Wheeled Mobile Robots
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Lecture - 15
Kinematic and Dynamic Models of a Mobile Base with Four-Independent Steerable Power Wheels

Welcome back to Wheeled Mobile Robot. So, this particular course is actually like so far what we have covered is actually like basic Kinematic Dynamics. And in the last class what I was ended one of the dynamic simulation along with forward dynamic simulation of few example we have taken. In that sense what I said, one of the special drive we can actually consider in the next class.

So, that is what we are trying to do. So, one of the special drive here I have taken is four independent steerable power wheels; so, the configuration which we has four independent steerable power wheels. So, if that is the case we will first start talking about kinematic model then we will go to dynamic model and then see how this differ from the mecanum or what you call Omni directional wheel ok.

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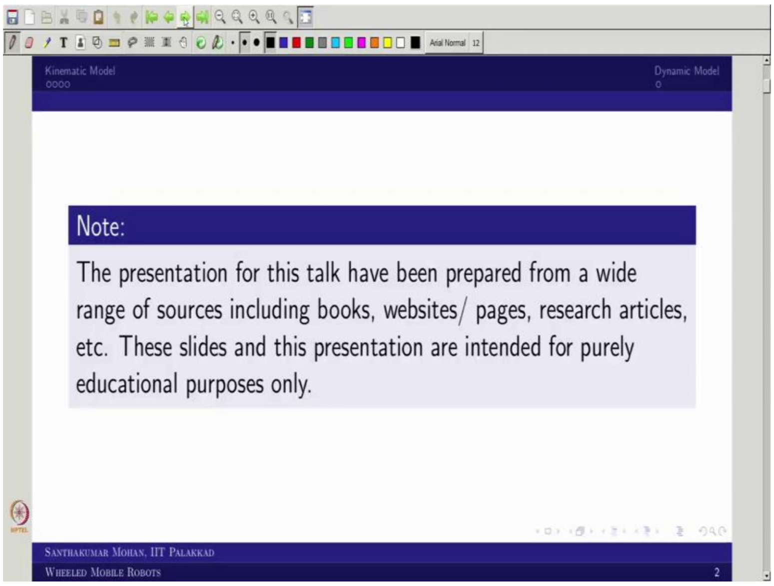

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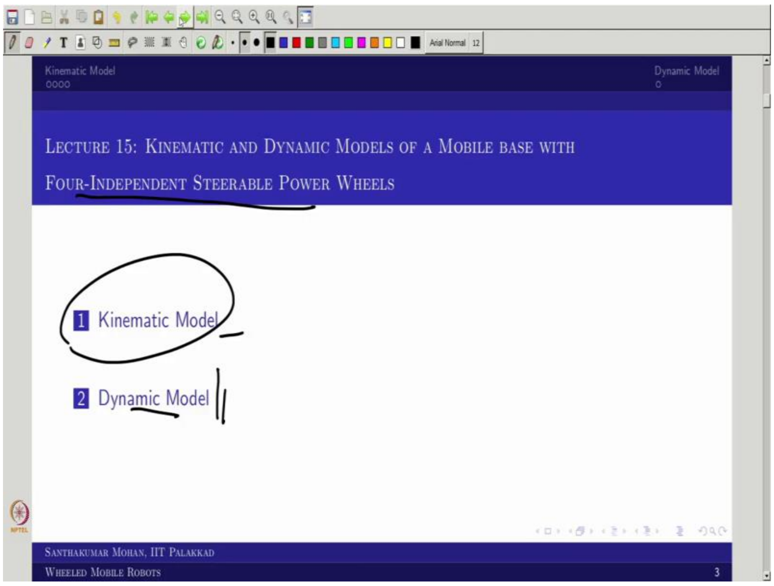

In the sense we will actually like move forward. So, where you can see like this particular lecture would be talking about the Kinematic model of four Independent Steerable Power Wheel Mobile Base and then the dynamic model derivation; you know already the dynamic model derivation is not much difficult; whereas, the kinematic model may be tricky and here we have to substitute not straight forward what you call wheel generalize model.

