Wheeled Mobile Robots
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Lecture - 12
Equation of Motion and Dynamic Simulation of a Mobile Robot

Welcome back to Wheeled Mobile Robot. So, last class what we have seen lecture 11 we have made it 2 part. So, where we were talking about a dynamic model of mobile robot that to we have taken a very general what you call land base mobile robot.

So, we have taken 2 popular method 1 as Newton-Euler the other one is Lagrangian-Euler, but at the end of the lecture 11 part 2 I said that the next class we will be talking about the same equation of motion what we obtained in these 2 methods we will write it in a you call state space form.

So, what that mean we will write it in a matrix and victor form. So, that is what we are going to cover in the lecture 12 in the beginning then we will move to a dynamic simulation using MATLAB for a generalized land base mobile robot. That is what going to cover in the lecture 12. So, what we are actually trying to cover. So, we will be trying to cover equation of motion then we would be actually like writing the equation of motion with respect to inertial frame.

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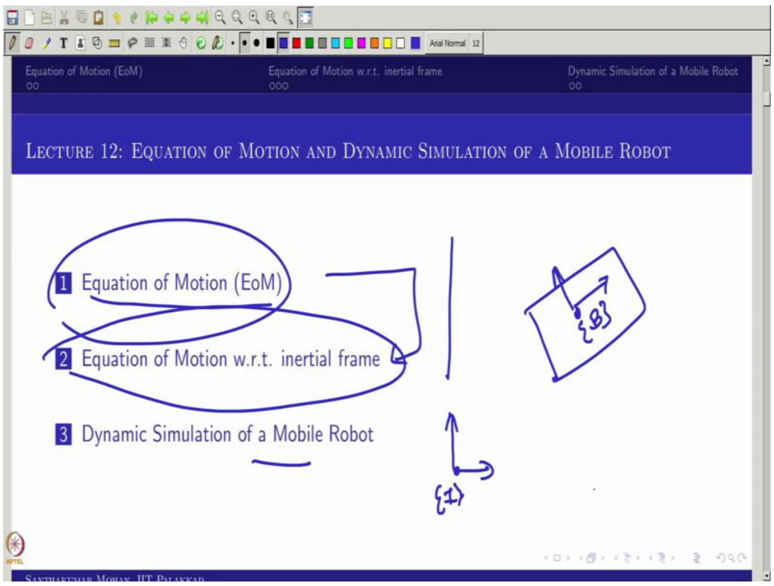

So, what that. So, you know like this is what the mobile robot the mobile robot having a B as the point and where you can say this is I as the another point. So, this is what you call actually like inertial frame. So, now this initial frame you can actually like write it and whatever we have derived that is actually like derived with respect to what you call body frame.

So, now, first we will write the equation of motion in what you call matrix and vector form which we simply call state space form thereafter actually like we will talk about the equation of motion with respect to inertial frame.

Once these things done then we will actually like move to the MATLAB environment and we will try to simulate. So, both forward and inverse dynamics certain extent and then we will move or we can actually close this lecture 12 and lecture 13 we will actually like talk about more on wheeled configuration ok.

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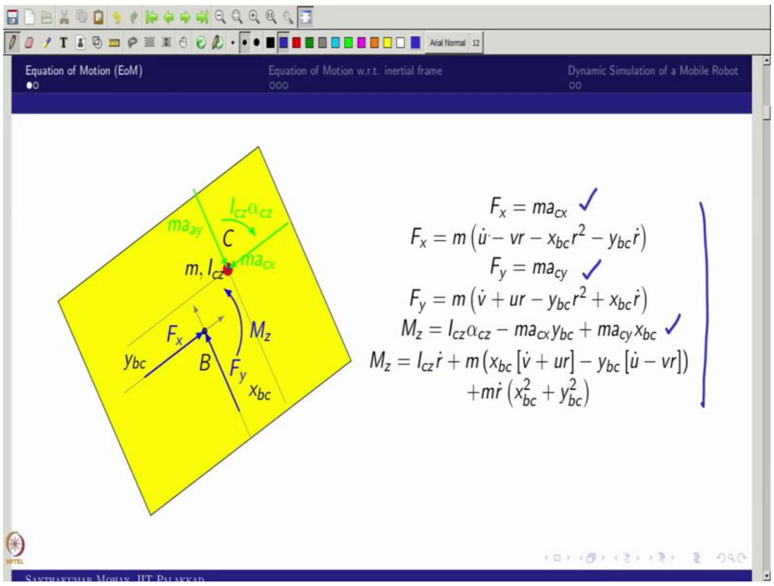

So, now, in that sense we will actually like begin. So, this is the you can say model we have shown in the last class where you can actually like say that F_x F_y and M_z we can actually like do it either Newton Euler or Lagrange Euler, but their generalized equation what you have actually like obtained what I mean to say generalize the equation.

If your body frame and the what you call the centroid is actually like away or it is not you can say same point they are distinct. Then you can actually like get the dynamic model in this form. So, now this is what we are actually like considering.

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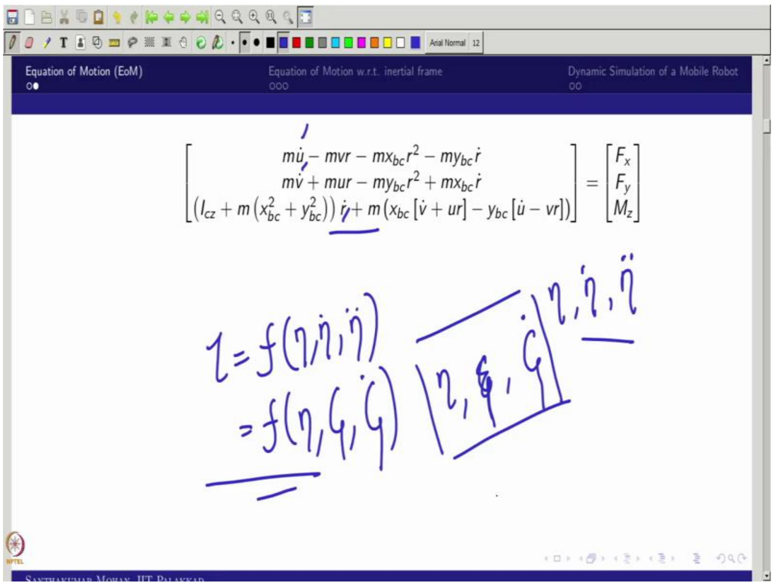

And we are writing in a very simple sense where matrix or not in this case it is actually like vector to vector we are writing. So, now, you can see that what we wanted is actually like something we can actually like integrable. So, what that means? So, you know the state variable which is η right and $\dot{\eta}$ and I can write $\ddot{\eta}$ this is one way or I can write η and ξ and $\dot{\xi}$.

So, this I can do it. So, in the sense you can see that I can write it either this or this form, but I wanted to have this form. Why? Because I told that the tau I can write as function of you can write in the sense $\dot{\eta}$ and $\ddot{\eta}$ or in the other way round if you write f(η, ξ, $\dot{\xi}$).

So, this is what I wanted. So, in order to do that what I am trying to do? So, here you know like $\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{r} \end{bmatrix}$ are the what you call the acceleration. So, which is actually like with respect to body frame whereas, $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$ actually like your body fixed velocity at instantaneous time and you know like the other things.

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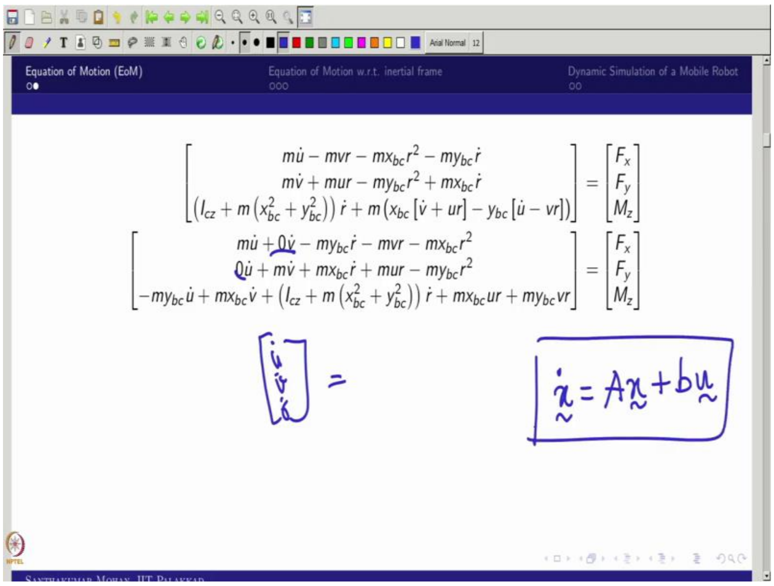

So, now what we are trying to do we are trying to rewrite that into other way round where we are putting every coefficient of the acceleration. So, what that means? So, you can see here there is no $\dot{v}$. So, I am putting that coefficient and similarly the second equation do not have any $\dot{u}$. So, that is also I am putting and the reminding equation I am rearranging in the such a way that. So, I can write it in terms of coefficient of $\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{r} \end{bmatrix}$.

So, this is what the overall idea and now what I can do? I can actually like segregate this $\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{r} \end{bmatrix}$ and the remaining I can actually like put it in one side and I can rewrite this equation this is what I wanted. So, in the sense what you can write $\dot{x}$ = Ax + bu. So, where this all vector.

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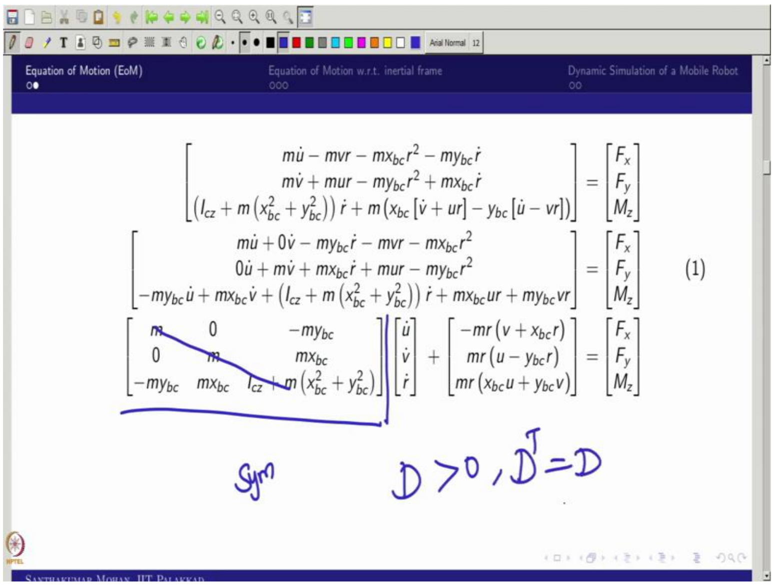

So, this is what we call state space form. So, this is what I also wanted in this way. So, in the sense what I can do? So, this I am rewriting in a matrix you can see like now these are the coefficient related to $\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{r} \end{bmatrix}$ and the remaining I am putting as a simple vector and equal to the forces which are actually I given.

So, now what you can actually like see. So, this particular matrix hereafter we are going to call inertia matrix why it is actually like purely based on the inertia, but this particular matrix has a speciality. What the speciality? You can see the diagonal value all are positive and you can actually like see that off diagonal is actually like similar of the bottom one or you can say the other way round. What that mean? So, it is actually like symmetric matrix.

In the sense if I call this as one of the matrix called D for example. So, this D matrix would be actually like positive and as well as what you can see the D matrix is actually like symmetric. So, this is what one supposed to know in this particular slide.

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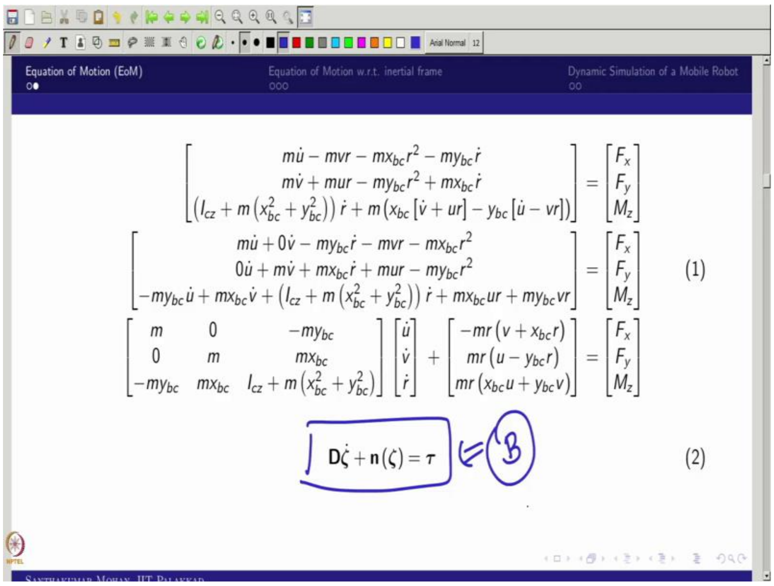

So, now this is what I am going to write as a equation of motion in a state face form. But what I said I wanted actually like everything in the form of what you call integerable form that we will see in the dynamics simulation right now we will move. So, this one you can see what this equation is in the form. So, this equation with respect to what you call body frame right, but what I wanted? I wanted everything in the initial frame.

