Wheeled Mobile Robots
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Lecture - 11
Mobile Robot Dynamics – Part 1

Welcome back to the Wheeled Mobile Robot course. So, far I hope you would be enjoying the you can say auspicious because you would have seen that the kinematic simulation we have done. So, now, you can see like you as touching something like very real right. So, as I already given a kind of teasing; so, what I said the next lecture we would be talking about more on the dynamic aspect where we will bring the real time aspect at least we start with the mass and inertia.

So, that is what we are trying to address. So, in the since this particular lecture 11 would be having two part. So, the first part what I am going to cover is basic dynamics what you call mobile robot dynamics in general then we would be deriving the equation of motion based on one of the approach in the part 2 we would be seeing the second approach which is one of the popular and we would be seeing how to write the equation of motion in a matrix and vector form.

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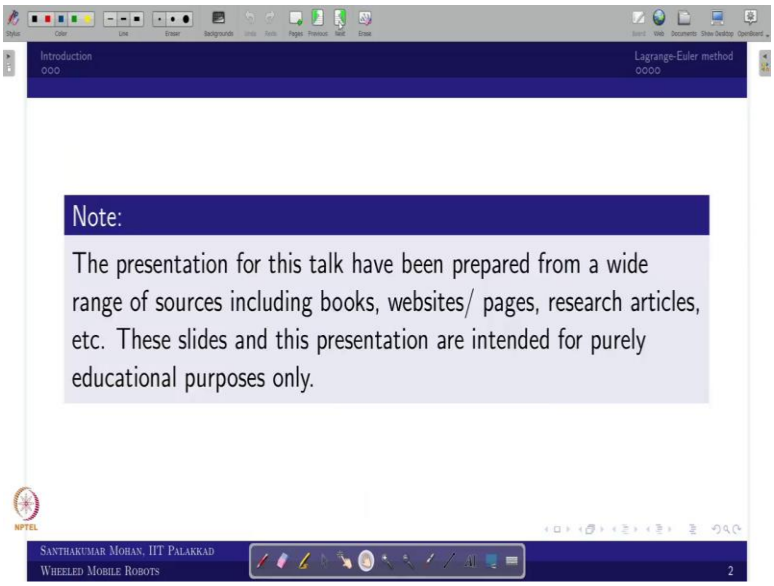

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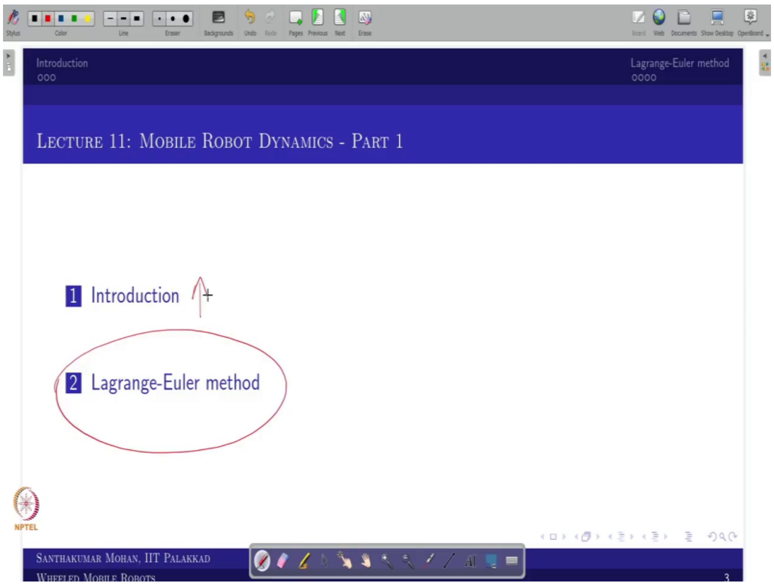

So, let us actually like begin the slide. So, where we can see like we are trying to see in this particular course. One of the popular method called what you call Lagrangian-Euler. So, where we can actually like see that this Lagrangian-Euler we are trying to focus, but before that we will see like what is actually like the overall aspect of dynamic model.

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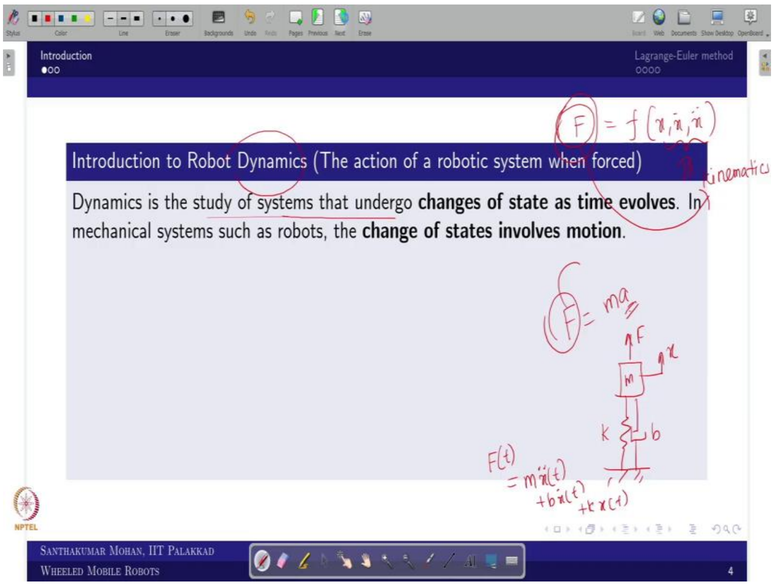

So, far that what we are actually trying to see what is dynamics. So, you know in physics aspect. So, dynamic is actually like very broad where you call actually like it is actually like contradict or you can say contemporary to the statics where the statics is actually like nothing but the you call there is a no motion or the motion with the constant right or you can say constant motion; whereas, the dynamics means it is actually like change with respect to time, but here robot dynamics means it is slightly different because the dynamics is already have a two classification which we used to do in physic.