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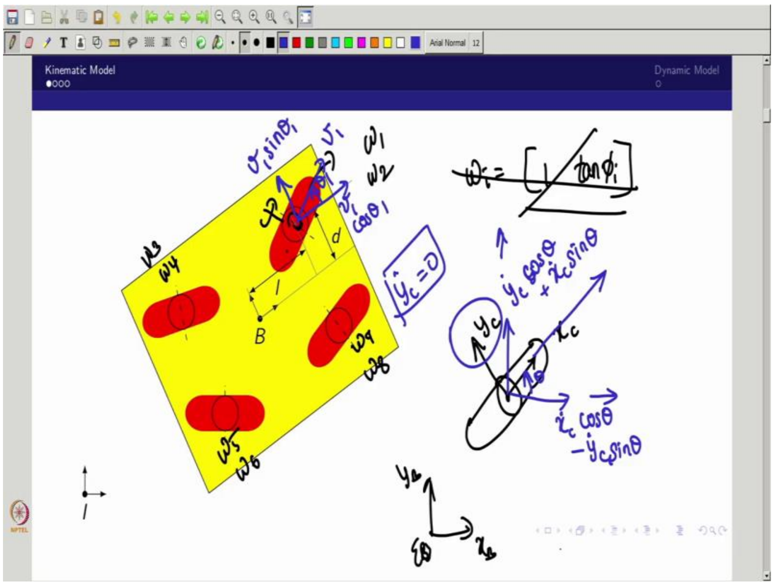

That is what the whole idea for bringing this. So, for understanding what I took the same way the yellow patches what you call the mobile base or you call mobile robot that as actually like four red wheels. So, in the sense you can see like these are the four wheels. So, these four wheels what one can actually like see very clearly.

So, this is actually like steerable in the sense it can do this and it can do this. So, in the sense it can roll and as well as steering about the vertical axes in the sense what are the inputs? So, you have actually like ω_1 and ω_2 in 1 and ω_3 and ω_4 in 2 ω_5 and ω_6 in the third wheel and ω_7 and ω_8 in the fourth wheel. And we can actually like understand what would be the coordinate and all.

So, now, what model we can use it? So, this particular model may not be useable. So, what that mean? So, what we have derived as actually like 1 tanϕ 1 that to like I put it is actually like a then it is actually like based on that. So, that particular model may not be valid. So, then what model we can look at it?

So, we can actually take this is what you call the steerable wheel. So, now, this steerable wheel would be having X_C and Y_C like this; whereas, I am actually like assuming that this is what the body frame B. So, this is X_B and Y_B. So, now what I can actually like see it what would be the velocity on this ok and what would be the velocity on this. So, if I know this is what I call ϕ or θ some angle. So, then you can see $\dot{X}_C$.

So, cosθ and Y_C I will just write it. So, $\dot{Y}_C$ you can say give it is actually like cosθ + $\dot{X}_C$sinθ ok. So, here actually like what it will come? So, $\dot{Y}_C$ sinθ right. So, this would be equivalent to this direction and this would be equivalent to this direction.

But by looking here so, the Y_C would be $\dot{Y}_C$. So, would be 0. Why? Because it is a fixed wheel and it is pure rolling. So, in the sense what one can see that the tangential velocity generated only along X_C direction.

So, if that is the case what one can actually like brought. So, assume that that is V_1 for understanding. So, this is actually like I call θ_1 then you can see that along X_B direction that would be $V_1\cos\theta_1$ along with Y_C sorry Y_B direction that would be $V_1\sin\theta_1$. Can I write anything like this? Yes I can write.