So, that I no need to actually like make this what you call body frame as a instant and I cannot actually like explained what are those things, but on the other way round the body frame equation would be benefit because you are what you call wheels are connected with the body.

So, whatever you are going to give as a wheel inputs those would be beneficial directly if you write the equation with respect to body frame that is why in the mobile robot community what we used to do we will write the kinematic equation which is actually like $\dot{\eta} = j(\Psi) \times \xi$, but the $\dot{\xi}$ we will write in terms of only body frame.

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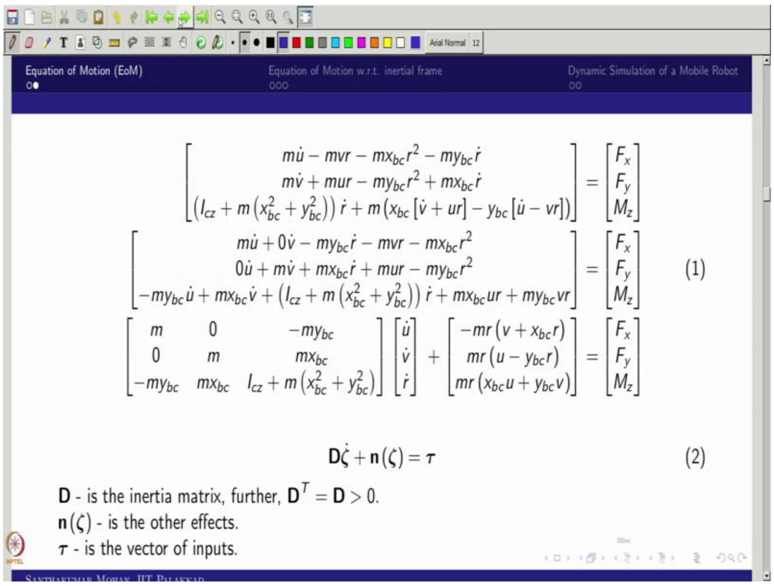

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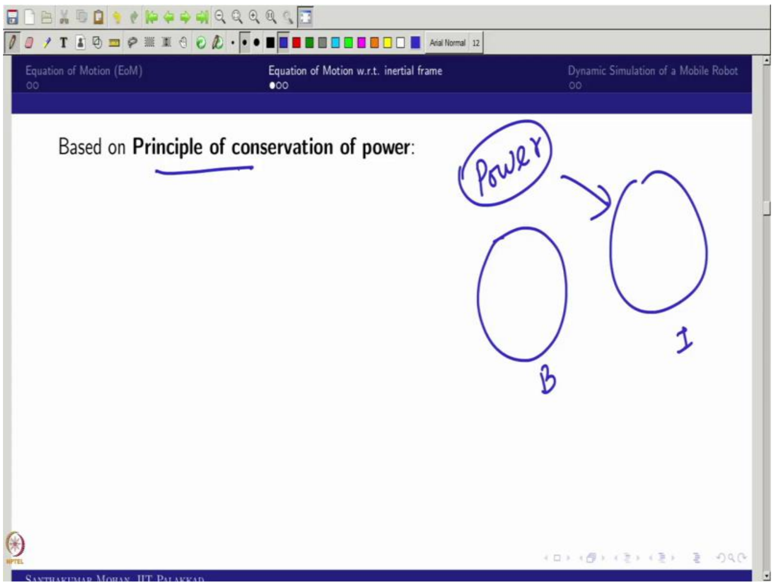

So, what that means, or we will actually like rewrite this. So, what we will do? So, we will actually like first take what principle it is. For example, now I call this is actually like with respect to body and this is what I call with respect to I. So, now, if the body is actually connected with a wheel and it is actual like a start getting a motion. So, now, what would be the power. So, something right. So, I am writing that is the power, but if I realize the same power in the inertial frame.

Do you think that the power would get changed? No, right because only thing you are representing with respect to one frame to another frame. So, in the sense that the total power you can say consumed by the system will not get changed. So, that is what we are trying to use as long we assume that there are no losses, then what we can bring? We can bring the principle of conservation of power.

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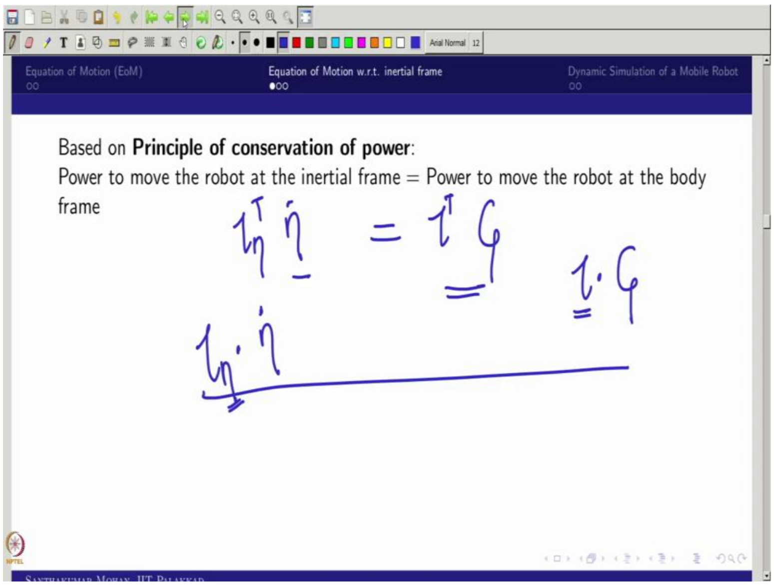

So, then what one can actually like right the power to move a robot at the inertial frame supposed to be equal to power to move the robot what you call at the body frame. So, then what you can write?

You can write what is a power relation. So, what you know the velocity and the forces or you can write the input vector and thee what you call velocity vector you know. So, in the body frame what would be the velocity vector? η Ψ ξ. So, what would be the inertial frame that is $\dot{\eta}$ right.

And what you can write here in the you call input with respect to body frame? That already we have written as τ right. So, now, the $\dot{\tau}(\xi)$ supposed to be equal to what I am writing τ $\dot{\eta}$ $\dot{\eta}$. So, what this $\tau\eta$? This is actually like the control input or you call input vector with respect to inertial frame where this tau is actually like inputs with respect to body fame that is all.

So, now you know this is we have written in a what so, called dot product this dot product we can write as actually like inner product you can rewrite as. So, tau $\eta^T \times \dot{\eta}$. So, the other way round $\tau^T \times \xi$ right, but what you know ξ and τ you can say that we know from the what you call dynamic model with respect to body frame and $\dot{\eta}$ to ξ you have already relation. That is what we are trying to use.

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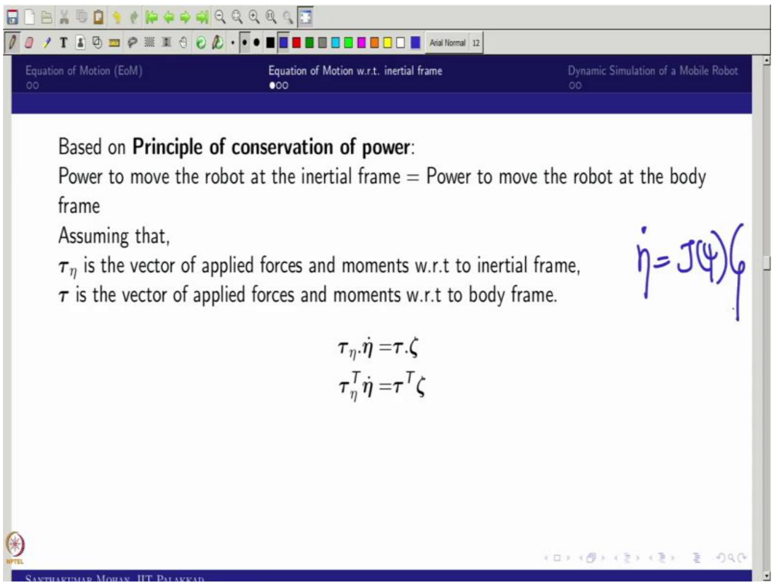

So, you can see like what we are doing that we are assuming that the $\tau \times \eta$ is the vector of applied forces and moments with respect to inertial frame and tau is actually like vector of applied forces and moments with respect to body frame. I am actually like bringing this equation.

So, now, what I am actually like trying to rewrite in the you can say inner and you can say inner product way. Then what you can write this equation you know. So, $\dot{\eta} = J(\Psi) \times \xi$.

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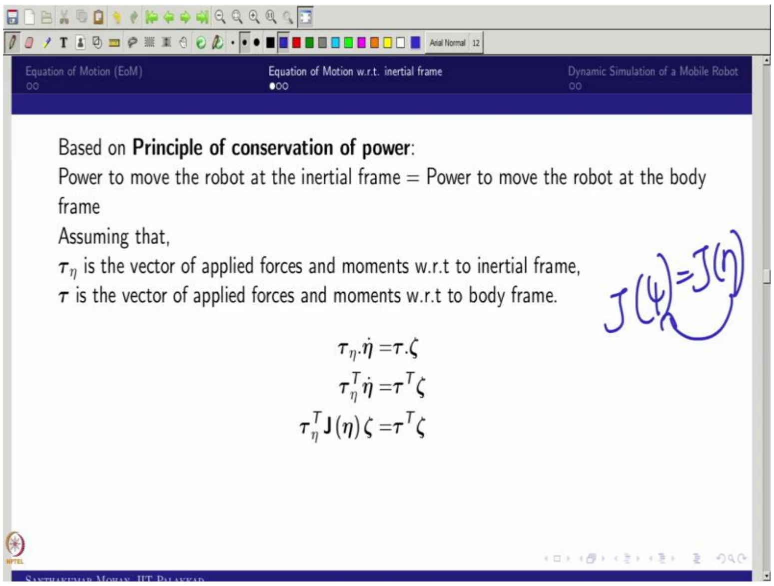

So, that is what we are actually like trying to substitute right. So, then what you can see like the ξ goes away and what remains τ $\eta^T \times J(\eta)$ ok. So, here this J(Ψ) I am actually writing as J(η) because this Ψ is actually like in the η form that is what only change we have made ok.

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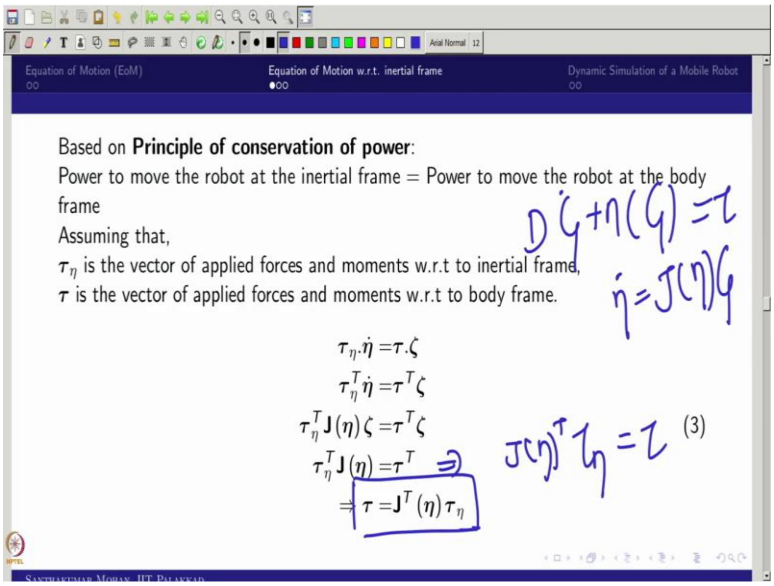

So, now, what 1 can actually like see I can rewrite this and take transpose throughout both side or you can say take transpose in both side. What you will get? So, you will get rewritten equation. So, that is what we are actually like writing. So, you can actually like rewrite. So, this would be $J(\eta)^T \times \tau(\eta)$ is z. So, you can see $\tau\eta = \tau$.

So, now you can see what you got it this relation also you got it. So, now, what you know? So, $D\dot{\xi} + n(\xi) = \tau$ and what else you know? So, $\dot{\eta} = J(\eta) \times \xi$. So, these 2 equation I am going to use and bringing the inertial frame form.

