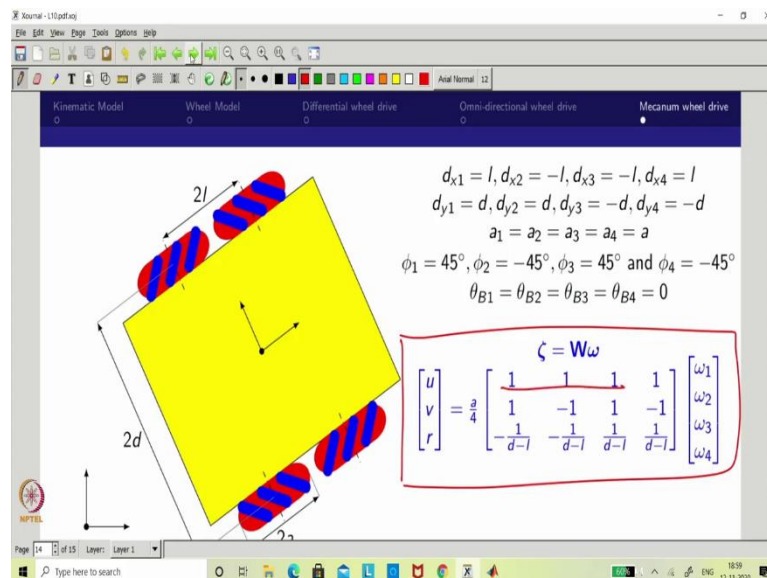


Wheeled Mobile Robots
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Indian Institute of Technology, Palakkad

Lecture – 10
Kinematic Simulation of Wheeled Mobile Robots Part 3

Welcome back to the lecture 10 part 3, where we are going to talk about the mecanum wheel drive. So, already I gave the introduction in the last part. So, now, we will actually like take the mecanum wheel drive and we will try to address first of all. So, what we wanted to do this simulation.

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So, for that we will take this equation what we have derived in the you can say lecture what you called 8. So, this is what we have derived in the lecture 8. So, this particular equation we will use it even the small glimpse I have given in the part 2. So, right now we will see like what our interest is.

So, we will actually like try to see if I want to move this vehicle in forward what supposed to be ω_1 to ω_4 . If I want to move this vehicle in lateral, what I suppose to do?

If I want to actually like rotate the vehicle what I suppose to do? So, for that, what I am trying to do? I am trying to take one blank page so I can actually like rewrite this equation.

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$u = \frac{a}{4} [\omega_1 + \omega_2 + \omega_3 + \omega_4] \Rightarrow \frac{a}{4} (\omega \times 4) = a\omega$
 $v = \frac{a}{4} [\omega_1 - \omega_2 + \omega_3 - \omega_4] = (-\omega) = 0$
 $r = \frac{a}{4} \left[\frac{-1}{d-l} \omega_1 - \frac{1}{d-l} \omega_2 + \frac{1}{d-l} \omega_3 + \frac{1}{d-l} \omega_4 \right]$

Case 1 $v=0$ $r=0$ $\omega_1 = \omega_2 = \omega_3 = \omega_4 = \omega$
 $\omega_2 = -\omega$ $\omega_4 = -\omega$ $r=0$

So, in the sense what I am trying to do is actually like you can see. So, if I write u in the form of $\omega_1 \omega_2 \omega_3 \omega_4$. This would be a by 4 in the sense $\omega_1 \omega_2 \omega_3 + \omega_4$ right.

So, then the other equation v is actually like a by 4 where $\omega_1 - \omega_2 + \omega_3 - \omega_4$. So, you can actually like cross check whether we have written right. So, this is the case right, 1 - 1, 1 - 1 right, that is what I am rewriting here.

So, similarly what I am trying to write the r which is I can write a by 4 which is actually like you can see here. So, what it is then it is actually like $\frac{-1}{d-1} \omega_1$, then what you can see that $\frac{-1}{d-1} \omega_2$, you can actually like cross check so this is what we have derived.

So, then what we have taken? I think I am going inclined so $\frac{1}{d-1} \omega_3$ then, $\frac{1}{d-1} \omega_4$. So, this is the 3 equation which we derived based on what you call this w into ω matrix.

But, what right now I am interested; I am interested is if I want to move only forward direction what would be the scenario? If I want to move in only you call lateral direction, what would be the scenario? Or I want to rotate clockwise or sorry anti clockwise or I want to rotate clockwise what are the options? Let us see the first one.

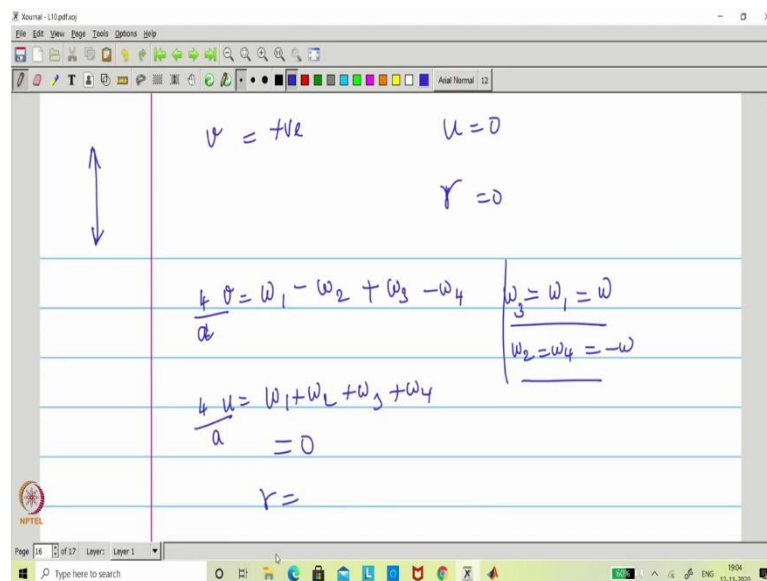
So, where u would be some value, but these two supposed to be 0. So, I am just using in a different colour. So, this supposed to be 0 for case 1. So, the case 1 what I am trying to see? So, the $v = 0$ and $r = 0$.

Then what one can actually like see if I actually like take ω_1 2 you can say all just for understanding I am taking all are actually like ω . Then, what you can see the u would be some positive which is actually like a by $4 \times 4\omega$ so; obviously, it is a ω right. So, this is what you obtain as a_u .

But when you think about v , so what would happen? So, ω_1 and ω_2 will go in the sense you can see $\omega - \omega + \omega - \omega$ in the sense what you will get 0 right. In the same sense you apply in the r equation.

So, this would be like positive say these two would be $-\omega$ and these two would be $+\omega$ so you will get $r = 0$. This is not a difficult thing where, ω_1 ω_2 and ω_3 and ω_4 all are positive and which are actually like same value then you will move in a forward right that is not a issue.

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Now, the issue is actually like if I want to actually like have something else. So, what; that means? So, I am actually like interested to see whether the vehicle is go either upward or downward direction. Now, we will take as upward direction. So, the v supposed to be positive where u would be 0 and r would be 0. You recall the equation what we have actually like taken. So, what the equation we have taken? So, we have taken these equation right.

So, in that case; so, this supposed to be positive means what you need to know. So, you

can actually like take 2 combination where, I am just rewriting that equation. So, v is actually like ω ; I am just taking a by 4 the other way around in the sense this is a. So, this is not the case, what I am actually like seeing is.

