Wheeled Mobile Robots
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Lecture - 13
Dynamic Models of Wheeled Mobile Robots with Wheel Configuration

Welcome back to Wheeled Mobile Robot. So, last class what we have seen is dynamic simulation of a LAN based mobile robot in general there we were talk about forward and inverse dynamics; this particular lecture is straightforward. So, you know like the last lecture I gave a teaser that. So, we would be talking about what is γ which is nothing but a wheel input matrix that how we can you can say find it.

So, in the sense you see it is not that complicated as compared to the wheel how can say, the wheel configuration matrix which we found in velocity kinematics this is straight forward; we will see how to do it; so, far that we are taking 3 example along with one complex example. So, in the sense totally 4 we are trying to attempt.

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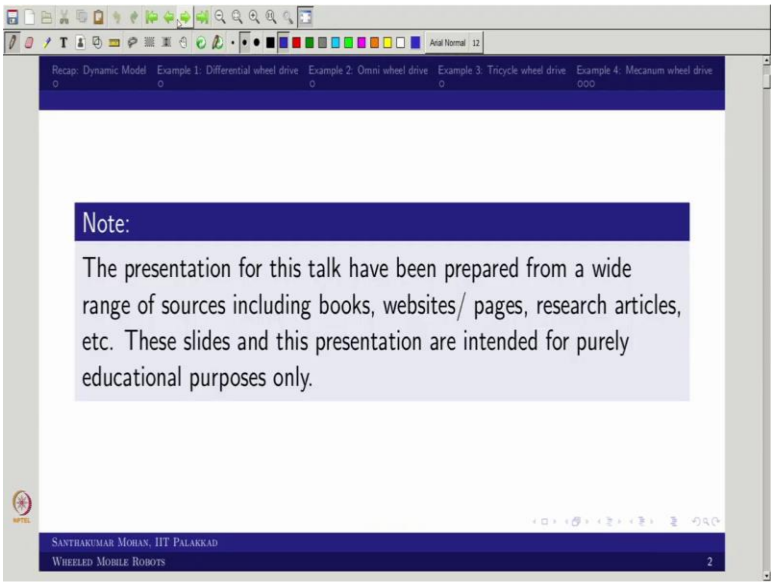

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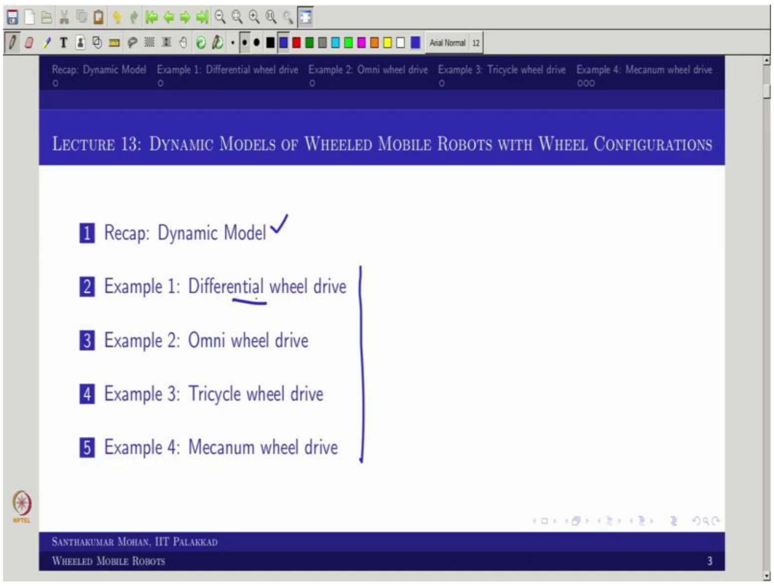

So, let us actually like moved to the slide side; so, this particular course we are trying to attempt 4 different variant of mobile robot with you can see variable of wheel configuration. So, in the sense we will recap is the just you can say matter of time what is we are trying to intent in this particular course.

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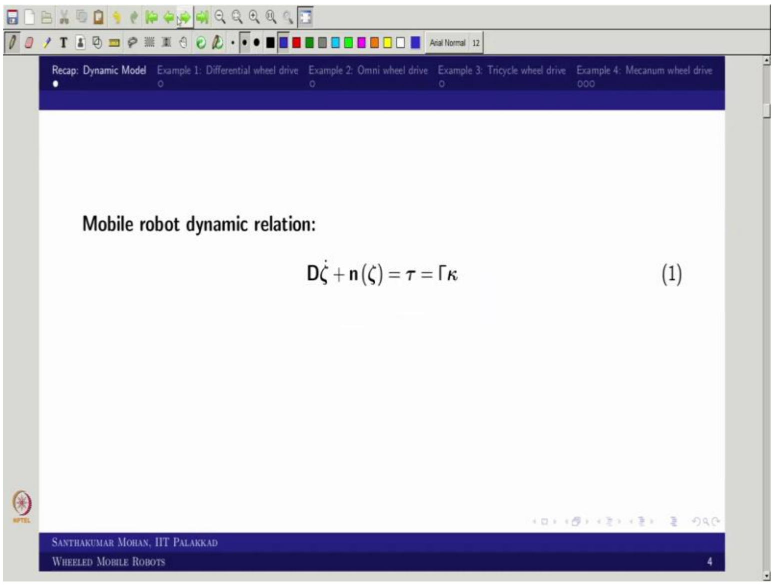

Then we will talk about differential wheel to mecanum wheel. So, these are the things we are trying to cover. So, I said this equation can be rewritten as you call γ into κ. So, that is what we are trying to find out what would be the γ matrix. So, that is what we are trying to interest or trying to find in this particular lecture so, with few example.