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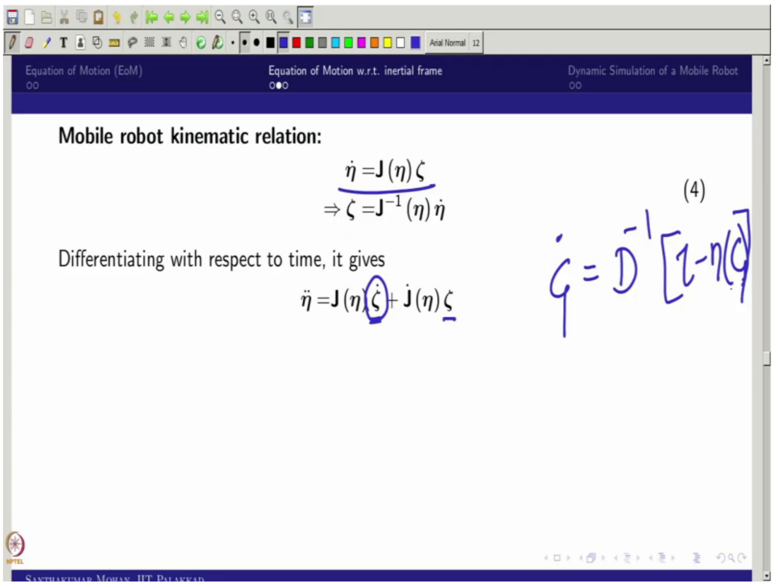

So, that is what we are actually like writing. So, this is the mobile robot kinematic relation we have obtained and that I can rewrite in this way and similarly what I am writing. So, differentiating this with respect to time what I will get $\ddot{\eta}$. So, the $\ddot{\eta}$ would be in the form of $\dot{\xi}$ and ξ and you $\dot{\xi}$ right. So, what is $\dot{\xi}$? $\dot{\xi}$ I can write as D^{-1} of you can say $\tau - n(\xi)$ I can rewrite with the dynamic equation right.

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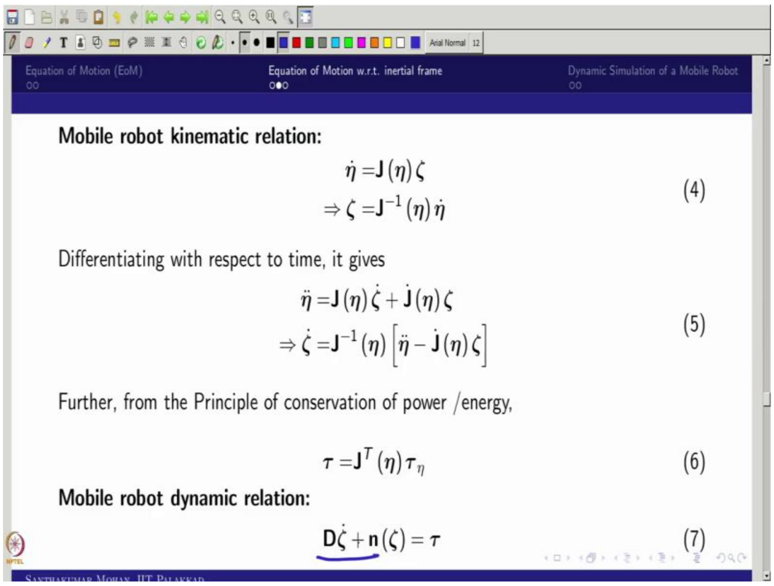

So, that I can actually like do it right now I am actually like writing $\dot{\xi}$ in this form I am going to substitute that in the general equation. So, this is also we have obtained. So, what that means? So, this equation I am taking and I am actually like substituting this here.

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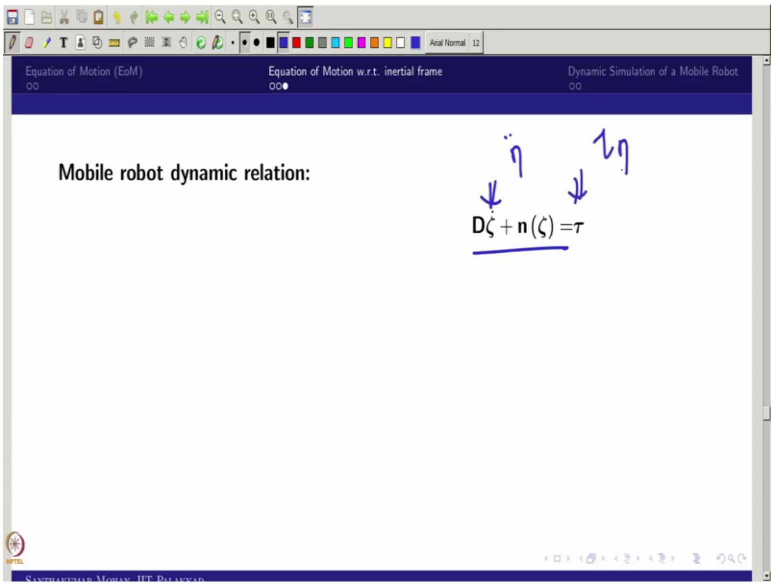

So, that is what we are trying to do ok. So, now, this I will substitute from the $\ddot{\eta}$ relation and this I am substituting with respect to this equation.

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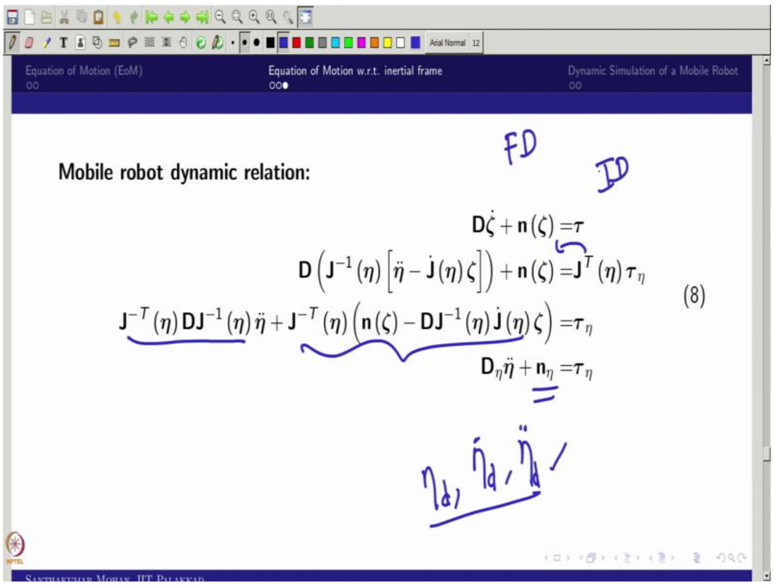

So, now, you can see like I substituted that. So, what I get? So, I get this is a bigger relation, this bigger relation I can regroup it with only acceleration. So, what that would be equivalent? That would be equivalent to a inertia matrix. So, that is what I am trying to do I regroup it only acceleration term and the remaining all I am putting in another group ok, which all put a other vector.

So, that is what I did. So, in the sense I first I take this inverse and then I have actually like regroup and make it. So, now, what happened? This is what you called Dη and this whole what you call nη ok. So, now, you can see like the entire equation you have written in the what so, called inertial frame. So, now, if you are actually like trying to control your overall system you can try a directly say that these are the inputs ok.

So, in the sense you can give the desired generalized coordinate, desired derivative of generalized coordinate and you can say desired double derivative of generalized coordinate you can give it in the sense you can give the vehicle position with respect to inertial frame velocity and acceleration, you can give then also you can do it right.

So, now you can see that the overall system you have written with respect to inertial frame. So, this is one side. So, now, what we wanted we wanted actually like see how the vehicle behaviour will happen. So, in the sense we can do the forward dynamics and if you have given this way can I actually like do the inverse dynamics.

So, for that what one supposed to know. So, you should know the dynamics simulation tool already I gave a small idea in the you can say kinematic simulation how we can one can use the Euler method the same Euler method can be extended here ok. So, that what we are trying to do in the dynamic simulation.

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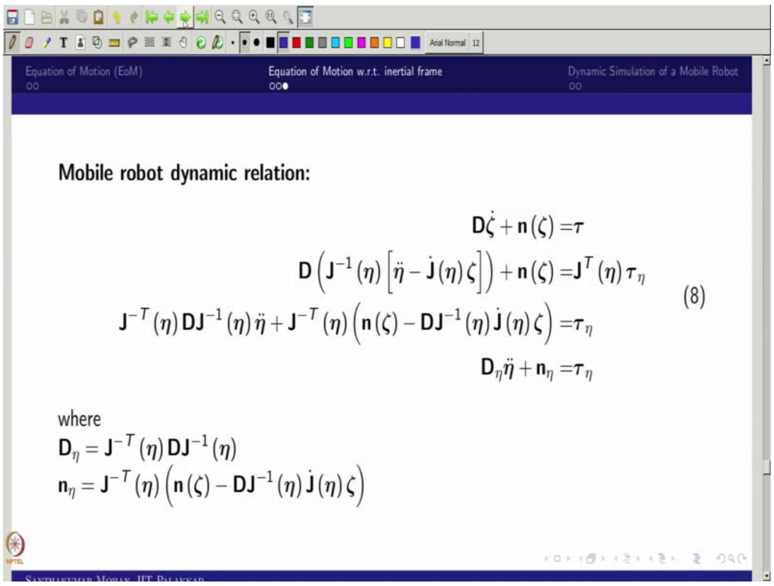

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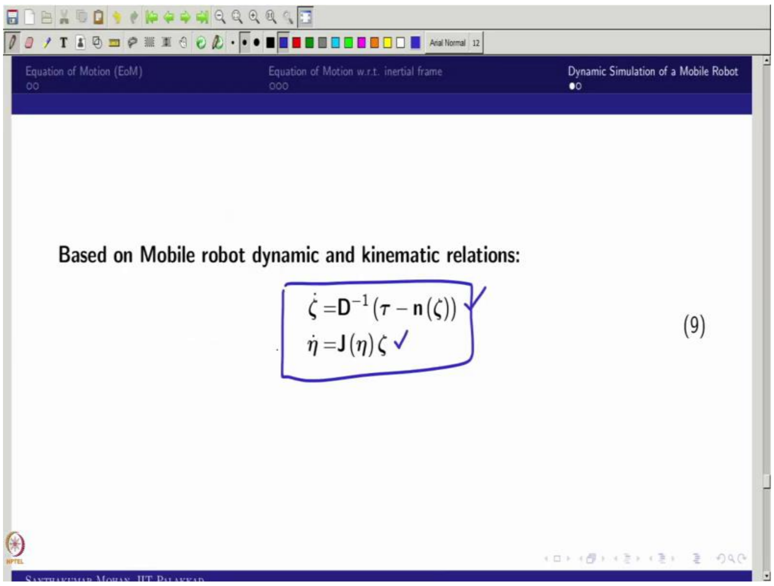

So, what we are trying to do. So, we are actually like trying to take one of the simplest equation which we have derived and this is the kinematic equation and this is the dynamic equation. I will be keeping the dynamic model as it is in the body frame because I told right my wheel forces or you can say my input forces would be with respect to body. So, I will keep it that.

So, that that would be you see then I will actually like make a another loop which will be doing the $\dot{\eta}$ integration. So, in the sense what I am trying to do? I am trying to do this with respect to what you call in MATLAB. So, now we will move to the MATLAB window then we can actually like see how that stuff works.

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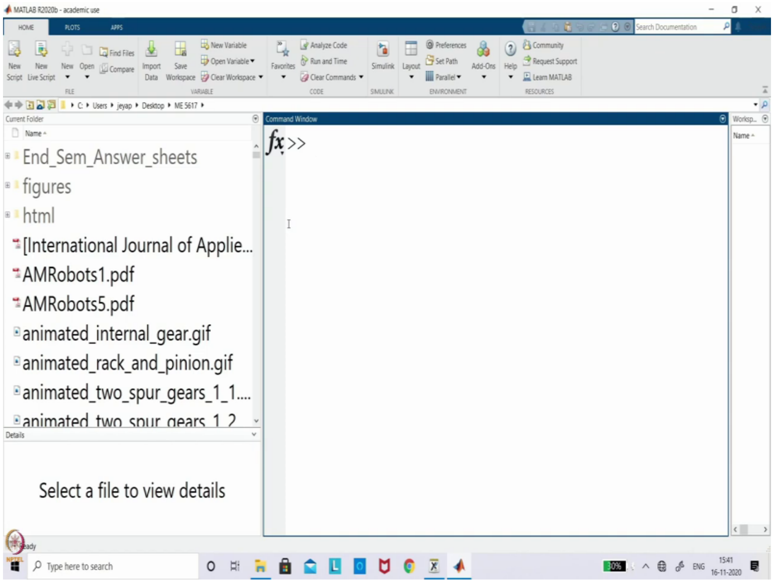

So, now, I am actually like opening one of the new scripts. So, we will actually like see how that takes place. So, you know how actually like we have done in the previous. So, we have actually like taken the simple Euler method and we have tried how to actually like simulate both forward differential kinematics and inverse differential kinematics. So, now the same thing we will extend to the land based mobile robot where you would be doing the dynamic simulation.

