Computational Fluid Dynamics Using Finite Volume Method

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Lecture – 05

Review of governing equations: General scalar transport equation

Over the last few lectures, we have seen the derivation of conservation of mass, momentum and energy right. We have kind of finished that. So, today we are going to list down all the equations and see if we kind of can find a common thread between them ok.

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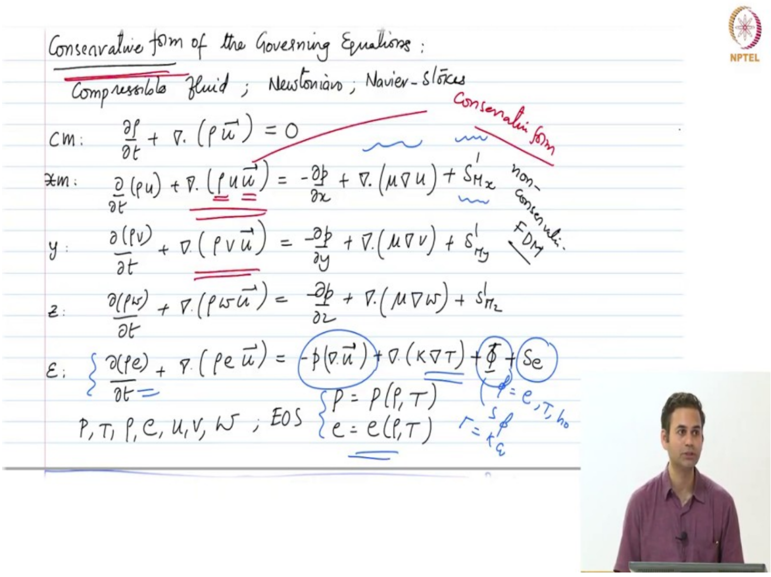

So, the conservative form of the governing equations, we will see what we mean by a conservative form little later. So, these are being written down for a compressible fluid with a Newtonian approximation ok. So, this is for a compressible flow with Newtonian fluid. So, we have this assumption. So, what does the, how does the mass conservation or continuity look like?

So, we have conservation of mass is or the continuity equation is what? $\frac{\partial p}{\partial t}+\nabla.(\rho \vec{u})=0$ 0 right that is our conservation of mass. So, let us call it cm and then our x momentum equation, if I expand the total derivative that is rho d u dt if I expand it into the local derivative and the convective term that would read like $\frac{\partial}{\partial t}(\rho u)++\nabla.(\rho u\vec{u})$ This is rho times d u dt equals on the right hand side we have minus partial p partial x plus. So, when I say x momentum equation we are actually writing down the Navier-Stokes equations ok

So, this see these are the Navier-Stokes equations ok. So, $\frac{\partial}{\partial t}(\rho u)++\nabla.(\rho u\vec{u})=\frac{-\partial p}{\partial x}+\nabla.(\mu\nabla u)+SMx$ yesterday, we have substituted the shear stresses in terms of the strain rates right and we have simplified the right hand side terms and that would read as what?, where we realize that this prime here, would be a source that comes from the body force plus a several terms that are gradients of velocity right or strain rates that are also absorbed into this ok.

In the case if this was incompressible then S Mx SM prime x would be same as S Mx. So, essentially whatever is the body force term that we get will only be there in that ok. So, that we have to kind of keep in mind about this source terms here ok. Now, what about the y component of the Navier-Stokes equation? So, the y component is $\frac{\partial(\rho v)}{\partial t}+\nabla.(\rho v\vec{u})=\frac{-\partial p}{\partial y}+\nabla.(\mu\nabla u)+SMy\cdot$

Similarly, we have the z component that is $\frac{\partial(\rho w)}{\partial t}+\nabla.(\rho w\vec{u})=\frac{-\partial p}{\partial z}+\nabla.(\mu\nabla w)+SMz$. Of course, now we have the energy equation as well. So, the energy equation is in terms of the internal energy that is dou by dou t of rho $\frac{\partial(\rho e)}{\partial t}+\nabla.(\rho e\vec{u})=-p(\nabla.\vec{u})+\nabla(k\nabla T)+\phi+se$

So, essentially we have all these five partial differential equations that are also coupled right. So, we have all these five equations and the unknowns are as we discussed before these are P T rho e u v w right.

So, these are all the unknowns right; essentially, seven unknowns. In addition to these equations we also have the equation of state right. So, the equation of state is given by P equals P of rho T as well as e equals e of rho T right. So, the internal energy and the pressure, both of them are related to the equation of state ok.

So, now, we have 5 plus 2, 7 equations and 7 unknowns right. So, the system is completely balanced and we can solve for this ok. Alright, questions still now on this part. So, by the way please feel free to ask questions ok. Any questions still now? Yes.

Yes, well I did not consider viscosity as a constant as such here right. Viscosity still could be varying, because I have mu inside the divergence operator right. I only said shear stresses are proportional to the strain rates right. The viscosity can still be varying with temperature or with space that is why I still have not left the mu outside the divergence operator.

So, usually the viscosity is known, because the properties of the fluid are known right. So, viscosity is not an unknown in these equations ok. It may vary, but you know how it varies with temperature or with space that is known to you of course, if you include viscosity as an unknown in these equations, then you would need a description for how viscosity varies.

Student: We already know.

You already know the viscosity of the fluid.

Student: (Refer Time: 06:25).