One is kinematics the other one is kinetics, but here the kinetics we are going to call more general as dynamics. Why it is so? Because dynamics means what the study of motion with respect to what you call change in you can say inputs or you can say by incorporating the input. In the sense we have seen the motion with the cause of motion. So, then you can see in robotics or in mechanical engineering, the kinematics will give the study of motion, but when you talk about dynamics the study of motion is by default incorporate.

For example if I write the equation F = ma. So, where the a is actually like required then only I can find the F right. So, in the sense if I give F, I can find a; in the sense, the study of motion I am relating with the kinematic aspect. For example, if you recall your second order system response for example, I am taking a mass spring damper system just for understanding I am actually like taking this is a mass, spring, damper.

So, what you use to write the equation? If I give a force F; so, this would get displace in x I assume that is one dimensional where it is only going to move in a you can see upward and downward direction I call this is a b is a damping constant and k is the you can say stiffness. So, you use to write the F is a input that you would be writing in the form of you can say time derivative of the displacement right.

So, 0th and 1st and 2nd derivative in the sense $b\dot{x}(t) + kx(t)$; so, what in that sense you will try to write you can see one side F. So, other side is actually like function of x, $\dot{x}$ and $\ddot{x}$, but what is this we have studied? We have studied this is nothing but kinematic aspect; this is what you have seen as a kinematics.

But now you are bringing the cause of motion. So, what we are at a trying to bridge? So, you are trying to bridge the cause of motion to the study of motion. So, that is what you have to see as robot dynamics. So, in the sense the action of a robotic system when forced, the force in the sense do not see only force.

So, forced means it is a general you are giving some input. So, in the sense what you are trying to see? So, the system will definitely will change of state as time evolves. So, in mechanical systems such as robotic what you are say that change of states involves motion.

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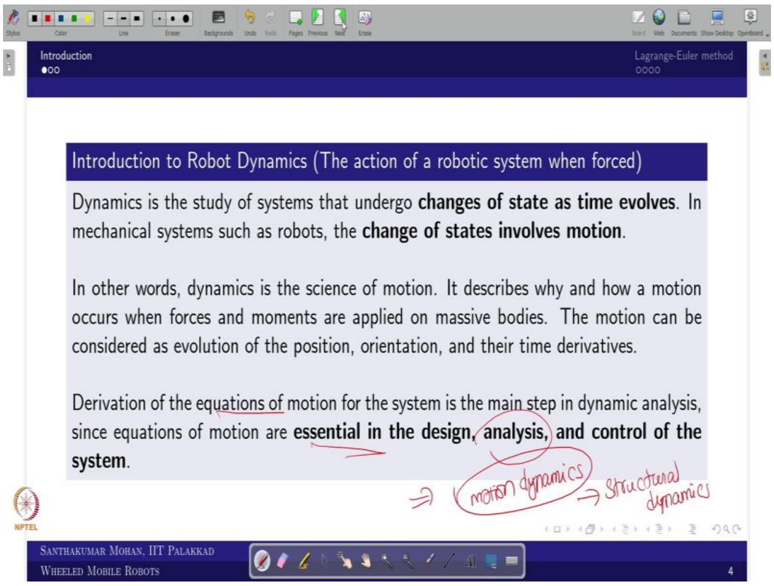

So, with respect to the change of you can say inputs that is what we are trying to see. So, in that sense what you can see this is also one kind of science, but what we are really interested to see this why you need to know the robot dynamics? I already gave a glimpse in the very beginning.

So, this is actually like going to help in design the overall system where you can design your mechanical you can choose or select your actuator or you can actually like design even the control system and along with navigational aspect. Further what you can see you can analyse that is what the aspect.

So, in the sense the dynamics is actually like one of the important case. So, here the dynamics would be broad into two. One is motion dynamics other one we call structural dynamics this particular classification not everyone will do, but most of the robotation will do because the motion dynamics what we are focusing in more in the you can say a robotics course, what that mean? Equations of motion.

So, when you talk about the motion so, the motion dynamics is actually like one side. So, the you can say contemporary would be the you can say structural dynamics. So, the structural dynamics is actually like where you take non moving entities, but the moving entity it is actually like going to give a set of equation that set of equations what we call equations of motion that is what we are trying to derive in this particular lecture 11 ok.

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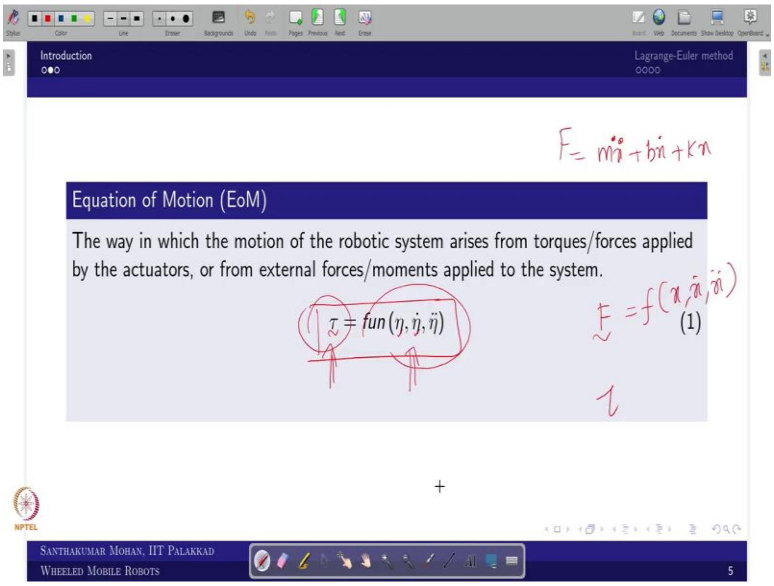

