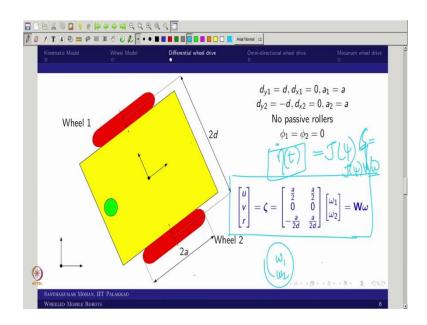
Wheeled Mobile Robots Prof. Santhakumar Mohan Department of Mechanical Engineering Indian Institute of Technology, Palakkad

Lecture – 10 Kinematic Simulation of Wheeled Mobile Robots Part 2

In the last part, what we have seen? In the same lecture 10, what we have seen is a differential wheel drive kinematic simulation with consideration of ω_1 and ω_2 . In this particular lecture a part 2, what we are trying to see the omni-directional wheel drive kinematic simulation where we will take one such configuration where the you can say three wheels are actually like orient with respect to 120° each.

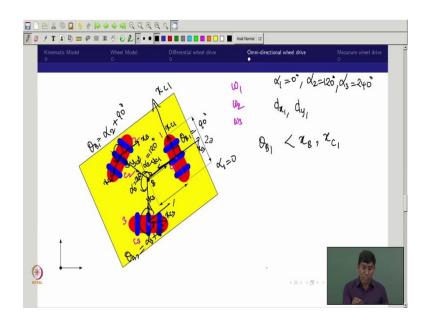
So then we can see like ω_1 , ω_2 , ω_3 comes. And on top of this omni-directional wheel having a passive roller, so definitely in the last lecture itself, I gave a hint that this can move in a lateral direction that we can see based on this particular kinematic simulation.

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So, I will actually like see the second example.

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So, which is actually like one of the phenomena which in the current trend people are bringing it, because most of the warehouse now they wanted autonomous vehicle either it is actually like automated guided or autonomous mobile robot. But these are actually like supposed to manoeuvre within the shop floor.

Within the shop floor, they do not want to actually like move as a conventional vehicle, because the floor is actually like smooth and actually like you can see engineered. So, even you can actually like allow the lateral motion.

But the same you call omni-directional wheel or mecanum wheel cannot be applied in the real environment. Why? The passive roller will not be giving the same performance if it is actually like non-smooth surface, so that is what the idea that is why the omnidirectional and the mecanum wheels are popular in the warehouses.

So, now, we will come back here. I just wanted to give little you can see background here. You can see like this is the point where we call C_1 , and this is the point what you call C_2 , and this is the point you call C_3 , in the sense this is the wheel 1, wheel 2, wheel 3. What we are trying to find; ω_1 , ω_2 , ω_3 equation.

So, now what one can actually see, you have to fix the frame, but what we usually take? Along the drive direction your X_{C1} ; then based on the right hand rule, you will take the Y_{C1} right. So, similarly you can see this is the X_{C2} . So, this is Y_{C2} , and this is what your

 X_{C3} ; and in that case, so this is what Y_{C3} . So, why am I actually like putting this? So, that you can bring a new variable called α . So, the α is actually like angle between your C₁ to the bay with respect to x-axis.

So, in this case, α_1 would be 0. Whereas, this one is actually like α_2 is exist. For simplicity, I am taking its identical in the sense symmetric about this point, so it is 120°. And this is another one this is actually like α_3 so which is 240°. So, in the sense, what I got it? $a_1 = 0^\circ$, α_2 is actually like 120 degree, and α 3 is actually like 240 degree.

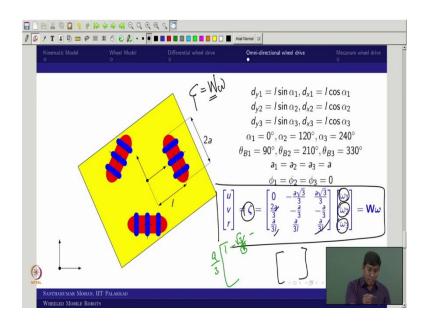
Why this is important? Because what I am trying to do? I am trying to find out what is d_{x1} , what is d_{y1} , similarly I will go d_{x3} to d_{y3} right. So, for finding that, I need these angles that is why I am actually like bringing it.

