Computational Fluid Dynamics Using Finite Volume Method
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Lecture – 42
Finite Volume Method for Fluid Flow Calculations:
SIMPLE – Co-located algorithm for Unstructured mesh

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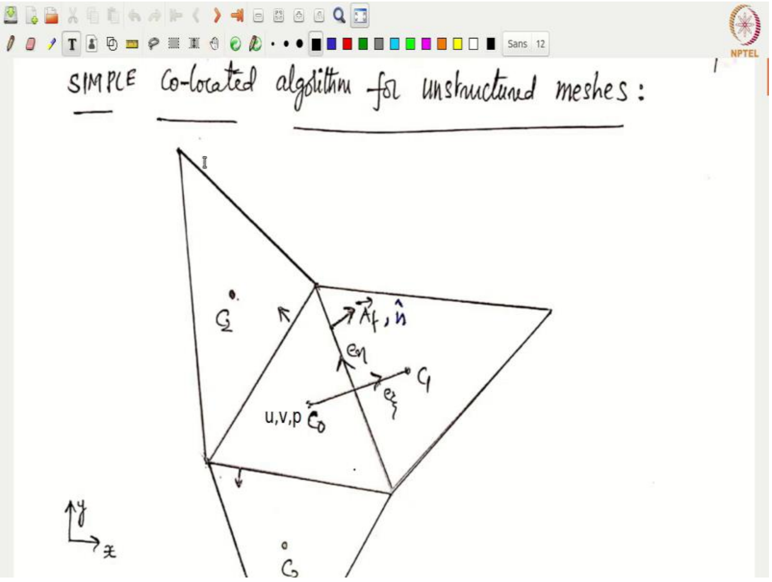

Let us get started. So, welcome to another lecture as part of our ME6151 Computational Heat and Fluid Flow course. So, in today's lecture we are going to kind of extend the SIMPLE algorithm on Co-located grids for unstructured meshes, right essentially. This is the ultimate thing that we wanted to do because in order to be able to solve fluid flow equations on either unstructured meshes or on non-uniform meshes, we have kind of had a motivation that the co-located grid approach would be most suitable.

So, essentially we kind of come back to this picture where we have a cell which is sharing it is faces with 3 other cells ok. So, we concentrate on the primary cell that is C_0 and we have a neighbouring cell C_1 with face f here and the line joining C_0 and C_1 would be e_ξ and the unit vector along the face would be e_η and then the face normal that is pointing outwards from the cell would be $\overrightarrow{A_f}$ and we also denote a unit vector of this face normal using $\hat{n}$ ok.

So, that is the idea here and then we have our x and y coordinates which are the Cartesian coordinates as usual. So, in the simple algorithm we are going to store essentially both the x-component of velocity and the y-component of velocity as well as pressure all at the cell centroid C_0, right. And, similarly, we are going to store a similar Cartesian components at C_1, C_2 and all other cells. So, that is the idea.

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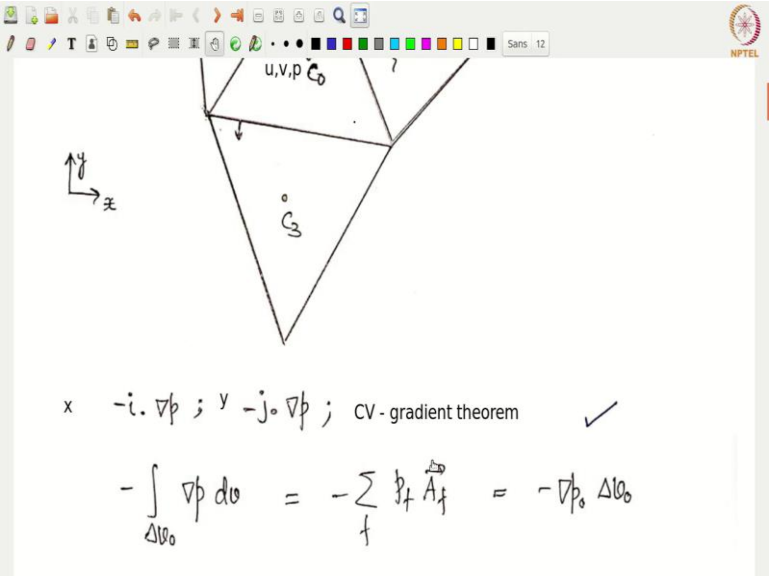

Now, if you look at the momentum equations we had in the x-momentum equation we had a $-\frac{\partial P}{\partial x}$. So, that was $-\hat{\imath} \cdot \nabla P$ and in the y-momentum equation we had $-\hat{\jmath} \cdot \nabla P$ that was $-\frac{\partial P}{\partial y}$, ok.

So, we had used these things we have integrated on a particular control volume and then we have also invoked the gradient theorem, right. And, then converted them into a surface integral of the pressures right and the essentially multiplication of p f times A f or all the faces.

Now, either these terms can be treated in that fashion using gradient theorem where the minus the ∇P dV or the control volume can be converted into a $P_f\overrightarrow{A_f}$. Or alternatively it can also be taken this can also be treated similar to a source term where grad p can be represented by using the cell centroid value that would be grad P_0 times once it is constant you can integrate this over the finite volume that would basically give you delta v naught ok.

So, both these expressions are equivalent. So, we are going to for now use this in the in the formulation instead of what we had used before in the context of the Cartesian meshes ok, alright.

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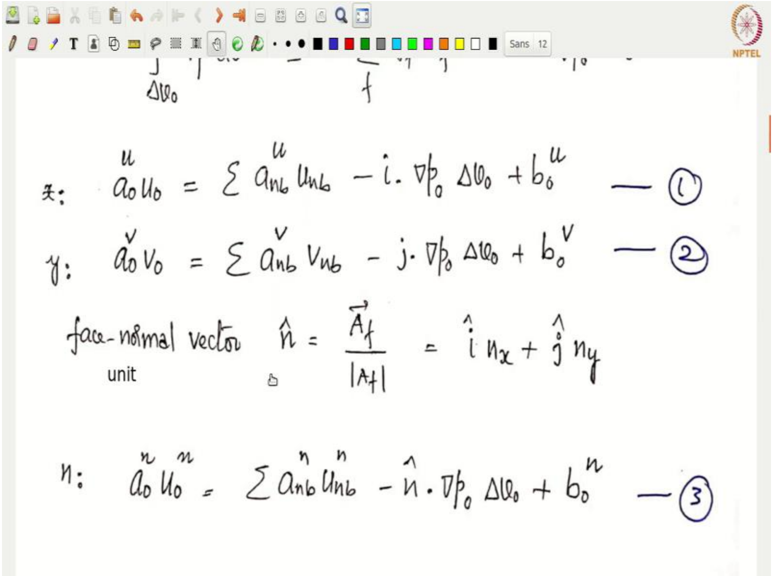

So, then essentially what we can do is we can we once we convince ourself that the pressure gradient term can be written as $-\nabla P_0 \Delta V_0$, then we can concentrate on the cell 0 here and we can write the x and y momentum equations in the discrete form for basically for the for this cell C_0 ok.