So, now you are clear what we are trying to see in this particular lecture let us move further what I said already the equation of motion. If there are several equations we call equations of motion. So, this is what we are trying to write. As I already given a glimpse. So, this is what the overall form where $F = m\ddot{x} + b\dot{x} + kx$ where you have returned this I can rewrite as F in the function of x, $\dot{x}$ and $\ddot{x}$. This is a scalar.

Now, we are bringing it as a vector then that vector I am calling as a tau, this is a vector where η is what you call system state generalized coordinate that would be having and their derivatives are given.

So, now, you can see like some aspects of come already. So, one side there is one input; so, the other side is actually like the response for the input in the sense this is action and this is the reaction. So, we are trying to see the relation between action what would be the reaction at the system level. So, this is what you call equation of motion.

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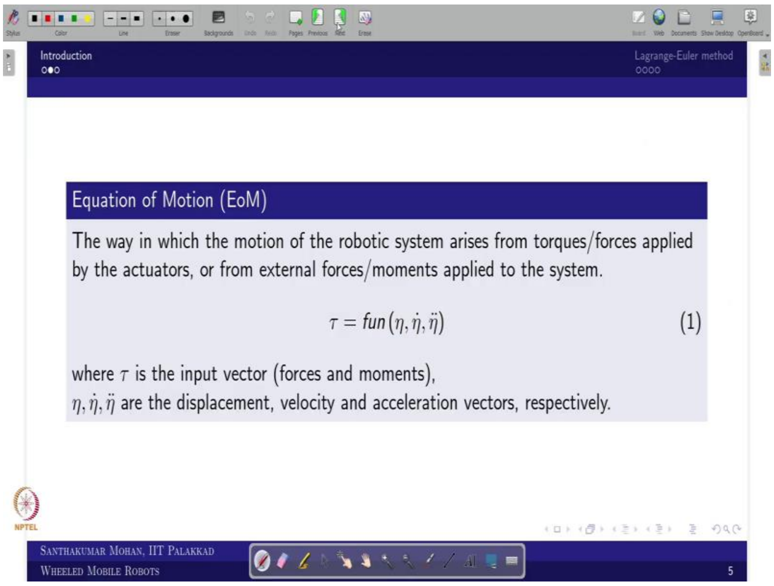

So, why this is so important? So, you can do two subset of the dynamics that is what we are trying to focus.

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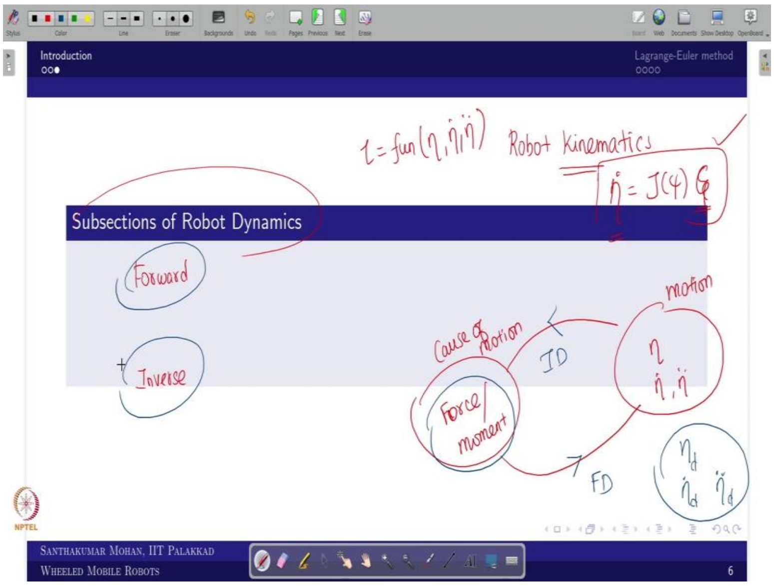

So, what that? So, you can see like the subsection of robot dynamics as what we have seen in the robot kinematics. So, what the robot kinematics aspect? So, we have seen that if you know the input and you are trying to see what are the study of you can say the motion that is what you have said as a forward differential kinematics; whereas, you need some desired system state, how you can actual like give certain input so that you can achieve that, that is what you call inverse differential kinematics.

There you have written you just recall the equation. So, you have actually like written $\eta^{\cdot} = j(\Psi) \times \xi$ right. So, this is what you have actually like obtained. But what you can actually like see that if I know this input come an, I can find the $\dot{\eta}$ or the other way around $\dot{\eta}$ decide is given I can find what would be the necessary input.

The similar direction what would be the 2 subsection. So, one would be forward ok. So, the other one is inverse. So, then by looking the equation itself you can see; so, this what you have written. So, I will write eat right. So, that is what we have actually like taken at. So, this is η, $\dot{\eta}$ and $\ddot{\eta}$ right.

So, but now looking at here this side right hand side we have given as a input based on that you have taken a forward and inverse, but here it is not like that. So, what do you have 2 spaces. So, one is actually like your force and moment. So, which as I call general as a cause of motion. So, these are the causes. So, the other side is actually like motion variable in the sense you call η, $\dot{\eta}$ and $\ddot{\eta}$ in the sense position in the sense displacement velocity and acceleration.