So, now, if you look at θ_{B1} , what is θ_{B1} ? The angle between you call x_{B2} , x_{C1} that is what θ , you can say B_1 . So, now, in that case, this is the x_B direction and this is what the y s you can say x_{C1} direction. So, what is the angle? So, θ_{B1} is actually like 90°. So, in this case, you can see, so this is what the case. So, you can actually like extend. So, I will just I erase this.

So, now what we have done this. So, now, you can see this is the x_B direction, and this is the angle so which is $\alpha_2 + 90$. So, similarly you can see here this is what your case. And you take always counterclockwise is positive in the sense $\alpha_3 + 90^\circ$ that is what θ_{B3} and this is θ_{B1} .

Because in the lecture what we have covered in lecture probably 8 which was actually like talking about example, I have directly given this θ_{B1} , θ_{B2} , θ_{B3} . So, you may have a confusion, so that is why I was actually like given this.

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So, now based on this what you have? So, you have obtained and the passive roller radius and as well as you can say the passive roller angle which is ϕ . So, what ϕ we have said? So, what is the slip direction? And slip direction with respect to y axis that is what ϕ_1 .

Or you can take so what you call the passive roller axis which is actually like angle between passive roller axis along with what you call x-axis. In this case omni-directional all are actually like aligned with x-axis in the sense the passive roller axis aligned with xaxis that is why we have taken ϕ_1 , ϕ_2 , ϕ_3 as 0.

So, if you do this, what you got it finally, the equation what you wanted. So, now, you can see that if you apply the 120° configuration, so you got the ξ in this form. So, now, what you can do? So, what we have did in the earlier case the same way, you can incorporate instead of what you have done earlier. So, this rectangular matrix you have substituted right. So, instead of that, you use this particular square matrix, and you do the same simulation.

So, I hope you can do this. Anyhow at the end, I will try to show something, but right now this is what the understanding. So, you know like what we have done. So, the $\xi = W \times \omega$. So, earlier case you have taken the W based on what you have obtained in the differential wheel this particular W I have given. So, now, you just replace, and you play this $\omega_1 \omega_2$ and ω_3 variation and you have actually like two variable a and 1. So, now you can actually like play accordingly, and then see what would be the required performance and all.

So, with that what one can see, like the second example also like done. So, I will move directly to the MATLAB after showing the third example, so then it would be easy. So, we will just you can say plug and play the W value.

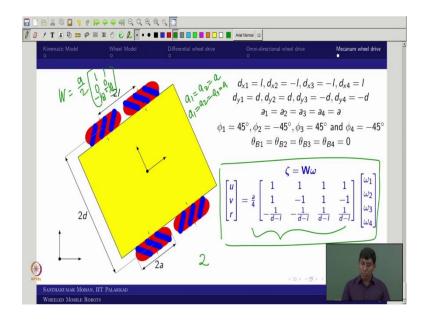
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So, now going to the third one. So, the third one you can see like this is a mecanum wheel drive. There are four wheels. So, there is actually like distance. So, the distance between two wheels in the y-axis, I call 2d; and the distance between two wheels along x-axis, I call 2L, and the wheel radius is 2a. So, further you know like the roller axis is like this and you are you can say x_{C1} is like this.

So, what would be this angle? This angle is actually like ϕ , and this is actually like counterclockwise so that is why ϕ 1 is positive. And here for simplicity, we have always taken as 45° mecanum angle in the sense ϕ 1 is 45. And you can look at this the wheel 1 and 3 are actually like similar in configuration in the sense ϕ_3 also like 45.

Whereas, if you look at this, the wheel you can say axis and the x_{C1} is actually like you can see it is non-counterclockwise; in the sense it is in clockwise, it is negative. So, that is why you can see ϕ_2 and ϕ_4 we have taken as - 45° in the lecture 8. Now, you got it right.

So, now, what we need? We need the distance. So, you can see like this is actually like symmetric. So, in the sense this is 1 and this is actually like - 1. So, this is actually like minus 1. So, similarly, this distance is actually like + d as per the you can say coordinate. So, this is, so this is actually like - d right.