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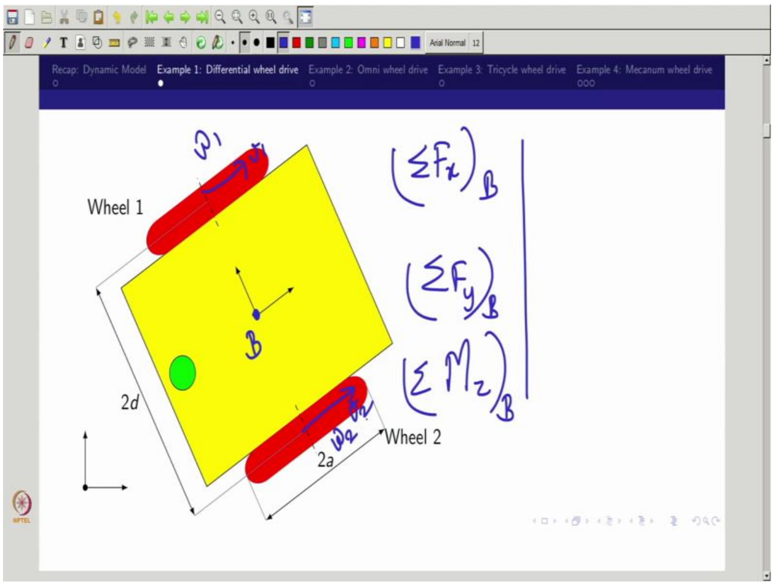

So, in the sense we will take the same differential wheel. So, what we have done earlier? we have done actually like this is what you call V_2 and this is what you call V_1 and then we say that this would be come from ω and this is come from ω_2 and then we equate and brought the what you call W bigger matrix right.

But here it is not that way. So, you know this is a point which is actually going to act as a body frame. This body frame you can say force and moments you need to calculate right. So, here what you need to calculate? You need to calculate $(\Sigma F_x)_B$ and $(\Sigma F_y)_B$ and M_z $(\Sigma M_z)_B$.

So, this is what we need to find out. So, far that what we can actually like bring? We can bring the wheel forces. So, here I told already the wheel forces we are going to talk in terms of as it is.

So, in the sense we are not talking about what would be the rolling friction of the wheel or we are actually like talking about F equal to something multiply with what you call angle velocity ω; that we are not discussing in this particular lecture. This particular lecture we call the wheel force whatever generate that we call F; so, in the sense F_1 F_2; so, how that F_1 and F_2 will map ΣF_x ΣF_y ΣM_z.

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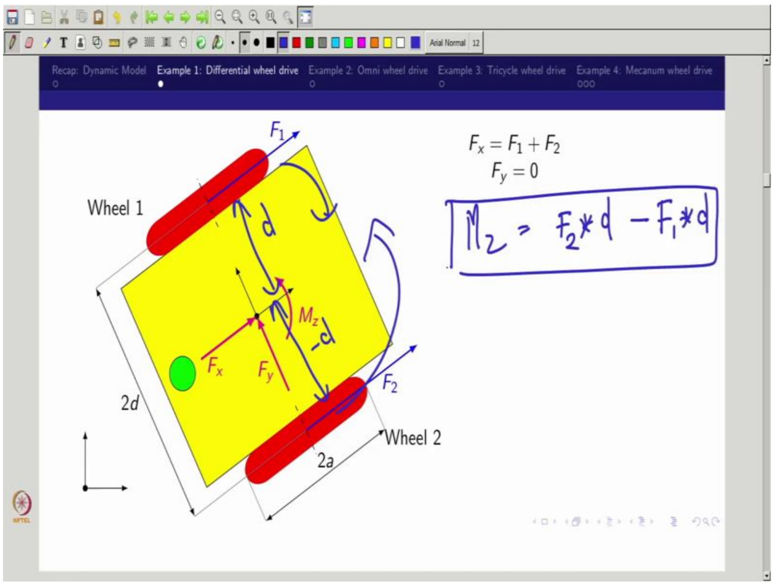

So, that is what we are trying to do. So, we see that this is the wheel force F_1 and this wheel force F_2. So, now what would be F_x? So, jut a sum of this right. So, that is what we are trying to write it right. So, what would be F_y? In this case you see both are actually like a parallel force that too like along with what you call you can say along with the x axis of the you can say vehicle frame. So, then what you can see F_y would be 0 straight away right.

So, that is what we are actually like writing. So, then what would be M_z? The M_z would be equal to this couple and then this couple resulted. In the sense, so one would be rotating anticlockwise other one would be rotating clockwise. So, in the sense you can write; so, F_2 into this distance.

So, what that distance? So, this distance is - d and this distance is actually like + d. So, in the sense what would be the torque would be generated? This would be generated F × d would be what you call counterclockwise and then the other one would be in the clockwise.

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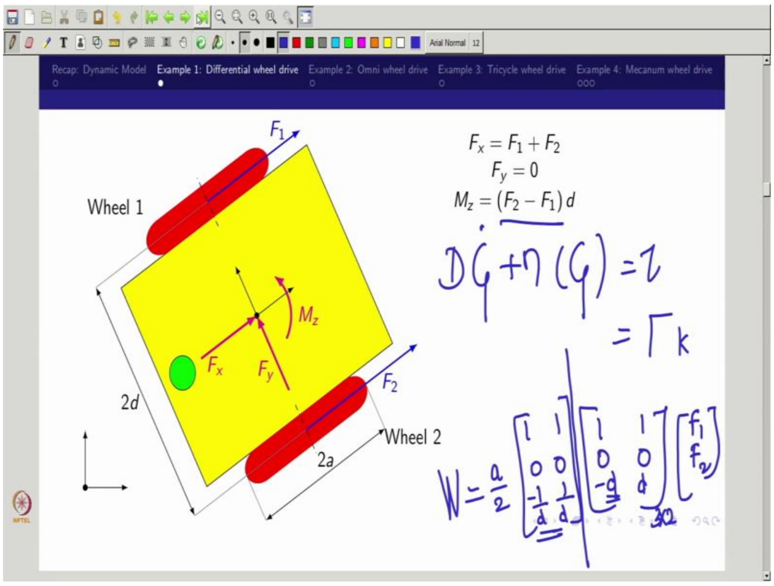

So, this would be what you call M_z. We can see so, whether that is what we are actually like obtained. Yes, you see. So, that d is common and F_2 and F_1 is actually like taken out. So, now, you can see that we have already obtained what we wanted? Already we know this. So, this we know. So, what we are writing this τ in the form of what you call $\gamma \times \kappa$.