So, that is basically going to give you your because it is cell 0 instead of $a_P u_P$ we have written $a_0^u u_0$ equals $\sum a_{nb}^u u_{nb}$ minus $\hat{i} \cdot \nabla P_0 \Delta V_0$ remember in the in the previous lectures this was a kind of coming out to be after integration this was coming out to be because there is an i dot here we got basically if it if would if it were let us say co-located approach then we got something like p west minus p east, right.

Essentially because of the minus we kind of absorbed it, but now because we are leaving grad P_0 as it is we still have this minus coming into play ok. So, that is what we have to keep in mind. This minus is associated with this ∇P_0 and in the previous lectures this minus was not there because that was absorbed into the when we have written the pressure as a summation of the faces plus we have some source term that is b_0^u.

Now, to distinguish these coefficients we have included a superscript u for a_0, a_{nb} and b_0. So, let us call this equation 1 and similarly, we can write an equation for the y component of velocity that is for v 0 which would be $a_0^v v_0$ equals $\sum a_{nb}^v v_{nb}$ minus $\hat{j} \cdot \nabla P_0 \Delta V_0$ plus b_0^v ok.

So, again essentially these coefficients might be different from the coefficients of the x momentum equation. As a result we wrote v to distinguish this right essentially we have $a_0^v v_0$ equals $\sum a_{nb}^v v_{nb}$ with v and so on let us call this y momentum equation as equation number 2. So, we know how to discretize these two equations and solve for them for a known guessed pressure field ok.

Because you know how to calculate the gradient this can be calculated and essentially this will a_{nb}'s will have components of convection and diffusion and you know how to calculate the central coefficient that is a_P and essentially we know how to solve for equations 1 and 2 for a guessed pressure value, alright.

Now, what comes next is basically we need to now come up with an equation for the face velocities right we need to come up with the face velocity because these velocities on the faces will be used in the discretization of the continuity equation ok. So, as a result let us look at an equation for obtaining face velocities ok, especially in the direction that is normal to the face that is in the n cap direction we would like to get an expression for the velocity which can be used in the continuity equation ok.

So, now, let us define the face normal vector essentially a unit face normal vector ah. Face normal unit vector can be defined as $\hat{n}$ equals $\overrightarrow{A_f}$ upon magnitude of A_f right that is basically constitute a unit vector; that means, this $\hat{n}$ can further also be written into it is Cartesian components as $\hat{\imath} n_x + \hat{\jmath} n_y$ ok, where n_x and n_y are the x and y components of the unit vector that we would obtain and this will be different for each face ok, but this can be computed and stored at one place, alright.

Then using the two equations we have that is equation 1 and equation 2, we can also write an equation for the velocity u_0 in the direction of $\hat{n}$ ok; that means, this u_0 is different from this u_0 ok; this u_0 is the x-component of velocity, v_0 is the y-component of velocity. Now, we are writing fall for cell 0 we are writing an equation in the $\hat{n}$ direction ok. We are writing the momentum equation in the $\hat{n}$ direction.

We already have it in the x direction and y direction. So, we can of course, write an equation in any other direction right using these x and y equations. So, that equation would be of course, we will see how to calculate these coefficients, but that equation would be $a_0^n u_0^n$ equals $\sum a_{nb}^n u_{nb}^n$

minus instead of i dot and j dot you have $\hat{n} \cdot \nabla P_0 \Delta V_0$ because this is in the direction of $\hat{n}$ in the direction of the face normal $\hat{n} \cdot \nabla P_0 \Delta V_0$ plus b_0^n ok.

So, that is basically is a momentum equation for cell velocity for C_0 cell velocity in the direction of the normal. So, basically in this direction we have written a a component of momentum equation in this direction from the equation we have in the x direction and the y direction alright.

So, now, how do we calculate of course, a_0^n and a_{nb}^n and b_0^n? We basically already have these two essentially we have to multiply an with n_x with this and n_x with the second equation and then somehow obtain what is a_0^n ok. So, that is what we would do.

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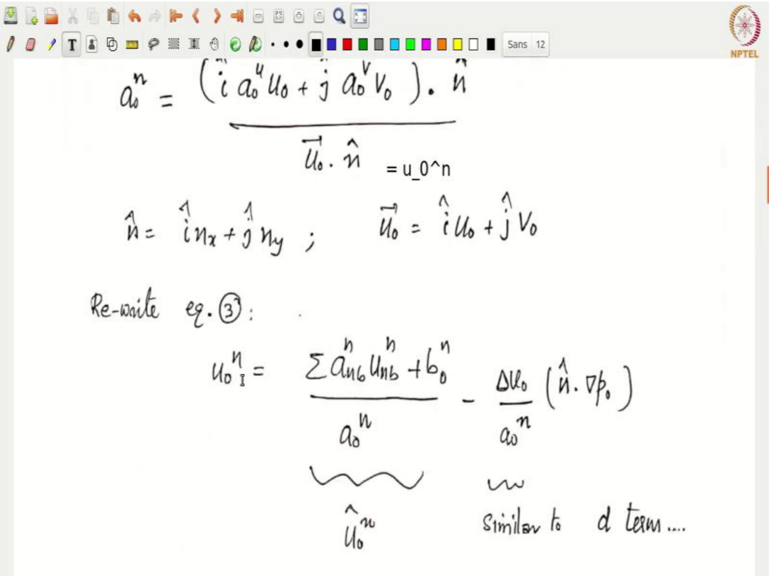

So, a_0^n would be you have $a_0^u u_0$ and $a_0^v v_0$ if you form a vector which is basically for the $\hat{i}$ component and the $\hat{j}$ components then you basically take a dot product with the $\hat{n}$. And. then you divide you know that the velocity in the direction of the face normal would be $\vec{u}_0$, this is this is basically $\hat{\imath} u_0 + \hat{\jmath} v_0$; if you take take a dot product with n cap you are going to get a velocity in the direction of the face normal right.

