; so, if you are study about the motion, so what one can see you can actually like see how to design that particular system. For example, if I actually like make a two wheel mobile robot, so how that two wheels supposed to be located whether this is what the length or you can say the distance between these two wheel I need to put it like this. For example, now you take it as a cycle, bicycle.

So, the front wheel and the back wheel if I have properly the length or you can say distance between these two wheel, so that parameter change the overall system study of motion, right. So, in the sense you can actually try to do. In that sense you can see the need of mathematical model come the broad way, so which is nothing but the to understand and design the system, right. To understand the behavior of the system and design the mechanical system that is what I mean to say here the locomotion system.

The second point is very straightforward. Since, I already told the kinematic model is deals about the, you can say relation between the control parameter and this the system parameter. In the sense, what you can see, his mathematical model can be used for you can say design the proper motion controller. Further what you can see since it is a mobile robot, even we can extend for navigational system design and there you can say performance tuning. So, these are actually like two broad category. Any mathematical model for, you can say mathematical model of robot, definitely these two are the prime most.

The third thing which is actually like very one of the important thing very, you can say specific we can actually try it to you can say predict or estimate the system parameter. For example, you take a car. The car I am assuming in kinematic or even you take a general thing, certain parameter you cannot actually like measure or estimate you can see accurately.

So, then what we can do? We can actually like use a mathematical model which is you predominantly based on the first principle and you can actually like adjust the parameters based on the real you can say output and as well as your model performance. You can see you can actually tune and you can actually like identify. In the sense what people call it is can be used for system identification or parameter estimation.

For example, you take in the other way around. So, you take a open system and give input and take output. So, what one can see from the input and output relationship? You can understand the system, right. So, what would be the system? So, that is what we are actually saying that to predict or estimate.

In the sense, the mathematical model definitely can be used for these 3 purpose; to design, to understand, this is the combined fact and to design controller this is another fact, and the third fact which we call to predict and estimate.

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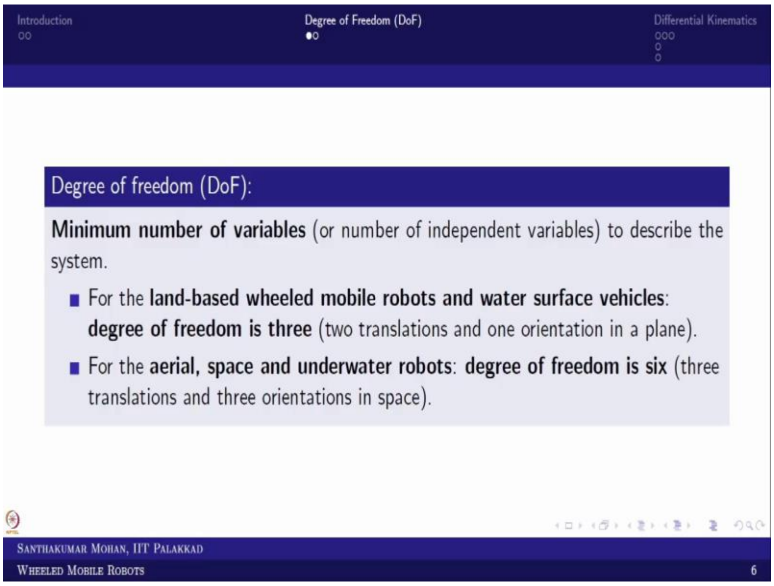

So, now we move a little forward. So, what that mean? So, we will talk the mobile robot kinematics. So, for that one of the important thing. So, what that? So, for example, now you are talking about the description ok that is what nothing but, right. So, what the description? So, you are going to describe the motion.

So, now, in order to describe the motion you have to see certain parameters, right. For example, you want to describe about me. Imagine, so you know like one of the faculty in your institute. He comes from probably IIT, Indore imagine. So, I was working in IIT, Indore, for example, 7 years, I was there.