So, here κ is actually like $\begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$ and the γ would be the matrix which is actually like mapping in this case it is actually like 3 × 2. So, what here? You can see this is $\begin{bmatrix} 1 & 1 \\ 0 & 0 \\ -d & +d \end{bmatrix}$. Very close to what you have obtained as a wheel configuration or you can say wheel configuration matrix what you call W.

So, if you recall W. So, what we have done? So, a by 2; so, $\begin{bmatrix} 1 & 1 \\ 0 & 0 \\ \frac{1}{-d} & \frac{1}{+d} \end{bmatrix}$. You recall this is what we have obtained and the same thing we are obtaining only thing it is actually like opposite.

I hope that disclaimer I already gave. So, here we are thinking about the angular velocity, but here we are seeing about the moment. So, moment is forces force into distance whereas, the angular velocity would be linear velocity divided by the distance. So, that is why it is coming $\frac{1}{d}$, but here it is d.

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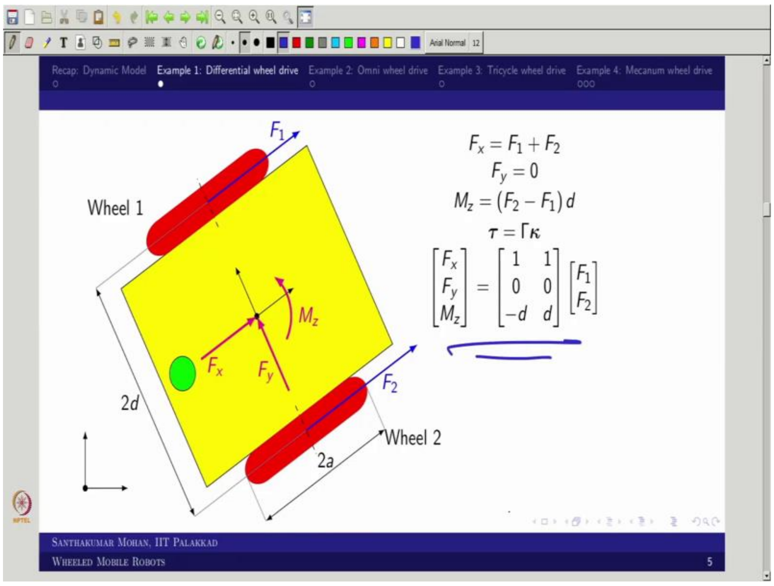

So, now what we obtained? We obtained the γ what we wanted; so, that is what we have taken ok.

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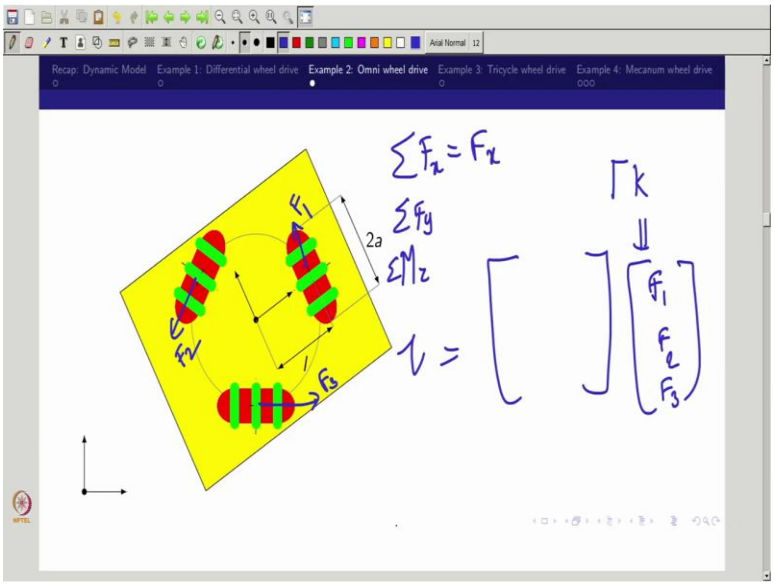

So similar way you can go with the further end where you can take a Omni directional wheel drive as the example. So, now what would be the direction of the you can say forward velocity? It would go like this. So, in the sense the same direction I am calling as a drive force which is F_1 and this is the direction of velocity that would be F_2 and this is the drive velocity direction so, this F_3. So, where $\begin{bmatrix}F_1\\F_2\\F_3\end{bmatrix}$ are the forces which is coming due to the wheel; so, in the sense what we are interested?

So, interest to find this the and the γ matrix; so, where $\begin{bmatrix}F_1\\F_2\\F_3\end{bmatrix}$ are known. How we can find out this Ţ? So, this is what the overall idea. So, here what you can see like the ΣF_x you can find ΣF_y you can find and M_z you can find that would be equal to what you call input force F_x right.