This is basically nothing, but this is what this is your u_0^n right $\vec{u}_0$ dot $\hat{n}$ would be your u_0^n, right. Essentially this is this is nothing, but your $\hat{u}_0^n$ right not hat essentially to the power superscript n ok; that means, we already have $a_0 u_0$ and $a_0 v_0$, this entire component divided by u that is going to give you the a_0^n the coefficient here, right.

Similarly, a_{nb}^{n} can be found by taking components of these two and b_0^n can be found by taking components of these two, right. So, that means, we can of course, calculate all these coefficients and write a discrete equation in the direction of the face normal for the velocity at the cell centroid C_0 ok, alright. Now, once we have this can we divide this entire equation with a_0^n yes, we can.

So, if I do that we are going to be left over left out with u_0^n on the left hand side that is basically u_0^n equals on the right hand side we have $\sum a_{nb}^{n} u_{nb}^{n}$ plus b_0^n all at in the direction n divided by a_0^n that is this coefficient coming here minus we have $\hat{n} \cdot \nabla P_0$ times ΔV_0 right, that is basically this component. This candidate divided by a_0^n that is nothing, but your $\Delta V_0 / a_0^n$ times $\hat{n} \cdot \nabla P_0$ ok.

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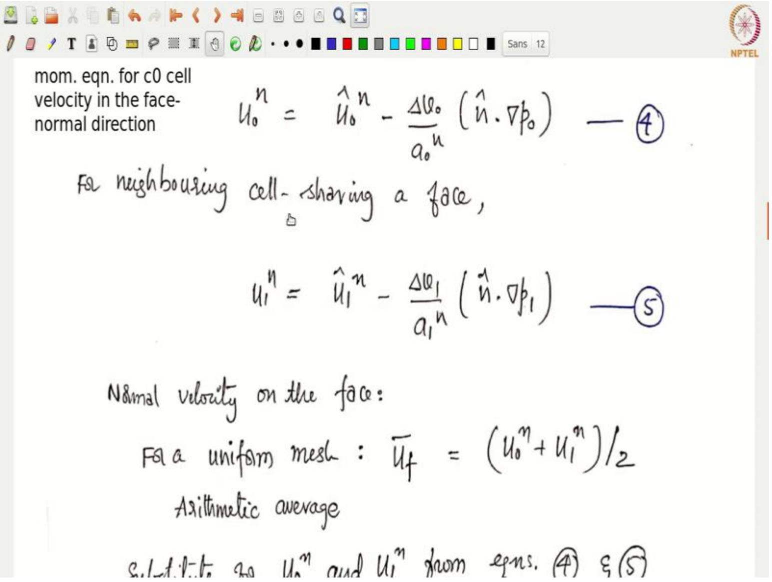

So, again we can of course, denote this using some $\hat{u}^n$. So, this is some $\hat{u}^n$ this term is basically similar to your d terms that you have written before right. You remember in the equations for the Cartesian x and y discrete momentum equations we had something like one by a e east right that we called as e east.

Now, that one was coming because of there was the volume was coming out to be getting cancelled out essentially you have ΔV_0 as the volume here right. So, that is why you got $\Delta V_0 / a_0^n$. So, this is similar to your d term. So, we will not substitute it at the moment we will leave it as it is.

Of course, we can rewrite this momentum equation for the cell velocity in the direction of the face normal as u_0^n equals $\hat{u}_0^n$ plus or essentially minus $\Delta V_0/a_0^n$ times $\hat{n} \cdot \nabla P_0$. So, this is basically if you know this is basically this is the momentum equation for C_0 cell velocity right in the face normal right direction right basically in the direction of face normal that is in the direction of n what will be the velocity component that is u n in the for cell C_0 ok. So, that is what this is.

This is basically momentum equation for cell 0 velocity in the direction of the face normal. That is u_0^n equals $\hat{u}_0^n$ minus ΔV_0 divided by a_0^n times $\hat{n} \cdot \nabla P_0$. So, let us call this equation 4. Then of course, we can write a similar equation for the neighbouring cell that is basically C_1 cell right.

So, because these two cells share a common face and that common face has one particular n cap direction we can write a component of velocity that is in this direction momentum equation and similarly for the velocity components here we can find a velocity component in the direction of n cap and write a momentum equation right.

So, if we do that essentially do the same process again for cell C_1 which is also sharing the same face with the cell C_0, then what we get is essentially the same equation with 0 replaced with 1, ok. So, what we get is u_1^n equals $\hat{u}_1^n$ minus $\Delta V_1/a_1^n$ times $\hat{n} \cdot \nabla P_1$, ok. Let us call this equation 5.

Now, this is basically if you try to connect to the previous Cartesian equations we had this is more like an equation for u p and this is more like an equation for u capital E right. Now, we are using p and capital E to calculate little e right, the face value that is what we do here.

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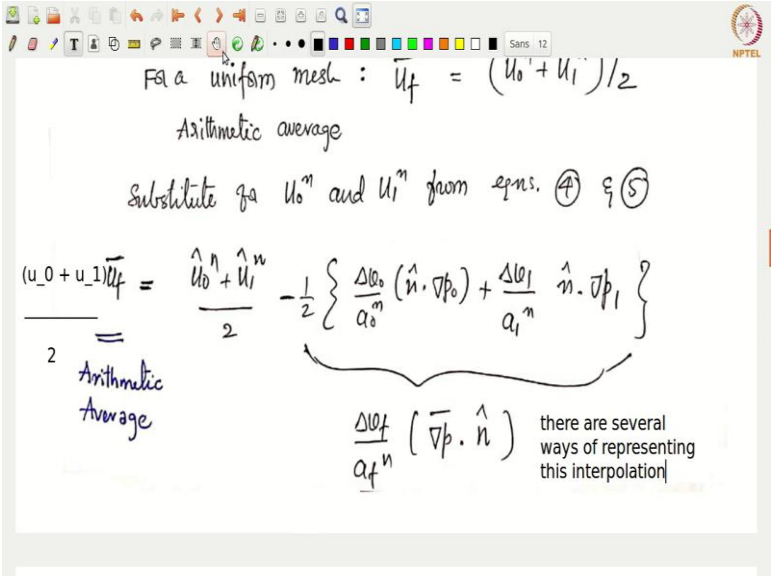

As well we want to essentially find we want to find the velocity on the face ok. Now, if again if you assume a uniform mesh we can take an arithmetic average. So, that will be if you have an arithmetic mean that will be $\vec{u}_f$ would be equal to $\hat{u}_0^n$ plus $\hat{u}_0^n$ by 2 this is basically similar to u little e equals u p plus u capital E by 2 right.