So, now, you forget my name somehow, but you are actually like know some of the credentials of me. So, the person who is coming from IIT, Indore definitely put that credential you can understand. Now, for putting that credentials you have to put the minimum number of you can say credentials.

For example, you can say that the guy actually like worked in mechanical; you want to describe me, ok. This is one credential. Second thing is he graduated from IIT, Madras. Then you can see that even in IIT, Indore currently there are 4 faculty working in mechanical engineering who graduated from IIT, Madras. These are not sufficient, right.

Then you put another key word probably he is working in robotics. Then, also you can see that in IIT, Indore there are two professors are working in robotics then that to like from IIT, Madras itself. Then you can actually like put one more credential. You can say that this particular person who was actually like in Germany for more than a year, then you can see that the person who is actually listening in the other side he can understand oh this guy is talking about Shanta Kumar. Ok, now I got it.

So, now what you put? You put you can say list of description in such a way that the opposite person can understand and identify the, you can say the correct person. So, now, the same way when you talk about you can say description, so when you talk about the study of motion the motion description. So, how many variable required to describe in a unique way? So, this is what we are calling as a minimum, number of variable to describe the system.

This minimum number of variable what we are going to call as degree of freedom. Why it is called degree of freedom? Because we are talking about the system which is in moving. So, what are the possible motions or what are the possible movement happened on the system that is what we are trying to give a idea. So, in the sense what in the other way around you can say, number of independent variable to describe the system in unique manner. This is what degree of freedom.

So, now you got a very general definition, right. So, now, we can actually like see if you talk about a mobile robot, so what are the possible motion? So, mobile robot mean it is a land base. So, you can see that the robot can move in longitudinal way, lateral way, and it can rotate about its own vertical axis.

In the sense of what are the possible motions? So, there are 3 motions. So, now, in order to describe these 3 motions, what are the number of parameter required? At least 3 parameter, right. So, that is what we are going to call as a degree of freedom. In the sense for land based you can say mobile robot in general wheeled, in specific wheel and whatever you call water surface vehicle all are actually like coming under a planner case. So, we are assuming that the off planner movement is not there.

So, in that sense what are the possible motions? Two translations and one you can say orientation in a plane. So, in the sense you can see there are 3 motion primitives are required. In the sense the degree of freedom is 3.

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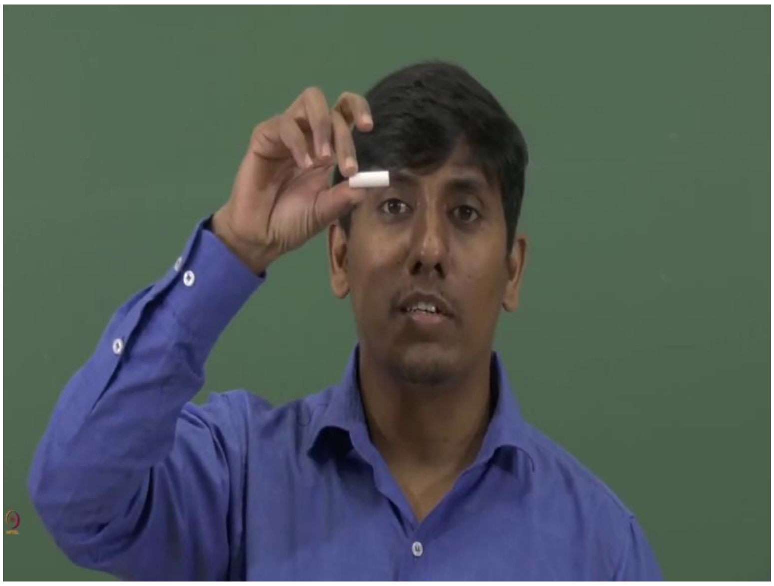

Now, you take a very general case you can take a space. So, now you take for example, this chalk. This chalk actually like I can put it on the space. So, now I want to describe this chalk motion for example. So, how many variables required? This chalk and actually like move in 3 dimensional space.