That with what you are solving for yes that is already known right. Now of course, if you like what we discussed yesterday, if you consider a constant viscosity and an incompressible fluid, then you can further simplify these equations in the source terms that we have right. Other questions? No, ok.

So, what we observe from this is we have several of these equations and there is a good amount of commonality between all these equations right. What do we see here is that all these terms are on the left hand have a transient term and there is a convection term and on the right hand side, we have some kind of a divergence term right del dot something and there are of course, some source terms ok.

So, the idea is can we write all of these equations with a single equation and then vary one particular quantity ok, such that we can get any of the equation that we like from that single equation ok.

So, essentially our life will be simple, because then we have to only worry about developing solution methods only for that equation and substitute the corresponding variables for that particular variable that we consider ok. So, what are we what I am trying to mean is essentially, what I am trying to say is we will now, replace all these equations with a single equation which we call it as say for a particular property phi ok.

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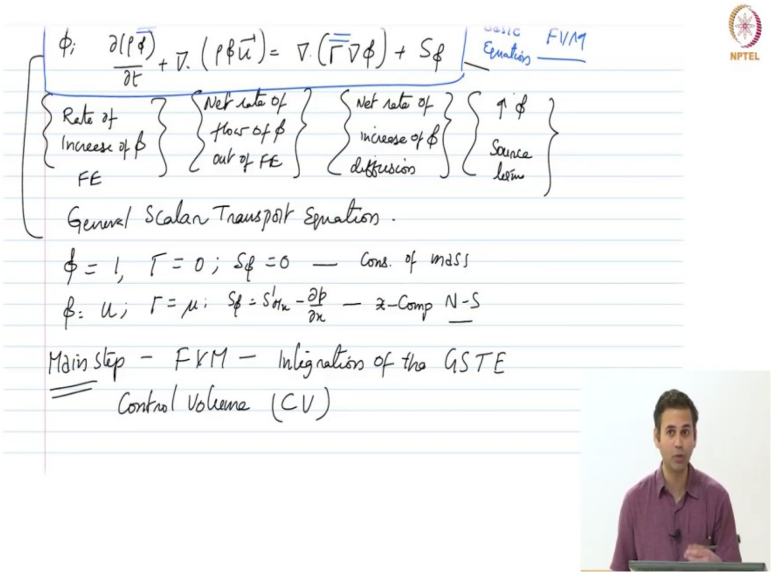

So, I would write an equation that would be $\partial((\rho\phi))/\partial t+\nabla.(\rho\phi u)=\nabla.(\Gamma\nabla\phi)+s\phi$. So, essentially what we have is we have an equation here, where the first term is again the rate of increase of phi right, rate of increase of phi for a fluid element right, plus the net rate of flow of phi out of the fluid element equals the net rate of increase of phi due to diffusion and the net of increase of phi due to the source terms ok. That is what we have; we have all these four terms.

Now, do you see that this equation can aptly describe all other equations? Is it in a similar form? Yes, it is right it is in a similar form only thing is that we have to somehow assign a particular value for phi as well as this diffusion coefficient gamma such that we can retrieve the corresponding equations that we want from these all the five equations of conservation of mass, Navier-Stokes equations and the energy equation right. So, that is are we going to do. So, this particular equation that we have written down in terms of phi is known as the General Scalar Transport Equation ok.

Of course, this general scalar transport equation can also model any other scalars that you have in a flow. For example, you want to track for pollutants or you want to track for any other species as part of your solution which will also have look similar to the equations that we have written.

So, you have to just assign phi to be equal to that species value and then you will be able to model that extra equation that you would get as part of this general scalar transport equation ok.

Now of course, we also realize that if I set different values, let us say phi equals 1 if I set phi equals 1; I am going to get on the left hand side here, if I set phi equals 1. I am going to get the conservation of mass right of course; I have to set the corresponding value for gamma which would be what? 0, right.

I would said $\Gamma = 0$ and $s_\phi = 0$ which would give me the conservation of mass similarly, if I set phi equals u right and gamma equals what? Mu and $s_\phi = SMx - \frac{\partial p}{\partial x}$ We have not yet figured out, how to put this term. So, I am going to kind of dump, this pressure gradient term into the source for now ok.

Then we are going to get if you plug in all these quantities, you are going to get the u x component of the Navier-Stokes equations, is not it ok. So, this would give you the x component of the Navier-Stokes equation ok.

Similarly, you can set phi equals v, little e or a species concentration and things like that and obtain any of the equations that you want by accordingly setting the values for gamma as well as the source terms S phi ok. So, those have to be accordingly assigned ok.

So, now this equation the general scalar transport equation is the fundamental equation that we will be working with throughout this entire course. This is the basic equation that we will use in the finite volume method that we are going to discuss as part of this course ok. So, this is the starting point for all the things that we are going to develop yes, how does the general scalar transport? The question is how does the general scalar transport equation show the energy equation?

So, essentially that is a good question. So, essentially what do you set? You have to set phi equals e right or T or h naught right. Now, the question is the confusion here is that the divergence operator on the right hand side has temperature right whereas, on the left hand side you have little e right.

So, essentially if you write an equation in terms of e you would write the K as cv times T. So, accordingly you have to set the gamma as cv times T or K by cv right, then you can essentially get the corresponding value for here right.