Of course, if you do not have a uniform mesh then you know you basically have to use a ∇u_0 ∇u_1 and the corresponding Δr_0 Δr_1 and calculate what will be the face values, right. Or else you can use some kind of a if you are dealing with Cartesian mesh, but with non uniform mesh then you would get some factor here right. You get f times u_0^n plus 1 minus f times u_1^n and so on, where f would be calculated based on the distance between the corresponding cell centroids and the face centroid, right.

So, that we know how to do. So, for now without loss of generality we will assume that the mesh is kind of uniform although it is unstructured. So, it will be kind of simple for us to do then what we can do is we can substitute for u_0^n and u_1^n from these two candidates, right. This is basically u P and u capital E. Substitute these two that is substitute for these two in this definition. So, this is basically your arithmetic average or linear average right.

So, linear interpolation; that means, what you have is $\vec{u}_f$ equals if you substitute for u_0^n in terms of $\hat{u}_0^n$ and this quantity and $\hat{u}_1^n$ and this quantity. What you get is you get $\hat{u}_0^n$ plus $\hat{u}_1^n$ by 2 minus you have these two quantities which are added up and then multiplied with the one half right essentially that is what you get essentially u f bar equals $\hat{u}_0^n$ plus $\hat{u}_1^n$ upon 2 excuse me minus 1 by 2 times $\Delta V_0/a_0^n$ times $\hat{n} \cdot \nabla P_0$ plus $\Delta V_1/a_1^n$ times $\hat{n} \cdot \nabla P_1$ ok.

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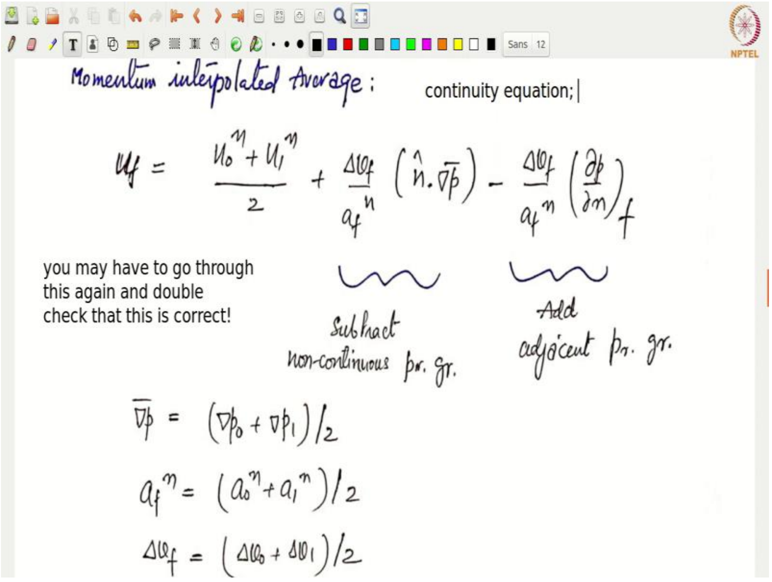

Now, let us call this entire thing this is half of this entire summation u with a new value that is represented on the face as $\Delta V_f/a_f^n$ times $\hat{n} \cdot \nabla\overline{P_0}$, where $\nabla\overline{P_0}$ is basically a face value of the pressure gradient $\nabla\overline{P_0}$ would be some combination of ∇P_0 and ∇P_0 and ΔV_f would be face value for ΔV_0 and ΔV_1 volumes.

Similarly, a_f^n would be some face value for C_0 and C_1 cells ok. So, that means, what we get is $\vec{u}_f$ equals $\hat{u}_0^n$ plus $\hat{u}_1^n$ upon 2 minus including this half we are calling this as minus $\Delta V_f/a_f^n$ times $\hat{n} \cdot \nabla\overline{P_0}$ alright. So, this is basically your arithmetic average, right. This is your arithmetic average; that means, we still have on the left hand side $\vec{u}_f$ is basically equals u 0 plus u 1 upon 2 right.

So, this is basically this value and then we have divided by 2 right. So, we have. So, this is what we have for $\vec{u}_f$ right in terms of hats this is what we get$\hat{u}_0^n$ plus $\hat{u}_1^n$ by 2 minus minus $\Delta V_f/a_f^n$ $\hat{n} \cdot \nabla\overline{P_0}$ right.

So, if I send if we send basically so, this is your arithmetic average, but what we want to do is we want to do we have to find a momentum interpolated average right we want to use momentum interpolation; that means, this pressure gradient we have we have to subtract this quantity and then add an adjacent pressure value ok.

So, but unfortunately here we already have a minus. So, subtraction means that you are doing a minus of minus. So, you are doing a plus ok. So, again we will not work with u 0 hat u 1 hat here we will we would like to work with u 0 and u 1 ok; that means, for this quantity we have to subtract this minus quantity and then add a quantity that represents an adjacent pressure ok.

So, that means, I can write I would not write u f bar anymore because u f bar denotes an arithmetic average I will just write u f which will denote a momentum interpolated value that is nothing, but your u 0 n these are all n's these are all n plus u 1 and by 2 and then we subtract off this quantity that is subtracting minus of this guy would be basically plus, right or else you can think of it as sending to the left hand side, right.

You remember in the last lectures we had we subtracted minus of this quantity and then added the addition pressure on both sides, right. So, you can also think of it like that because there is already a minus when you subtract it off on the left hand side direction what you get is a plus, you get a plus $\Delta V_f/\mathrm{a}_\mathrm{f}^\mathrm{n}$ times $\hat{\mathrm{n}} \cdot \nabla\overline{P_0}$ and then you. So, this is subtracted and then you add.

So, when you say add you already have that minus in there right because we did not use the expansion for grad p and then you add the adjacent pressures, right. So, now adjacent pressure this is where we would add it as a d times that is basically $\Delta V_f/\mathrm{a}_\mathrm{f}^\mathrm{n}$ times $\frac{\partial P}{\partial n}$ on the face ok. So, this is your subtract the non continuous pressure gradient and then add the continuous pressure gradient ok.

So, this may require you to basically you may have to go through this again and double check that this is correct. So, whatever I have done here you basically have to just double check it is correct you just have to make sure you understand it, fine alright.

So, essentially what we have done is we have subtracted off minus $\Delta V_f/\mathrm{a}_\mathrm{f}^\mathrm{n}$ $\hat{\mathrm{n}} \cdot \nabla\overline{P_0}$ and added on both sides and added this quantity that is minus $\Delta V_f/\mathrm{a}_\mathrm{f}^\mathrm{n}$ $\frac{\partial P}{\partial x}$ on both sides, and we are not working with hats anymore. We are working with for now; we are working with the original values ok; that means the u 0 and u 1 ok.