So, this is what the case right. So, now there are several choices right. So, where ω_1 and ω_3 are actually like a positive in the sense you can see like they are positive and ω_2 and ω_4 if I actually like take the other direction, in the sense I am taking all are actually like equal, but $\omega_1 = \omega = \omega_3$.

So, ω_2 and ω_4 , I am taking $-\omega$. Then you can see like it is happening right in the sense u also like you see. So, I am saying this is 4 by a so this is $\omega_1 + \omega_2 + \omega_3 + \omega_4$ this is also like fulfilling right this becomes 0 right. So, that is actually like straight forward you will get actually like this become 0. So, in the sense, but what happened to r equation you can actually like recall here.

If I put ω_1 is actually like ω ; ω_3 is actually like you can say ω whereas, $\omega_2 - \omega$ and ω_4 is actually like $-\omega$. So, in that sense if I see that way. So, what I will get these two would get cancelled right. So, this would be $-\omega$ right. So, in the sense this become plus and this would be minus ω and these two would be again getting cancelled.

So, in the sense what one can see from this. So, if I actually like take this configuration ok, I will actually like say the vehicle move upward or downward. Now, one wanted to have a rotation about its own clockwise or anti clockwise you look at this. So, you look at this what you wanted? You wanted all in same direction right. So, in the sense ω_3 and ω_4 , I put actually like $-\omega$ these two are actually like $+\omega$.

What you can see this would be positive in the sense whole negative in the sense it would be rotating clockwise, but in the other case you can see like ω_3 and ω_4 are actually like you can say is same.

So, this will get cancel this also would cancel and this is just ω_1 and ω_4 all are simple addition. Anyhow two are minus and two are positive, in the sense you will get it right. So, now, you want the other way around you want actually like rotate the you can see anti clockwise direction. So, you put you can say ω_1 and ω_2 are actually like $-\omega$ and ω_3 and ω_4 are positive.

So, these are the cases, now we are trying to simulate in the MATLAB ok. So, you can actually like try to see that. So, I am just moving to the MATLAB window ok.

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```

28     0, 0, 1];
29
30     %% inputs
31     omega_1 = 0.5; % left wheel angular velocity
32     omega_2 = 0.5; % right wheel angular velocity
33     omega_3 = 0.5;
34     omega_4 = 0.5;
35     omega = [omega_1; omega_2; omega_3; omega_4];
36
37     %% Wheel configuration matrix
38
39     W = [a/2, a/2;
40          0, 0;
41          -a/(2*d), a/(2*d)];
42
43     % velocity input commands
44     zeta(:,i) = W*omega;

```

So, now what we are trying to see? So, we are trying to see as, I will just use it a new file just for our benefit. So, I am just writing this as mecanum wheel.

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```

39     % velocity input commands
40     zeta(:,i) = W*omega;
41
42     % Time derivative of generalized coordinates
43     eta_dot(:,i) = J_psi * zeta(:,i);
44
45     %% Position propagation using Euler method

```

So, now mecanum wheel would be having $4\omega_s$, so ω_3 and ω_4 right. So, now obviously, you have to see the left right and all so, I am not writing that way just for my understanding I am writing ω_3 equal to point so 0.5. So, properly I can take all the things

are same at the initial ok.

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```
Editor - C:\Users\yyp\Desktop\MPTEL\MAR\kinematic_model_robot_w_mma.m*
34 omega = [omega_1;omega_2;omega_3;omega_4];
35 %% Wheel configuration matrix
36
37 W = [a/2,a/2;
38      0,0;
39      -a/(2*d), a/(2*d)];
40 % % velocity input commands
41 zeta(:,i) = W*omega;
42
43 % Time derivative of generalized coordinates
44 eta_dot(:,i) = J_psi * zeta(:,i);
45
46 %% Position propagation using Euler method
47 eta(:,i+1) = eta(:,i) + dt * eta_dot(:,i); % state update
48 % (Generalized coordinates)
```

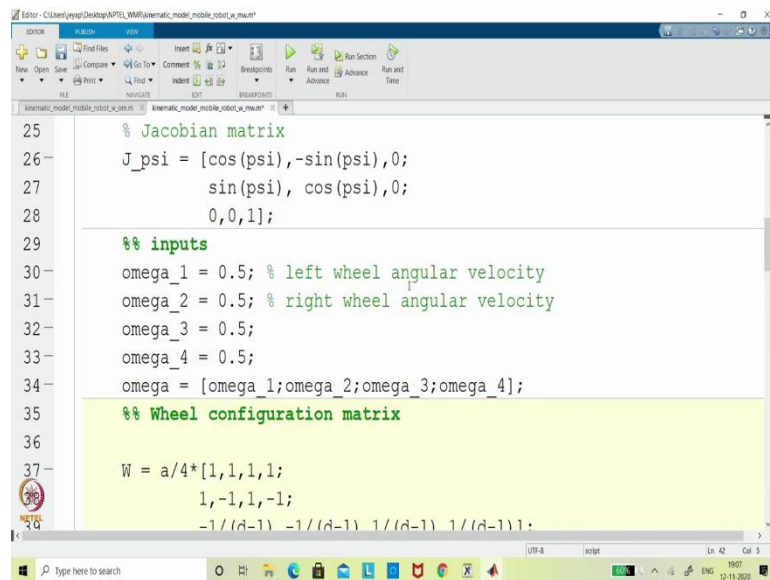
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```
Editor - C:\Users\yyp\Desktop\MPTEL\MAR\kinematic_model_robot_w_mma.m*
34 omega = [omega_1;omega_2;omega_3;omega_4];
35 %% Wheel configuration matrix
36
37 W = a/4*[1,1,1,1;
38         1,-1,1,-1;
39         -1/(d-1),-1/(d-1),1/(d-1),1/(d-1)];
40 % % velocity input commands
41 zeta(:,i) = W*omega;
42
43 % Time derivative of generalized coordinates
44 eta_dot(:,i) = J_psi * zeta(:,i);
45
46 %% Position propagation using Euler method
47 eta(:,i+1) = eta(:,i) + dt * eta_dot(:,i); % state update
48 % (Generalized coordinates)
```

So, now what would be the W? W I can rewrite here. So, what would be W? This is a by 4 right. So, this one is actually like 1 comma 1 comma 1 comma 1 that is what you got it and the other one is actually like 1 , -1; you can just recall so I will just to show you after typing this. Then what you got it? This is $\frac{-1}{d-1}$ right. So, the similar sense I will actually like take it so d - 1. So, you can actually like see the purpose right so that is what the whole idea here.

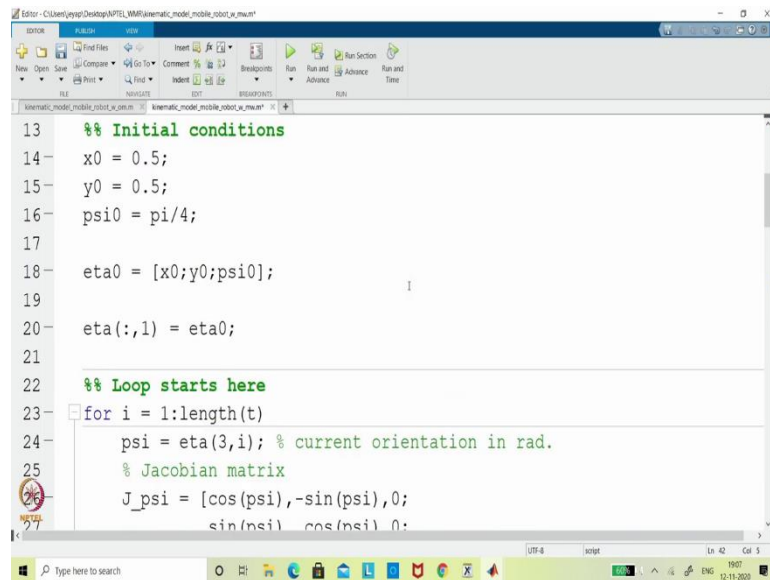
So, now even you change the you can say the mecanum wheel configuration so you can actually like rewrite this equation and come back right. So, that is what. For example, now we have taken diagonally both you can say are in the same axis right. So, I hope; I have seen there is a multiplication missing ok. So, now, this is done.