So, in the sense 3 translation and 3 orientations required. In the sense the degree of freedom for a general case in 3 dimensional space, it required 6. So, now you take a aerial robot or underwater robot or space robot these are all required actually like 6 parameter described this is the degree of freedom is what you call 6. So, 3 translation and 3 orientations are what you call actually like a degree of freedom for this particular system.

So, this is what we are actually like mentioning. So, now, you know what is degree of freedom. It is nothing but minimum number of variable to describe the system in unique sense. In that case, we are focusing on the land base; in that sense the degree of freedom is 3, that is what we are focusing.

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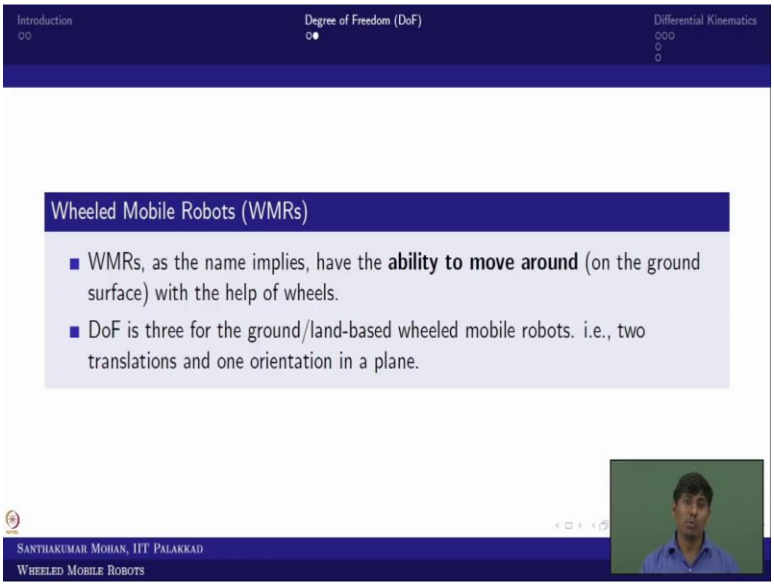

So, in the sense what we are doing in this particular course? This particular course is all about wheeled mobile robot. In the sense, I am putting the same keyword again. So, ability move around with the help of wheels. So, in the sense the degree of freedom in this case is 3, right. So, now, two translation and one orientation we need to describe in a plane.

So, now we are taking that into a general case. So, now, imagine I have a magnetic duster and there is a steel board in the behind. So, now, I put that magnetic duster on the steel board, what will happen to that? That steel board will not allow the magnetic duster come away. But if you actually like move around what happened? The two translation and one orientation is possible.

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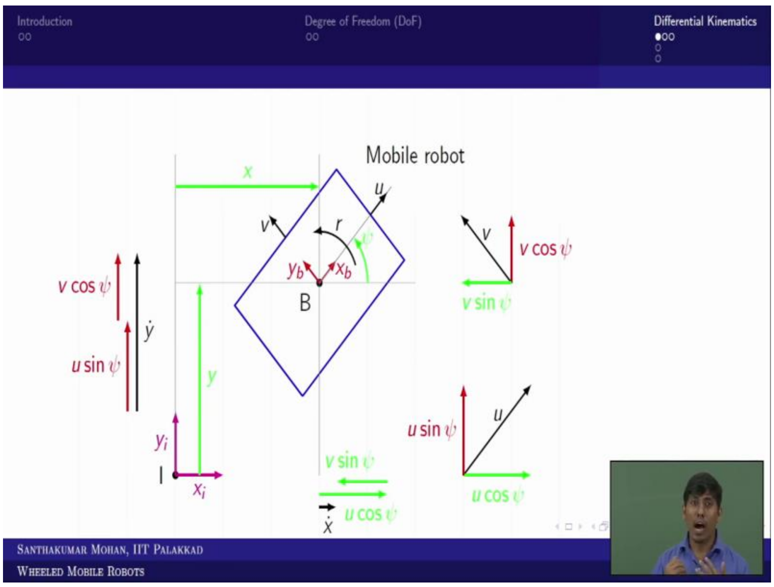