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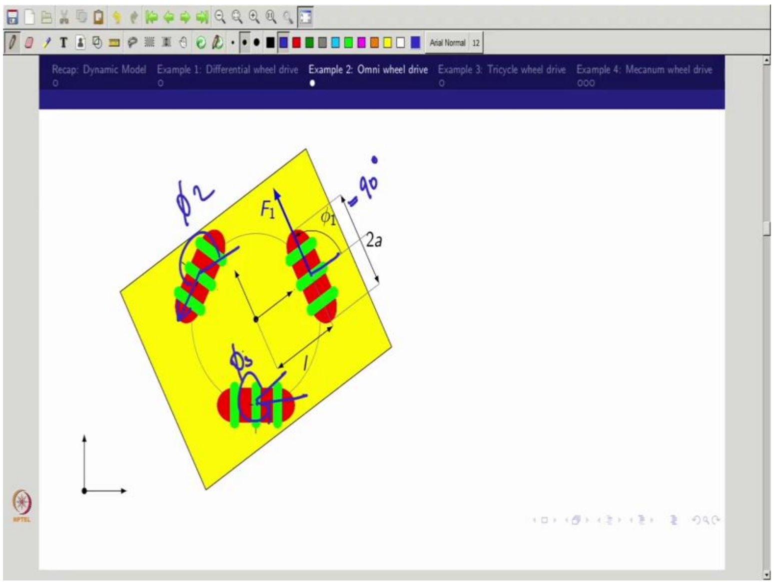

And F_y and M_z that is what we are trying to find out. So, for that we will recall. So, whatever we have seen earlier. So, there would be some angle that angle I am calling as ϕ_1. So, here ϕ_1 would be 90°, but we will take it as a general, then the second one would be. So, this one would be ϕ_2 and so, this would be ϕ_3 right. So, these three angle we are actually like trying to calculate first, then we will actually like apply.

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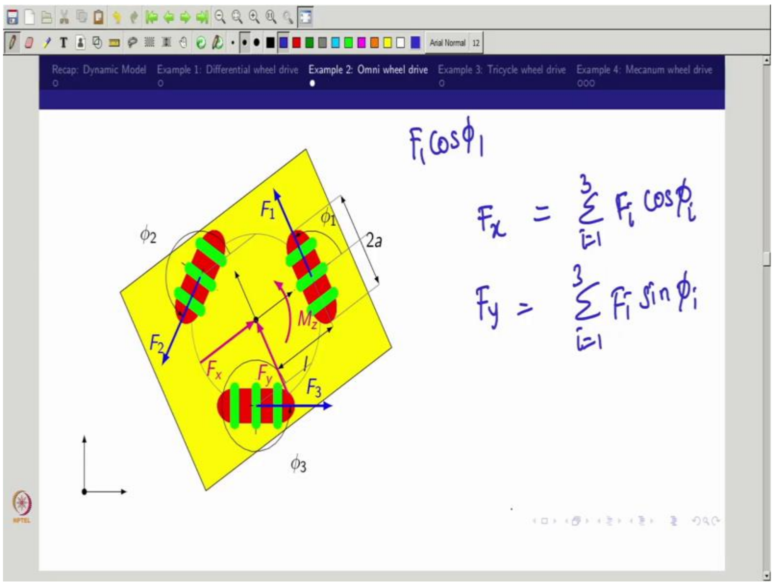

So, that is what I am actually like doing it, ok. So, now, what one can actually write? So, the F_1 x axis component would be. So, $F_1\cos\phi_1$; so, I am rewriting as actually like F_x is actually like written as. So, $\sum_{i=1}^{3} F_i cos\phi_i$ right.

So, that is what we are writing similarly F_y would be the other way round. So, this is F_y and $sin\phi_i$ right. So, now, the based on the quadrant the you can say $sin(F_x)$ component of that F_1 would be positive or negative, but what one can see we can make it a general.

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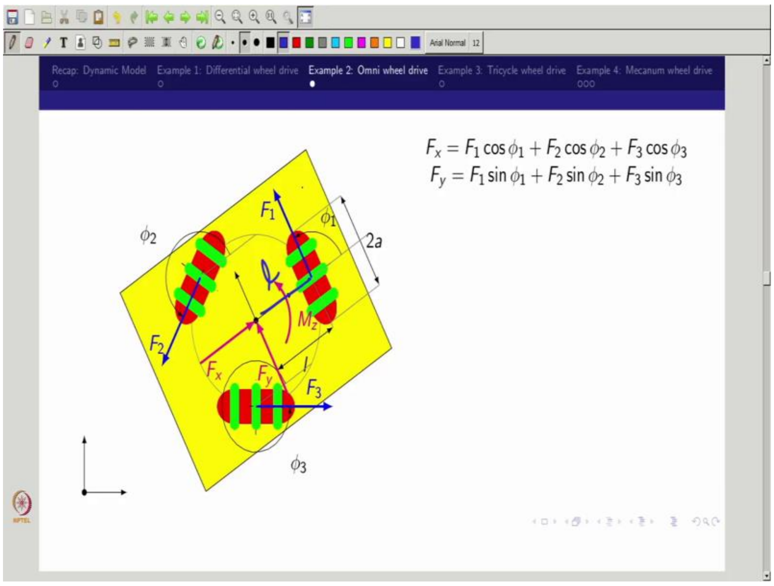

So, that is what we are trying to do. So, if you make it a general. So, what would be total sum of F_x? That would be F_1 component, F_2 component, F_3 component of you can say x component of $\begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$ would be the sum. And similarly you can take the F_y and you can take M_z. How you can write? So, M_z is in this case it is actually like all the 3 forces are actually like equal distance of l and that is all actually like rotating in the you can say counterclockwise moment.

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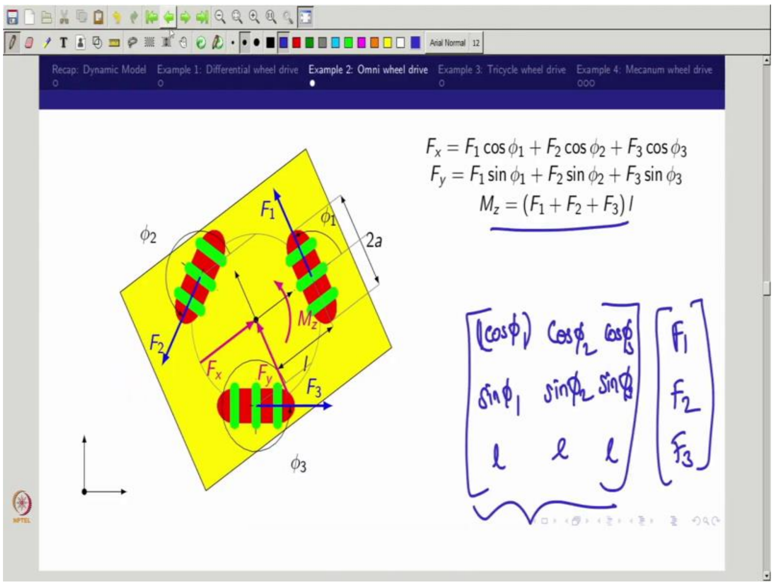