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```
25 % Jacobian matrix
26 J_psi = [cos(psi), -sin(psi), 0;
27          sin(psi), cos(psi), 0;
28          0, 0, 1];
29
30 %% inputs
31 omega_1 = 0.5; % left wheel angular velocity
32 omega_2 = 0.5; % right wheel angular velocity
33 omega_3 = 0.5;
34 omega_4 = 0.5;
35 omega = [omega_1; omega_2; omega_3; omega_4];
36
37 %% Wheel configuration matrix
38
39 W = a/4*[1, 1, 1, 1;
40         1, -1, 1, -1;
41         -1/(d-1), -1/(d-1), 1/(d-1), 1/(d-1)];
```

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```
13 %% Initial conditions
14 x0 = 0.5;
15 y0 = 0.5;
16 psi0 = pi/4;
17
18 eta0 = [x0; y0; psi0];
19
20 eta(:, 1) = eta0;
21
22 %% Loop starts here
23 for i = 1:length(t)
24     psi = eta(3, i); % current orientation in rad.
25     % Jacobian matrix
26     J_psi = [cos(psi), -sin(psi), 0;
27             sin(psi), cos(psi), 0;
```

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```
Editor - C:\Users\jyng\Desktop\MPTEL\MAR\kinematic_model_mobile_robot_u_msim.m*
25 % Jacobian matrix
26 J_psi = [cos(psi), -sin(psi), 0;
27          sin(psi), cos(psi), 0;
28          0, 0, 1];
29
30 %% inputs
31 omega_1 = 0.5; % left wheel angular velocity
32 omega_2 = 0.5; % right wheel angular velocity
33 omega_3 = 0.5;
34 omega_4 = 0.5;
35 omega = [omega_1; omega_2; omega_3; omega_4];
36
37 %% Wheel configuration matrix
38
39 W = a/4*[1, 1, 1, 1;
40         1, -1, 1, -1;
41         -1/(d-1), -1/(d-1), 1/(d-1), 1/(d-1)];
```

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```
Editor - C:\Users\jyng\Desktop\MPTEL\MAR\kinematic_model_mobile_robot_u_msim.m*
4 %% Simulation parameters
5 dt = 0.1; % Step size
6 ts = 100; % Simulation time
7 t = 0:dt:ts; % Time span
8
9 %% Vehicle (mobile robot) parameters (physical)
10 a = 0.2; % radius of the wheel (fixed)
11 d = 0.5; % distance between wheel frame to vehicle frame (along y-axis)
12 l^1 = 0.3;
13
14 %% Initial conditions
15 x0 = 0.5;
16 y0 = 0.5;
17 psi0 = pi/4;
18 eta0 = [x0; y0; psi0];
```

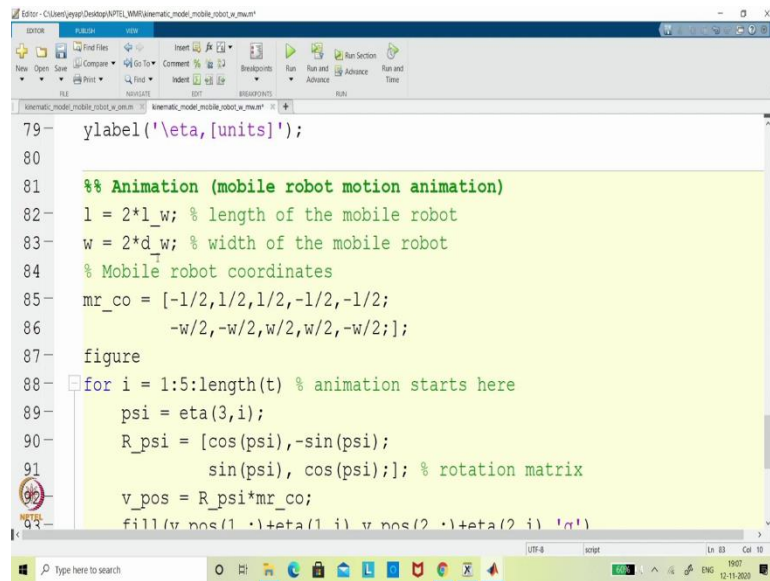

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```
Editor - C:\Users\jyng\Desktop\MPTEL\MAR\kinematic_model_mobile_robot_u_msim.m*
34     omega = [omega_1;omega_2;omega_3;omega_4];
35     %% Wheel configuration matrix
36
37     W = a/4*[1,1,1,1;
38             1,-1,1,-1;
39             -1/(d_w-1_w),-1/(d_w-1_w),1/(d_w-1_w),1/(d_w-1_w)];
40     % velocity input commands
41     zeta(:,i) = W*omega;
42
43     % Time derivative of generalized coordinates
44     eta_dot(:,i) = J_psi * zeta(:,i);
45
46     %% Position propagation using Euler method
47     eta(:,i+1) = eta(:,i) + dt * eta_dot(:,i); % state update
48     % (Generalized coordinates)
```

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```
Editor - C:\Users\jyng\Desktop\MPTEL\MAR\kinematic_model_mobile_robot_u_msim.m*
52     % set(gca,'fontsize',24)
53     % xlabel('t,[s]');
54     % ylabel('x,[m]');
55     %
56     % figure
57     % plot(t, eta(2,1:i),'b-');
58     % set(gca,'fontsize',24)
59     % xlabel('t,[s]');
60     % ylabel('y_t [m]');
61     %
62     % figure
63     % plot(t, eta(2,1:i),'r-');
64     %
65     % figure
66     % plot(t, eta(3,1:i),'g-');
```