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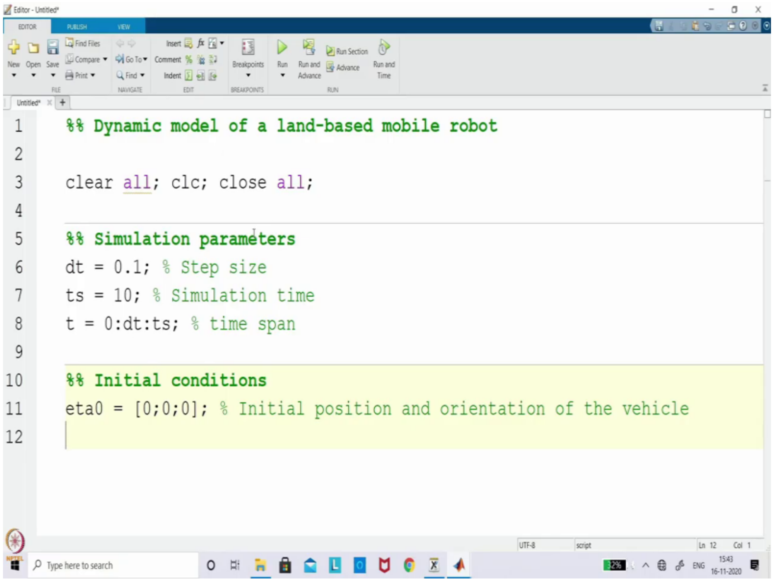

So, for that I will actually like write in the same way. So, dynamic model of a land based mobile robot. So, why I am actually like saying the land based? Because we are restricted with only one play. So, the half planar movement we are not discussing ok. So, now what you have to do? So, I am just taking I will be running so many things. So, better I will actually like put the mandatory things.

So, we will come back. So, what we did? So, the simulation we have started. So, the simulation parameter I will take it from the previous case. So, what we have done? We said δ_t we denote as dt that would be 100 milli second which we have written as Step size. So, I will write it that. So, then we write a total Simulation time as ts.

So, I can take it any, but right now I am taking 10 second, this is the simulation time. So, which are actually like same, but you can see that something is going to change ok. So, then the t also like a times span where it start from 0 and it increased by the δ_t time or you can say the increment by δ_t up to ts. So, this is what you are at total time. So, in the sense this is I put time span, but what we have done earlier. So, earlier we have done as the initial condition. So, that is only first order derivative, but here second order derivative.

So, then the condition would change. So, in the sense what are the things we have to do? So, η you have to define. So, η_0 I am actually like defining. So, which is x_0 y_0 and Ψ_0 I am taking that as 0; 0 just for my convenient ok, but we will change it later on. This is initial position and orientation of the vehicle. So, then what you can actually like see.

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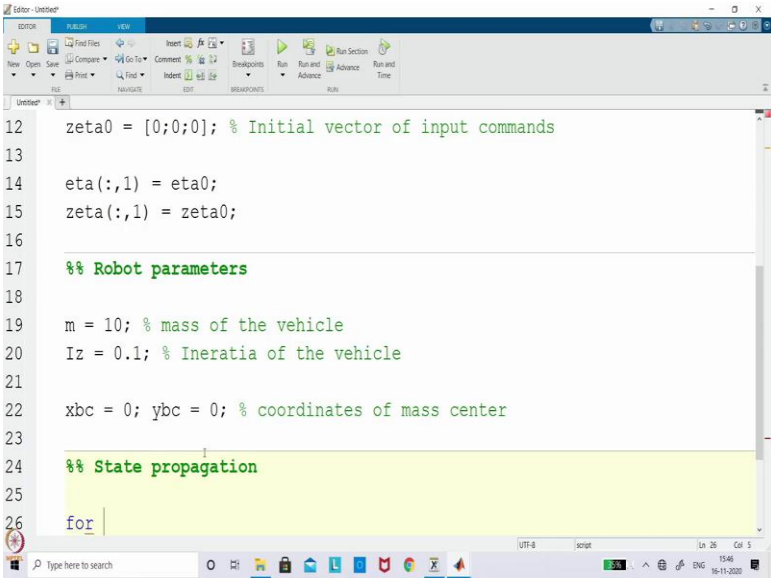

So, I will put it ξ. So, because this is second order derivative right the $\dot{\xi}$ we are trying to integrate. So, in the sense ξ_0 also I am taking. So, which is what you call the input command so vector of. So, I will write initial you can say vector of input command. So, which is nothing, but the body fixed velocity.

But what we are trying to do? We are trying to propagate where η would be getting propagate up to you call tenth second. So, in the sense I am actually like taking the first segment is η_0 because every loop iterating. So, this η would be increased by 1.

So, in the sense I am taking the ξ_0, then the ξ also like the same way. So, that is also like going to iterate every loop. So, I am assuming that this is 0 ok. So, now, what we have done? We have taken the initial condition now we will go to the vehicle parameters I will write robot parameters because I always teach vehicle dynamics on that way. So, robot parameters. So, here what are the robo parameters we a supposed to use? You are to use what would be the mass of the vehicle right.

I assume that the mass of the vehicle is 10 kilogram. So, mass of the vehicle then what we have taken the inertia of the vehicle that to like is z axis inertia I am taking just a small one, we will take it original when we are doing in real time. So, this is the Inertia of the vehicle or inertia of the robot whatever you can call because this I assume that vehicle because mobile robot. So, is there any other things that required? You can actually like open the slide. So, where we were talking about what you call the stuff. So, I will just show you.

So, here you can see is there anything else is coming? No right so, but if you look at the CG and the body frame is actually like away then you have to bring the coordinates of C with respect to B right. So, that also like I will bring it here. So, in the sense X_{bc} is actual like 0 I am taking, but I will bring it that ok. So, in the sense these are actually like what you call.

So, coordinates of mass center I will just put it mass centre ok. So, what is required then that is all. So, we will actually like move to the what you call system or state propagation. So, I am just putting State propagation where I will be starting with the for loop. So, you may think that we are doing a second like a we would be solving 2 derivative, then there may be 2 for loop no because we are actually trying to do in a single instance.

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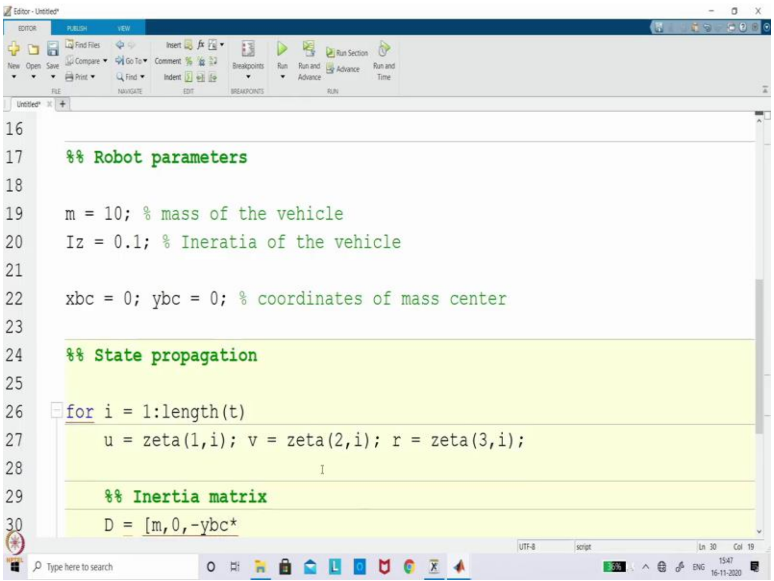

So, in the sense only one for loop where it starts from i = 1 : length(t). So, but what are the stuff you required? You see like we have written everything as generalized. So, where we have written u. So, u is actually like what ξ of 1 because that is what the case right and v is actually like what ξ (2, i) the i^{th} instant that what you call and r is what you call ξ(3, i) right.

So, then what else you required? You required X_{bc} we have given Y_{bc} we have given m we have given i is that given. So, then that is straight forward right you can actually like recall here is there anything else is there no right. So, I will just use them what you call this one. So, this matrix form I will actually like use. So, that would be easy. So, you can see like this is diagonally it will come and as well as off diagonal you can see it is going to give a symmetricity.

So, I am just using that ok. So, what that would be? So, the Inertial matrix I call D I will write that is Inertia matrix. So, Inertia matrix which is what you call D as a matrix. So, that would be. So, mass right. So, then 0 and $-Y_{bc} \times m$ right. So, I can actually like cross check right. So, this is what the case. So, I will take this equation and put it here. So, that it is easy for you to recall what we are trying to write ok.

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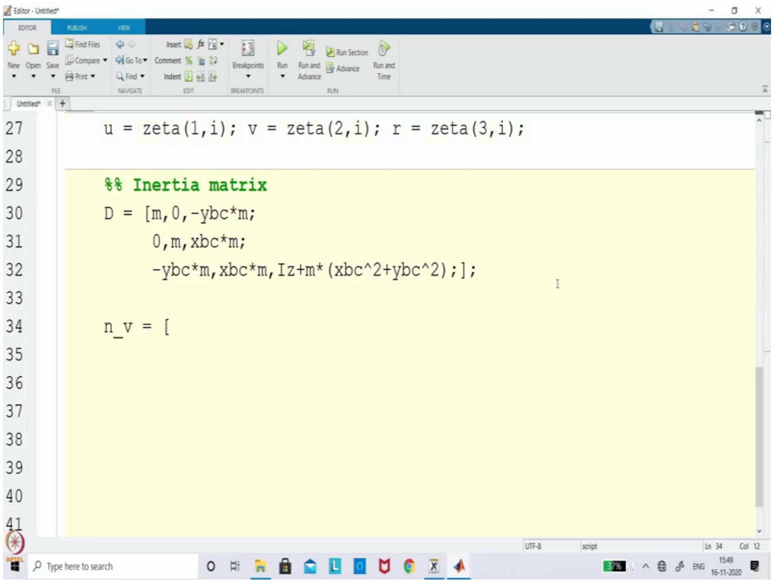

So, this is what we are doing it then the second one is 0, m then $m \times X_{bc}$ into because we have taken in the earlier is like that then you have actually like $-Y_{bc} \times m$ then. So, m sorry $X_{bc} \times m$ then the final one is actually like I_{cz} so, which we have written $I_z + m \times (X_{bc}^2 + Y_{bc}^2)$ ok. So, although it is not there in our case, but we are actually like taking it that as a general case. So, now, you can see that the inertia matrix which as 3×3 we have actually like taken.

So, now what is required? So, you need to have the $\vec{n}$. So, I put n vector as small one. So, $\vec{n}$. So, that also like would be having 3 components. So, I will use it here that vector is ok. So, this is the $\vec{n}$ where you can see there is a coriolis down and as there you call the radial term is there. So, I will actually like use it that.

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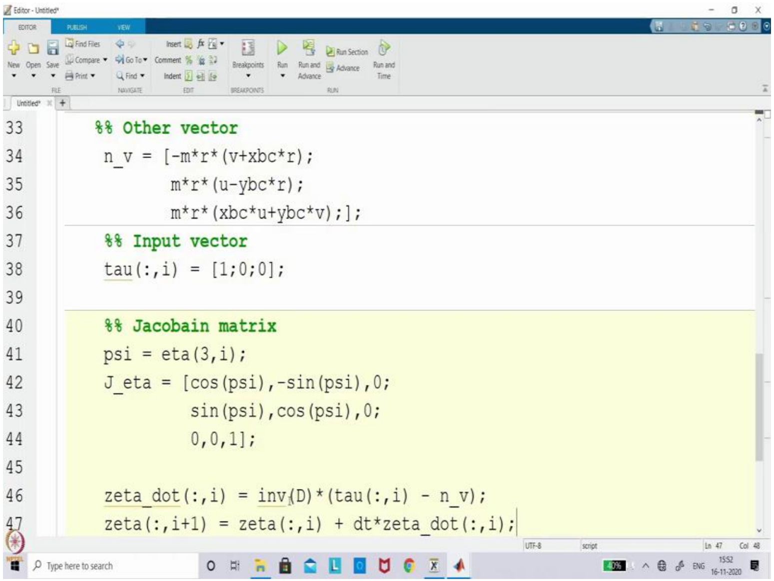

I hope you are able to see my screen. So, which is actually like m × r. So, multiply with what you call v + X_{bc} × r. So, then the second term which is what you call m × r. So, u - Y_{bc}.