In the sense what one can actually like find out? so, this way. So, now, you rewrite in the matrix form. So, what would be that? So, $\begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$ you can rewrite in these form. So, what you will get? This would be 1 right, but what happened? So, $\begin{bmatrix} cos\phi_1 & cos\phi_2 & cos\phi_3 \\ sin\phi_1 & sin\phi_2 & sin\phi_3 \\ l & l & l \end{bmatrix}$ ok.

So, now what you have done? You have already found the γ matrix right. You recall what we have actually like struggled right we have struggled because there would be a slip and all, but here we are not bothering about the slip; because the case here is actually like how that force is generated. When you are having a wheel force when your wheel is activated that wheel would be definitely generating a force along the x axis.

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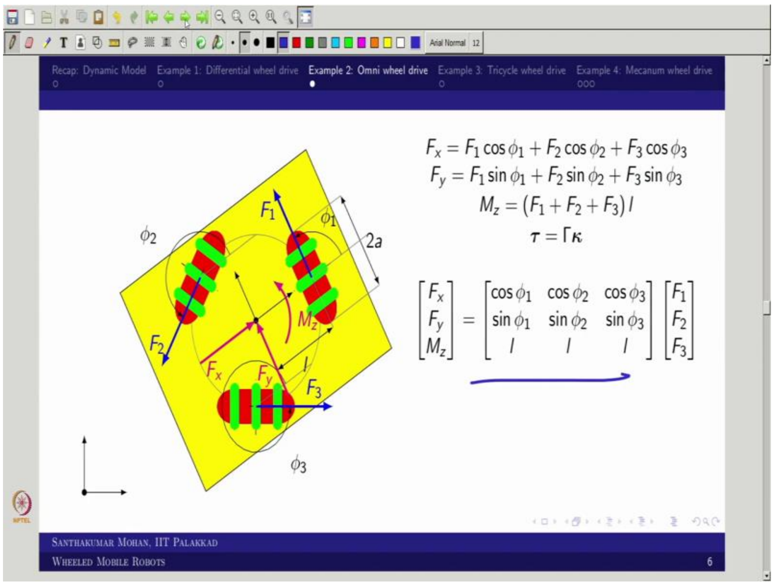

Because the passive roller in the y axis direction, that would not create any additional force input, that is why we are able to do this. So, that is what we are actually like done you see. So, now, this is what we are actually like going to use in the dynamic simulation later on.

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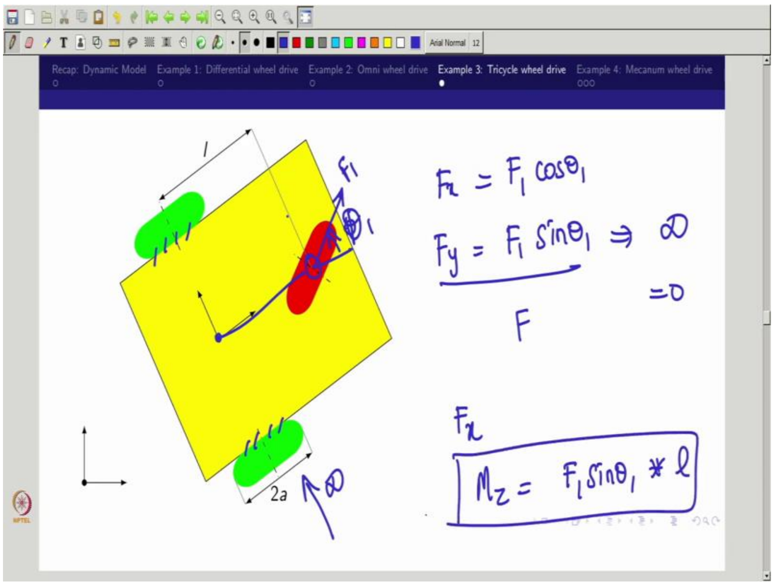

In the same way you try for a tricycle model where this is actually like a powered and as well as steerable wheel. So, then you can see that your force would be in this direction. So, this forced composition would be based on this ϕ_1 or θ_1 whatever you write θ_1 so, this F_1. So, in this sense what one can write? So, $F_1\cos\theta_1$ would be F_x F_y would be like $F_1\sin\theta_1$, but what one know because of the fixed wheel; fixed wheel in the sense it is fixed with the vehicle base, but this is actually like fixed wheel in the sense conventional.

The lateral resistance is actually like infinite. So, in the sense although you have force F_y the resistance would be actually like infinite to that. So, where you can equate this $F_i = 0$. So, that is what we are going to do later on. So, now, in that sense what would be the F_x component create moment? That would be 0; because there is only one distance. What would be the F_y component create moment here?

So, that would be M_z is actually like $F_1\sin\theta_1 \times l$ so, that would be M_z right. So, that is what we are actually like writing as 3 equation ok. So, these two are actually like we made it and you see so, this one. So, now, as I already said this would be 0.