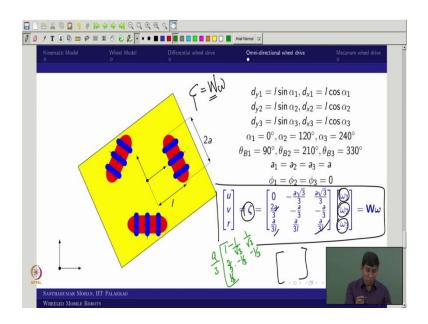
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So, now you can actually like get that is what we did in the lecture 8. So, these are the cases. So, now, if you substitute, what you will get? You will get this particular, you can say final model. So, now again you can actually like plug this, and actually like you can play.

So, now, one easily you can notice. So, when you have two wheels that too like identical shape in the sense, so the wheel radius is actually like a₁ and a₂ are a, what you got it? So, the W matrix is actually like something like a by 2 as a common factor right. So, I can rewrite that.

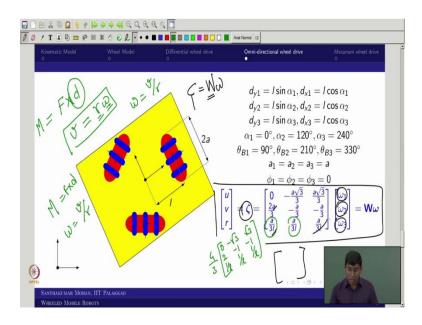
So, this is the common factor. Now, you assume that $a_1 = a_2 = a_3 = a$, so what you have obtained in the mecanum wheel to omni? So, omni you can see that the a by 3 is a common out. If you take it out, you can see that the a by 3 will common out, and you can see this is actually like $-\frac{\sqrt{3}}{3}$. And you can see this is actually like in the other way around I can right that is easy for me to get it.

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So, what that? This is $\frac{1}{\sqrt{3}}$, this is $\frac{1}{\sqrt{3}}$. So, this is actually like $\frac{2}{3}$, and this is $-\frac{1}{3}$, this is $-\frac{1}{3}$, and this is $\frac{1}{3}$, 3 has come right out, so that is why we are actually like interested.

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So, in that sense, so this is actually like straight forward, so this is 0, this is $-\sqrt{3}$, this is $\sqrt{3}$, and this is 2, and this is - 1, this is - 1, and this one would be $\frac{1}{L}$, and this is $\frac{1}{L}$, and this is also $\frac{1}{L}$. Why we are taking this kind of thing that is what I wanted you to explore?

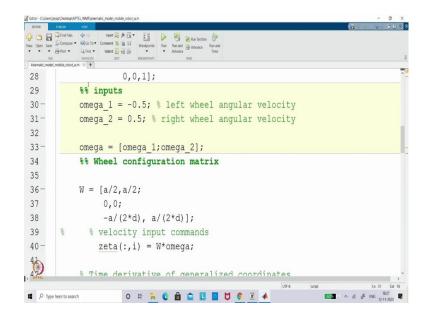
So, now I will give you a small hint. So, what the hint? If you have two powered wheel, so definitely the combination will actually like make it. So, when you write the force into distance, what that? That is moment right. So, there the distance would be multiply. Now, you recall what you written as a linear velocity?

The linear velocity you write $r\omega$; r is the radius, and ω is the angular velocity. In that sense, the linear velocity you can write it in this form. But right now this is the last row what it looked like? The last row would is actually like ω component, this is ω I can write as $\frac{\nu}{\omega}$.

So, now, you have to be very clear. If you talk about moment that would be force into distance. Whereas, the angular velocity that would be linear velocity divide by the distance or the radius, so that is why you can see that so this is coming as 1 or you can take it in the other way around.

So, d all are actually like in the denominator rather than the numerator. The same phenomena you can actually like see it even in the end. You can see that whatever the distance that is coming in the denominator. So, now, you got it why this a by 4, and a by 3, and a by 2 is coming, similarly why the last row is coming the distance is you can say something like a reciprocal or the distance is in the denominator that is what the whole idea.