So, now we are trying to map these two if I do the mapping from the known forces and moments in the sense I connect the wheel, I give a desired task in the sense I take a proper motor and giving a definite torque, what would happen to the motion? So, that I do then that is what you call forward dynamics; whereas, I am actually like looking the other way around I am giving everything is desired in the sense earlier what we have seen position trajectory, but now you are giving actually like what you call that general trajectory if I actually like do this given and I can find the forces and moment.

So, that is what you call inverse dynamics. So, you can actually like by looking itself this is nothing like analysing and this is like you can say controlling.

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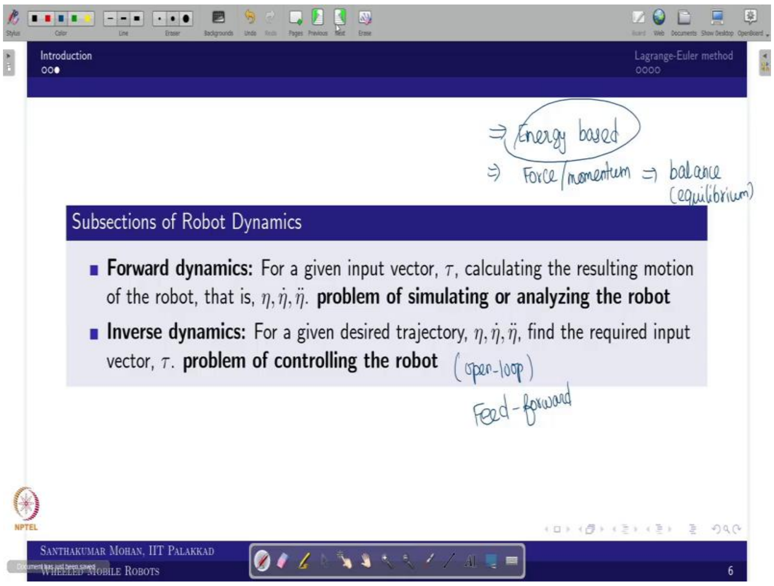

So, that is what I put it in this particular slide you can see forward dynamics means for a given input vector tau, calculating the resulting motion of the robot that is nothing but η, $\dot{\eta}$ and $\ddot{\eta}$ whereas, the inverse dynamics is nothing but. So, for a given desired trajectory where η, $\dot{\eta}$ and $\ddot{\eta}$ all desire and tried to find out what would be the input vector tau.

So, this is what we are actually like looking at. So, one what we call problem of simulating what we did in the you can say kinematics the same way it is simulating, but in a you can say dynamic level. So, the other one is actual like controlling the robot in dynamic level again I am giving a disclaimer. So, control means do not see it is a feedback it is a open loop. So, very specific it is a feed forward. So, I do not want to give a straight forward terms now, because we are trying to cover this all in the what you call the motion control.

But this is a feed forward control ok. So, similarly what we did in the inverse differential kinematics the same thing feed forward where you are not taking any feedback you were just giving whatever you calculated that you are giving as a input. So, now, we will actually like see how to derive this. So, there are several way ok there are several methods. So, there are two popular methods in robotics which we commonly use, one is actually like based on energy ok.

So, I will just make it. So, one is actual like energy. So, energy based. So, the other one is actually like a simple what you call force moment base. So, I can actually like say that

that is a momentum. So, force momentum base in the sense it is a balance approach or you call equilibrium base. So, the other one is energy base in this particular lecture 11 part 1 we are trying to see the energy base.

So, you know energy equation. So, we are actually like modified the energy equation by Lagrangian or you can say Lagrange. So, that equation modified equation what you call Lagrange and this energy equation given by Euler modified one equation. So, the Lagrange has modified the Euler equation into further. So, that is why it is called Euler Lagrange equation or Lagrange Euler method.

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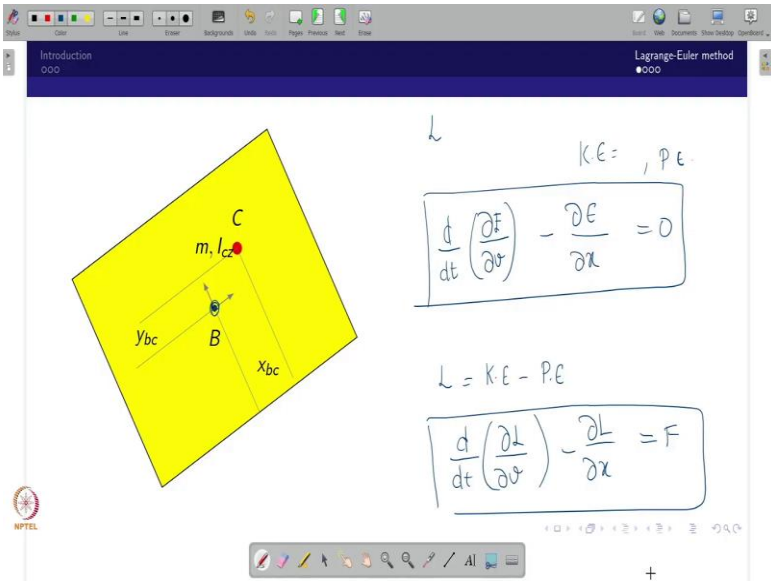

So, that is what we are actually trying to use for that again I am bringing back to the original. So, the mobile robot I am assuming as a rectangular box where this is the body frame because we cannot always assume that the body frame and the wheel you can say the vehicle centroid would be aligned.