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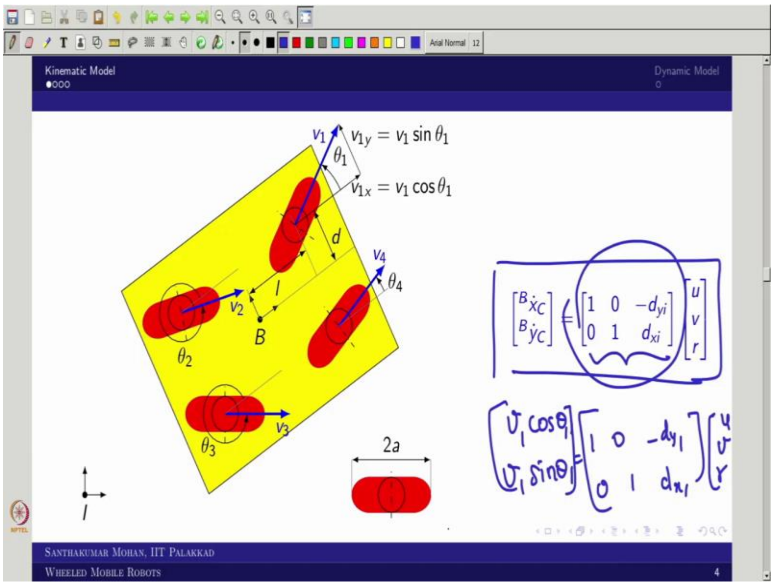

So, you can actually like see. So, this particular relation what we have obtained ok; so, I am actually like writing this with all the wheels ok. So, what I am actually like bringing it all the wheels as θ_1 to $\theta 4$, but what model I am actually trying to use? So, this is the equation we have obtained.

But what model I am trying to use? So, based on this ok; so, the ${}^{B}\dot{X}_C$ in this case only tangential velocity then I can write in this form right. So, if I know this form from the previous derivation itself. So, what I need to substitute? Only I need to substitute here actually like $\begin{bmatrix} V_1 cos\theta_1 \\ V_1 sin\theta_1 \end{bmatrix}$.

So, this would be coming as $\begin{bmatrix} 1 & 0 & -d_{y1} \\ 0 & 1 & d_{x1} \end{bmatrix}$ relation ok. So, this relation you can actually like use it. So, then you can see that that would be written in $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$ form right. So, now, similar way I can write it for all four wheels. what I can see? I can collate that right.

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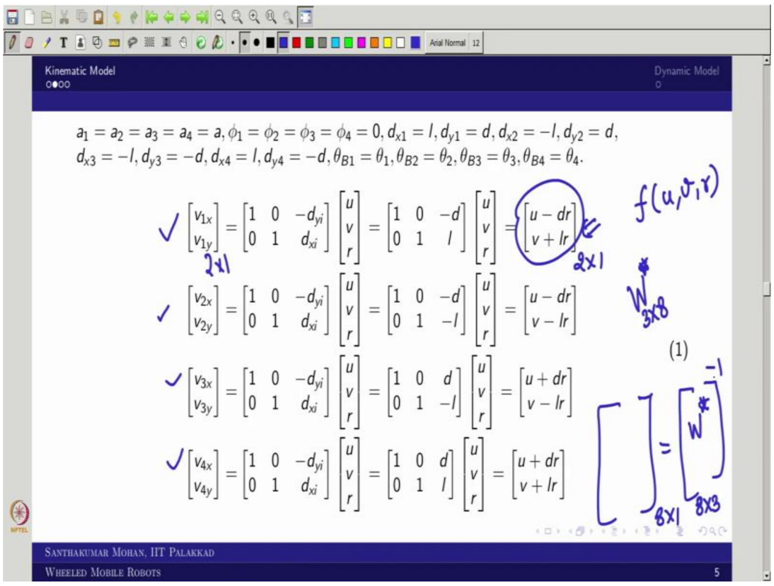

So, that is what I am trying to do. I am trying to do for wheel 1 I have shown; the same way I will do for all four wheels. So, one can see here. So, although it is actually like 2 × 2 in the sense the final matrix, but here you can see it is a 2 × 1 final answer also 2 × 1, but what one can see here. So, it was combination of $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$ right, all four wheels. So, in the sense what one can see you can actually like make it the left hand side as 8 × 1.

So, then you can see this would be. So, you can write 8 × 3 ok. So, that is what we are trying to make it as a kinematic model; so, in that case. So, your W in this case; so, I put W^* because not straight away W in that case it would be 3 × 8. So, here what you got? This is W^{*-1} ok.