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```
79 ylabel('\eta, [units]');
80
81 %% Animation (mobile robot motion animation)
82 l = 2*d_w; % length of the mobile robot
83 w = 2*d_w; % width of the mobile robot
84 % Mobile robot coordinates
85 mr_co = [-1/2, 1/2, 1/2, -1/2, -1/2;
86         -w/2, -w/2, w/2, w/2, -w/2];
87 figure
88 for i = 1:5:length(t) % animation starts here
89     psi = eta(3,i);
90     R_psi = [cos(psi), -sin(psi);
91            sin(psi), cos(psi)]; % rotation matrix
92     v_pos = R_psi*mr_co;
93     fill(v_pos(1,:) + eta(1,i), v_pos(2,:) + eta(2,i), 'g')
```

So, now if I actually like run this it definitely work, but I need to actually like change the parameter here because here d has given, but the l is not given I am saying that l is probably 0.3, in the sense here I will write just for benefit, I am writing this is 2 d and this is l is actually like, this is $2 \times l$ you can say v ok. This I will write as I will just for consistence then I have to write it everything as same way.

You see this animation part also like we have used l so I do not want to change. So, that it would be easy for you. So, now, this is what we derived in the lecture 8 and this is what we discussed even in the part 2.

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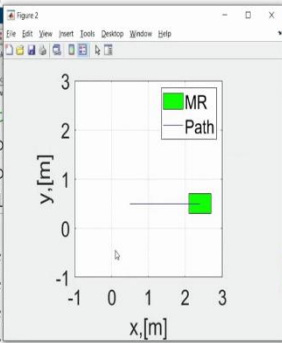
```
Editor - C:\Users\jyng\Desktop\MPEL\MPEL\kinematic_model_mobile_robot_w_mma.m
kinematic_model_mobile_robot_w_mma.m
4 %% Simulation parameters
5 dt = 0.1; % Step size
6 ts = 100; % Simulation time
7 t = 0:dt:ts; % Time span
8
9 %% Vehicle (mobile robot) parameters (physical)
10 a = 0.2; % radius of the wheel (fixed)
11 d_w = 0.2; % distance between wheel frame to vehicle frame (along y-axi
12 l_w = 0.3;
13 %% Initial conditions
14 x0 = 0.5;
15 y0 = 0.5;
16 psi0 = pi/4;
17 eta0 = [x0;y0;psi0];
```

So, now I am actually like redoing just once. So, I am just taking this as 0.4 is the width and 0.6 is the length just assuming it ok. So, now, if I actually like run this so it would actually like go in a $\pi/4$ direction.

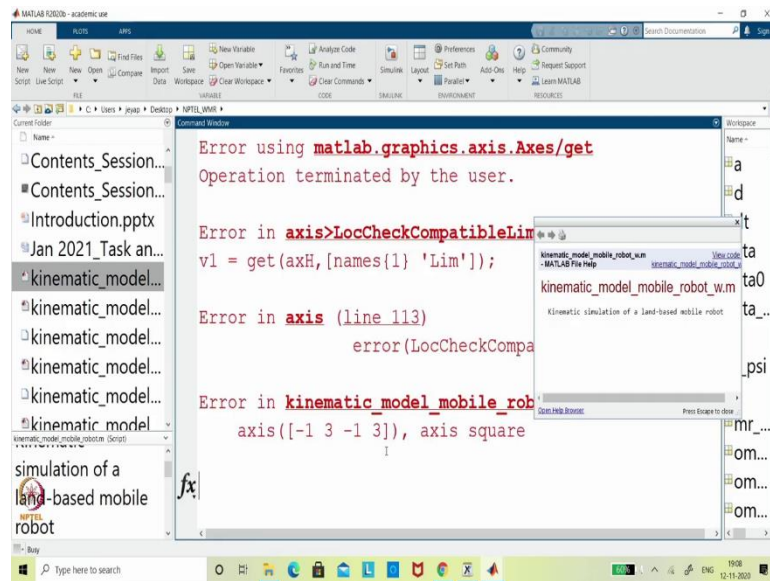
So, just to show that it is going forward just for your understanding I am putting that 0. Now, all 4 are actually like same speed. So, then what we have seen in the you can say discussion this if I give all 4 are same speed and same direction then that would go forward right you will see whether that is happening, it is going right.

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```
Editor - C:\Users\jyng\Desktop\MPEL\MPEL\kinematic_model_mobile_robot_w_mma.m
kinematic_model_mobile_robot_w_mma.m
25 %% Jacobian matrix
26 J_psi = [cos(psi);
27          sin(psi);
28          0,0,1];
29 %% inputs
30 omega_1 = 0.5;
31 omega_2 = 0.5;
32 omega_3 = 0.5;
33 omega_4 = 0.5;
34 omega = [omega_1;omega_2;omega_3;omega_4];
35 %% Wheel configuration matrix
36
37 W = a/4*[1,1,1,1;
38          1,-1,1,-1;
39          -1/(d_w-l_w) -1/(d_w-l_w) 1/(d_w-l_w) 1/(d_w-l_w)];
```

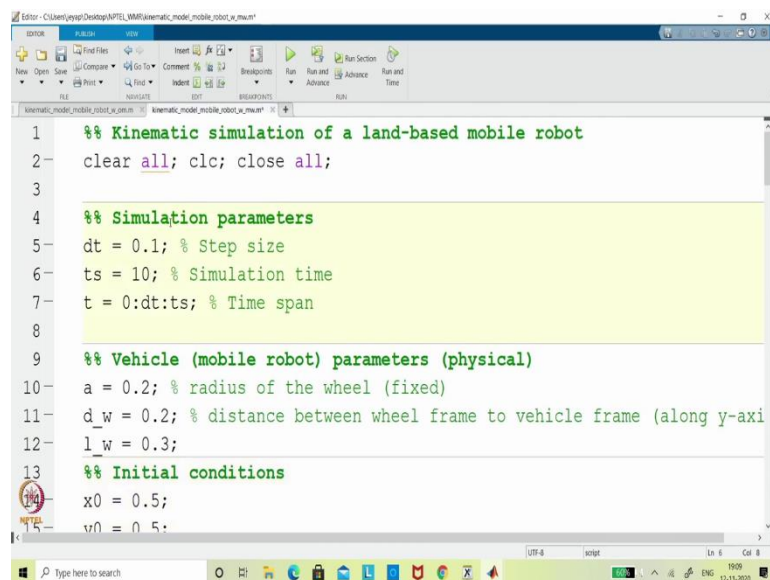


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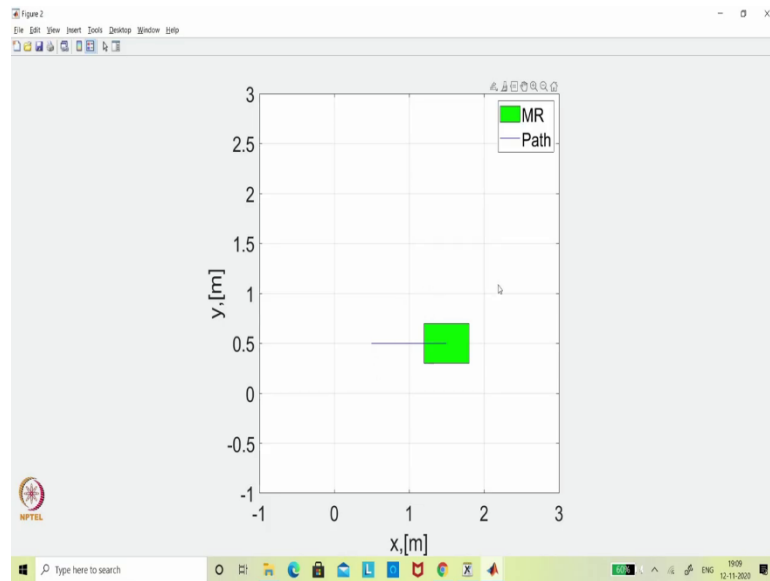
So, this is actually like very good. So, you are actually like achieved. So, and it keep on going because we have given the time is actually like 100 sorry 100 second so that is why it is keep on going I will just reduce it.