So, that is what I am trying to put it in this particular slide. You can see that this blue color box what I drawn imagine as a magnetic duster. Now, this magnetic duster can actually like move lateral, in the sense in this case it is a longitudinal lateral and as well as rotate about it, right.

Now, I call that is as a mobile robot in a land, land based systems this is a mobile robot. Now, in order to describe this system what I need? So, first of all in order to describe some standard thing is required. For example, you are actually like describing about me for example, brown color skin, you can say long hair, short height. So, these are description.

But now the same description if I go in a different country, for example, if I go a Mongolian country I may not be a short height, right. If I go probably in a Korean country I may not be having a longer. If I go probably Africa, I may not be a you can say brown fair, right. So, in the sense you can see that there is some reference you have to maintain. So, what that? So, you need to have some reference.

So, in that case here we are going to describe about the motion. So, what would be the motion? Motion can be described in two translation and one orientation that is what here. So, in the sense what you need to have? You need to have a frame. So, in the sense you need to have a coordinate system.

So, there are several coordinate systems are possible, but we are taking one of the simplest one which is what we call Cartesian coordinate system. There are 3 mutually perpendicular axes which is connected with a single point that is what we call Cartesian coordinate system. Since, it is in a plane we can take one of the Cartesian coordinate system here. And now you take even Cartesian coordinate system.

Still I cannot define the system, right. For example, if I take a duster on the board. So, I need to define the position of the duster with respect to something. So, for that what I am bringing? I am bringing one of the point which I call I. So, why I call I? This is inertial. So, I am going to call I means inertial, in that sense it is fixed on the wall or fixed on the board, it is fixed.

So, now, I am actually like putting the Cartesian coordinate system where Z_i in the sense Z axis that is actually coming out of the screen, ok. So, that is what we are going to use. In that sense, we are going to use the, right hand coordinate or right hand rule, so where thumb finger goes x, you can say forefinger goes y and Z axis would be represented with the middle finger. So, now, this is fine.

So, still you may have a doubt. Sir, how can I represent the motion of the robot here? Because this is one of the coordinate, but the mobile robot is having infinite number of points, right. So, how I will define? So, for that what we are bringing? One more coordinate which is we are going to call body fixed coordinate.

So, this is I am putting one of the convenient point here which is nothing, but B. That B is having another coordinate system call you can say x_b, y_b and a Z_b. Since it is in a plane, so I am not showing a you call Z_b and Z_i.

So, now, you can see that I put one point B, another point is I. So, definitely in a Cartesian way I can define the point B with respect to i. So, what that would be? So, I can actually like a draw imaginary line. I can actually like write it x would be the x axis displacement or x axis position with respect to I, right. So, and y would be the y axis position of b with respect to i. And I am putting another variable called psi which is nothing, but the rotation about Z axis.

Now, you can see that I have already done, right. What I have done? I have actually like represent the point B with respect to the fixed frame I. But what the issue comes? The mobile robot is actually like moving system, and mostly what happened the mobile robot would be connected with a wheel or leg that would be energizing the mobile base.

For example, even you take a duster the duster will not be having any input with respect to i, the input would be with respect to the B frame in the sense what we know from the body frame. So, there would be a longitudinal velocity which I am representing as u and there would be a lateral velocity which I am representing as a v, and there is a angular velocity which I am representing as r. So, these are related with a body, ok.