Now, this is this u f is our momentum interpolated face value ok. So, this is our momentum interpolated face value. Now, once you have the momentum interpreted face value where would you use this thing? You will use this in the continuity equation, right.

So, we need this in the continuity equation because only then continuity equation will not support checker boarding then we do not care if the momentum equation supports checker boarding because as one of the equations are not going to support it is not going to be there in the final converged solution alright.

So, then of course, we did not introduce how do we calculate the average values that we have used here right that we have said these two would be represented using some face values. Now, this is one particular way of doing things ok. So, there are several ways of in fact, representing this interpolation ok.

So, we are using one particular way; that means, we are using a arithmetic average where we are saying grad p on the face that is $\overline{\nabla P}$ would be equal to arithmetic average of the neighbouring cells that is ∇P_0 pus ∇P_0 by 2.

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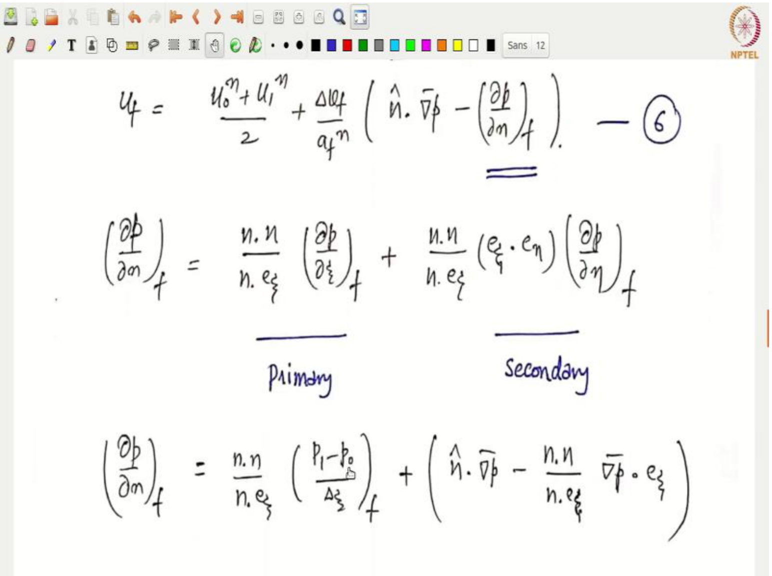

Similarly, the coefficient a_f^n that we basically got here would be we say again it is an arithmetic average of a_0^n and a_1^n. And, the volume value kind of a representative volume value for the face that shares two cells we will again take it as ΔV_0 plus ΔV_1 by 2 this is not going to cause much of a difference, but there are several ways of doing this thing, ok. You can also use a scaling factor based 1 or linear interpolation and things like that, alright.

Then if we go back and look at this equation what we have is u_f equals u_0^n plus u_1^n by 2 plus we have this $\Delta V_f/a_f^n$ common in these 2 terms. So, if we take it out we have $\Delta V_f/a_f^n$ times $\hat{n} \cdot \overline{\nabla P}$

minus we have partial $\frac{\partial P}{\partial n}$, ok. So, that means, this is our non contiguous pressure and this is our continuous pressure right adjacent pressure term.

Now, what do we wish to do? We wish to do essentially we want to represent this pressure gradient in the direction of the normal to the face that is $\frac{\partial P}{\partial n}$ on the face f using the cell centroid values, but we know that for in general for an unstructured mesh this gradient right cannot be that is in the normal direction cannot be just simply represented using the cell centroid values you would get two gradients, one in xi direction one in the eta direction.

You remember you get these quantities basically in the context of diffusion equation for unstructured meshes we got A dot A by A dot e_ξ right, similarly we got A dot e_ξ A dot A by A dot e_ξ times e_ξ dot e_n. So, similarly we get these two components one is the primary gradient and the other one is the secondary gradient.

Now, we know that the primary gradient can be expressed as p 1 minus p 0 by delta xi whereas, the secondary gradient term if we do not have a particular way of solving it then what we can do is we can express this entire secondary gradient as the total minus the primary gradient right. So, if I somehow know the total gradient on the face that is grad p bar then this entire secondary gradient can be written as total minus the primary gradient.

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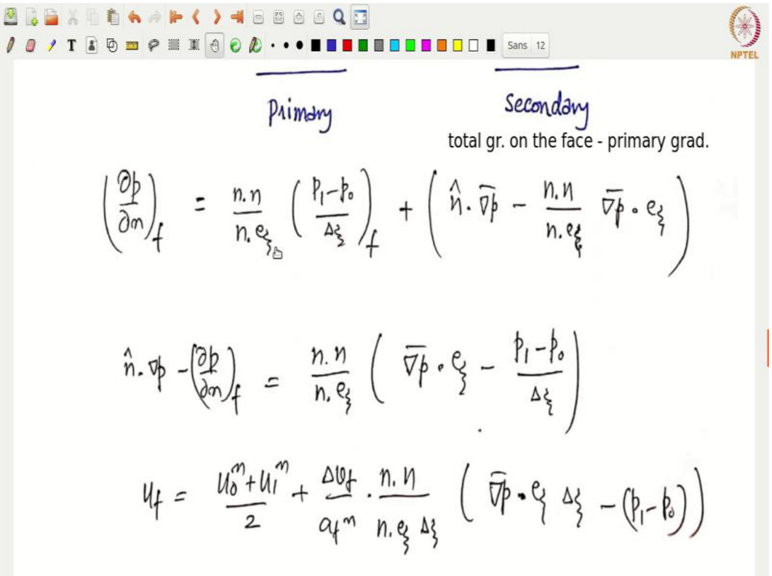

That means, $\hat{n}\cdot\nabla P$ this is the; this is the total gradient on the face right minus we have the primary gradient right that is basically n dot n by n dot e_ξ ∇P dot e_ξ, ok. Now, what you have to pay attention here is that the $\overline{\nabla P}$ this is somehow known ok. So, that is the idea here, right. We have gone through this discussion before in the context of a diffusion equation. So, we will not again go in detail here. But, of course, now we have replaced the secondary gradient as total minus primary.

Now, what do we want essentially we want $\hat{n} \cdot \nabla P$ minus $\left(\frac{\partial P}{\partial n}\right)_{\mathrm{f}}$ that is what the quantity we are looking for; that means, from here from this equation I can write $\hat{n} \cdot \nabla P$ minus this guy that is if you bring this quantity to the right hand side and send these to the left hand side, then what you get is $\hat{n} \cdot \overline{\nabla P}$ there is a bar missing minus $\left(\frac{\partial P}{\partial n}\right)_{\mathrm{f}}$ that is essentially this quantity equals, what does it equal?