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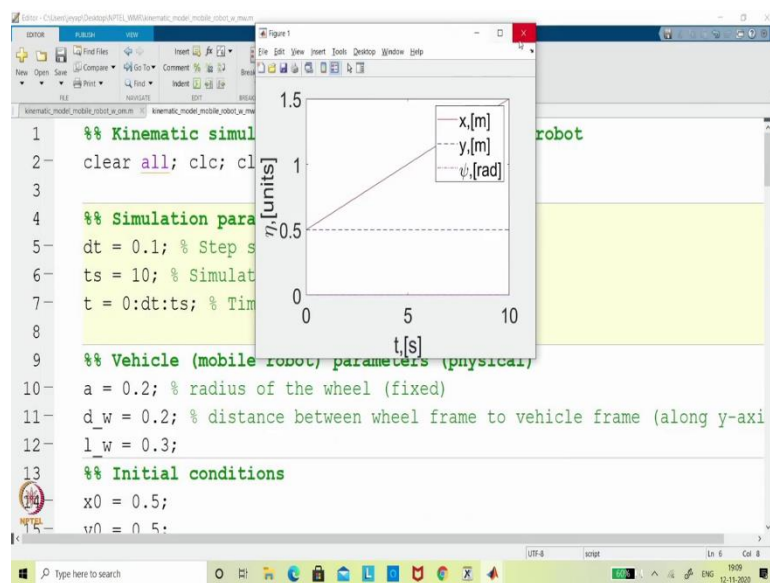


So, now I just try to show this again my resolution still not changed.

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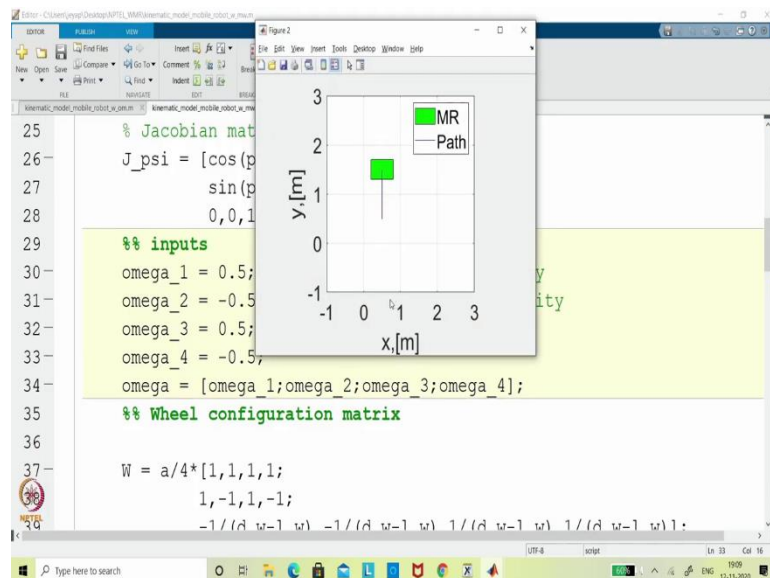
So, you can see that this is a go vehicle going forward right. So, now, what we have seen if vehicle wanted to go in you can say lateral direction the diagonal wheel supposed to have same speed, but opposite direction right.

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```
Editor - C:\Users\jyng\Desktop\MPEL\MMR\kinematic_model_mobile_robot_w_msim
kinematic_model_mobile_robot_w_msim.m
25 % Jacobian matrix
26 J_psi = [cos(psi), -sin(psi), 0;
27          sin(psi), cos(psi), 0;
28          0, 0, 1];
29 %% inputs
30 omega_1 = 0.5; % left wheel angular velocity
31 omega_2 = -0.5; % right wheel angular velocity
32 omega_3 = 0.5;
33 omega_4 = -0.5;
34 omega = [omega_1; omega_2; omega_3; omega_4];
35 %% Wheel configuration matrix
36
37 W = a/4*[1,1,1,1;
38         1,-1,1,-1;
39         -1/(d*w-1*w) -1/(d*w-1*w) 1/(d*w-1*w) 1/(d*w-1*w)];
```

So, in the sense I am putting it these two are actually like opposite. So, now, what one can see this can actually like give you can say lateral direction that is what you have seen earlier; we will see whether that is what you obtained right.

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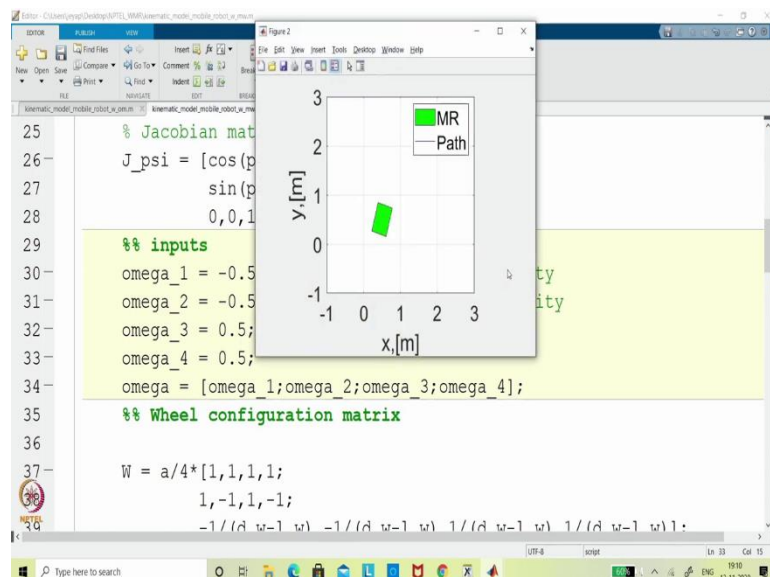


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```
Editor - C:\Users\jyng\Desktop\MATLAB\kinematic_model_robot_u_msim.m
25 % Jacobian matrix
26 J_psi = [cos(psi), -sin(psi), 0;
27          sin(psi), cos(psi), 0;
28          0, 0, 1];
29
30 %% inputs
31 omega_1 = -0.5; % left wheel angular velocity
32 omega_2 = -0.5; % right wheel angular velocity
33 omega_3 = 0.5;
34 omega_4 = 0.5;
35 omega = [omega_1; omega_2; omega_3; omega_4];
36
37 %% Wheel configuration matrix
38
39 W = a/4*[1,1,1,1;
40         1,-1,1,-1;
41         -1/(d*w-1*w) -1/(d*w-1*w) 1/(d*w-1*w) 1/(d*w-1*w)];
```

So, now if I actually like change the direction of these 2 wheels so this I put plus and this I put minus so that would go downward right. So, now, you can see like this also done. So, now, I want to have a clockwise rotation. So, then these two are actually like negative and these two are positive right so that is what going to happen. So, then it would be anti clockwise rotation I hope; we will see. No, its a clockwise rotation ok.

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So, that we can actually like cross check, I think the here only we were discussed right. So, you can see like ω_1 . So, ω_1 and ω_3 are same, but ω_2 and ω_4 in the opposite direction

that would give lateral, but in the other way around if you take it so this will actually like give clockwise and anti clockwise.