So, here the set phi equals so, I would set phi equals let us say e right or I can also set phi equals T or h naught to get the energy equation. If I work with phi equals e, then my left hand side is retrieved right whereas, on the right hand side. I have these two terms which is minus p times del dot u capital Phi and Se, all of these will go into the source term right. We have not yet so, essentially this part, this part and this part will all go to the S phi term right and then what about this guy? How do you, how much do you set for gamma here?

It has to be in terms of T right. So, K cv right that would give you your temperature here or if you write in terms of temperature, it should be K upon cv right. Accordingly, you have to set these values and you can retrieve the energy equation also ok. There are only two unknowns.

So, essentially density and temperature are the unknowns. So, instead of writing this equation for temperature we have written in terms of internal energy e right. So, I could even write this equation in terms of temperature right.

If I write for, if I have a perfect gas right e equals cv T right, in which case I can rewrite this equation as dou partial T of rho cv T right. So, the energy equation will be an equation for temperature right. If you have a perfect gas, if you do not have then you have to use the additional thermodynamic relations given by the equation of state that are given here to substitute for those values. Is that clear ok?

Other questions? Alright. So, now, we all agree that we can work with a general scalar transport equation in terms of a discretizing it and we can use these different terms that we have and later on at any point we can retrieve whatever equations we want from by substituting the corresponding values for phi as well as gamma as well as the source term S phi ok. So, this is the fundamental equation that we will be working with in finite volume method which is known as a scalar transport equation or a general scalar transport equation ok.

Now, the key step or the main step in the finite volume method is the integration of the general scalar transport equation ok, which I write it as GSTE that is the General Scalar Transport Equation. So, the main step or the key step in finite volume method is always to integrate this or any equation that we get on a control volume ok. So, we would choose a control volume and we would integrate the differential equation that we have on this particular; on this particular control volume ok.

So, that is the first step in finite volume method. Now, we are going to do the same thing ah. So, I am going to integrate the general scalar transport equation that is given here on a control volume ok. So, we are essentially performing a volume integral ok, so that it is going to give me on a control volume.

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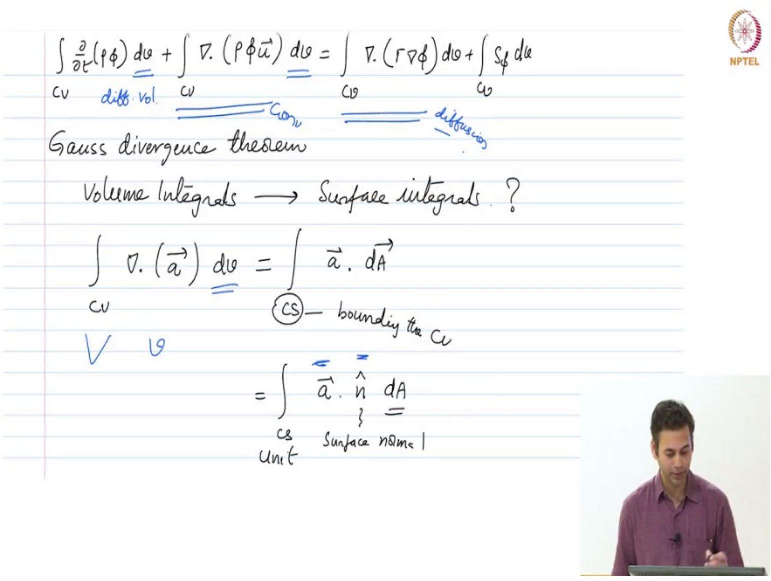

The first term is the transient term that is $\int_{cv} \frac{\partial}{\partial t}(\rho\phi)\,dv + \int_{cv} \nabla.(\rho\phi\vec{u})\,dv = \int_{cv} \nabla.(\Gamma\nabla\phi)\,dv + \int_{cv} s_\phi\,dv$ on the right hand side we have Here, a little v is the, dv is the differential volume of the control volume we have chosen. So, this is the differential volume all right. Now, we have all these terms. Now what we do is; we invoke the Gauss divergence theorem and we are going to replace the volume integral that is the convection term as well as the diffusion term.

These two volume integrals so, we are going to use the Gauss divergence theorem and then we are going to convert the volume integrals to surface integrals ok. Now, why do we do this stuff? I leave it to you to understand or else we can discuss later on why do we do this fine.

So, if I convert this thing, what does Gauss divergence theorem say? Gauss divergence theorem says that if you have a control volume right if you have divergence of a vector a bar on a particular volume dv, this is equal to integration of this particular quantity on the entire surface right in the direction of the surface areas, over the all the surface area that is bounded that is bounding this control volume right.

So, essentially this is a control surface a bar dot d A bar is what we have right. So, this is your Gauss divergence theorem, where the divergence of a vector and integration on a particular control volume would be equal to the sum of the a right in the direction of the surface areas. This control surface is bounding this control volume. Now.

Student: (Refer Time: 18:27).

Which one?

Student: (Refer Time: 18:30).

What is not matching, dimensions? Why are there not matching? So, the left hand side is a scalar or a vector? Velocity where is velocity? This one dv to dv is a volume, this little v is a volume ok. So, I would use this v for velocity, this one for volume.

So, essentially we are talking about divergence of a vector right, summed over the entire volume by taking these differential volumes right. That is what we have done here. These are all these dv's are the differential volumes not the velocities right.