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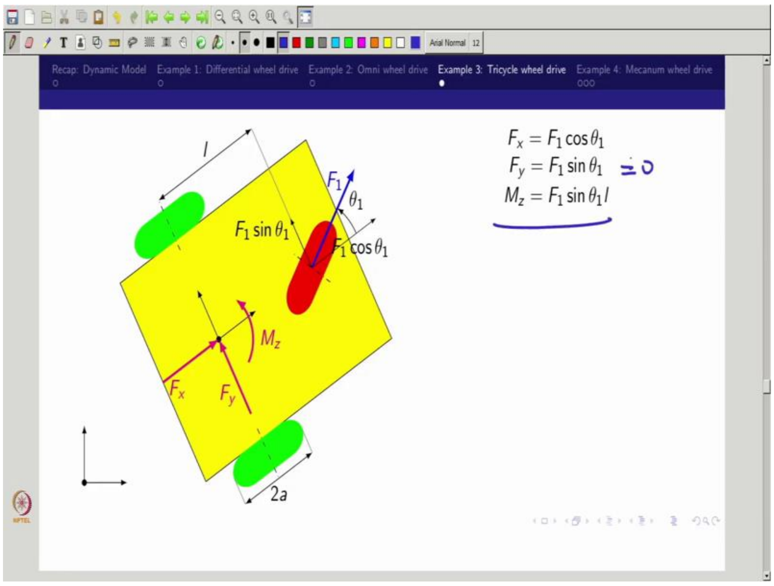

That is what we are actually like putting it, $F_y = 0$ due to infinite lateral resistance.

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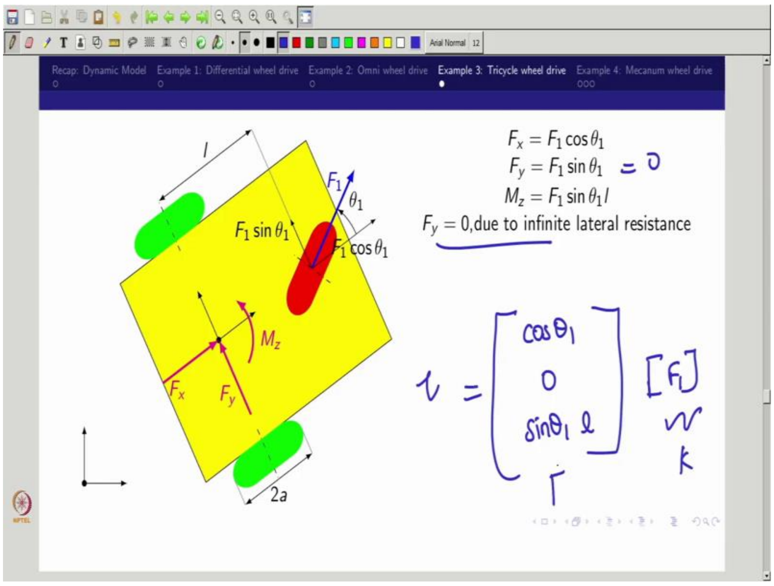

So, in the sense what would be your γ matrix? It straight forward right. So, what would be the γ matrix? So, since this is going to be 0. So, then this would be simply $\begin{bmatrix} cos\theta_1 \\ 0 \\ sin\theta_1 l \end{bmatrix}$.

So, this would be simply F_1 right. So, this is what you can write as a Ʈ. So, this is what your κ vector and this is what your γ right.

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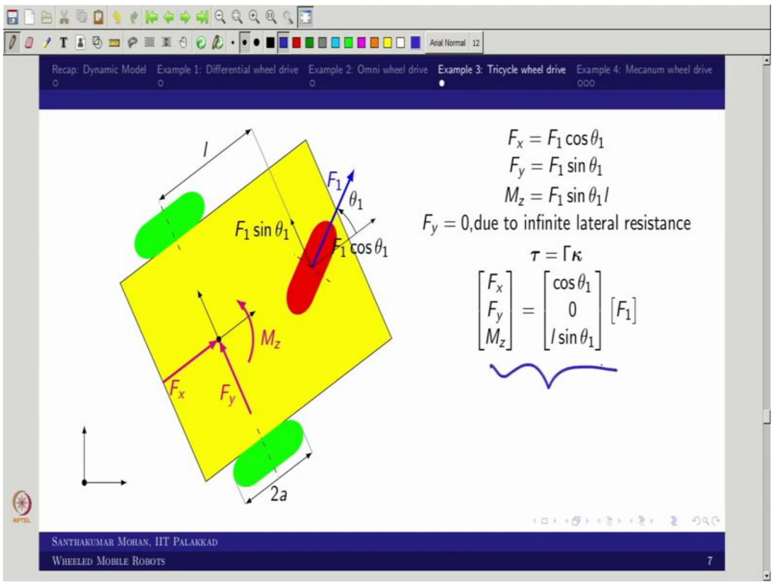

So, that is what we are actually like trying to do, you can see. So, now, this is what the third example which we have taken.

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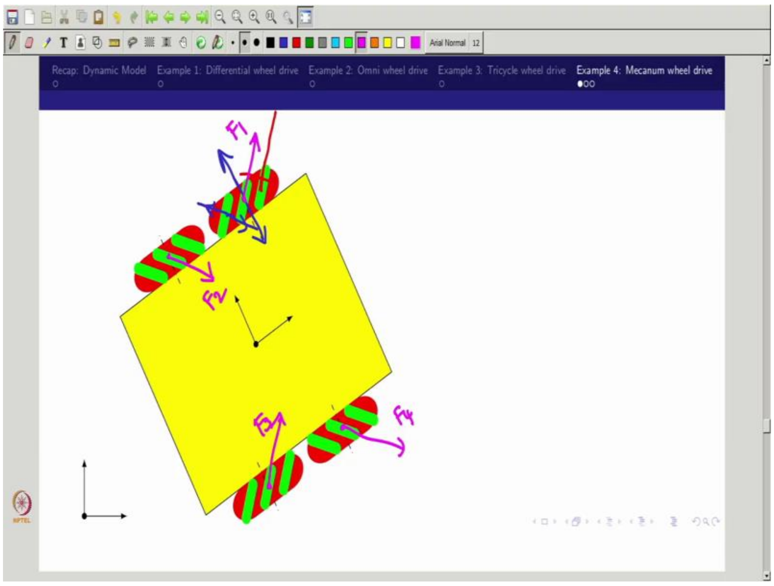

Now we will move to the fourth example where the mecanum wheel; so, now, the mecanum wheel is having what you call this is the hub axis and this is the roller axis, ok. So, I am putting this as a roller axis and this is the hub axis; so, now, where the force would generate?