So, I will actually like type it faster. So, that it would be beneficial. So, this is X_{bc} × u + Y_{bc} × v then you can see this is what the last term and this is what we have actually like done. So, then what else required? So, remaining all we need to do it ourselves.

So, that tau what you call that is actual like every instant is going to change. So, this is I call other vector ok. So, this I call input. So, the Input ok. So, the Input vector what we are going to write F_x F_y and M_z, but right now I am taking that is simple only F axis is having one Newton y axis is not having anything and z axis is also not having anything just for simulating. Then what you need you need actually like Jacobian matrix because you are going to do 2 things. So, I am just taking a Jacobian matrix.

Why we are doing 2 things? Because it is 2 you call second order system you would be integrating twice. So, the first one you are trying to get the ξ then the ξ you would substitute in the $\dot{\eta}$ equation that is why we are writing the Jacobian matrix. So, $J(\eta)$ I can

write as. So, $\cos(\Psi)$ ok. So, then – $\sin(\Psi)$. So, then 0, then what you call $\sin(\Psi)$, then $\cos(\Psi)$ a 0 then what you call 0, 0, 1 ok.

So, this is what the case so, far that what one supposed to know. So, you should know what is Ψ. So, Ψ I am writing as. So, η of third element so, that also I have written ok. So, now, we can actually like do the integration. So, the first one is actually like I am writing $\dot{\xi}$ at the i^{th} instant ok. So, this would be what $D^{-1} \times \tau$ ith instant - $\vec{n}$ right.

So, this is what you do $\dot{\xi}$. So, then what you want? ξ you want $\xi(i) + 1$ iteration. So, that

what you call $\xi(i) + dt \times \dot{\xi}$ right. So, $\dot{\xi}$ of i^{th} one right, this is what we have taken as a

Euler integration. So, till now you can see right this is only for a velocity update, but you know like this is actually like second order system. So, we need to actually like do little more. So, we will actually like come back to the other you can say state which is you call η.

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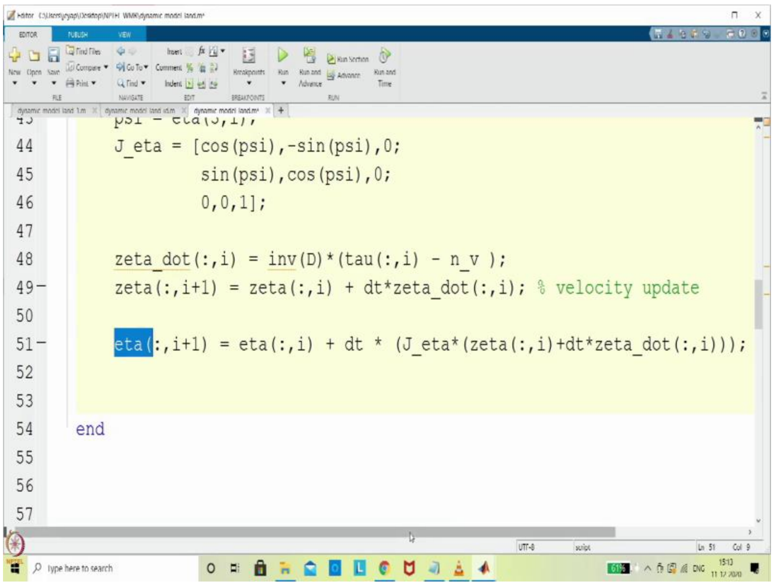

So, we are actually like updating that η with the help of η ok. So then you can see like what usually we does because the $\dot{\eta}$ we write it in a ξ form where J(η) × ξ. So, this is what we usually say that that would be equivalent for a first order system, but if you look at this η this η is actually like dependent on not only initial velocity it is dependent on even initial acceleration.

So, in the since we can actually like add one more fact. So, which is what we call you can see this fact we can actually like add what that mean actually like you can see we are adding the acceleration part what; that means, $\dot{\xi}$ also becoming. So, in the sense I am actually like making it.

So, $\dot{\xi}$(i). So, now, you can see like this is actually like dependent on the initial velocity and initial acceleration. So, now, if we actually like close this then you can see like this is what the η of update the sense I can actually like write this state update ok.

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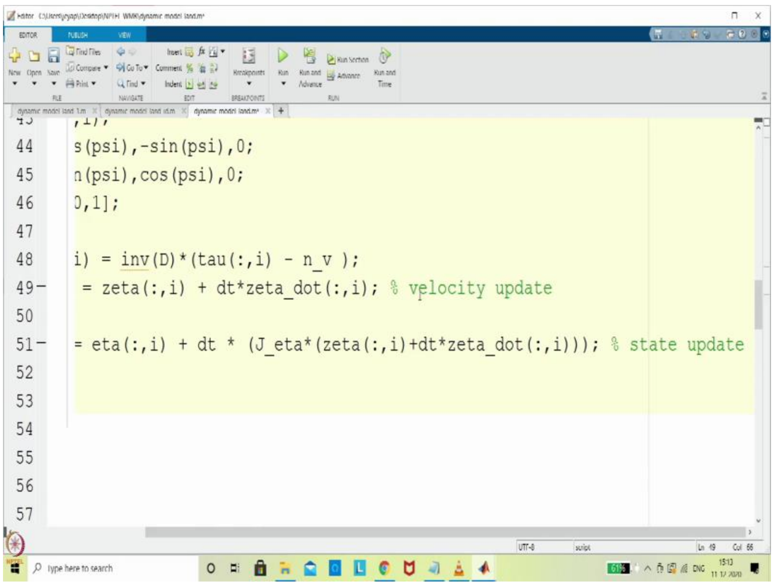

So, this is I call the velocity update. So, only issue is actually like here we are multiplying this j(η) because the ξ and $\dot{\xi}$ is in the other domain where it is actually like with respect to body what we call instantaneous velocity and instantaneous acceleration with respect to body frame, but we are looking for the generalized coordinate which is actually like you call eth fixed. So, that is what we are actually like trying to find.

So, now if we apply this way then you can see like whatever we have done we have assume that the tau as the input and actually like we are taken J(η) actually like calculated based on the Ψ and then we are actually like going again with $\dot{\xi}$ and then ξ we calculated based on the velocity update then η. So, now we can actually like bring the you can say the file which we have usually uses for the animation

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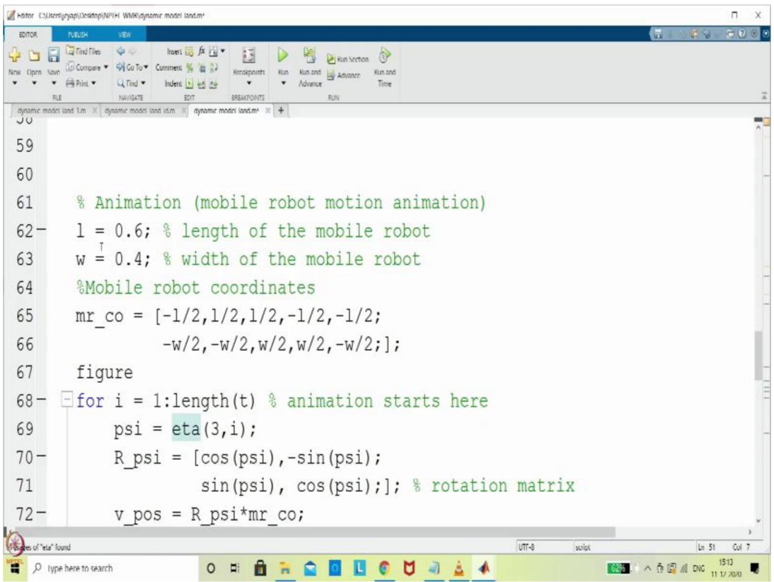

So, that I have already copied here. So, you can see like this is the animation part. Now, if I give any Ţ. So, in this case I modified to 1 and 0.5 in the sense 1 Newton in x and 0.5 Newton in y.

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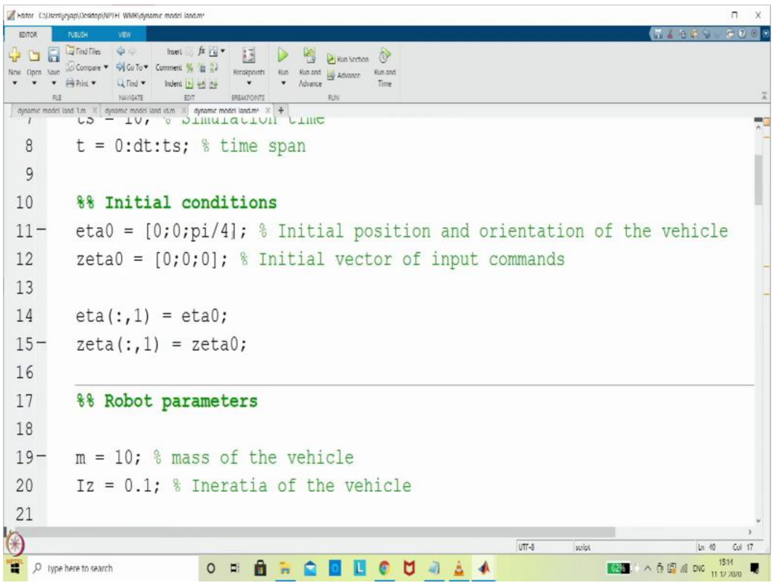

And you can see like I modified just for a clarity. I will start with 0 initial condition. So, now, if you here what it is giving? This F axis force and the y axis force.

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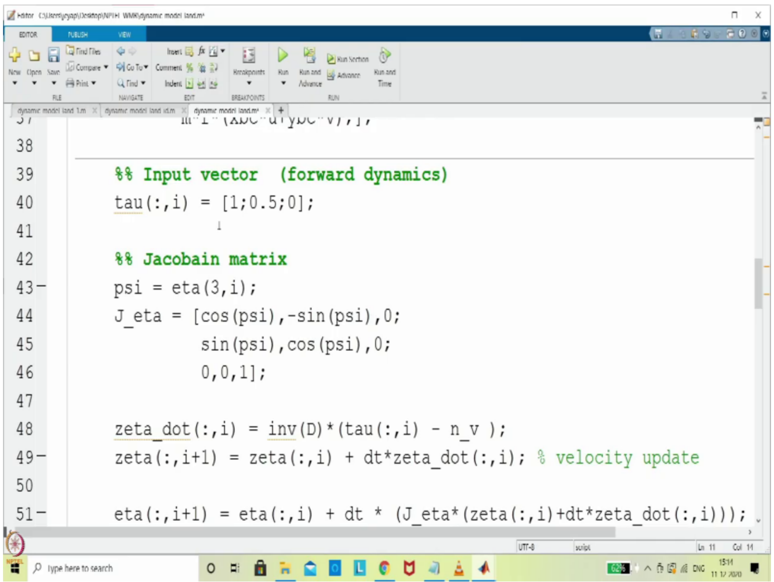

So, now the vehicle will or you can say drag in the lateral direction in a inclined way that we can actually like explore by looking the animation.