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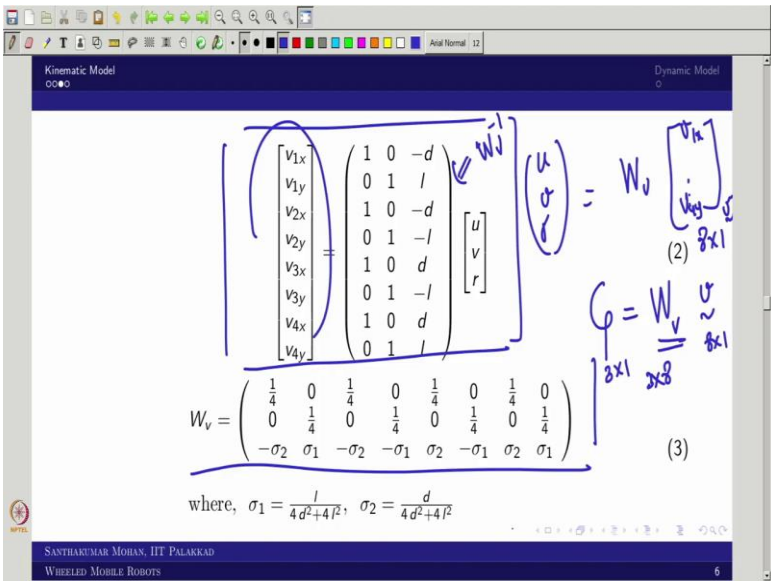

So, that is what we are actually like obtain. So, we will actually like rewrite that into the equation form ok. So, now what you got? This is actually like W_v^{-1} because I did not put what you call W^*. So, here I have written as W_v is the representation. So, then this is equal to W_v^{-1}. So, now, if I actually take the inverse what I will get? The W_v; so, now, you can actually like rewrite. So, the $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$ I am writing as W_v into.

So, 8 × 1 which is actually like V_{1x} to V_{4y} ok. So, this is what I call some vector call small v vector. So, this small v vector I can write instead of W_ω I can write as $W_v \times v$. So, in the sense I can write the ξ. So, naught or dot; so, this is actually like $\xi = W_v \times \vec{v}$ ok. So, now this is actually like in this case it is actually like you can write 3 × 8 and this would be 8 × 1 and this is 3 × 1. Now imagine it is having only what you call.

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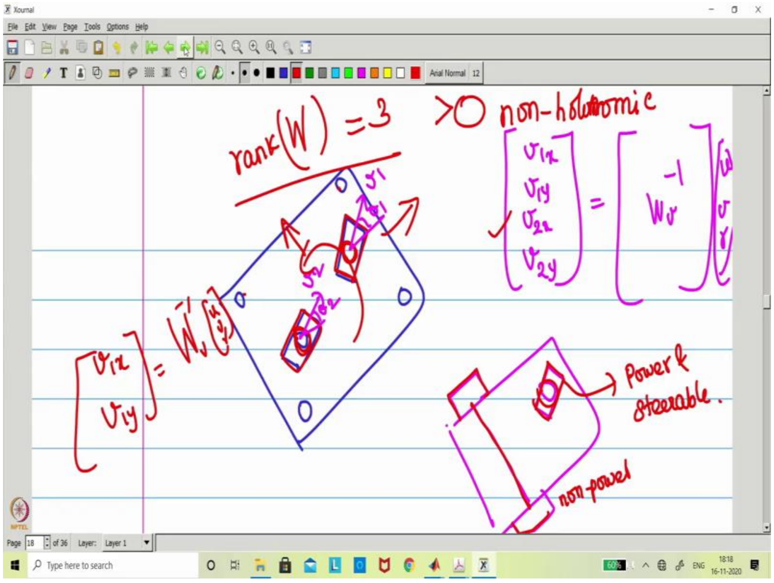

So, now I am actually like adding one page. So, you imagine this is having only 2 wheels right. So, in the sense you have one steerable wheel here and another steerable wheel is here ok. So, just for stability I am putting four casters. So, in the sense these are actually like what you call steerable wheels.