So, now, if I take B is instantaneously frozen, so now, what would be the instantaneous velocity of B along with x_b y_b? That would be u and v and what would be the instantaneous velocity of B with respect to is Z_b which is the angular velocity that is r. In the sense, I have a instantaneous velocity $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$, which I am assuming that this information is known with respect to frame B. But what I wanted? I wanted $\begin{bmatrix} x \\ y \\ \psi \end{bmatrix}$.

So, how I will get? This is what the motion variable, right. So, how I will get? So, you know the equation, right a = b when this would be valid. So, there are two variable I am writing a =

b. When this equation is valid? Both a and b dimensions are, right and units are, right. So, in this case you can see that $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$ is one set of variable.

So, $\begin{bmatrix} x \\ y \\ \Psi \end{bmatrix}$ is another set of variable, but you can see $\begin{bmatrix} x \\ y \\ \Psi \end{bmatrix}$ or you can say positional variable, there are linear position and angular position; whereas, $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$ is the velocity information in the sense linear and angular velocities. Both are actually like distinct. So, these are not straightaway same.

So, then what one can see? Either you can actually like integrate that velocity bring it to you can say positional domain or you can actually like elevate the position by differentiating and bring it to the you can say time derivative. In the sense, what one can see, so you can actually like do either one, so which is easy the instantaneous velocity is already instantaneous, I cannot get the instantaneous position, right.

So, obviously, one can actually like see, so I can elevate the $\begin{bmatrix} x \\ y \\ \Psi \end{bmatrix}$ into $\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\Psi} \end{bmatrix}$. Whatever I put x top of dot that represent $\frac{dx}{dt}$, ok. So, this is what we are trying to see. So, now, you can see that the $\dot{x}$, $\dot{y}$, and $\dot{\Psi}$ would be along with the x axis that would be $\dot{x}$, along with y axis that would be $\dot{y}$, and above the Z axis rotation that is what $\dot{\Psi}$, but what we have $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$.

So, now, the u can be actually like represented on the frame of I. So, how we can represented? So, we can project, so you know law of cosine. So, now, I am actually like taking the u and projecting on the frame you can say I.

So, now what would be there? The angle already I know Ψ, so if I projected on the x axis that would be equivalent to ucos(Ψ), if I projected on the y axis that would be equivalent to usin(Ψ). So, now, similarly I take $\vec{v}$, so if I actually like projected on x axis and y axis, it is vsin(Ψ) and vcos(Ψ). So, now, I projected. So, then why you projected? So, this is one of the question will come.

Now, since the system is you can say static at instantaneous point, in the sense whatever the motion happening on the B, if I actually like see with respect to I that supposed to be same, right. So, in the sense you can see that whatever the longitudinal velocity which is ucos(Ψ)and vsin(Ψ), the vector addition supposed to be equivalent to $\dot{x}$ that is what we are doing it. So, I am taking ucos(Ψ) and vsin(Ψ), I am doing in the vector addition what would be that is what $\dot{x}$.

Now, similarly you can see y axis vector addition vectors, so usin(Ψ) and vcos(Ψ). So, if I take vector addition that would be equivalent to $\dot{y}$. And what one can see easily the $\dot{\Psi}$ and r, are actually like in the same you can see direction and as well as it is parallel. So, in the sense r is straightaway equivalent to $\dot{\Psi}$.