We are not integrating with respect to velocity ok. So, is that clear right, everybody agrees, this is correct? Gauss divergence theorem ok. Now of course, I can also write it in a different way, I can write this as integral over the control surface if the surface has n cap as the surface normal this would be equal to a bar dot n cap times d A right, where d A now, you say scalar right. So, where n cap is the surface normal or a unit surface normal that is what we have either I can write it as a dot d A bar or a bar dot n cap times d A ok.

What this says is that the divergence of a vector summed over the entire control volume using differential volumes is nothing, but the component of a bar right, the component of a bar in the direction of the surface normal right, summed over the entire surfaces that bound the control volume ok. So, that is what we have from the Gauss divergence theorem. So, we are going to use this and replace the convection term and the diffusion term of the general scalar transport equation ok.

So, let us do that. So, that would give me, I would leave the first term as it is that is control volume. So, essentially this is I am also making one assumption here, I am assuming that the control volume is not changing with respect to time ok, so that I can take out the time derivative outside this control volume. So, I am assuming that the time derivative and the control volume commute.

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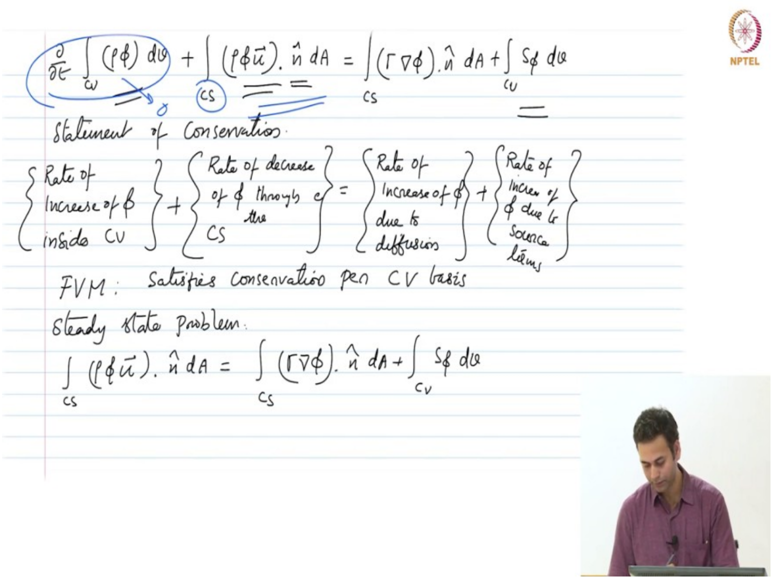

So, that will be $\frac{\partial}{\partial t}\int_{cv}(\rho\phi)dv+\int_{cs}(\rho\phi\vec{u}).\hat{n}\,dA=\int_{cs}(\Gamma\nabla\phi).\hat{n}\,dA+\int_{cv}S_{\phi}dv$ this is the differential volume plus over the control surface. Now, what is the; what is the a bar here that we have; rho phi u bar is my a bar now right. So, I could write this as a del dot rho phi u bar dv.

I can write it as control surface. This particular term, I am writing it as gamma grad phi is a vector right, gamma grad phi dot n cap d A right.

So, this will alright everybody, with this equation right, we have just used the Gauss divergence theorem.

So, we are going to do this for in the entire course ok. So, for each of the equations we get. Now, what we see is that we see something very interesting ok. Now, we have performed an integration and what we see is that the integration has resulted in a kind of a conservative or a statement of conservation right.

It is essentially it resulted in a statement of conservation ok. Why do we say it is a statement of conservation? Because if you look at the terms, what does the first term indicate? The first time indicates that for the control volume that we have the rate of change of rho phi right or the phi inside the control volume is the first term.

So, this is rate of increase of phi inside the control volume is the first term plus what does the second term mean? Second term means that the rate of phi that is going through the surfaces right, going out right, because n cap always let us say if you assume n cap always points in the outward direction of this control volume then all this is leaving the control volume through the surfaces.

So, this is rate of let us say decrease of phi out of the control surfaces right of the volume or we can say out of the control surfaces of the boundary or we can just say control surfaces that is fine, control surfaces equals, then what we have here; diffusion term. So, this is again rate of increase of phi due to diffusion right, plus we have again here rate of increase of phi due to source terms right. So, that is what we have. So, essentially this is a statement of conservation.

Now, you may not be surprised, because what is the big deal in this, we have started off with a similar term right, but we have ended up with a similar conservation equation right. Initially, we started off with without the control volume terms, we started off with one equation, which was also a conservation statement, but now again we ended up with a statement of conservation for the control volume ok.

So, this is important which is the main characteristic of the finite volume method, in which for each of the control volumes you choose, the conservation will be satisfied for the phi ok, for the property phi ok. So, this is the characteristic of the finite volume method. So, it essentially satisfies conservation per control volume basis ok. So, every control volume that you take; that means, every cell or you know the mesh cell that you take is going to satisfy conservation ok.

So, there is a physical reasoning behind the solution that you obtain ok. Now, this was all possible, because in the first place we have left the equation that we had before, if you go back we had these equations right. We wrote the equation as del dot rho phi u bar right. We have written this as del dot rho phi u bar, as a result of which we can use the Gauss divergence theorem and we could get rho phi u bar dot n cap right.