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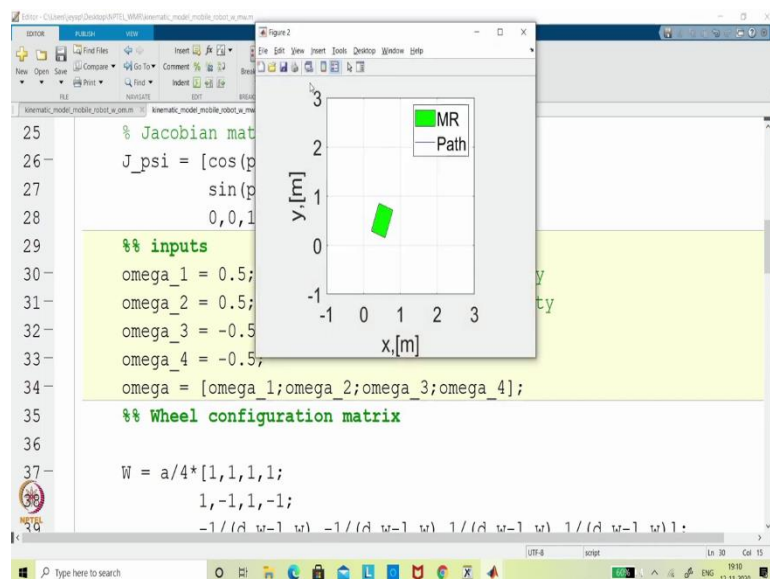
```

25 % Jacobian matrix
26 J_psi = [cos(psi), -sin(psi), 0;
27          sin(psi),  cos(psi), 0;
28          0, 0, 1];
29 %% inputs
30 omega_1 = 0.5; % left wheel angular velocity
31 omega_2 = 0.5; % right wheel angular velocity
32 omega_3 = -0.5;
33 omega_4 = -0.5;
34 omega = [omega_1; omega_2; omega_3; omega_4];
35 %% Wheel configuration matrix
36
37 W = a/4*[1, 1, 1, 1;
38         1, -1, 1, -1;
39         -1/(d*w-1*w) -1/(d*w-1*w) 1/(d*w-1*w) 1/(d*w-1*w)];

```

So, now this is actually like what we have done as a clockwise, you want to see even anti clockwise you can actually like run it. So, you can actually like run this so now it will actually like rotate in the other opposite direction right.

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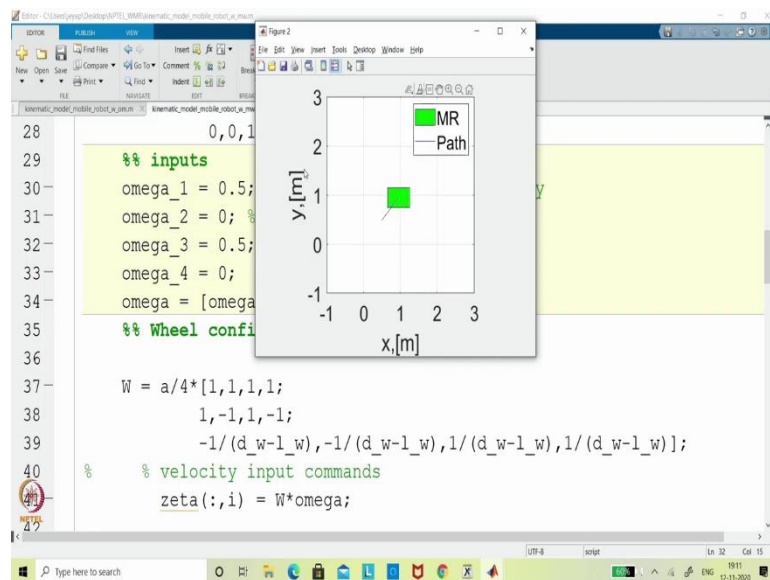
So, now it is actually like rotating in the right. So, like that you can actually like get it. So, now, what you wanted you want the diagonal movement then you can see what you

wanted. So, u and v only supposed to be there. So, you are what you call the r should be 0.