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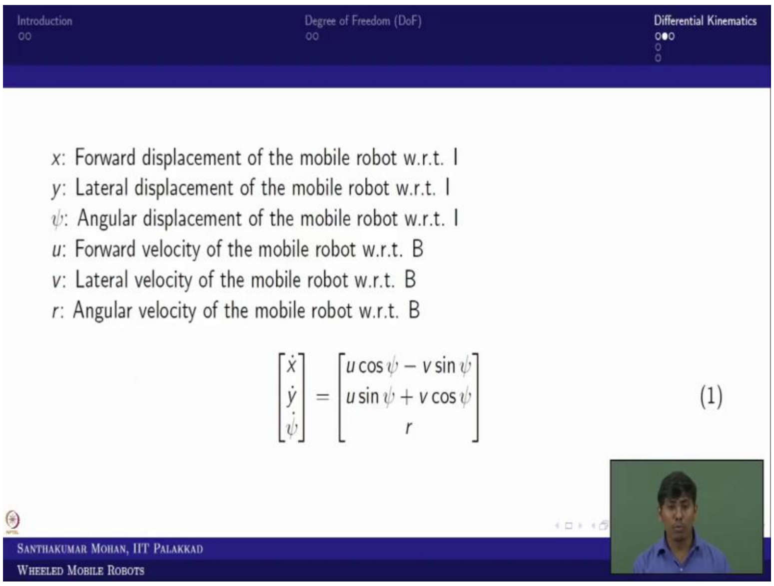

So, in that sense what one can see, so $\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} u\cos\psi - v\sin\psi \\ u\sin\psi + v\cos\psi \\ r \end{bmatrix}$, that I am writing in a vector form. You can see this particular slide, so $\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\Psi} \end{bmatrix}$ I am writing as one of the vector that later on I am going to call as eta dot, so where eta is vector of $\begin{bmatrix} x \\ y \\ \Psi \end{bmatrix}$. So, $\dot{\xi}$ is actually like $\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\Psi} \end{bmatrix}$ that is equivalent to $\begin{bmatrix} u\cos\psi - v\sin\psi \\ u\sin\psi + v\cos\psi \\ r \end{bmatrix}$.

So, now, what one can see? I can actually like group it this $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$ into one side and $\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix}$ is another side. So, whatever the in between that is what you call mapping. So, now, what you can see $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$ is the body fixed instantaneous velocity and $\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix}$ is the initial fixed time derivatives of the generalized coordinate.

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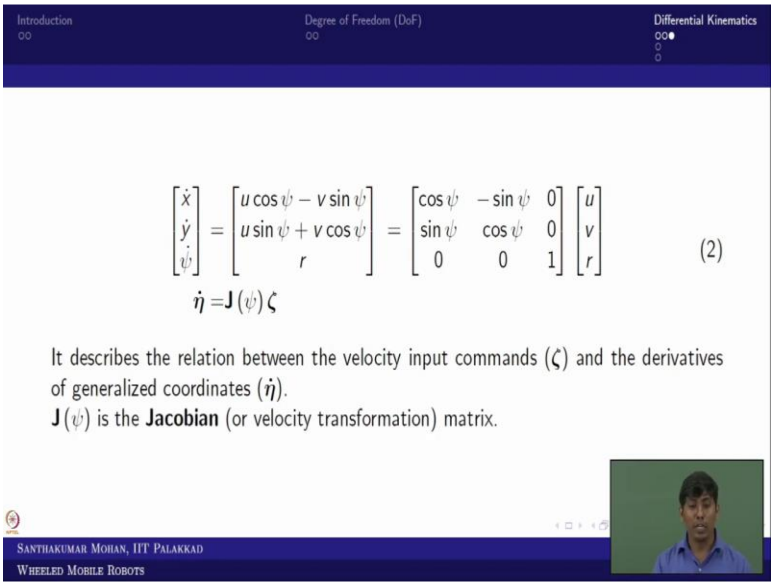

Now, I can map. So, this is what the next slide is telling. You can see this is the relation I got it and I am rewriting into a matrix and vector form. So, what one can see, I can map $\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix}$ with respect to $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$. So, this is what we call kinematic relation. So, this is what the kinematic model.

So, now, this particular matrix I am going to call as one of the generalized matrix call mapping matrix, some people call kinematic transformation matrix, but this kinematic transformation as actually like given in a time domain in the sense time derivative domain given by Jacobian, so this particular matrix called Jacobian matrix. So, now J(Ψ) what you call Jacobian matrix.