So, now I am taking a very general case where the body frame is here, but the centroid of the vehicle in such a way that you assume that the mobile robot is consists of several batteries and the payload the cgs actually like move away from the body frame. So, then what would be the scenario? So, now, you see that now we brought the mass in the sense you brought the inertia. So, the linear and the rotational inertia you brought it. So, then what happen? The centroid point will have as a lumped mass and a lumped inertia we can realize.

So, I am just giving as a disclaimer. So, the I is you can say a tensor, but here I am taking since it is moving in a x y plan I am taking only the c_z in the sense I c_z only I am taking. So, now, what one can see the energy base approach what Lagrange said? So, the Lagrange said that if there is a energy difference then there is a motion exists then the motion exists means can we use a energy method? Yes.

So, what the energy method says that. So, the $\frac{\delta e}{\delta v}$ whole this is a actually like going to give a momentum equation. So, this would be $\frac{\delta e}{\delta x} = 0$. This what the energy equation, but what Lagrange modified is? So, this would happen when there is a energy difference. So, the energy difference here it is a rigid body we assume.

So, in the sense what are the energies would be there? Only kinetic and potential energy. In that sense you take kinetic energy minus potential energy this energy difference exists then there is motion exists. So for example you take a simple pendulum. So when the simple pendulum start oscillate you assume that it is a regime body there is no spring attach. It is a simple rod attach with the you can say hinge point; so now when the pendulum will start oscillate you take the point away from the equilibrium.

So then what you are trying to induce? You are giving a potential energy or what you can see that the bob which is hanging. You just hit it, what you are trying to give? You are trying to give a kinetic energy. Either one or combination of this, if there is energy difference then there is a motion exists. This is what Lagrange gave.

So now what Lagrange modify this Euler equation. So he modify this $\frac{d}{dt}$ so δL he brought and δv still exists. So $\frac{\delta L}{\delta x}$, this is equivalent to the applied force or torque. So, this is what we have actually like see. So, this particular equation we are trying to use. So, in the sense what you need? You need kinetic energy of the system, then you need a potential energy of the system fortunately the mobile robot or land based system we assume that the potential energy is constant.

And that is we even assume 0. Why it is so? Because it is moving on the land; so, the gravity effect would not be there. If we bring the suspension that to like we assume that multibody approach we need to bring, but right now we assume it is a rigid body that too like a lamped mass system then this all will go as it is.

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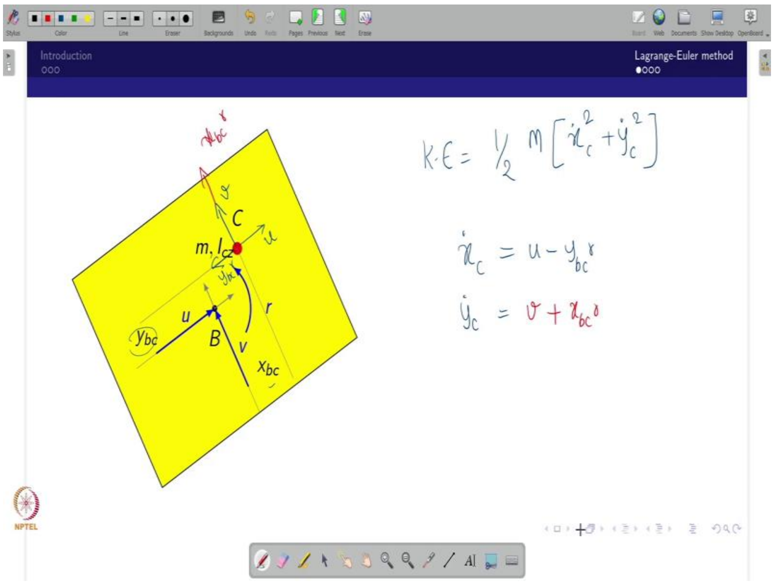

So, in the sense what one can see the body frame would have a linear velocities $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$ this is what we have seen in the kinematic aspect in addition to that what we brought it? The displacement or you can say the distance between B to C is X_{bc} in x axis Y_{bc} in the y axis. So, now what we are trying to see? We are trying to find out the kinetic energy. What would be the kinetic energy? Half, ok.

So, the mass so, there are two kinetic energy would be there one is translational kinetic energy which is due to the mass there is a you can say rotational kinetic energy due to the rotational inertia which is we call second moment of inertia. So, in the sense it is supposed to be $\dot{X}_c^2 + \dot{Y}_c^2$ where X_c and Y_c we need to define and $\dot{X}_c + \dot{Y}_c$ need to be defined.

We have already taken this is actually like instantaneous velocity $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$. So, now, we can actually like find what would be $\dot{X}_c$? So, that would be you can see like the u would be there in the x axis in addition to that you can see that this $Y_{bc} \times r$ would be opposite direction right. So, this u and the other side is actually like $Y_{bc} \times r$.

So, in the sense what you will have? This is. So, similarly $\dot{Y}_c$ what would be there? So, you actually like see that this is v and in addition to that what you can actually like see?

So, there would be additional velocity which is due to $X_{bc} \times r$. So, in the sense what you can see $v + X_{bc} \times r$ right. So, now, these two velocities are there; further I said potential energy we no need to bother because it is in the lane.