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```
Editor - C:\Users\yngsi\Desktop\MATLAB\MMR\kinematic_model_mobile_robot_w_mmu.m*
kinematic_model_mobile_robot_w_mmu.m
28     0, 0, 1];
29
30 %% inputs
31 omega_1 = 0.5; % left wheel angular velocity
32 omega_2 = 0; % right wheel angular velocity
33 omega_3 = 0.5;
34 omega_4 = 0;
35 omega = [omega_1; omega_2; omega_3; omega_4];
36
37 %% Wheel configuration matrix
38
39 W = a/4*[1, 1, 1, 1;
40         1, -1, 1, -1;
41         -1/(d_w-1_w), -1/(d_w-1_w), 1/(d_w-1_w), 1/(d_w-1_w)];
42 %
43 % velocity input commands
44 zeta(:, i) = W*omega;
```

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So, then what combination you can put it. So, you can actually like see even if I take only 3. So, wheel 1 and 3 is having you can say angular velocity and these are 0's so what one can actually like expect. So, it will actually like do something right I will just try that combination here, you can see right it is going diagonally. So, like that you can actually like play your variation and then actually like you can move ok.

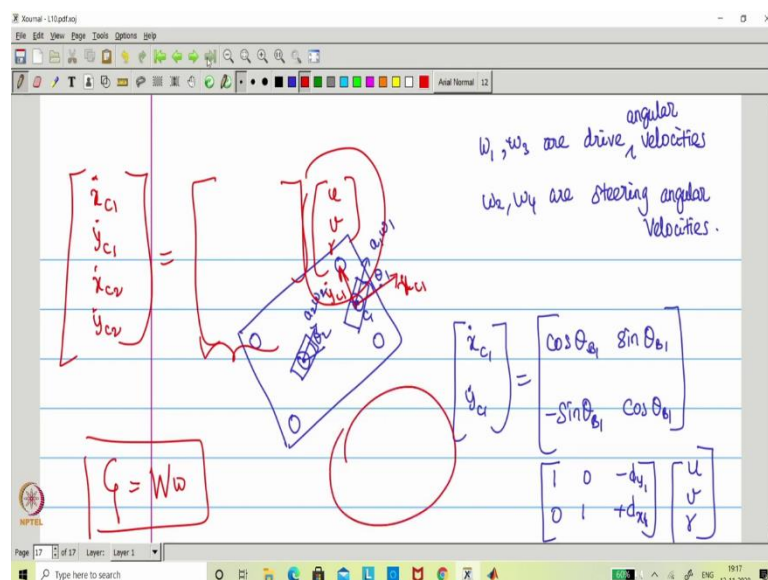
So, now we will actually like come back to the you can see the slide what we were actually like discussed. So, now, based on the you call this particular configuration you have seen like how the kinematic simulation can be benefited, because these 4 wheels are actually like what you can see.

So, these are actually like mecanum wheel, you cannot directly get the understanding. So, that is why if the kinematic simulation is exists then what one can actually like see it you can actually like make more beneficial by varying the parameter and do it. Right now we have taken l and d even you vary l and d you can get it.

Similarly, you change the wheel radius you will definitely get it and now you will just replace these wheels; I have interchanged this, what will happen? So, if I interchange this what will happen? Or I take all are actually like same directional wheel what will happen? These all combination you can do in a kinematic simulation. So, that is what the whole idea about this particular you can say the lecture 10.

So, now we will actually like move forward I said already. So, I will give 1 simple example where the special drive so I will just give that example and then we will actually like move forward. So, I will actually like see that how we can do it.

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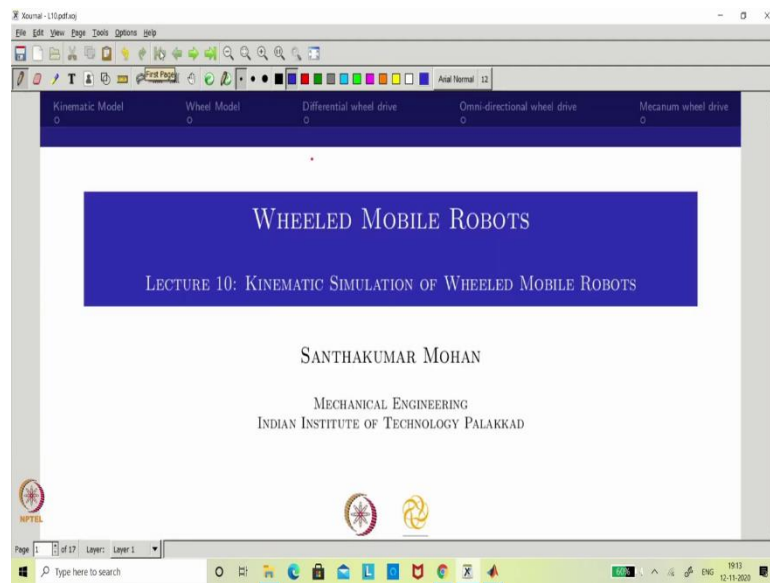


So, I am taking one of the simplest configuration. So, which is actually like I take as you can say a rectangular box as the vehicle and I am taking a 4 caster wheels which are

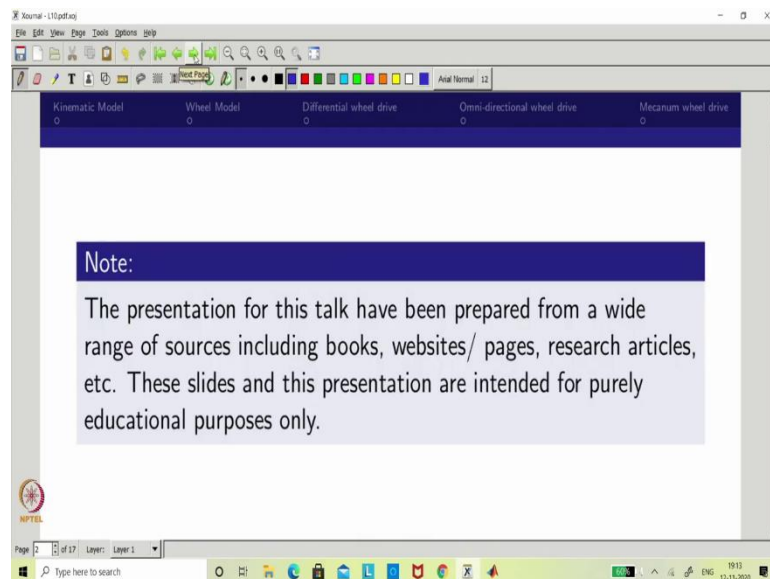
actually like spherical caster, but what I am trying to do there is a you can say powered and as well as you can say rotatable way ok. So, similarly I am taking this way.

So, there are 2 wheels which are actually like powered and as well as rotatable, in the sense you can see this θ_2 and this θ_1 are actually like playable; in the sense this wheel what it is giving a_1 and ω_1 this is giving a_2 and ω_2 , I can decompose into what you can see x and y. If that is the case what you can see. So, this particular model what I have shown that may not be workable.

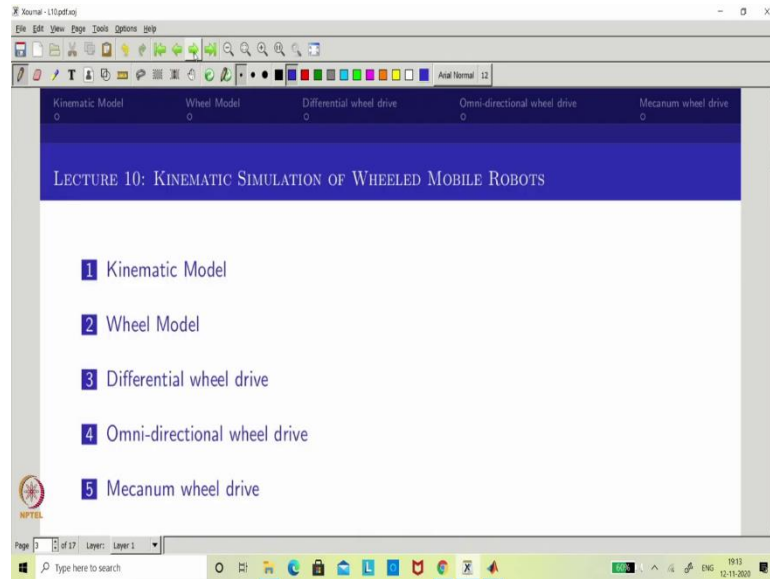
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The known mobile robot kinematic model, as:

$$\dot{\eta} = J(\psi)\zeta$$

Based on wheel configuration

$$\zeta = W\omega \quad (1)$$

$\dot{\eta}$ - is the vector of time derivatives of generalized coordinates.
 $J(\psi)$ - is the Jacobian matrix which maps the input velocity commands to derivatives of generalized coordinates.
 ζ - is the vector of velocity input commands.
 W - is the wheel input or configuration matrix.
 ω - is the vector of wheel angular velocities.

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The slide displays a kinematic model equation for a wheel:
$$\omega_i \begin{bmatrix} 1 \\ a_i \tan \phi_i \end{bmatrix} \begin{bmatrix} \cos \theta_{Bi} & \sin \theta_{Bi} \\ -\sin \theta_{Bi} & \cos \theta_{Bi} \end{bmatrix} \begin{bmatrix} 1 & 0 & -d_{yi} \\ 0 & 1 & d_{xi} \end{bmatrix} \begin{bmatrix} u \\ v \\ r \end{bmatrix} = \begin{bmatrix} \dot{x}_{ci} \\ \dot{y}_{ci} \end{bmatrix}$$

Variables defined on the slide:

- ω_i - Angular velocity of the i^{th} wheel.
- a_i - Radius of the i^{th} wheel.
- θ_{Bi} - Angle between the vehicle frame (B) to the wheel frame (c_i).
- d_{xi} and d_{yi} are the position coordinates of c_i with reference to B .
- ϕ_i - Angle between roller axis to the x_{ci} axis.
- u : Forward velocity of the mobile robot w.r.t. frame B .
- v : Lateral velocity of the mobile robot w.r.t. frame B .
- r : Angular velocity of the mobile robot w.r.t. frame B .

Handwritten notes on the slide include a circled diagram of the wheel geometry and a vector diagram for $\begin{bmatrix} \dot{x}_{ci} \\ \dot{y}_{ci} \end{bmatrix}$ with arrows pointing to ω_3 and ω_4 .

So, what that model we have seen? So, you can actually like recall this particular you can see the file. This model would not work why there is no passive roller further there is no need of actually like finding ω , because the ω_1 would be actually like drive, but ω_2 would be actually like steering angular velocity. In that sense this cannot become so similarly ω_3 is drive and ω_4 is actually like steering.

So, this combination I cannot incorporate in the generalised wheel model, but what I can do, I can bring it this as \dot{X}_{ci} and \dot{Y}_{ci} . So, then what you can see this is you actually like merged this. So, this equation would not come. So, now, this is equal to what you call \dot{X}_{ci} and \dot{Y}_{ci} right.

So, now this is what important. So, now, if I know this equation I can actually like do it. So, I will come back to that slide where. So, you can see that ω_1 and ω_2 , I will write ω_1 and ω_3 are drive angular velocity I call drive velocities ok.

So, that angular is actually like by heart, but still for understanding. So, ω_2 and ω_4 are steering angular velocities. So, now, we will actually like see further what the equation we have, I assume that this is c1. So, \dot{X}_{ci} and \dot{Y}_{ci} would be equal to; so $\cos \theta_B$ here $1 - \sin \theta_{B1}$. So, $\sin \theta_{B1}$ and \cos .