What this particular relationship is giving a idea? Where you consider $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$ as one of the vector called ξ that is what nothing but velocity input. So, now, you assume that there is a command.

So, now, velocity input command is mapped to the you call the time derivatives of the generalized coordinate $\dot{\xi}$. So, now you can see that this is mapping between these two, right. So, that is what we call mobile robot kinematics.

Now, what you obtain this equation nothing but the robot kinematic equation. So, as I already told robot kinematics means not only study of motion it is mapping further, right. So, now, you can see that the mapping between happening either actually like velocity input command to the time derivative of generalized coordinate. So, now, based on that it can be classified into two, ok. So, what that mean?

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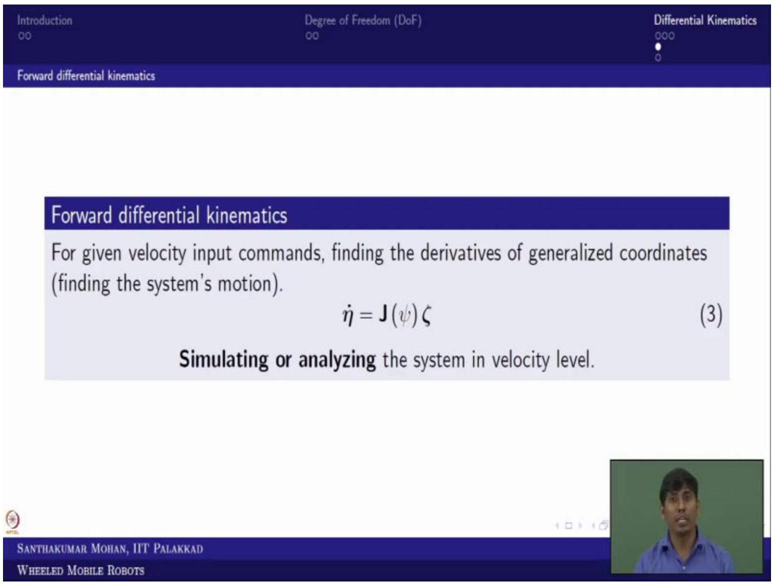

So, one is actually like you give input command velocities are input velocity command you give it, in the sense you would take the mobile robot, connect the wheel, and connect the motor and run the motor with certain speed. So, what you will see? You will see how the robot moves, right.

So, in the sense what you are trying to understand? You are giving the input command and seeing the you can say the time derivative. And if you integrate that what you will get? That generalized you can say generalized coordinate. So, this is what we are trying to see.

For a given velocity input command finding the derivatives of generalized coordinate nothing, but finding the system motion this is what you call forward differential kinematics. So, why it is so called forward? Because your input is straightforward given and you are seeing the motion variable how it moves.

So, this is nothing, but one scenario. If you are doing mathematically it is tried to simulate, you take a real robot and do it nothing but analyzing, right. So, that is what we call, so simulating or analyzing the system in velocity level that is what forward differential kinematics. So, now, what would be the next case? You just imagine; you just imagine, so you are having some desired derivative of generalized coordinate you want to see what would be the input.

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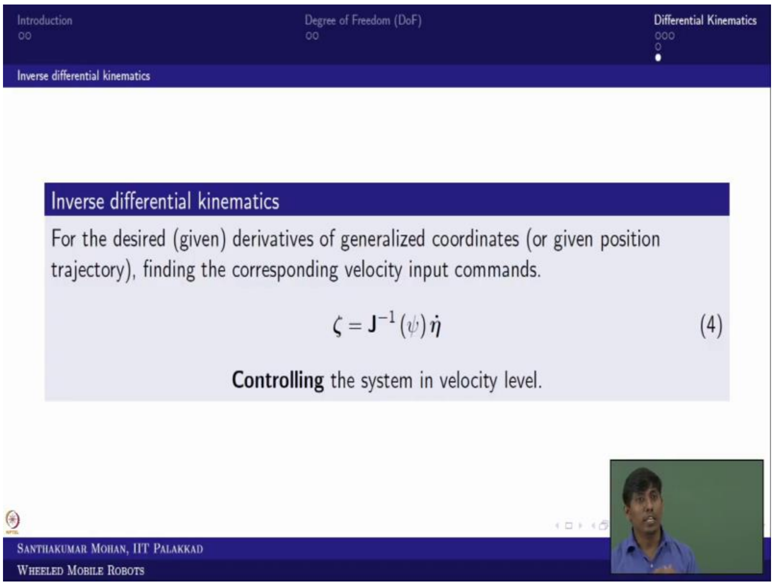