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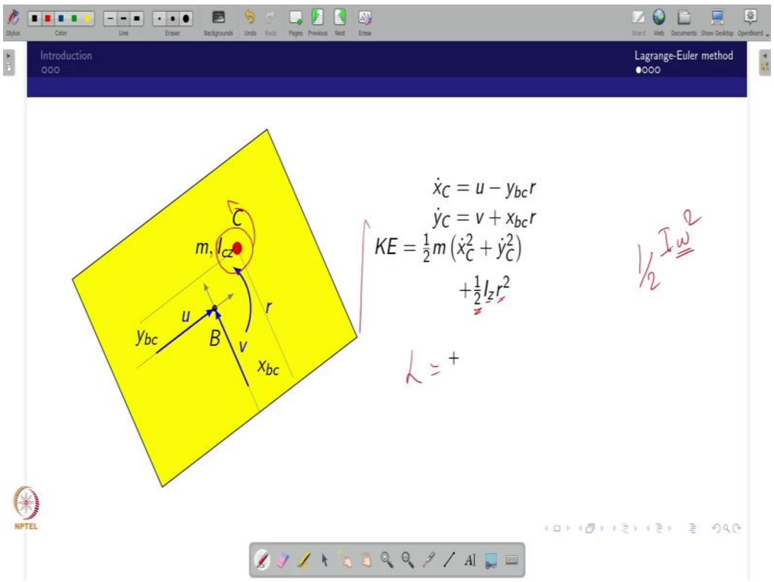

So, now we will actually substitute that. So, this is the equation we obtain. So, now, you actually like put the kinetic energy for the translation mass and what would be the rotational kinetic energy thus there is one inertia which is possess and there is a rotational speed. So, I $C_z \times r^2$ would be the equivalent you can say kinetic energy along with half right.

So, it is actually like half I ω^2 in general. So, since the ω here is r and I is actually like a vector, but here sorry tensor, but here we assume that is only one dimensional we are bothering. So, in the sense I_z comes and the remaining is coming right.

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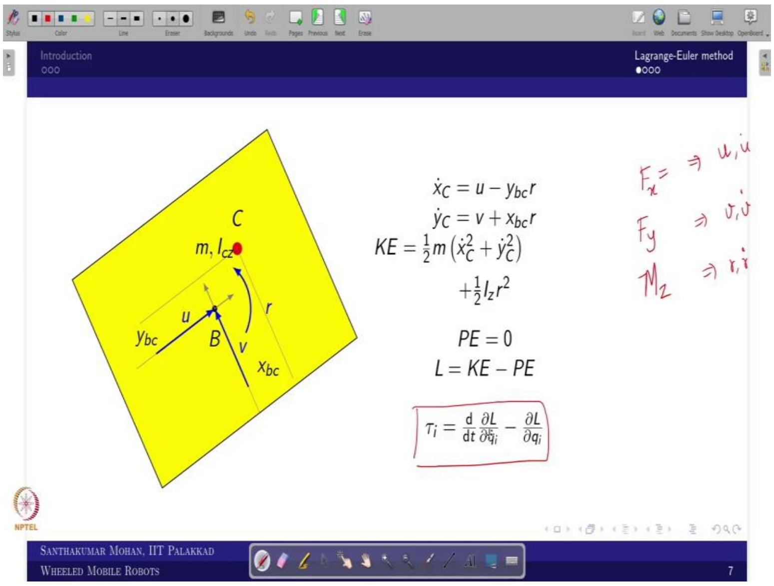

So, now, what you got it? The kinetic energy you got it now you substitute in a Lagrangian equation; what would be the Lagrangian equation? So, kinetic energy minus potential energy; so, this is the equation. So, in the sense the kinetic energy is the nothing but your Lagrangian and you take the Lagrangian Euler equation and you do it.

So, what are the equation you wanted? So, what would be equivalent in the x axis force? What would be in y axis force? What would be in the z axis moment? Right. So, now, this would be actually like related to the u and $\dot{u}$ and this is related to v and $\dot{v}$ and this is related to r and $\dot{r}$.

So, what dot? It is nothing but acceleration right with respect to body frame. So, this is what we are trying to derive it here. So, for that the first one is we will actually take it this. So, we will actually like move forward. So, what that means, actual like we will actually like take this.

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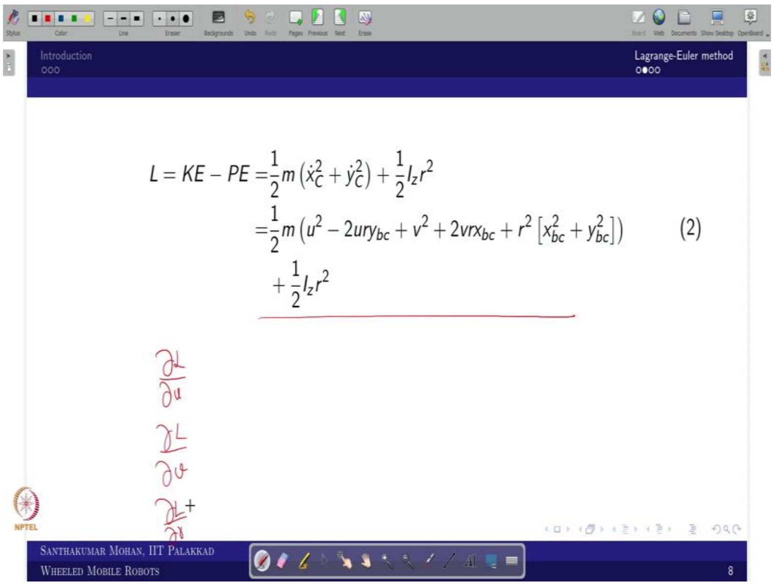

And we will actually like partially take the differentiation with respect to you call u then we will take v and take r and we will actually like try to take the time derivative that is what I am doing it one by one. First I write the bigger equation in the sense I expand this $\dot{X}_c + \dot{Y}_c$ in the form of $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$.