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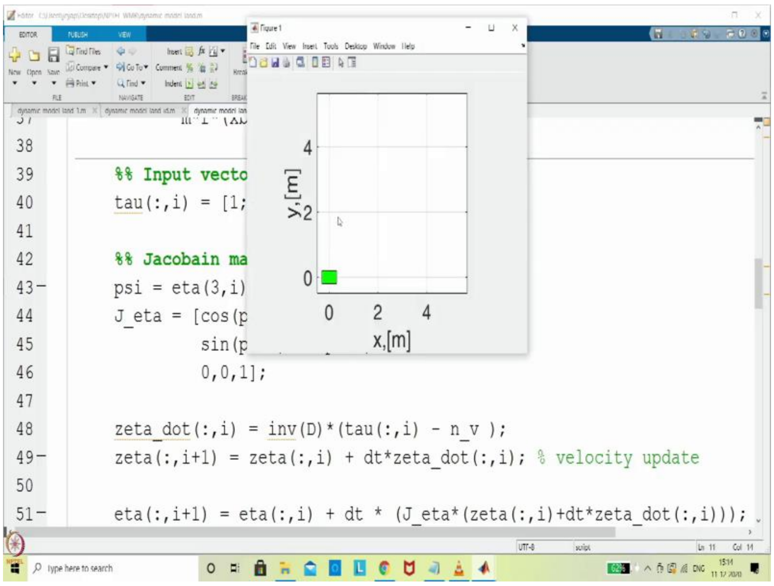

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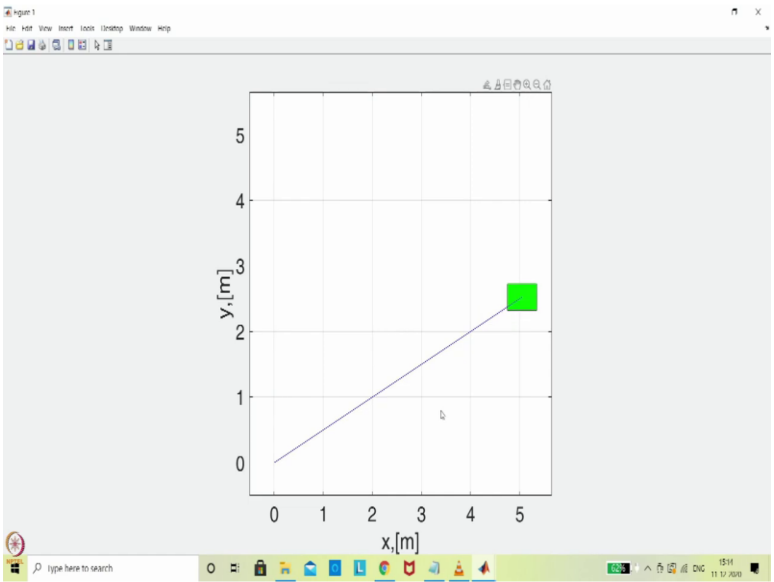

So, now, you can actually like see the animation. So, you can see like the 1 Newton force is acting on the body in the forward and 0.5 Newton is acting on the y direction. So, now, the vehicle is actually like moving both a longitudinal and lateral in the sense it is actually like moving in a inclined fashion.

So, now, the same thing you want to actually like explore little more. So, what in the sense I will give some non zero initial condition; for example, the vehicle state already it is actually like rotated something.

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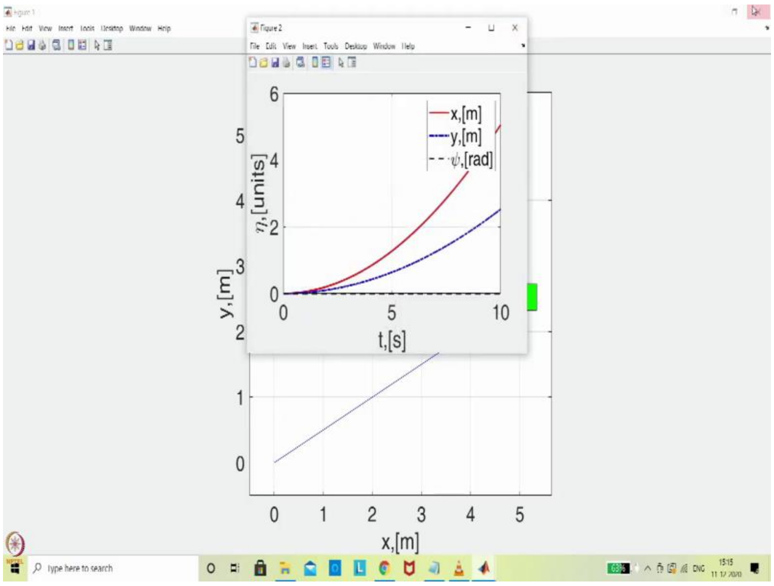

So, then you can realize what is forward dynamics is all about. So, these are the system states will come back. So, now I am actually like seeing that instead of you can say the η_0 as 0 in the angular.

So, I am saying that this is actually a 45°. So, now, if I actually see that what happened the vehicle itself is actually like orient 45°. Now, if I actually like push the force in F you can say F_x and F_y you can see like from that 45° again it would be inclined not actually like from 0.

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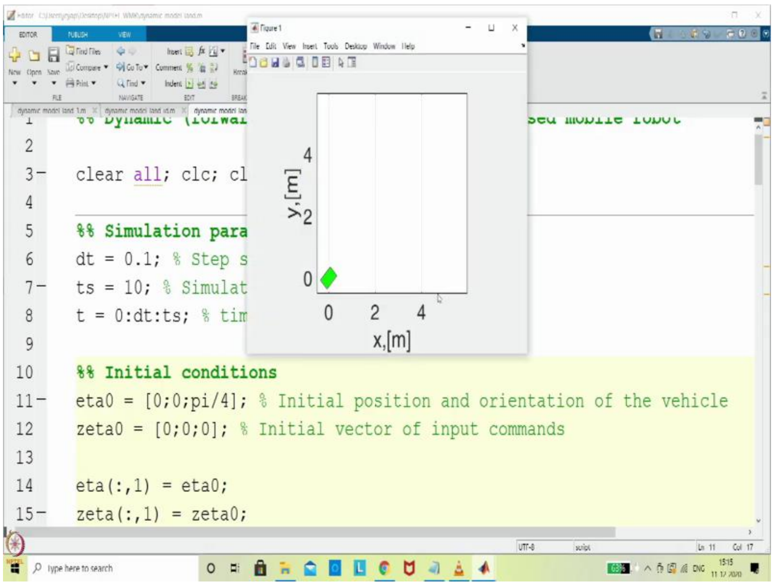

That you can realize from this particular simulation you can see already the vehicle is in 45°.

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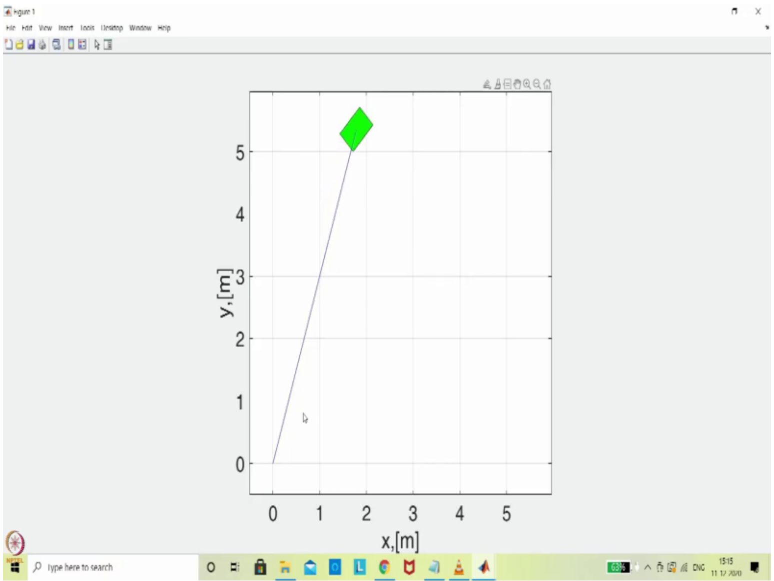

Now, we are actually like inducing the force in x direction and y direction of the body fix. So, that is what you can realize. So, now, you can see the vehicle is actually with respect to body whatever the motion earlier you have seen that is exist, but since it is the system state from the generalized coordinate is modified. So, that is why you are actually like getting you can see like plus 45° added in the vehicle.

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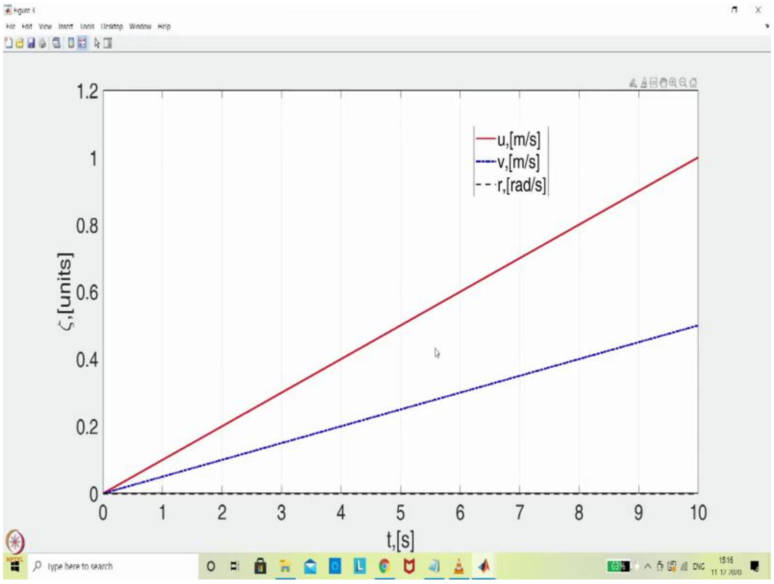

So, similarly you want to see what is the system state for example, body fixed velocity how it is increased we do not have any friction right. So, that is why you can see the velocity keep on increasing.

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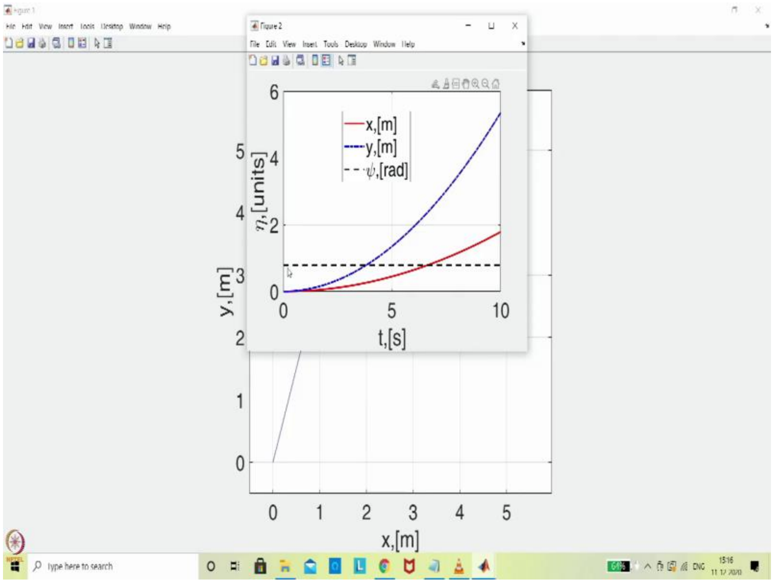

And similarly system state also like keep on increasing and we assume that the Ψ is actually like 45° constant. So, that is what you can actually like see, but when you see it initially 45, but after all after that you can see like it is a gradually actually like exchanging small variation right, but it supposed to be 45° throughout that you can actually like realize at the end.

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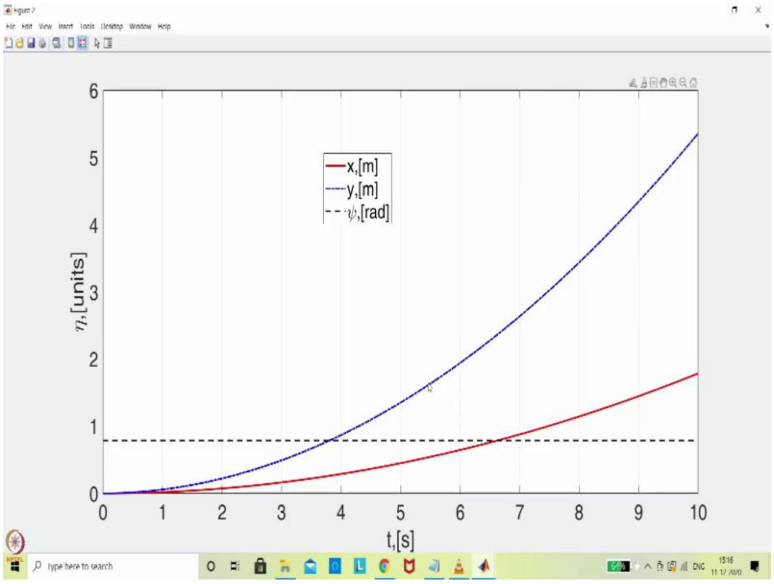

So, now, I am actually like looking at the other round other way round. So, what the other way round? I am actually like inducing some friction.

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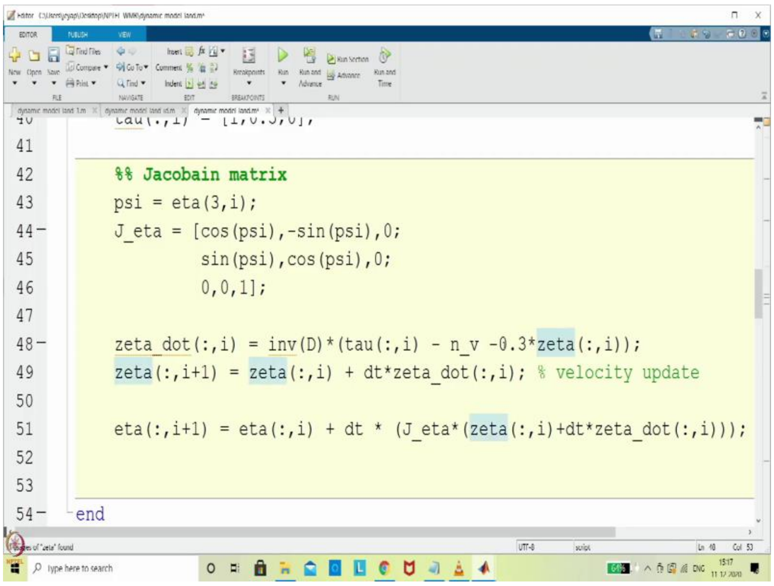

So, for example, I am assuming that it is a viscous friction. So, what that means? So, the viscous friction coefficient is known I assume that that is 0.3 and that is actually like in each and every state there is actually like the direct relations. So, in the sense ξ of actually like i. So, in the sense b v right when we write a second order system.