So, this red color these are steerable wheel. So, now, what you can actually like rewrite this would be having. So, V_1 and this would be having V_2 and I assume this is θ_1 and this I assume θ_2. So, then you can see. So, this would be again V_{1x} to V_{2y}. So, this I can actually write it as the W_V^{-1} into $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$ right.

So, what benefit I am getting here? So, the benefit I am getting is here. So, everything I am writing in the form of what you call the velocity component linear velocity component; so, now, the same thing you take it even the other model which I have already discussed. So, for example, you take you can see a simple what you call the powered and as well as what you call steerable in the only front and these two are actually like single axle and it is actually like non power.

So, these are non power and this is actually like power and steerable. So, then what you can do it you can write this equation in this way. So, $\begin{bmatrix} V_{1x} \\ V_{1y} \end{bmatrix}$ and you can write as W that is

W_v^{-1} into $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$ right. So, now, in the sense even you bring n number of you can say steerable power wheel independent power wheel then you can actually make it.

So, now one question will come whether this is actually like what you call; so, the holonomic or not ok. So, for that what we said the W rank. So, the rank(W) = 3 then you call holonomic if not then what you call it is non holonomic right in the sense if it is actually like less than 3, so then it is what you call non holonomic ok.

So, that is what we discussed in this case you see this is actually like going to give a rank you can say in this case 3. Why it is? So, because it is actually like going to give all possible motion in the sense it can go longitudinal, it can go lateral and as well as it can rotate. Why it is so? There is no you can say fixed wheel.

It is a rotatable conventional wheel is there and there are four casters. So, whereas, you look at even the previous configuration what we obtain. So, this is also the same thing none of the wheel like fixed; so, where it is fixed with the base none of the wheel like this. So, in the sense what one can see? It can actually like provide the lateral motion as it is. So, in that sense what one can actually like see. So, we can actually like use this as a Omni directional mobile robot; that means, it is a holonomic mobile robot.

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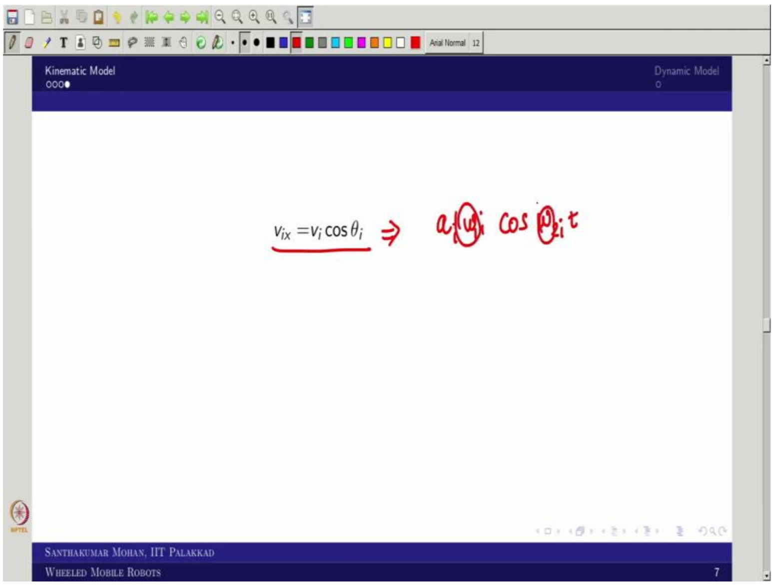

Now, we will come back to the dynamic model derivation for the same thing ok. So, this is what we were actually like written further where the V_{1x} I can write as $V_i \cos \theta_i$ where this V_i is actually like coming from the drive. So, that I can write as $a_i\, \omega_i$ where this θ_i is coming from. So, this one I put $\cos \theta_i$ is coming from $\omega_{2i}t$. So, now, I can rewrite in the sense ω_1 and ω_2 I can relate on the first wheel ok.