And I am actually trying to substitute and actually like square it and I am making you this equation. Now, what we are trying to do? We are trying to take the $\frac{\delta L}{\delta u}$. So, $\frac{\delta L}{\delta v}$ and $\frac{\delta L}{\delta r}$ right.

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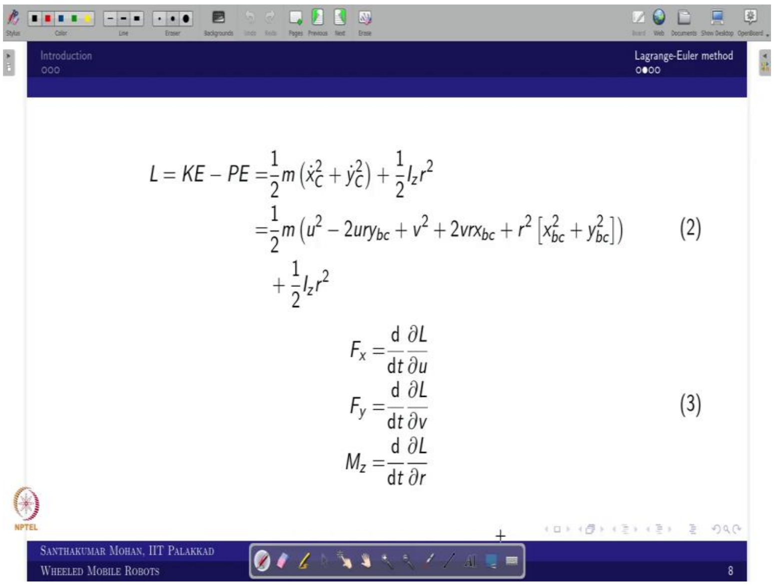

So, this is what I am trying to do. So, first I am trying to take F_x. So, since $\frac{\delta L}{\delta \int u}$ not there. Why it is not there? Because we are seeing everything in the body frame; when we are seeing with the inertial frame that is what we are trying to cover in lecture 12.

So, then everything will come. Right now, we are taking everything with respect to body frame. Why it is so? Because the wheels are attached with the body right; so, that is why we are trying to take directly as F_x with respective to body. So, now, this is what the equation similarly F_y and M_z you can get it; already you know like what is $\frac{\delta L}{\delta u}$.

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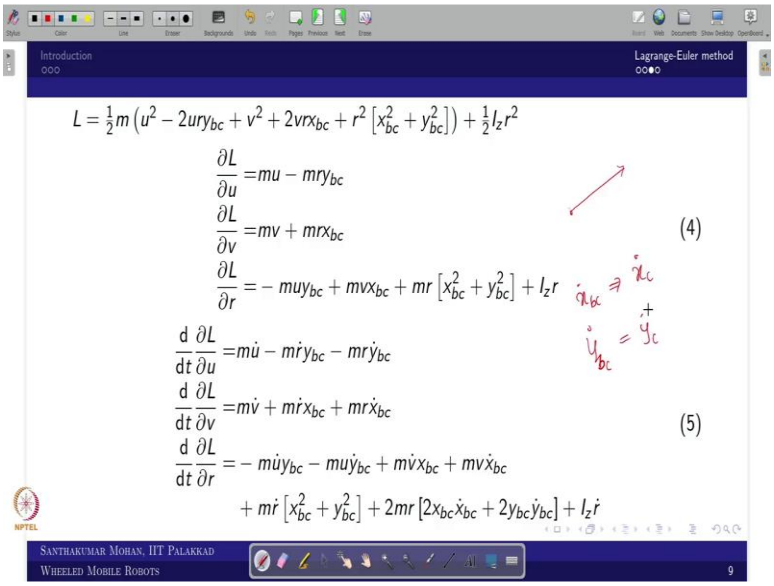

So, that is what we are trying to do in the next coming slides. So, you can see like this is what happening. You take $\frac{\delta L}{\delta u}$; so, this term will there, this term is there. So, remaining all the terms are 0, right. So, similarly you take $\frac{\delta L}{\delta v}$ you can see these two terms are not having any v term only these two are there. So, then you can take $\frac{\delta L}{\delta r}$ you can see r is here and here right and here also right.

So, now what you got it? You got it $\frac{\delta L}{\delta u}$, $\frac{\delta L}{\delta v}$, $\frac{\delta L}{\delta r}$. So, now, you take the time derivative that is equivalent to F_x F_y and F you can say M_z. So, that is what we have done right. So, you have taken time derivative of this. So, u also function of time, r also function of time and Y_{bc} also like function of time. Why it is so? Because that is actually like moving with respect to you can say body.

So, the body is having a longitudinal velocity. So, now, this $\dot{X}_{bc}$ and $\dot{Y}_{bc}$ are equivalent to $\dot{X}_c$ and $\dot{Y}_c$. So, why it is so? Because if you look at the picture then you will get idea.