Now, let us actually like move to the you call a MATLAB screen and we can actually like try to identify this particular case. So, for that, I am taking the mecanum wheel first, then we will actually like move forward. So, I will actually like move to the MATLAB screen. (Refer Slide Time: 11:53)



mecanumSo, I will just change it as a name just for my understanding. I just put this omni. So, I will put omni as o m. So, now, this would be having 3 ω right.

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28	0,0,1];	
29	%% inputs	
30-	<pre>omega_1 = -0.5; % left wheel angular velocity</pre>	
31-	<pre>omega_2 = 0.5; % right wheel angular velocity</pre>	
32	omega 3 = 0.5;	
33-	omega = [omega 1;omega 2;omega 3];	
34	<pre>%% Wheel configuration matrix</pre>	
35		
36-	W = [a/2, a/2;	
37	0,0;	
38	-a/(2*d), $a/(2*d)$];	
39	% velocity input commands	
40-	<pre>zeta(:,i) = W*omega;</pre>	
41		
	% Time derivative of generalized coordinates	
		>

So, now you would be writing ω_3 . So, I will just take it this is also 0.5.

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7 - 8	t = 0:dt:ts; % Time span	
9	<pre>%% Vehicle (mobile robot) parameters (physical)</pre>	
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11-	1 = 0.5; % distance between wheel frame to vehicle frame (along y-axis)
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13	<pre>%% Initial conditions</pre>	
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15-	y0 = 0.5;	
16-	psi0 = pi/4;	
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18-	eta0 = [x0;y0;psi0];	
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So, now this would be having two things. So, what two things? So, the distance is actually like between the wheel frame to the vehicle frame, now I call it as 1. So, now, this is what we have used.

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29	%% inputs	
30-	<pre>omega_1 = -0.5; % left wheel angular velocity</pre>	
31-	<pre>omega_2 = 0.5; % right wheel angular velocity</pre>	
32-	omega_3 = 0.5;	
33-	omega = [omega_1;omega_2;omega_3];	
34	%% Wheel configuration matrix	
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36-	W = [a/2, a/2;	
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42	% Time derivative of generalized coordinates		
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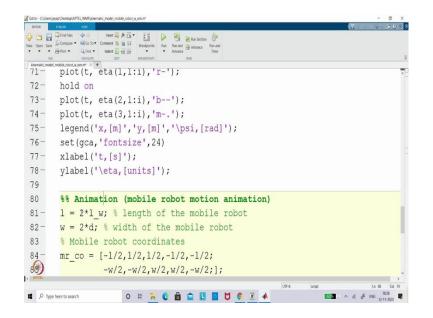
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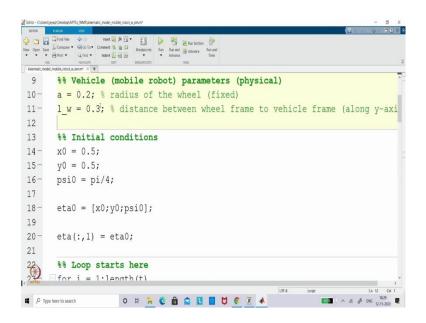
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So, in the sense, I call it here this I is actually like I will put it as two times I just for simply, here also I have given I right. So, I will put this is length or I put I. So, here that is fine. So, the same I you can take it as. There I will change it that is actually like easiest. So, I will just change it as that is wheel configuration.

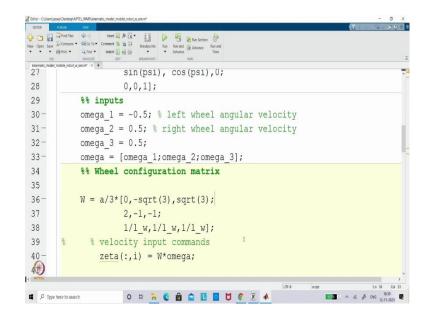
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So, now what I am trying to do here is Lw, so that would be probably 0.3. So, now, what we are trying to do? We are trying to incorporate this W; this W is no longer you can say 3×2 . This would be 3×3 . And if you look at it, what we have done, the first wheel

would be in the you can say vertically upward.