It equals basically your n dot n by n dot e_ξ that is common between this term and this term and it equals ∇P dot e_ξ minus P_1 minus P_0 by $\Delta\xi$. So; that means, we can replace for this quantity that is right here with ∇P dot e_ξ minus P_1 minus P_0 by $\Delta\xi$ ok.

Now, you see there is a difference between these two this is not equal to 0 right because P_1 minus P_0 are the cell values and $\overline{\nabla P}$ is a gradient of pressure evaluated on the face ok. So, this is not 0 alright. So, that we understand; that means, if we plug in back into the u f equation then what we have is you have $\mathrm{u}_0^{\mathrm{n}}$ plus $\mathrm{u}_1^{\mathrm{n}}$ by 2 u 0 1 plus $\Delta V_f/\mathrm{a}_{\mathrm{f}}^{\mathrm{n}}$.

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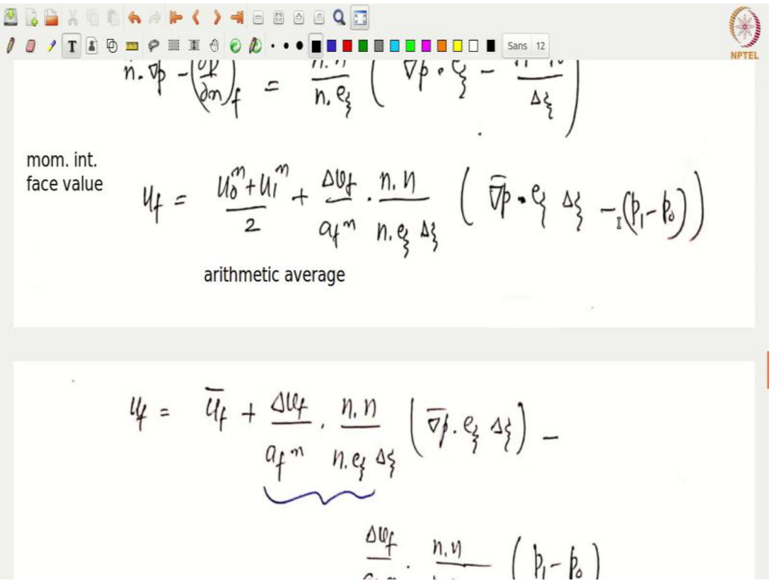

And, we are replacing this quantity that means, $\Delta V_f/a_f^n$ by replace this quantity with this; that means, what you get is n dot n by n dot e_ξ. And you multiply this with $\Delta\xi$ then you have $\Delta\xi$ in the denominator, then what you have is a $\overline{\nabla P}$ dot $e_\xi \Delta\xi$ minus P_1 minus P_0, alright.

Now, we got a very long expression for the for what is this one? This is basically momentum interpolated face value. This is basically your momentum interpolated face value for which we got a big expression, but nonetheless.

We can rearrange this thing we can write this as u_0^n plus u_1^n by 2 can be written as $\overline{u_f}$ right basically this quantity is your arithmetic average right. So, this can be written as $\overline{u_f}$ plus we have $\Delta V_f/a_f^n$ times n dot n by n dot e xi delta xi times this quantity minus this entire quantity times P_1 minus P_0 ok.

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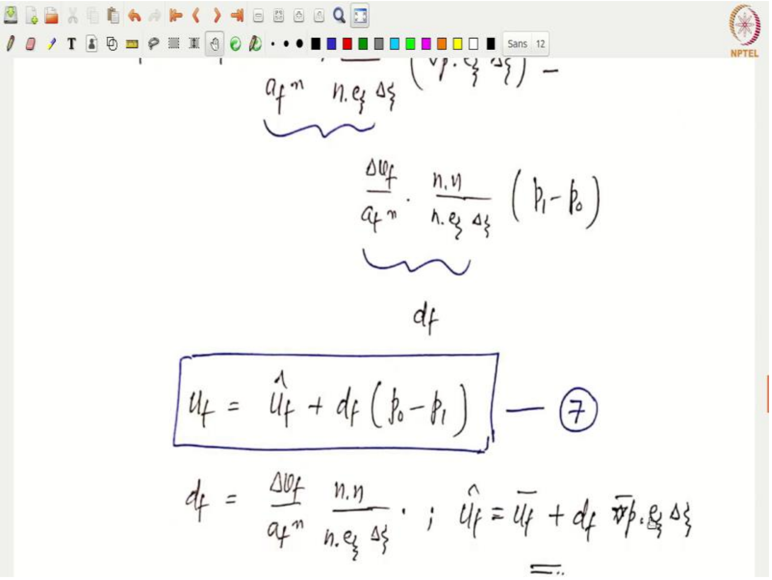

So, that is what we have now if you denote this entire quantity which is the same for both as some d f. So, that is $\Delta V_f/a_f^n$ times n dot n dot n by n dot $e_\xi \Delta\xi$ as some d_f this is similar to your d_e or d_w in the Cartesian context. Then what we have is, you have u_f equals we can write simplify this entire expression as if you if this is d_f then this $\overline{u_f}$ plus this entire quantity can be written as $\hat{u}_f$ plus we have d_f times.

So, if you want to get rid of this minus I would switch the P_1 and P_0. So, we say P_0 minus P_1 where of course, d_f is this quantity $\Delta V_f/a_f^n$ n dot n by n dot $e_\xi \Delta\xi$ and $\hat{u}_f$ would be equal to $\overline{u_f}$ plus d_f times this quantity that is $\overline{\nabla P}$ dotted with $e_\xi \Delta\xi$ ok.