Now, if you go little further up, we started off with saying ok. We are going to talk about conservative form ok. Now, conservative form refers to this particular convection term that is this del dot rho u u or del dot rho v v and so on. All these things when you have divergence of something as one term this is the conservative form ok. Now, you may ask ok, then what is a non conservative form? A non-conservative form is one where you do not write like del dot rho u u bar right.

For example; now, you can expand this del dot rho u u bar as two terms right, like what we have done before. You could consider rho u bar to be together and you could write this as rho u bar right dotted with del dot u and so on right. So, you can write this as a two expressions in which case you would get, you can use again continuity equation to simplify and so on. So, if you expand it out like that then it is not a conservative form ok, that is a non conservative form ok, where in you would not write the equations as del dot rho u u bar ok.

So, but you usually work with a non conservative form in the context of finite difference methods, because a finite difference method does not involve integration on a particular control volume ok. So, you never have to invoke Gauss divergence theorem at all ok. So, as a result finite difference methods would not result in a statement of conservation on a cell by cell basis, but of course, you cannot solve for something which does not satisfy the physical laws.

So; that means, finite difference method would definitely satisfy principle of conservation on the entire domain that you choose, but not on a element by element basis whereas, study satisfied on a cell by cell basis in finite volume method, because you are now using this conservative form to integrate on a particular control volume and arrive at a statement of conservation ok. Is that clear?

So, that is the difference between a conservative form and a non-conservative form. You do not write this del dot as this thing. You expand this out as two terms, then you get a non conservative form ok. So, it is only that so, if it is only that the final solution will be the same, but in a finite difference method you would be going through you may not be going through a conservative set of solutions.

So, for example, if you had stopped your simulation halfway between then a finite volume method although may not be correct at that point would still give you a conserved solution on every cells whereas, the finite difference method may not give you a conserved solution at that iteration ok, but eventually essentially it is only difference between. So, the final solution will the same whether you use finite difference or finite volume it is only that the path you to the solution is different ok.

The path through the solution goes through sequence of conservative solutions for finite volume whereas; it need not be going through the same path if you take a finite difference method that is the only difference. We will probably do one problem in the assignment, where you can see the difference between conservative form and the non conservative form and appreciate how they are different.

And in fact, if you will again go through this little later if you discretize these forms, you see that the resulting equations you get to solve both from finite difference and finite volume method will look the same, as long as you have as long as you have linearities in your problems ok.

So, if you have the moment you start having nonlinearities you will see that the conservative form is results in a very different equations than finite difference method ok, that is what you will see in terms of these differences. Other questions ok fine, then let us move on ok.

So, it kind of net rate of decrease of a phi essentially through the boundaries, I would not say out of so, essentially decrease of phi through the control surface right. So, we say decrease, because it is leaving right del dot u. So, this essentially is leaving the control volume that is why through the surfaces that is why I call it as a decrease, right.

Student: (Refer Time: 29:41).

Phi, that is right. So, essentially it is the net rate of decrease, because it is kind of leaving the it is going out of the control volume right, because it is positive ok, but of course, if you have a negative term that is coming inside and that will be there as well.

Well if you have a rotational flow, you can write it as a you know gradient of a potential function right and so on for the diffusion equation, but in this context when we say the conservation is satisfies, we are doing is you have a balance equation, you know you have a essentially, you have something that is created in the cell something that is leaving the cell and something there is an accumulation in the cell. So, that balance equation is what we call it as conservation here.

But of course, you can of course, write this as a gradient of a potential function and so on all right, other questions ok?

Student: (Refer Time: 30:31).

Right.

Student: (Refer Time: 30:34).

That is the question, I asked you to think about it in your asking back ok. So, this is my question back to me. So, why did I do the ok, you would probably be clear little later. So, the thing is ok, if I do not do the Gauss divergence theorem what so, for example, if you see kind of look through this thing what we kind of did is we kind of have one term for the control volume right, one term for the control volume and these two terms, we have converted into control surfaces right.

So, if I do not have do not convert them into control surfaces, I just leave it as control volumes, what will happen to the equation? I would get eventually; so, essentially I have some terms, I am integrating them on a control volume right or the control volume could be represented with one cell centroid right. Everything will become dependent on this cell centroid that is all. So, essentially you do not have any connection to your neighbors right.

So, essentially you have a set of equations which are all disconnected right. Essentially, you have for every control volume, you will have one equation. There is no connection between the cells at all. Now, to bring in the connection between the cells, we are saying that we are replacing this convection, which is going through the surfaces in a more physical way, where I would say, write it as that is going to the surface.

Now, when we model this particular thing, this particular control surface we would again make use of the corresponding control volumes that share this control surface right. As a result we are bringing this, bringing in this connection between the control volumes through this particular control surface ok. As a result, the equations you would get will be a set of linear equations right, which are all connected, they kind of start depending on the neighbors ok. That is how it is.

Otherwise, you would have one equation which is all in terms of one particular cell right, that is what we did which you cannot do anything with it right it is so, we did ok. I thought you will tell me the answer, but I have told the answer fine. Any other questions? No, ok. So, this is kind of the fundamental equation.

Now, of course, if you have a steady state problem; so, if we have a steady state problem then the first term drops out, then all we have is control surface rho phi u bar dot n cap d A equals control surface gamma grad phi dot n cap d A plus integral control volume S phi d little v ok, that is a steady state problem which is obvious.

Because we have set essentially this term to go to 0 ok. Now, in general you would probably have transient problems as well ok, you will have unsteady problems as well in which case just like we have.