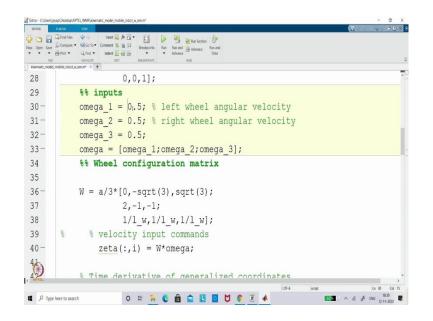
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So, in that sense, this would come. So, I will take it as a common out a by 3 as common out. So, this is 0 and $-\sqrt{3}$; it taking time, $\sqrt{3}$. So, then you have actually so I will just check what the equation we have use, so 2, then - 1, and - 1. So, I will just use it that so 2, - 1, - 1. The last one would be just $\frac{1}{1}$ Here I is I have written as I underscore w. So, this is what $\frac{1}{1}$ w.

So, now what we have done? We have actually like incorporated the wheel configuration matrix based on the omni-directional wheel. So, there are three omni-directional wheels which are actually like having 120° apart. So, where the first wheel is actually like exactly aligned with y-axis, but it is distance away from l, so that is what we have tried. And now you can actually like run it.

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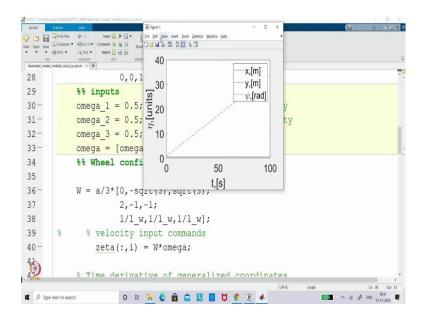


So, I will just give initially like what we did in the earlier case, we have taken all are equal right. So, now if you are looking in that way, if you put ω_1 , ω_2 , ω_3 , all are equal what one can expect? So, it will move forward right as per the differential wheel configuration. But if you look at in this configuration, it would not do that way.

Why? Because all the wheels are actually like aligned in some such a way that it is a circumference way, and you look at the configuration. So, you just look at in this configuration. So, the first wheel is actually like giving the velocity in y direction, but the second and third is giving you can say other way around.

In that sense, what you can see all three are actually like trying to give a tangential velocity in a you can say counterclockwise direction. So, in that sense, if you run this code, so what you can see? The vehicle is actually like rotating about its own that is what you can feel it. So, I will actually like try to see.

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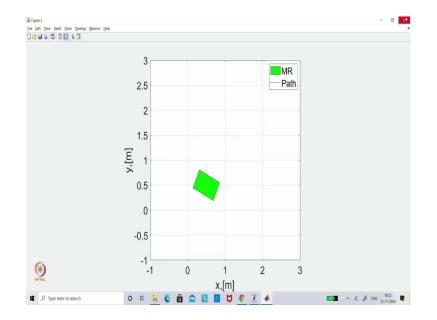


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So, I think, so the distance I made it. So, this is not 2d. This I am putting as 0.4, because the d we did not define right.

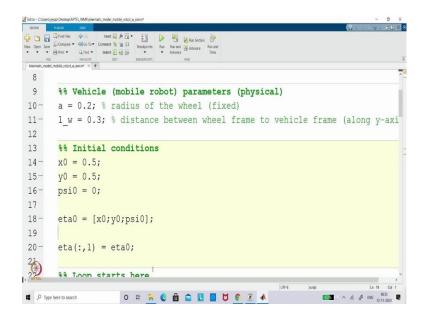
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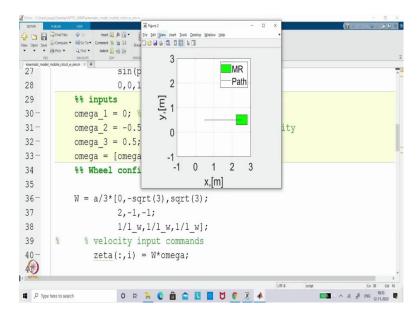
So, now you can see like it is much faster, because there are three wheels and it is actually like making it rotating. So, that is why you could not see any path because that is actually like making it in the same point. So, I am really sorry for the resolution. I am just tried, but it did not come up. So, now, we will actually like see it is keep on going because we have ran for 100 second.