So, in the sense you are trying to do the reverse way. So, that is why it is called inverse differential kinematics, where you can see that for a given you can say it desired derivative of generalized coordinate you are trying to find out, you can sees that corresponding velocity input command.

In the sense, what you are trying to see? For a given position trajectory you are trying to see what would be the you can say body fixed velocities. So, this is what we call you can say inverse differential kinematics. You know already the equation. You have see eta dot is straight away Jacobian of you can say $J(\Psi)\times\xi$.

Now, we are trying to find out the ξ, so obviously, what one usually you see this is known, and this is actually like known and this is what unknown, so you take the inverse of this. So, that is what we are doing it. So, you can even imagine that this is the way you can actually take, remember that inverse means J^{-1} will exist, right this is a blind it is not, right, but I can actually like give a idea.

So, zeta is $J^{-1}(\Psi)\times\dot{\xi}$. So, now, what we are trying to see? The given I already mentioned give and decide. So, in the sense what we are trying to see? I want the robo supposed to move in this manner, I want certain motion this way, can I actually like make it.

In the sense, I am trying to navigate in particular way. So, in the sense what you are trying to do? You are trying to do a controlling action, right. So, that is what you call controlling the system in velocity level. So, this is what the inverse differential kinematics.

Further on, this inverse differential kinematics what we call control; the control can be open loop, in the sense you just take $\dot{\xi}$ and $J^{-1}(\Psi)$. This is actually like very open, but you can actually like even make a closed loop, you take a feedback and make it then that is actually like feedback control, but these all we would be seeing in the end of this course.

But, right now what you understood is actually like what is mobile robot kinematics? It is nothing but mapping between two you can say velocity spaces. So, one is actually like input you can say velocity input commands, so the other one is derivative, that to time derivative of generalized coordinate. You are trying to map why it is mapping because in mobile robot it cannot be directly find it. So, you have done with the mapping. So, this is what.

The mapping further divided into two. So, one is forward differential kinematics another one is inverse differential kinematics. In the next lecture, we will see how the ξ is obtained because the ξ is actually like a slightly different, right because the ξ is actually like what you call velocity input commands.

The velocity input commands is actually like coming from the wheeled configuration. So, then we have to bring one more mapping where the ξ would be coming with you can say angular velocity of the wheel. So, that is what we are trying to address in the next lecture.

So, before that probably will see the one of the important aspect you should know like what are the types of wheels which are used in the mobile robot, how that can be classified, and how we can actually like make it. In the sense, the next slide or next lecture would be talking about more about a real mobile robot. So, right now we talk about degree of freedom, but since it is a mobile robot the maneuverability will come into a picture.

So, the next lecture would be talking about degree of maneuverability and we will see how to obtain that. So, based on that the further lecture will come about the wheeled you can say locomotion in the sense, you will be bringing the kinematic relationship between the angular velocity of the wheel to the input command velocities and then you can make it to the time derivative of generalized coordinate.

So, with that you can see that this particular lecture 3 is over. So, then lecture 4 would be talking about types of wheel and then lecture 5 would be talking about the kinematic simulation. So, with that we would be finishing this course here. So, we will see in the lecture 4 later. Bye.

Thank you.