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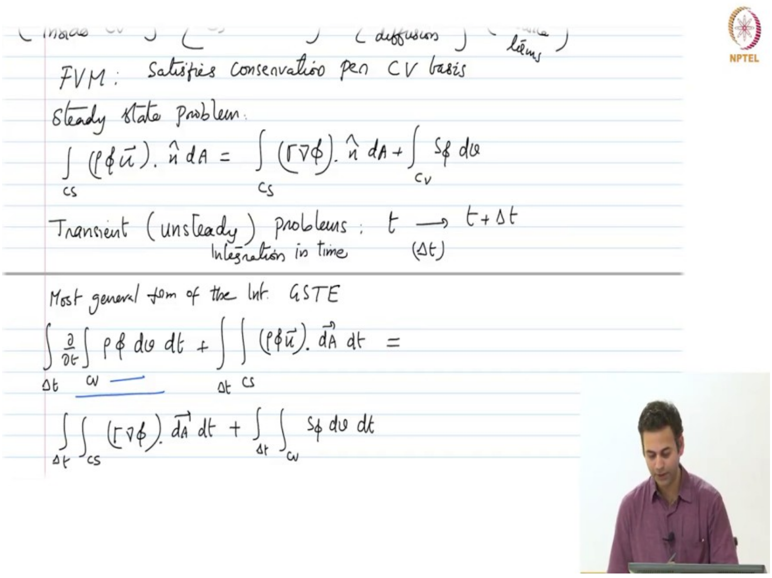

So, if you have a transient or unsteady problems, then just like we have integrated on a particular control volume we have to also integrate this partial partial t on a particular time interval ok.

So, essentially we have to integrate from a known time t to another known time another time t plus delta t by an interval delta t ok. So, we have to introduce a integration in time ok, as well just like we have integrated in space for the control volume, you are integrated in time as well for this equation, because we cannot leave partial partial t in there ok.

So, if you integrate then you have to integrate each and every term that you have in the equation ok. So, that is going to give you the most general form of the integrated general scalar transport equation ok.

So, that is essentially now, you have integral delta t, then we have a

$$\int_{\Delta t}\frac{\partial}{\partial t}\int_{cv}\rho\phi dvdt+\int_{\Delta t}\int_{cs}\left(\rho\phi\vec{u}\text{¿¿¿¿}\right).\overrightarrow{dA}\,dt=\int_{\Delta t}\int_{cs}\left(\Gamma\nabla\phi\right).\overrightarrow{dA}\,dt+\int_{\Delta t}\int_{cw}s_{\phi}dvdt\text{¿¿¿¿}$$

We are integrating with respect to space here and then we are integrating with respect to time ok. So, we have this. So, we have double integration here plus integral over delta t integral over the control surfaces right.

We have that is your most general form of the transient or the integrated general scalar transport equation ok.

So, we will be using this for unsteady problems fine. Now, of course, we will make some assumptions in terms of how do we evaluate the surface integrals, the volume integrals, and how do we evaluate this time integration ok.

All these things we will make certain assumptions, we will make use of some profile assumptions and then integrate them right, because we are trying to solve for these terms and of course, we do not know the terms themselves, how can we integrate them right.

So, essentially you make a profile assumption, assuming that the these unknowns that we are solving for will vary in a certain manner and then we are going to say introduce that and integrate these equations that is what we are going to see in the next lecture. So, we will work with all this integration of time and space for all the diffusion equation convection, diffusion equation and so on fine.

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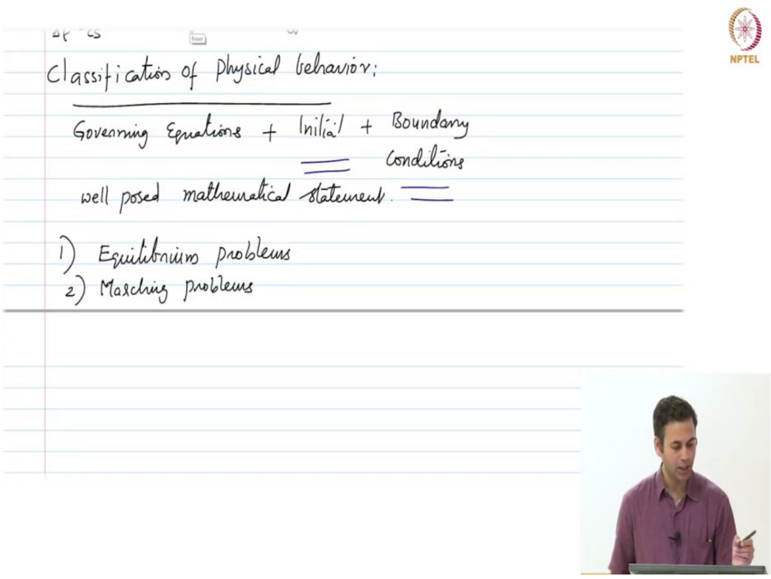

So, let us kind of move on to the classification of physical problems. Our classification of physical behavior, because in order to solve a problem we not just need a governing equations. We also need to know together with the governing equations we need to know what? Boundary condition; so, we need to know the initial and are the boundary conditions ok, only then we can form a well posed mathematical statement ok.

So, now, the requirement of the initial and boundary conditions kind of stems from the kind of the physical behavior or the type of the equation we are looking at ok. So, we will only know do we need an initial condition or do we need a boundary condition that depends on the type of the physical problem we are solving and also the based on the classification of the problem itself ok.