The same way it is equivalent it is a very close to a damping coefficient this is damping coefficient and this is the damping overall term this is a velocity base. So, now if I actually like heard what happened the velocity would be getting you can say streamline that you can actually like see.

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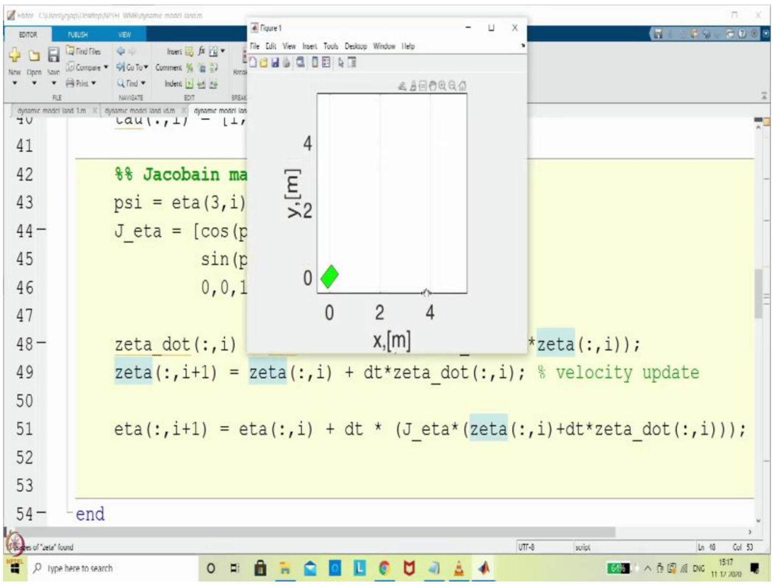

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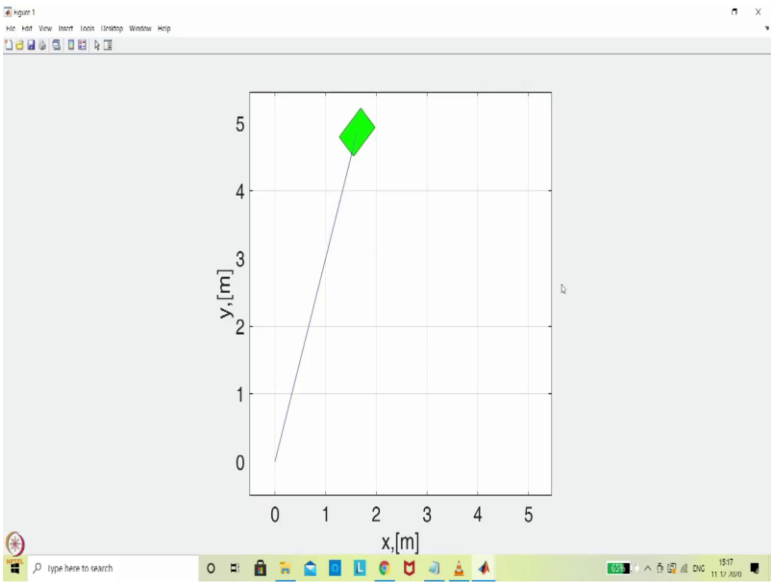

So, the same scenario only think the velocity would be getting somewhat actually like getting settle. So, earlier it was keep on increasing. So, now, you can see like some kind of saturation you can expect at the end of you can say simulation probably.

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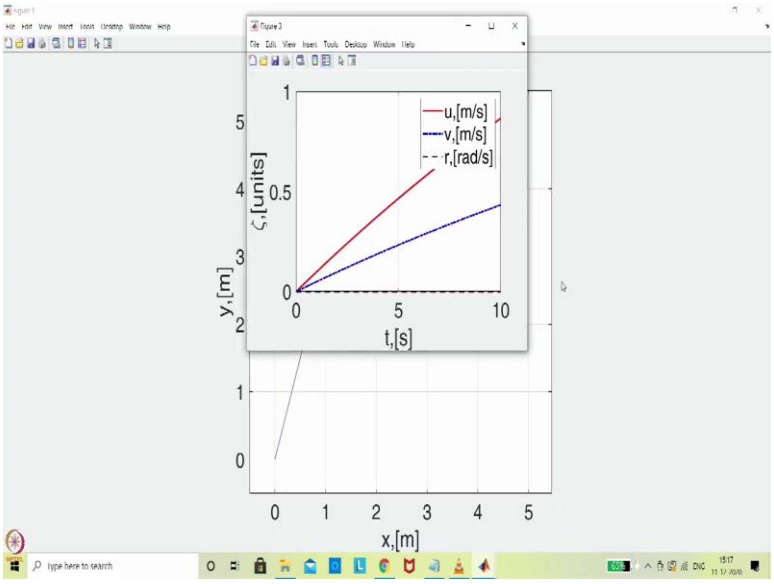

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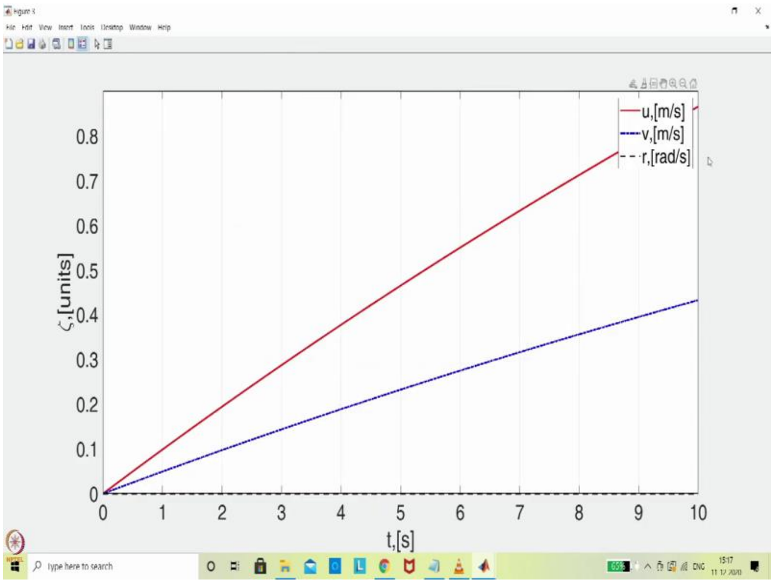

So, now, you can see that the system is actual like a getting curve, but if I increase the time step you can actually like see that increasing the time step in the sense span time. So, right now it is actually like 10 second.

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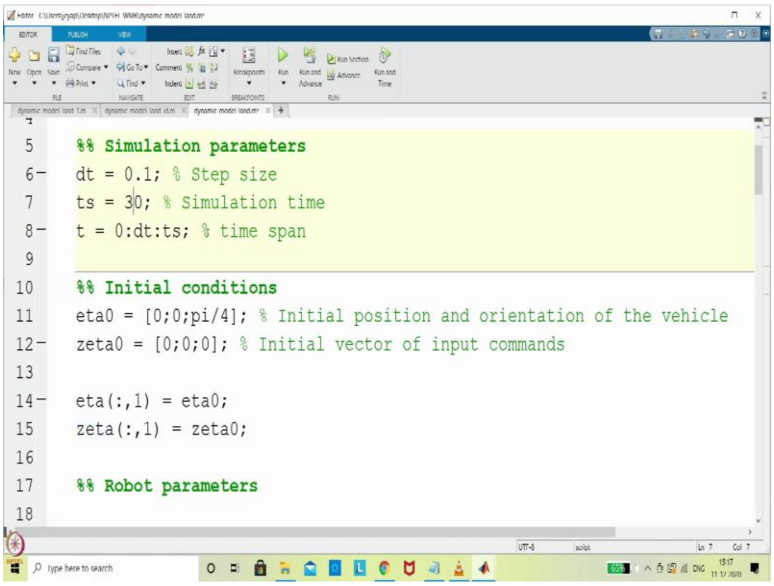

If I actually like increase probably 30 and I actually like pass the simulation I do not want.

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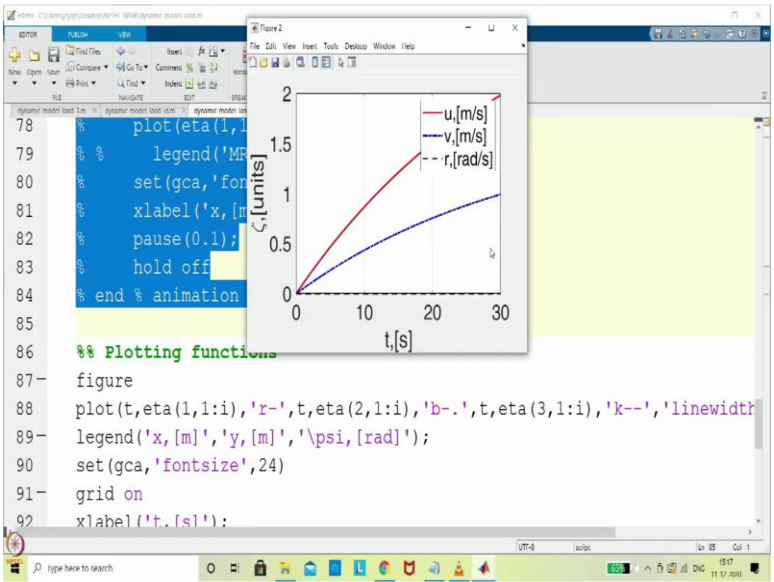

I just want to plot the you call all the system states. So, you can see like the velocity would be getting a streamline.

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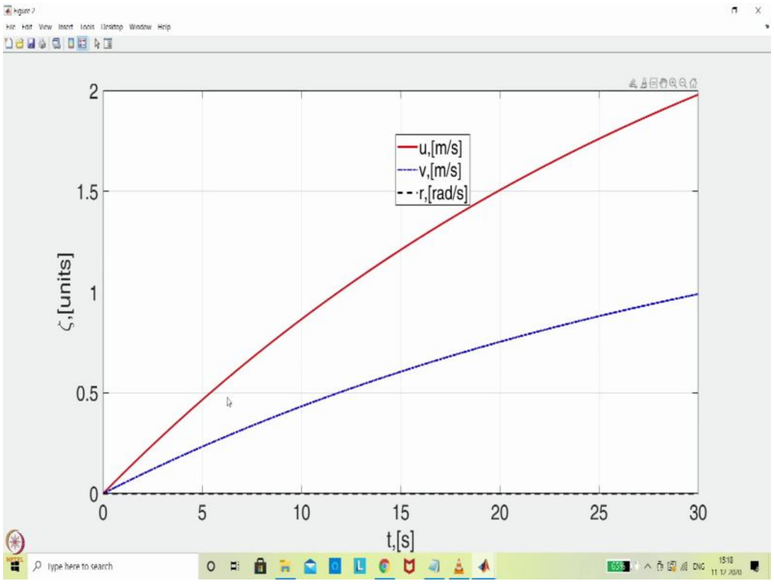

So, still it is not because the friction is actually like not really actually like is sufficient to drag this because it is actually like one Newton and it is a frictionless surface earlier. Now, you can see like it is not like a straight line it is curved. So, in order to check that probably we will increase the time again.