So, but now you can actually like see if you want to move forward, so what one can expect by look this configuration? By looking this configuration, what one supposed to see? The first wheel whatever angular velocity I give that will try to move you can say lateral direction. So, you take it second and third opposite direction in the same velocity. What one can see, you can see that that is moving forward.

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So, just to understand that I just make it for that, so I am taking that I am taking it the configuration also like instead of moving it, I am just showing it otherwise all the time I am showing inclined way right. So, I am just changing.



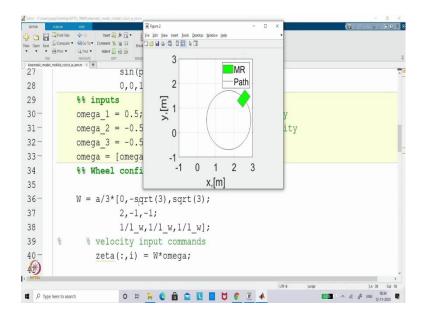
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So, now, this is so this is actually like opposite direction, and this is having a 0. So, now, in the sense ω_2 is actually like rotating in the sense the tangential velocity will be towards x, and this is also towards x. Whereas, the y component would be counteract, and then it go away that is what we can see.

So, now if you are actually like looking at, you can see right it is moving forward. So, now, you want to actually like move upward, then do not tell that only ω_1 I give and ω_2 and ω_3 , I give 0 then it would go like that, no, not like that. So, for that, what you have to do? So, both are actually like supposed to be same.

And now you actually like try to see, what would be the case? So, you take the picture then you can actually like understand. So, this is going like this, and this is actually like going like this. But what you want? You want actually go upward right. So, in that sense, you can see this is giving the velocity component y axis is downward, here also it is giving downward. So, then what you need to know? So, you actually like put it the other way around. So, both are actually like negative.

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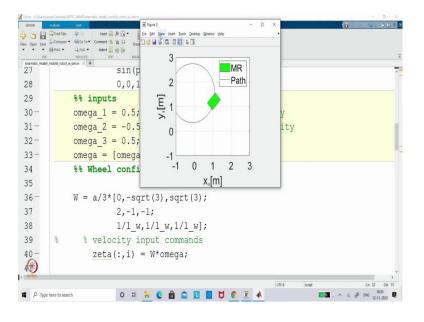
So, then what one can actually like see if I give something here also, so you can see some action, so you can actually like feel it I am just giving a random you can see right some rotation is happening. Why? So, you have to actually like see the square root that is why the wheel configuration matrix is required. You see randomly we have tried it is not working right.

So, now, you do in the mathematical form. What the mathematical form? I take the w, you can say case. So, what you want? You want only v right. v is actually like written as so $\frac{\alpha}{3} \times 2\omega_1 - \omega_2 - \omega_3$. Now, you just substituted only in the other way around right. So, in

the sense, what you have tried. So, these two are actually like equal to this, so that is what you did right.

So, in the sense it is actually like giving 4ω , but what you are looking at here you did not look at it, and here also you did not look at it. So, in the sense, it is supposed to be opposite direction, so that you can actually like make it. So, now you actually like try.

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So, I will just give this, and then we can actually try it. So, you can actually like see it. So, what you are trying to see, you are not getting it again right. So, why it is so? Because you have to have one thing, so for x-axis, you can actually like make it; for yaxis, you have to actually like get it in the other way round you take this equation. So, you want to make it u = 0. So, then this supposed to be opposite.

But when you are actually like putting it here it is actually like giving. But what happening here, the third equation? For example, I will take it in a PDF also so what happening these three? So, still there is a moment which is exist right, so that is why we are actually like unable to get this. So, if you want to do this, so we can actually like make it further. Similarly, we can do the mecanum wheel.

So, in the sense the lecture you can say 10 - part 3 would be addressing the you call kinematic simulation of mecanum wheel and as well as the one of the special case. So, with that the part 2 end here, where we talk about the omni-directional you can say

mobile robot kinematics simulation. So, we will see in part 3 where we would be seeing mecanum wheel and two-independent rotatable power wheel configuration. With that saying.

Thank you.