So, let us kind of classify the physical behavior of the fluid flow and heat transfer problems that we get often. So, we can kind of broadly categorize them into equilibrium problems and marching ok, marching problems ok either equilibrium problems or marching problems. Let us look at what is and what we mean by equilibrium problems ok.

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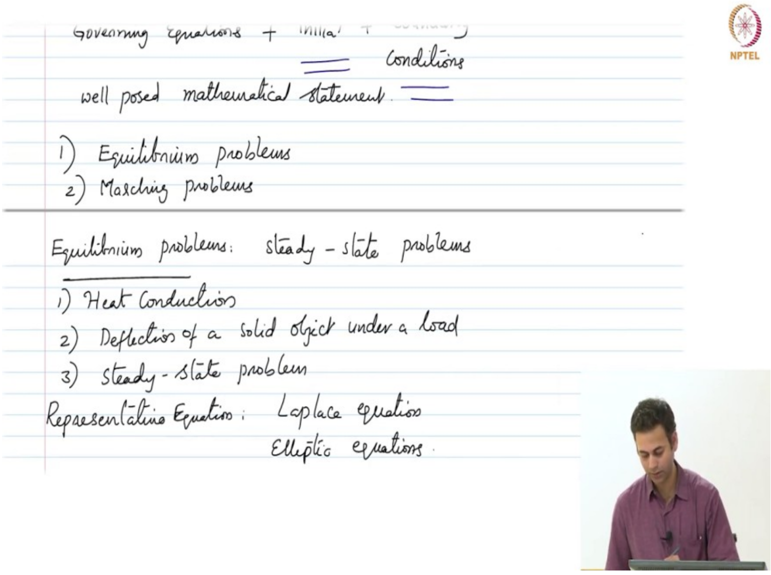

So, few examples are the steady state heat conduction. So, all the steady state problems we would call them as equilibrium problems ok. So, all the steady state problems for example, steady state heat conduction problem or we have a the deflection of a solid object under a load and so on or any other steady state problem ok.

Now the governing equation for all these equilibrium problems or we have a kind of a representative equation is the Laplace equation and these are also known as elliptic problems these are also known as elliptic equations or problems in the literature ok. So, all these are governed by a representation that is known as a Laplace equation what is a Laplace equation? So, a typical Laplace equation would look like.

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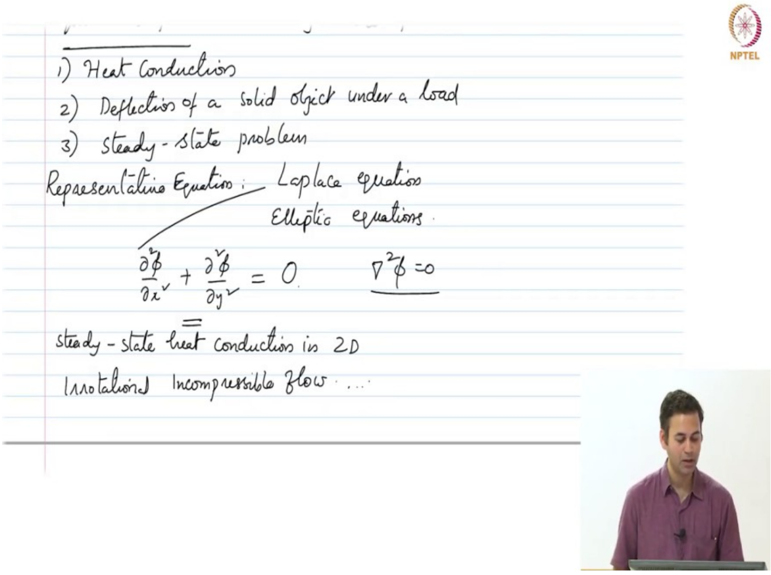

Student: (Refer Time: 40:54).

Divergence of gradient of some scalar right, that is what we have so, I can rewrite this as $\frac{\partial \phi}{\partial x^2} + \frac{\partial \phi}{\partial y^2} = 0$ So, that is a typical equation for this kind of problems. Now, what does this represent? This represent of course, as we discussed the steady state heat conduction in 2 dimensions, any other physical behavior this can be used to describe that we have come across in; fluid mechanics at the Laplace equation, irrotational flow right.

We had $\nabla^2 \phi = 0$ in irrotational flow, if you have del square phi equals 0 this is the governing equation for irrotational potential flow, essentially irrotational and incompressible flow right and so on. So, we could use it to kind of represent, these steady state problems ok. So, one example could be if you consider a, if you consider a 1 D problem.