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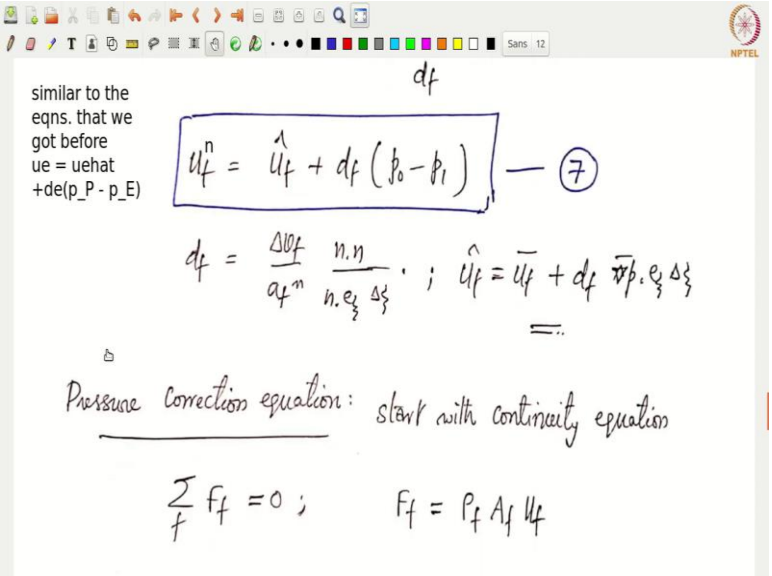

So, we have now simplified it by defining two coefficient that we call it as $\hat{u}_f$ and d_f and we got this equation 7 which is basically similar to the equations that we got before, right. So, basically this is more like your u_e equals right $\hat{u}_e$ plus d_e times something like P_P minus P_E right something like this remember. So, this has very good resemblance to the Cartesian components. So, what we have is u_f equals $\hat{u}_f$ plus d_f times P_0 minus P_1.

So, what you notice is you got a pressure difference of the neighbouring values that is P_0 and P_1 and you got hat velocities and here we do not have the superscript, but u f itself denotes the velocity vector on the face in the direction of the face normal ok. So, this is already in the direction of n.

So, we have not been writing this right. So, that means, if you multiply this with the area magnitude and you multiply with density then you are getting the flow rate already so that we can use this directly in the continuity equation ok, alright.

So, then we have now computed a value for the velocity in the direction of the face normal from momentum interpolation with the help of adjacent pressure gradients, let us call this as equation number 7 ok.

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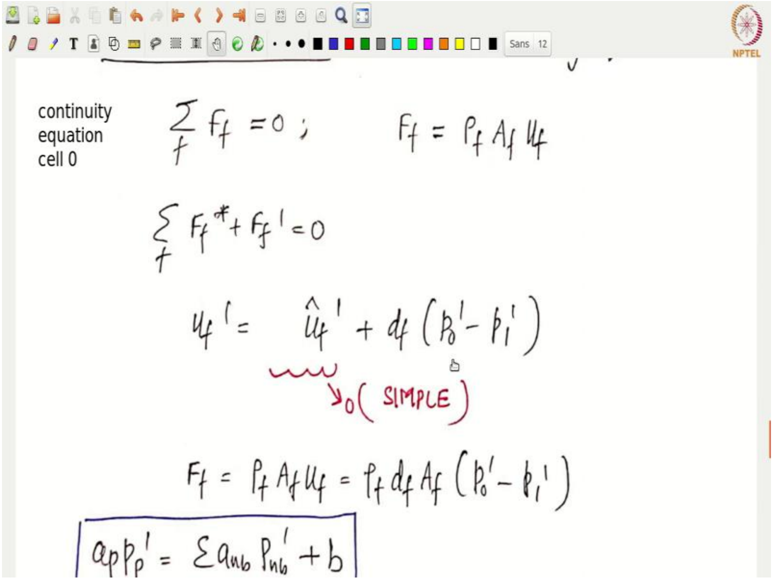

So, once you have this the final step is basically to come up with a pressure correction equation pressure correction equation is basically where do we start for that? We start off with the continuity equation. So, the continuity equation would be if you have several faces then the continuity equation is $\sum F_f$ equals 0 right, where your F_f is ρ_f this is a kind of a flow rate that is $\rho_f A_f u_f$ right if $A_f u_f$ is always pointing outwards.

So, if u_f is going out this quantity would come out to be positive, if it is coming inside this will automatically come out to be negative ok. So, that will take care of the plus minus signs, then what we have is then the continuity equation is basically summation over f, F_f. So, this is your continuity equation right for the cell ah 0 ok.

Then what do we do we split this into a star value and a prime value. So, we can write this as $F_f^* + F_f'$ equals 0, then of course, the F_f'. So, the stars are known values these are the guess values that are known ah. Of course, they do not satisfy a continuity equation.

So, this sigma F_f^* would not be equal to 0, but F_f''s is what we want to express them as in terms of u_f''s right. So, now, we need an expression for u_f', how do we get it? We basically get it from the momentum interpolated value. So, if you have u_f if you want to take a prime here basically subtract this off from the star equation. So, then what you have is u_f minus u_f^* that will give you u_f'.

So, this equation, equation 7 if you rewrite for primes what you get is u'_f equals $\hat{u}'_f$ plus d_f times P'_0 minus P'_1 ok. So, this is what we get again keeping with the simple approximation then contribution of the neighbours is neglected. So, what you get is u'_f equals u_f times P'_1 minus P'_1. This is more like your minus P'_P minus P'_E right.

Then you plug in there is a star hat missing here so, that means, your F'_f prime equals $\rho_f A_f$ u'_f equals $\rho_f A_f$ times d_f times P'_0 minus P'_1 essentially substitute for u'_f from the equation above then you got an equation in terms of pressure corrections for flow rate correction, right.

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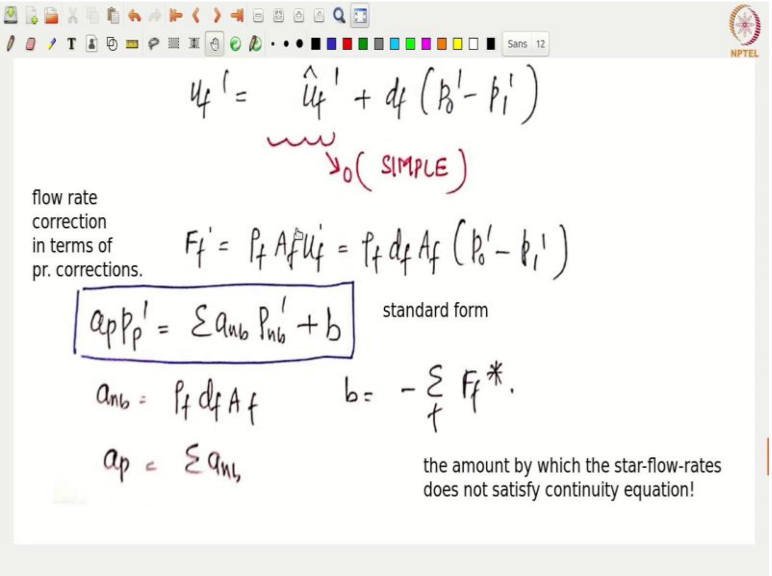

So, this is basically flow rate correction in terms of in terms of pressure corrections fine then what do you do you basically substitute for F'_f in terms of pressure correction into this equation and rearrange them into our favourite standard form that is basically this is our standard form which is basically $a_P P'_P$ equals $\sum a_{nb} P'_{nb}$ plus b of course, you can write this as $a_0 P'_0$, either of them is fine.