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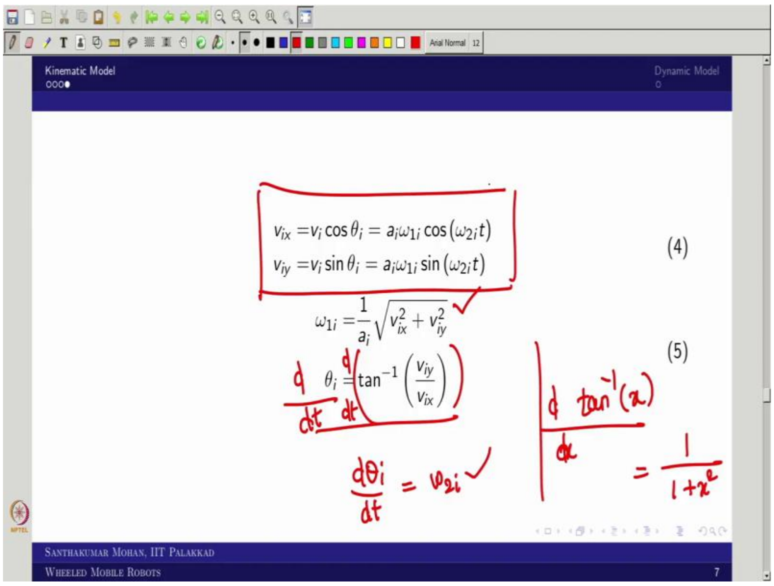

So, like that I can write every wheel. So, now, I know these two from the what you call the kinematic model which we derived. So, now, we are actually like using this relation can we find out ω_i in the sense ω_{1i} and ω_{2i}? Yes, I can do it you can write this equation bigger. And then you can actually like find it ω_{1i} then ω_{2i} is not straightforward.

What one can do? You can actually like first calculate what is θ_i and then you differentiate this ok. So, what it will give? So, this you differentiate. So, you know like $\frac{d\tan^{(-1)}(x)}{dx}$ I am just writing. So, that would be $\frac{1}{1+x^2}$ right.

So, if you substitute that relation you can find out this $\frac{d\theta_i}{dt}$ = ω_{2i} that you can write it based on the derivation what you are obtaining. In the sense what one can see what you wanted? So, that ω also you can obtain once you know this individual $\begin{bmatrix} V_{1x} \\ V_{1y} \end{bmatrix}$. So, that is what we have done in this particular what you call special drive model.

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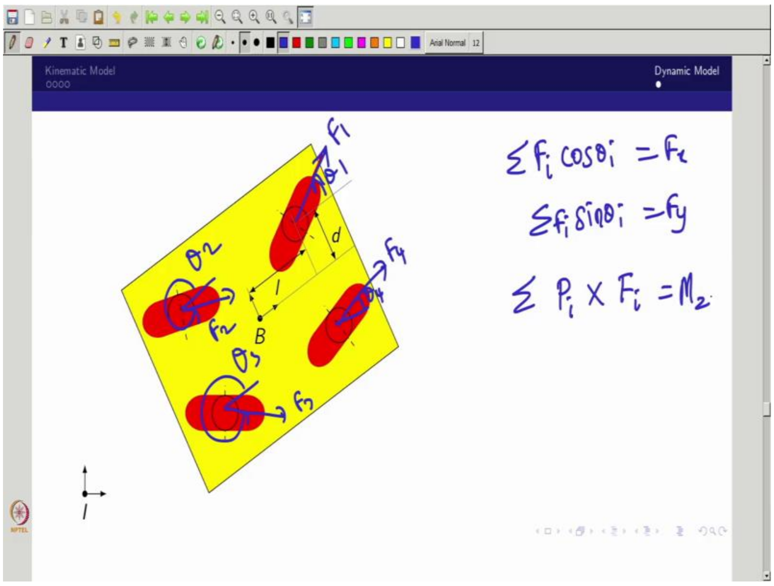

So, now we will move to the dynamic model it is actually like as I already told it is straightforward it is very very simple you take the force, which direction? So, this is F_1 and this F_2 and this is F_3 and this is F_4 which is very close to what you did in a mecanum wheel right. So, the same thing you can substitute this is actually like θ_1 and this is substitute as θ_2 and this substitute as θ_3 and this angle you call θ_4.