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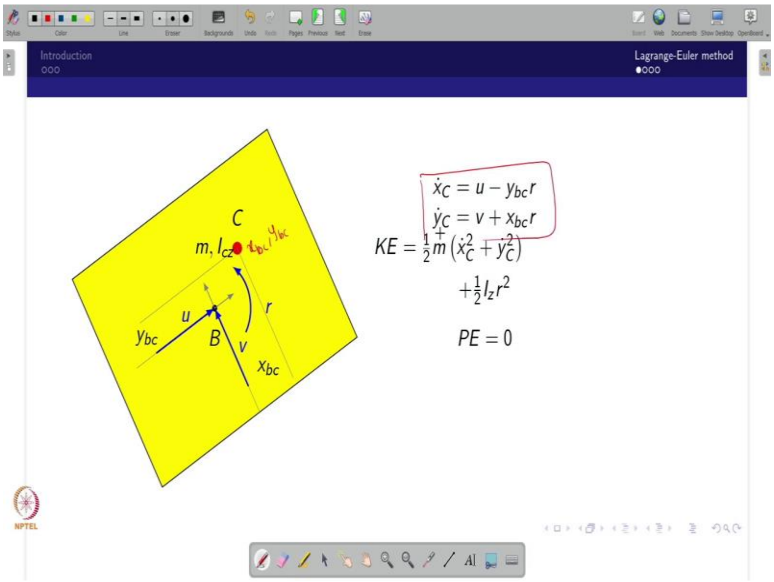

So, I will you can see like what this equivalent this is what you call X_c X_{bc} right , Y_{bc}. So, if you take a time derivative what it is equivalent? That is equivalent to this right. So, that is what we have actually like taken here.

So, now, you can actually like see if you take a time derivative. So, these all actually like going to come and you know already what is $\dot{X}_{bc}$ $\dot{Y}_{bc}$. So, then you substitute, you will get the final equation. So, that is what you call you can say F_x, F_y and M_z.

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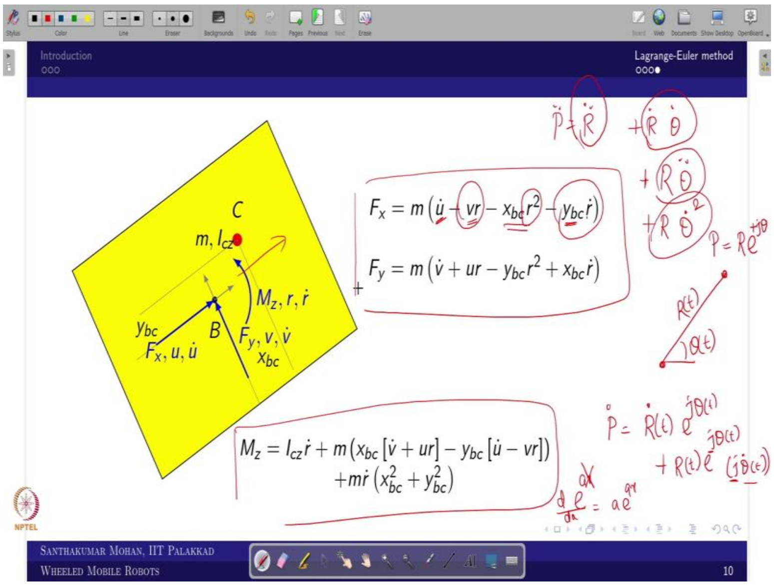

So, now you can see that this is the equation we obtained for x axis force and y axis force and this is the moment. So, once you obtain you can just recall. So, what you have studied? So, you have studied in you can say simple case I assume that. So, this is actually like R and this is θ. So, this point I am calling P ok, the P I can write as R $e^{j\theta}$ right.

So, now, you assume that this is also varying with respect to time this is also varying with respect to time what would be the $\dot{P}$? So, $\dot{P}$ is actually like $\dot{R}(t)$. So, $e^{j(\theta)}$ + R (t), then you can say like $e^{j\theta(t)}$ and then j$\dot{\theta}$ right in the sense what you will get?

First you will actually like get this value then it is something like $\frac{d\,e^{ax}}{dx}$. So, what you will write? So, a e^{ax} right. So, that is what we have written, but right now it is a big X. So, then you will differentiate that. So, in the sense what one can see this is actually like j is coming right.

So, now, you take the $\ddot{P}$ term. So, what you will see. So, $\ddot{R}$ also will come then $\dot{R}$ term along with you can say $\dot{\theta}$ will come then you can see like R × $\ddot{\theta}$ term will come and R× $\dot{\theta}^2$ term will come.

So, this is what you call slip acceleration, this is what you call Coriolis and this is what you call tangential acceleration, this is what you call radial acceleration; whether you are getting all the component in F_x competent or there or not you can see. So, this is you can say along with that direction that is nothing but a slip acceleration and this is you can see the tangential velocity in the sense longitudinal velocity multiply with angular velocity.

This is nothing but a Coriolis component and here you can see r squared. So, what that? So, this is a radial component. So, here r dot multiply with the position what that is equivalent to you call the tangential acceleration. So, now, these are the component you can actually like cross check when it comes to the moment. So, what you call the Coriolis and other things we will actually like rewrite in a gyroscopic effect that we will discuss in later; right now you can see like these three equation we obtained.

Now, we can see that the same equation can be obtain in the other method because I told there are two popular method right one method gave this equation whether the other method also like will give the same thing or not. So, this is what we are going to cover in the lecture 11 part 2 where we are actually like trying to address what we call the other method which is nothing but the Newton Euler method where the Newton equation and Euler axiom we will merge it and then we will derive the same equation of motion whether we are getting it or not.

So, that is what we are trying to see as lecture 11 part 2. Once we obtain you have already obtained these equations right. So, then we can rewrite in the matrix and vector form. So, that is what we are going to see in the lecture 11 part 2.

In the lecture 11 part 1 you can actually like see what we have seen what is robot dynamics, why it is so, why it is so important in the mobile robot community, how we can obtain with one of the popular method call Lagrange Euler. So, with that we will see you in the lecture 11 part 2 until then bye.

Thank you.