So, when you are actually hitting this ground. So, due to this roller is actually like rolling with respect to this, this is it is try to slide in this way. So, in the sense what would be the force acting at this point? So, the force would be inclined to this. So, in this case the roller is moving this way. So, the force would act here ok.

So, now you can see that this would be I call F_1, F_2 this is F_3 and F_4. Here although the wheel hub is actually like generating, but the roller which is hitting on the ground that would be generate the friction force; in the sense the F_1 would be along the roller axis that is what the simplicity on this mecanum wheel.

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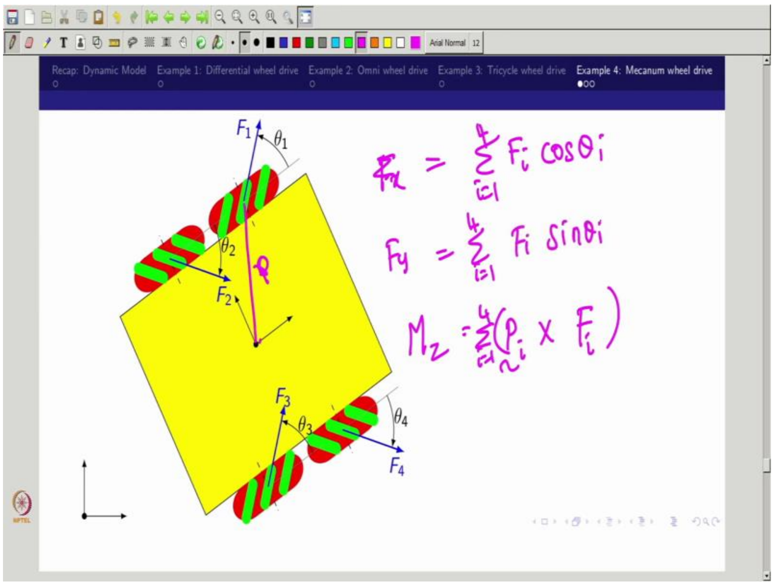

If you do this way, so what one can see? We can rewrite this based on what you call the known value. So, the θ_1 θ_2 θ_3 θ_4 all known, because that is actually like what you call passive roller axis already we have seen, the angle between passive roller axis along with what you call wheel x axis coordinate. So, these all we have already got it.

So, if that is the case F_1, F_2, F_3, F_4 component again you can make it this what you call. So, $F_i cos\theta_i$ where i = 1:4 this would be what you call F_x and F_y you can write in other way round and M_z. So, since this is actually like a way where I call this is r vector or some vector P.

So, then this would be P vector multiply with your F vector. So, the P_i and F_i I can put. So, this is what the sum. So, like that I can rewrite this overall case. So, the overall case here $\sum_{i=1}^{4} F_i sin\phi_i$.

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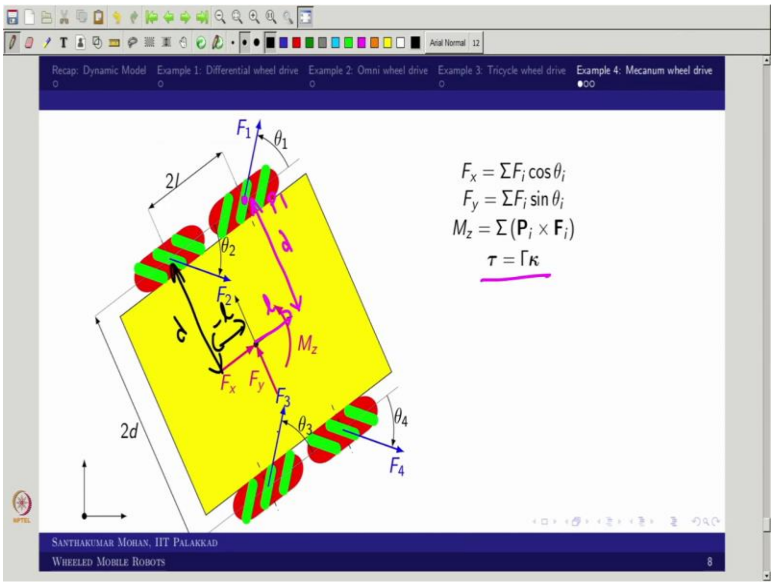

So, now, if I write this what I will get? I will get the bigger picture right. So, now this is what I wanted now I put that and make it the other way round. So, I actually like substitute the θ_1 θ_2 θ_3 θ_4 and I substitute the position vector where you can recall.

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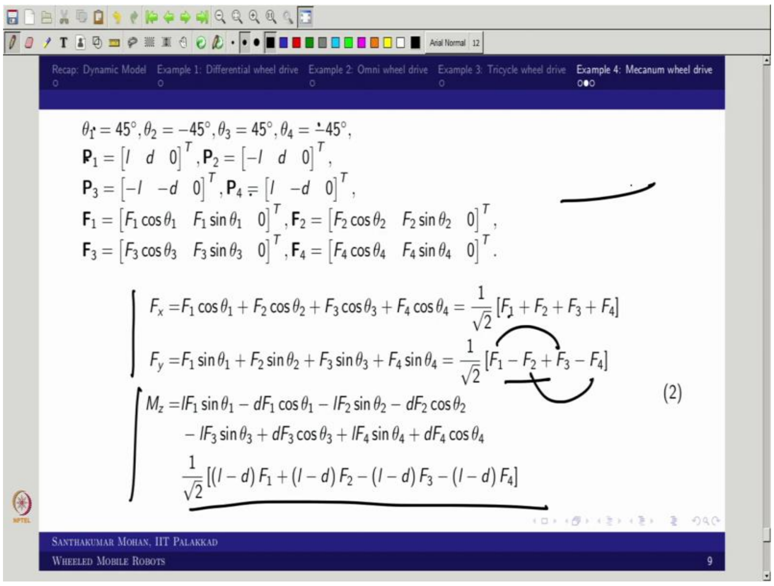

So, this is actually like the position victor P_1 that would be having small l in x axis and small d in y axis, ok. Similarly, this one you can see this is d and this is - l. Like that you can actually like keep right everything where θ_1 to θ_4, P_1 to P_4 you can find; then your F_1

component also would be having you can see x and y z and you can actually like now take that you can say simple sum.