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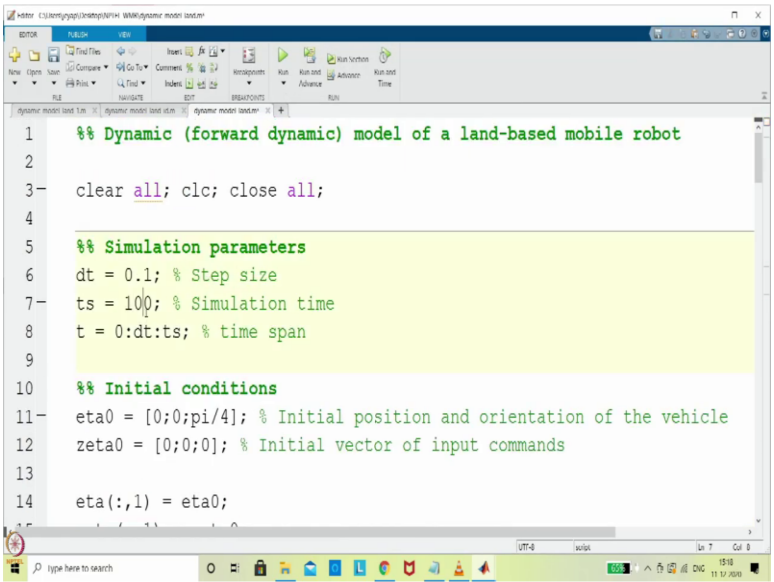

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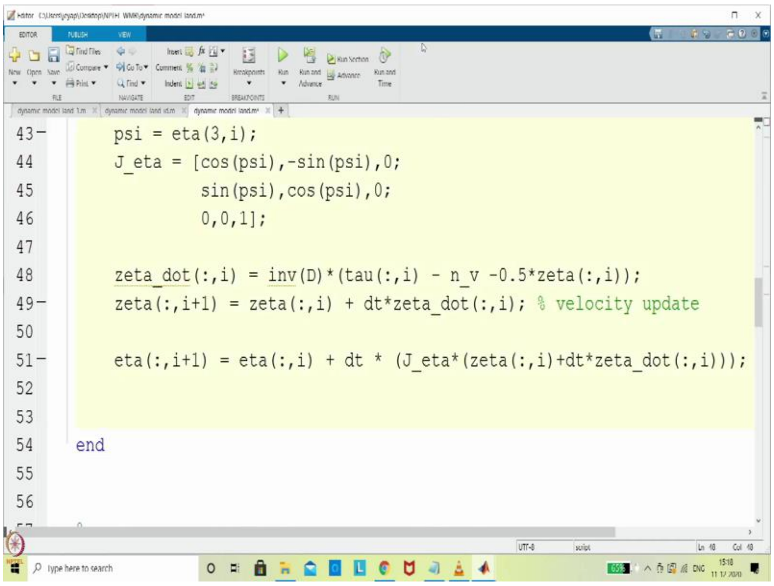

So, probably I will just give 100 and I will just increase the frictional coefficient probably 0.5.

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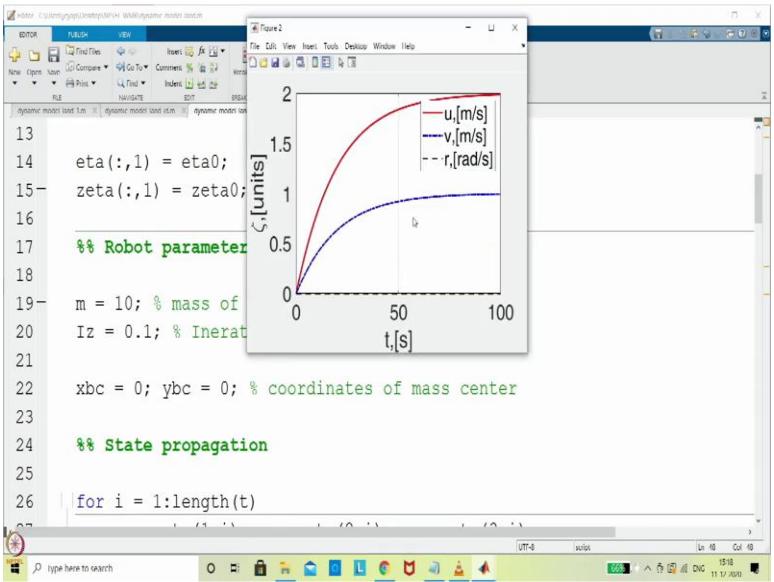

Just for because the mass is here is 10 kg right. So, then one Newton is actually like very small.

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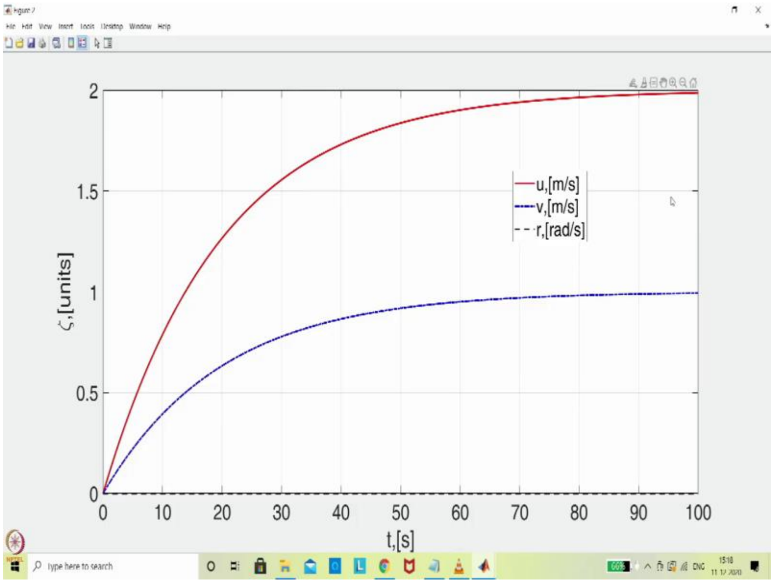

So, now you can see like it is getting saturated right it is very closed to 1 meter per second in v and almost 2 meter per second in you can say u. So, this is what we call actually like forward dynamics where you know the force tau which is actually like known to you and you deploy your actual like equation of motion and then you can verify.

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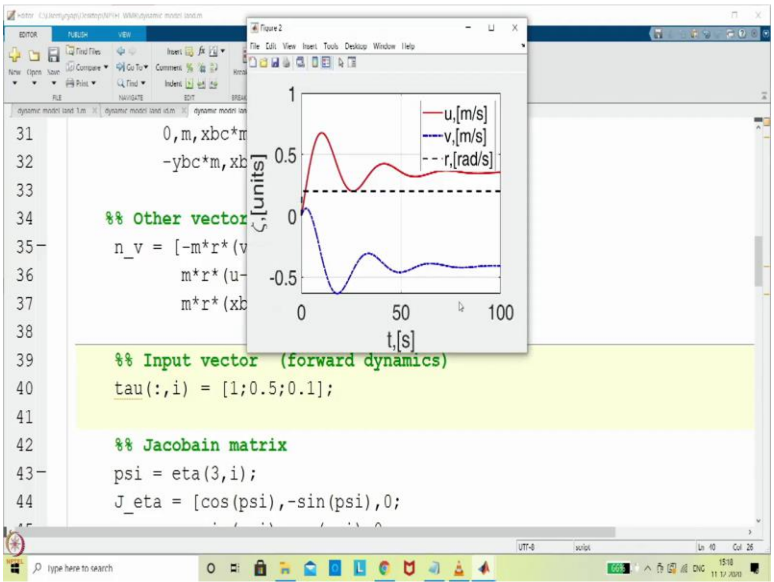

For example now if I give even the angular torque. So, then you will get actually like the angular force also like in the sense it is getting changed. So, now, that is what you can actual like realize.

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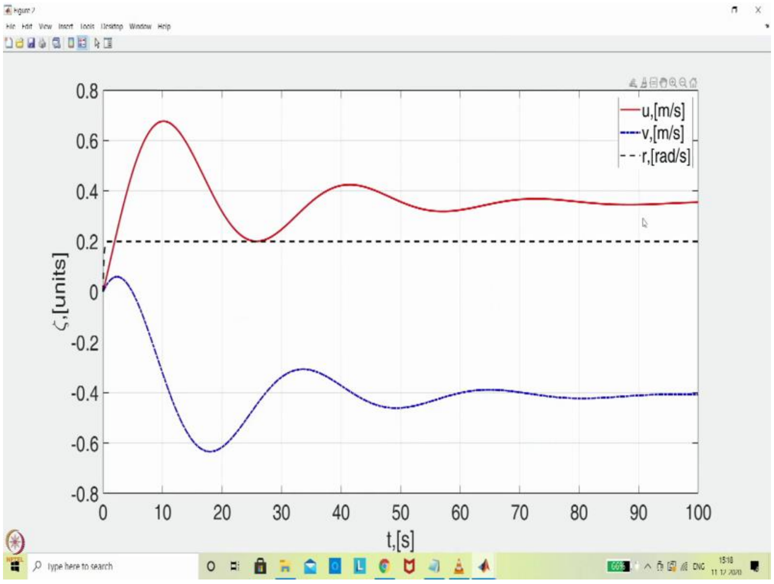

So, what that means, actually like you can see initially it is 0 velocity in you can say angular velocity with respect to body frame, but now it is actually getting approximately 0.2 radian per second. So, because of that what happened? The vehicle is actually like start you can see rotating. So, that is why you can see x and y is actually like making a curvy way.

So, for example, x is actually like getting increase after that you can see it is something like a sinusoidal form. So, now, you can actually like bring the simulation part just to give probably. So, you can actually like get a idea how it look like. So, I am just bringing the simulation part.

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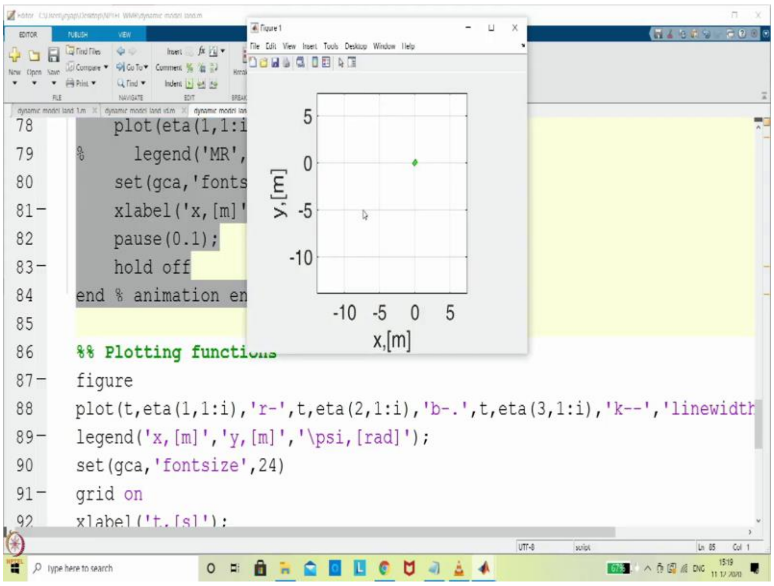

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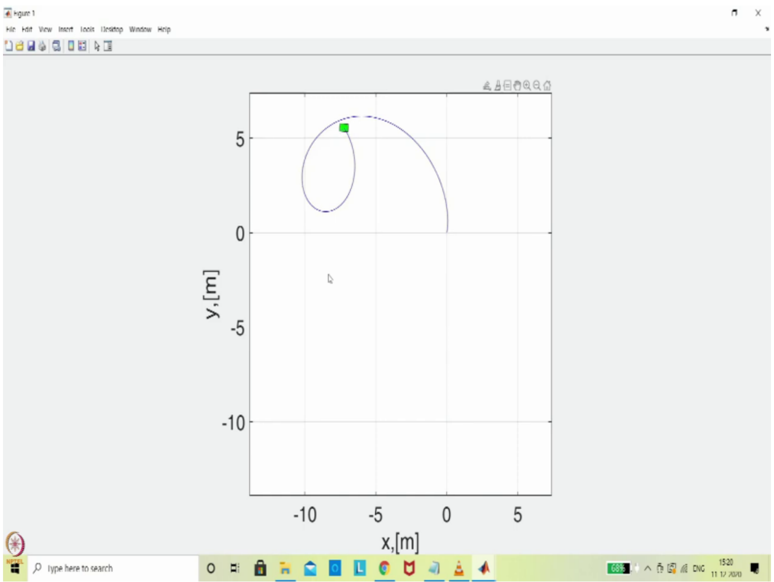

Then you can see that vehicle is getting actually like increase. So, here I scale down or scaled up according to the vehicle overall coordinate you can see like it started from 45°,

but it is keep on increasing.

So, it is actually like properly getting a certain constant velocity in x and y. So, after that you can see it is something like a sinusoidal form it is making a circular or you can say very close to a circular form. So, these all the cases what we have seen as a forward dynamic. So, now, you can change your X_{bc} Y_{bc} you can change your vehicle mass and you can say the a inertia and then you can actually like play all those things, but this is what we called the forward dynamics.

So, now in the next lecture we will see how this forward dynamic force what you call Ʈ the Ʈ = $\gamma \times \lambda$. So, that we can bring and then we can actually like see how that would be beneficial in the simulation side where you can actually like think about how to optimize your you can say mechanical parameter how to increase your performance all those things we can understand there ok.

So, now you can see the vehicle is actually like getting into one kind of streamline way. So, with that I am actually like ending this particular lecture and then we can see you can say in the next lecture. See you then bye.