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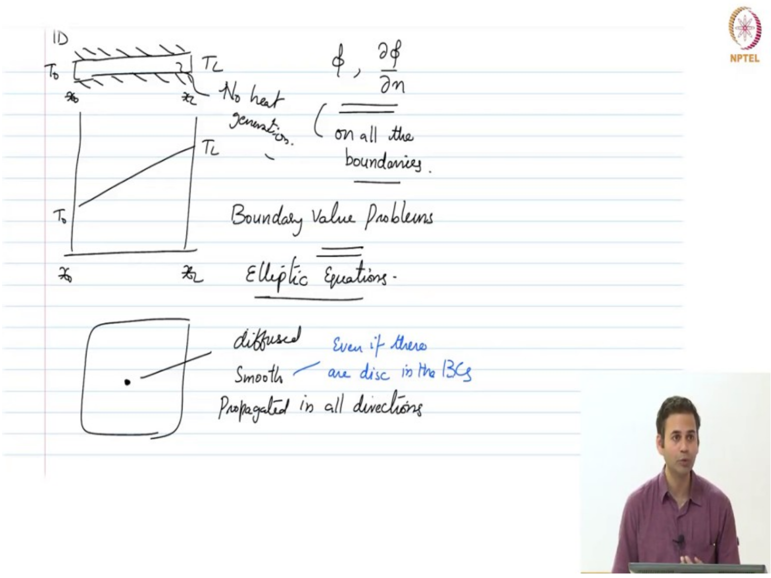

For example let us say we have a solid rod, where we keep 1 and at T naught another end at a higher temperature T L and if we insulate the all the sides of it right. So, in a steady state and if you assume that there is no source as part of this thing so, if we have a steady state heat conduction the solution would look something like this, if you have, if there is no heat generation right fine.

So, this is your x naught and x L alright. Now, what kind of boundary conditions do we need to specify or initial conditions do we need to specify for solving these problems? Do we have to specify a condition on the solution variable that we are solving for let us say for example, phi right by the way we have to specify either phi or it is derivative partial phi partial n right, needs to be specified on, do we have to specify on all the boundaries or only on few boundaries?

Student: (Refer Time: 43:29).

You have to specify on all the boundaries right, in order to; in order to be able to solve this equations right. So, because for example, here in the 1 D problem we have to specify T at x equal to 0 and T at x equal to L right so that we can solve for this 1 D heat conduction problem. So, these kind of do we have to specify a transient do we have to specify a initial condition also?

Student: No.

For this we do not have to, because there is no over time derivative in this thing. So, there is no need for an initial condition, but we have to specify a boundary condition on all the boundaries that we have in terms of either you specify the variable itself or a derivative of that right ok. Now, these kind of problems are known as what boundary value problems, which require values to be specified on all the boundaries of the domain ok.

Now, these are these boundary value problems of Elliptic equations have a, elliptic equations have certain characteristics. So, these characteristics are kind of important to look at. For example, if there is a sudden increase of temperature in anywhere in the solution domain ok. If you are let us say solving for a 2D problem and if there is a sudden increase of temperature, because of a source or something like this.

Now, this sudden increase would be propagated in all directions ok. It will get; it will get diffused in all directions ok, as a result the solution you are going to get will be always smooth. So, because any disturbances you have in the flow field would be propagated in all directions. So, the solutions obtained are smooth even if you have discontinuities in the boundary conditions.

So, even if you have even if there are discontinuities in the boundary conditions ok, even then the solutions obtained are smooth and they have to kind of propagate in all directions. Now, this is a kind of very convenient thing to device numerical methods to solve for elliptic problems ok; that means, the numerical methods that you would device have to take this into account wherein.

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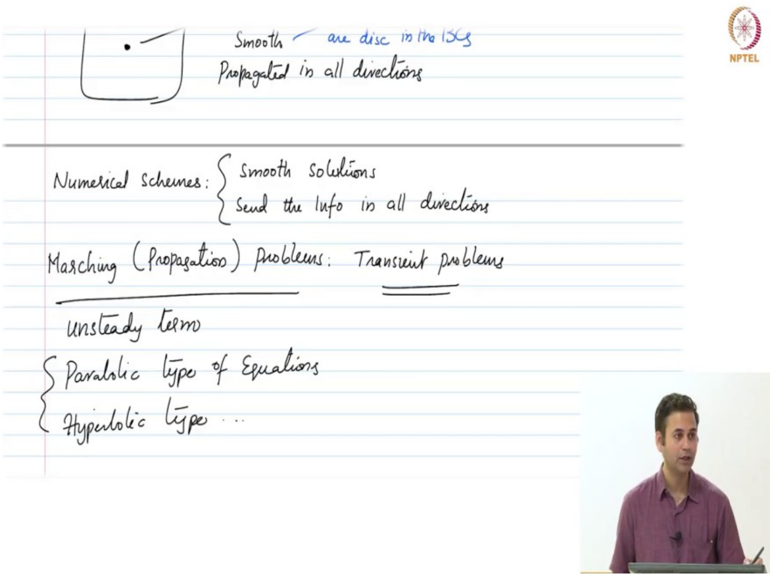

So, the numerical schemes that you want to use have to be such that they produce these smooth solutions right and also they should be able to send the information in all directions.

So, if you have a numerical scheme that does not satisfy these things, then the solutions you are going to get out of the elliptic solution would not be correct ok. So, that is; so, that is why we are learning the physical classification of these problems ok, such that we can devise better numerical schemes for our solution methods fine. So, these are kind of the insights from an elliptic equation.

Now, what about the marching problems, marching or propagating problems; propagation problems? So, all the transient problems are all considered as marching problems ok. So, we have all the transient problems. So, where we have an unsteady term, these are known as marching problems ok.

Now, in the marching problems we can kind of classify further into two different things; one we call it as parabolic type of equations, the other one we call it as hyperbolic type of equations, but both of them kind of belong to the transient flow problems ok. So, we are going to see how these two kind of differ and a working equation for it and the characteristics etcetera in the ok.

Thank you.