So, this is the first matrix and then you have actually like so $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$ ok. So, then what you

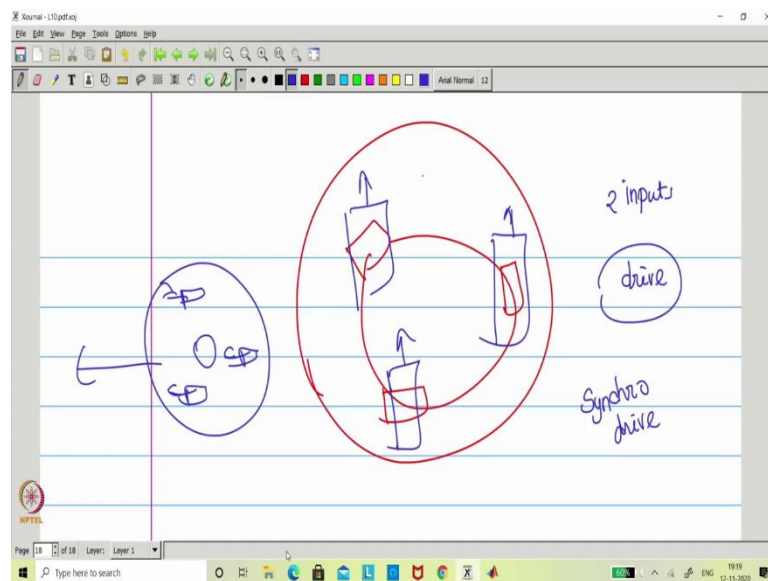
can see this is $1 - d_y$, I will actually like cross check because sometime the notation I might have used differently. So, this is the case, yes I have used rightly only. So, then what you can see this is $0 \ 1 + d_x$ right. So, these are actually like i^{th} one so this is 1 so this is the case.

So, now, we are actually like doing it, what you will get? So, the you can say projection velocity in the sense you can see this. So, I call \dot{X}_{ci} and this is actually like \dot{Y}_{ci} we will get. So, now, what you can see so I can make this equation as like this. So, \dot{X}_{c1} and \dot{Y}_{c1} \dot{X}_{c2} and \dot{Y}_{c2} I can write. So, now, this I can write as a bigger matrix into what you call $\begin{bmatrix} u \\ v \\ r \end{bmatrix}$. Now, you I take this inverse so that is equivalent to what you call W into ω equivalent not exact as a ξ .

So, the $\xi = W \times \omega$ that way I can write, but here it is not direct, but I can equate very close to that. So, this is what I know, and this is what I wanted, I can actually like use this equation right. So, this is what I was saying as actually like one of the special case.

So, what I can actually like do it in the next you can see, when we are doing a dynamic simulation I can show this kinematic simulation also altogether ok. So, then you can actually like understand where I will bring even the synchro drive motor. So, where the synchro drive in the sense you have a; I will just add it one more page.

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So, where there is a mobile robot there is actually like 3 wheels I assume that there are 3 wheels ok; all the 3 wheels are actually like connected together and whenever I am actually like moving in particular direction all the wheels are actually like go in to configure at that same way ok.

So, in the sense all the wheel are actually like aligned in that direction. So, in the sense this would be having 2 inputs ok; one input for the drive, so the other input to orient all the wheel in the same direction of orientation.

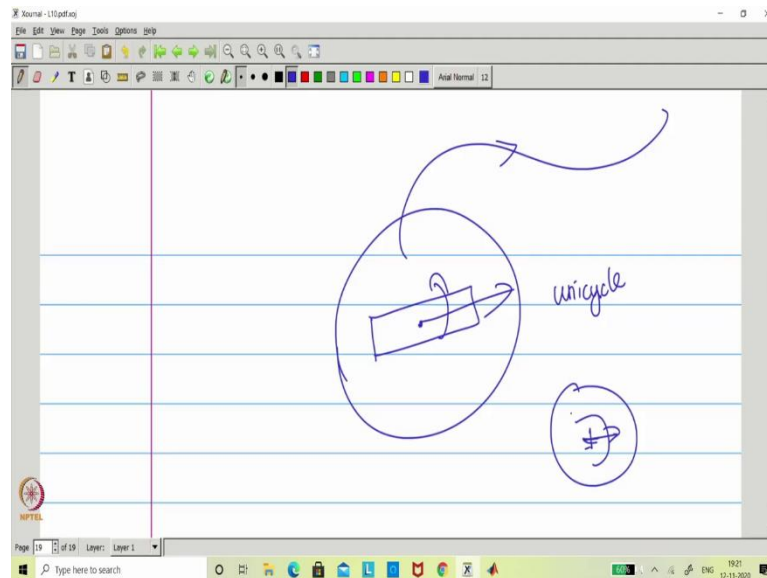
So, in the sense what happened this is very close to your you call the rotating chair. So, when you drag the chair you can see that by looking the bottom all the wheel would be aligned in the direction what you are applying the force. So, that is what we call synchro it is synchronised so synchro drive.

So, synchro drive actually like will come with minimum 3 wheel even it can go 4 5 6 and all, but this is one special case which I cannot actually like derive straightaway the model; because it is having a two things one is actually like the drive other one is actually like synchro drive, which is actually like try to synchronise all the three and if you look at it the same motor would be used for both that is the speciality on the synchro drive mobile robot.

So, at a time for example, now you want to take a lateral turn or you can say you want to move in a lateral direction, what the synchro drive will do? First it will actually like rotate all the wheel in that so the same motor will actually like rotate all the wheel after that what it will happen it will actually like come back so now, the drive will run. So, in the sense so, for example, you want to move in the lateral so the all the wheel will actually like align like this.

So, now the second part or second step you will give energy and then it will move forward, but there is only one motor that motor would have a switching. So, that is why I call it is one of the special case, which is what we call synchro drive.

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So, similar way what one additional special drive I can just add it here which is what you call unicycle; where only one wheel, but that wheel would be actually like can rotate and as well as you call move forward. Since, its in the one wheel drive so you can see like it can actually like rotate and move. It is very close to you call differential wheel, but in the sense differential wheel drive the other wheel will actually like give a infinite resistance, but it is steerable right. So, then it can actually like maneuver wherever you want. So, that is the idea so which you call unicycle.

So, you know bicycle, but this is a unicycle. So, unicycle is only 1 wheel and it would be maneuver wherever you want. You would have seen in the circuses and all where one of the you call you can see like clown. So, you would be like moving in a 1 wheel cycle and you will maneuver wherever you want right, he wants.

The same way what you have done in a mobile robot that particular thing what you call unicycle, but the unicycle usually come with a spherical wheel in order to give the you call 0 friction on the lateral direction.

So, which is nothing, but a spherical wheel where there is a 2 you can say power involved. So, one will actually like powered in the forward direction the other one is just to rotate the wheel. So, this is what for this particular lecture 10 where the part three also done.

So, what we have seen in the lecture 10 overall from part 1 part 2 part 3 all the kinematic simulation. Why this kinematic simulation is important? You can understand already you might be knowing. So, how the individual parameter will play in the overall performance? That is what you call simulating or analysing the system right.

Now, you take the wheel size as probably given in the catalogue, but you are not getting that particular velocity as the longitudinal velocity then you can go to other you can say wheel or you change your motor. In that sense you can see like you are coming into a design aspect. So, this is what important.

So, that is why this lecture 10 is actually like mean to put it here. So, now we will actually like move real side in the sense we will bring the inertia. So, far we did not brought the mass or you can say material of the vehicle. So, now we will bring the inertia; that means, we are still in the rigid body, but dynamics.

So, that is what the lecture 11 all about; you would be actually like covering all the dynamic aspect of the wheeled mobile robot in the sense land base. So, that is what we are going to cover in lecture 11. So, we will see more aspect in lecture 11 until then bye.

Thank you.