And then you can use it whatever relation you took right. So, this is what you written as F_x and this you have written as. So, what you call F_y and then you know the position vector then you write. So, $P_i \times F_i$ ok. So, that sum would be given as M_z. So, that is what we have done already in number of example where mecanum wheel or you call Omni direction wheel we have done. So, we can use it.

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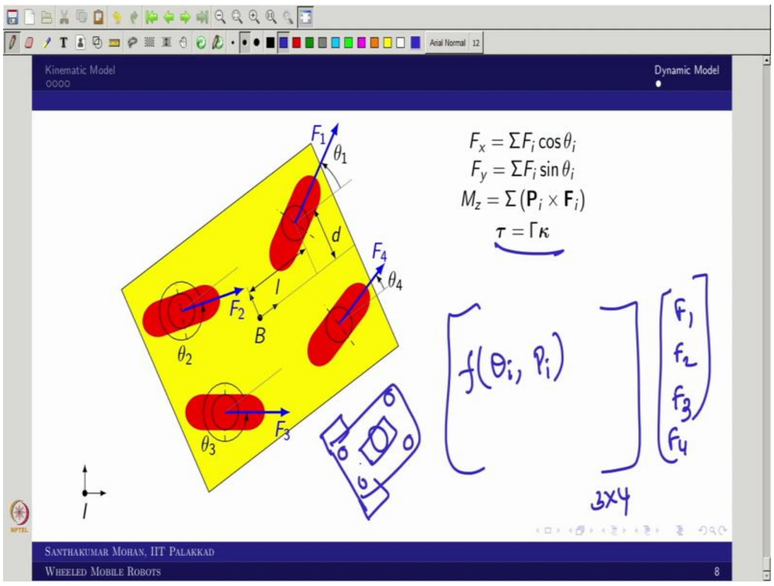

So, that is what I am trying to show here just for clarity you can see like I have taken $\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$ with respect to the θ angles. So, now, this is what the sum that would be equal to F_x.

So, this sum equal to F_y and this sum is equal to M_z. So, now, if I actually like substitute this will come.

So, you can do it further I am actually like left it as your own homework you can actually substitute all the component then whatever you got it as actually like mecanum wheel base you can get it only thing there you call you use as θ_1 and θ_3 as +45 and θ_2 and θ_4 are -45.

But here you will be taking as θ itself θ_1 θ_2 itself. So, in the sense what you will get? So, you will get a bigger matrix. So, where it would be $\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$, but this is actually like what you call 3 × 4, but that would be actually like function on θs. Then you call I put P_i's ok. So, these are the things which will come. So, now, you are very very clear about this.

So, I hope actually like with that I can actually close at this particular section. So, you can do the simulation by yourself because you know once you know the γ matrix and then you can substitute replace the γ which we did in the last class and then do it and you play your variable θ and what you call P_i then you can actually make it even you would take 3 wheel configuration, 2 wheel configuration, 1 wheel configuration all those things and then you can play it.

So, now, imagine one addition just for understanding you have only one powered and steerable wheel. So, what this look like? So, this is actually like what we call unicycle right. So, the unicycle model also straight forward only thing if you take a what you call the tricycle model where you have a 2 fixed wheel right.

But in this case there would not be any fixed wheel in the sense it is actually like very free even you want some stability you can actually like make it. So, this is what we call unicycle model. So, the unicycle model also can be realized from the same example what we have seen now.

So, that is why I said in the last class I was giving a idea. So, we can take a special drive vehicle the special is unicycle tricycle or 4 independent or 2 independent or 6 independent wheels and all steerable power wheel those all configuration you can make it in this way ok. So, with that what one can actually like do it?

So, we can actually like do the simulation based on the γ and κ and then you can actually get idea. So, now, you have got a clarity on your vehicle dynamics. So, now, what left? So, the left is actually like how to make the vehicle in complete the task given to the robot right. So, then what we are thinking about doing in the next week portion. So, we are trying to do the control aspect.

So, the next lecture we will start from the basic control in the sense introduction to robot motion control then we will go stage by stage. With that we will see you in the next lecture.

Thank you and see you then bye bye.