And then you can take the vector you can say cross product and then you can actually take all the sum. Finally, what you will get? You will get these bigger values. Now, you can see that $\frac{1}{\sqrt{2}}$ if you take it common out.

So, what it is actually like in x axis all 4 wheel are actually like contributing equally; whereas, y axis it is actually like opposite because you have seen the configuration is actually like slightly different. These two are actually like similar and these two are similar right.

So, that is why you can actually like see that F_1 and F_3 and F_2 and F_4 are opposite each other. The same way you can actually like see that the upper wheel would be having a similar trend and lower wheel would be giving a similar trend.

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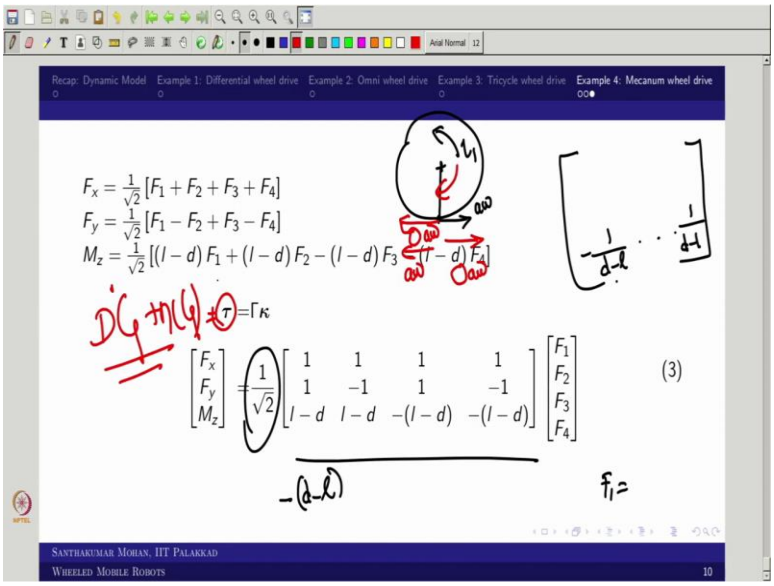

That you can actually you can like relook at it. So, that is what we are trying to find out. So, now, you can look at in a matrix and vector form. So, this is what the case. So, now you relook at it what you relook at it? So, you can actually like recap. So, this is what we have done.

So, the last one is actually like this. You recall when we do the same mecanum wheel for the you call W matrix or you can say the wheel input configuration matrix when we try to find out the last row was like this. So, now you rewrite this. So, what it is? It is very similar to that right.

So, this would be d - l and this is actually like d - l whole multiply with negative sign right; so, in the sense minus sign. So, now, this is actually like very close to what you have done. Only thing here you can see that it is only wheel force and moment we have calculated in the sense we are actually like the wheel force F_1 to F_4 we have given not angular velocity form.

So, that is why there is no you call radius of the wheel involved, but you know this F_1 I can write as. What you can write? So, you know this torque of the wheel I am just plotting this is what the wheel and the torque τ_1 the τ_1 is actually like going to hit here. So, what it can give? It can give actually like some kind of force right.

So, now, you assume that it is actually like rotating clockwise or anticlockwise whatever way. So, now, it is actually like giving a tangential velocity which I call a ω in this way, but the same time what would be the friction force for this? The friction force would actually like generated opposite direction ok. So, what that would be? That would be equivalent to some friction coefficient multiply with a ω right.

Now, you assume that the wheel is rotating in a clock wise. So, then what would happen? This would be opposite direction. So, this a ω would be opposite direction, the something multiplied with a ω would be the opposite direction where you can assume that this what the traction force.

So, now here there would be a you can say closed question in the sense close proximity of this you know like we do racing right. So, we do racing for 2 way. So, one we do like a cycle race so, the other one we do a motor race. So in the sense manual race and then you call motor race. For example, you take a car race the car tire the width is actually like very large; whereas, you take the bicycle racing the cycle wheel would be actually like disc type very thin disc right.

It is like I can call a disc it is a thin not like a thicker wheel. Why it is so? You can actually like get idea right. So, now, based on this you can actually like feel it. So, whereas, one case you are actually trying to increase the frictional effect because whatever your energy that is you are trying to import on the ground, we are trying to do a local motion based on that where the vehicle you take anything motorized.

For example, you take a formula race car the width of the wheel is actually like very big or you can say very broad the same scenario you can take a car and then SUV. The SUV tire always like broader width only because of this; whereas, you take a cycle you are putting a effort where there is friction suppose we minimum ok, that is what the whole idea. So, there would be a compromise.

So, you can discuss among yourself and yourself you can think and take out all the notes and then you can actually like you even if you are ready, you can do something like internet browsing and find it whether whatever I am explaining is actually like right or wrong ok. So, with that what we have done? So, we have done the dynamic model derivation along with wheel configuration we have done because, you know like a before τ there is another thing. So, this we have already done this is based on what you call the rigid body dynamics.

But this τ is actually like what based on what do you call wheel configuration and then the external forces; right now we did not brought the external dissipative forces, but you can call this is based on the wheel forces. So, now, you got it all the aspect. So, in the sense that next lecture what one can expect?

So, definitely can we incorporate these all γ into a dynamic simulation and see what we did in our earlier case in you call kinematic model the same thing can be seen in dynamic level. So, with that I am actually like ending this lecture and see you with the dynamic simulation of these in the next lecture. See you then bye.