Then what we have is a and b would be equal to $\rho_f A_f d_f$ right this the multiplication coefficient for P'_1 that is your a n b and there will be the same contribution to the cell P_P or P_0. So, as a result your a_P would be summation of all the neighbouring coefficients; that means, a_P equals $\sum a_{nb}$ then your b term would be because it goes to the right hand side will become minus F^*_f sub f ok.

So, this is again basically the amount by which the star flow rates does not satisfy continuity equation ah. So, that is what we have then we basically solve for the pressure correction.

Once you have the pressure corrections you basically put them here and calculate what is the velocity corrections and flow rate corrections once you have the velocity corrections for the face velocities you go back and correct your face velocities as well with the corrections ok. So, that is what we do.

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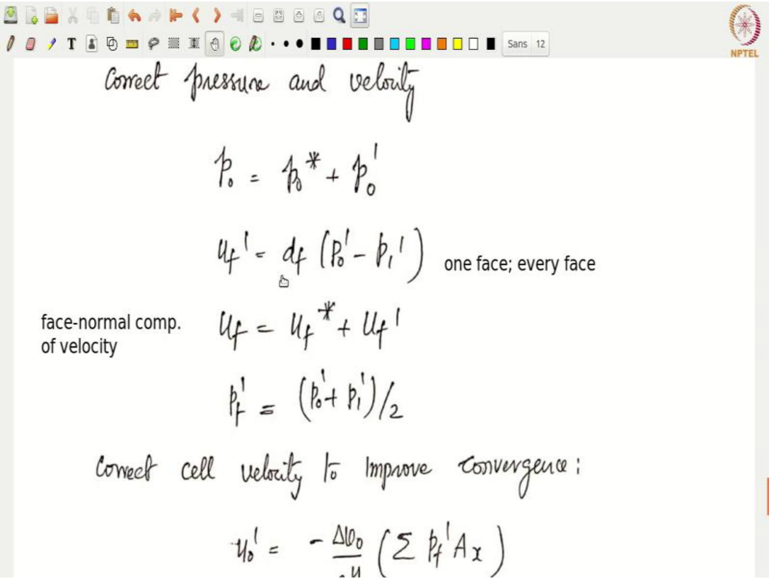

So, once you have the pressure corrections you correct the cell pressures $P_0 = P_0^* + P_0'$. u_f' equals. So, basically obtain the velocity corrections u_f' equals d_f times P_0' minus P_0' of course, see this is for one face is what we have written, right.

So, this is for one face now this has to be done for every face that we have right. So, that means, P_0' minus P_0', P_0' minus P_2' and so on right for each of the u_{f1}', u_{f2}', u_{f3}' prime and so on depending on how many faces that the cell C_0 has ok, alright.

Similarly, once you have the face velocity corrections you can essentially correct the face velocity itself that is $u_f = u_f^* + u_f'$ right. So, this basically would be correct the face-normal component of velocity which will be later on again used in the continuity equation ok. And, then you have P_f' this is the correct the face pressure as the arithmetic average of the cell pressures P_0' plus P_1' by 2.

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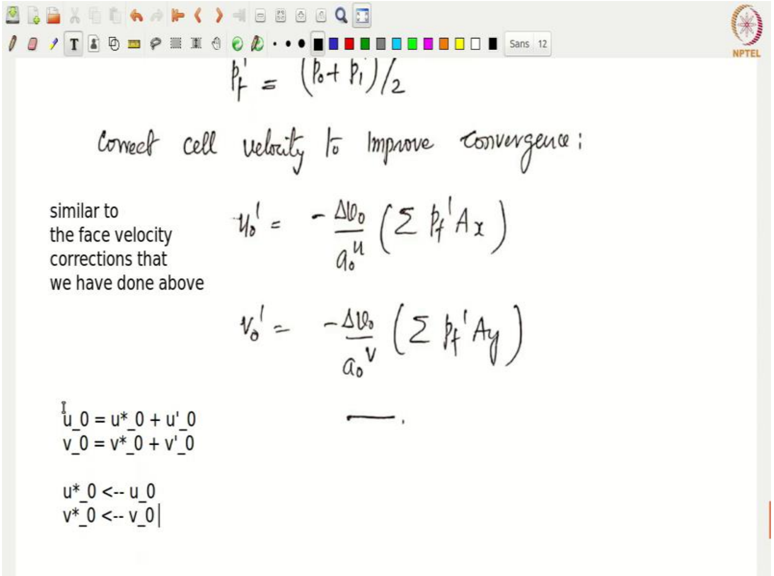

Now, in order to improve convergence we not only kind of correct the face pressures, but also correct the cell pressures. So, cell pressures are u_0' equals. So, if we follow a similar approach instead of $\Delta V_f/\mathrm{a}_f^{\mathrm{n}}$ we have $-\Delta V_0/\mathrm{a}_0^{\mathrm{u}}$ times here I am using similar to this I am using a you remember we get this $\sum P_f' A_x$, but because this is x component with only the if you have this i dot p of a f only the A x component survives.

Similarly, u_0' would be equal to $-\Delta V_0/\mathrm{a}_0^{\mathrm{v}}$ times $\sum P_f' A_y$ ok. So, this is basically correct the cell wall velocities to improve convergence ok. This is basically similar to the face velocity corrections that we have done above purely to improve convergence ok. So, once you have this thing then essentially you know what is the face velocity correction, then the cell velocities can be updated as $u_0 = u_0^* + u_0'$.

Similarly, $v_0 = v_0^* + v_0'$ ok. Then basically you update your new guess u_0^* would be your u_0 and v_0^* would be your v_0 and then you go back to the previous step that is basically solution of the discrete momentum equations this one is right, equation 1 and 2 discrete momentum equations with the new guess values ok.

So, that is the idea. It is basically very much similar to what we have done for the co-located mesh on a Cartesian arrangement and of course, you know how to discrete how to solve for the momentum equations 1 and 2 anyway right that is basically similar to the general scalar transport equation ok.

So, that kind of finishes the this chapter on the on the computation of fluid flow equations, wherein we have seen the staggered mesh simple algorithm and the simple algorithm on co-located meshes, then simple algorithm extended on co-located meshes to unstructured meshes and everything ok.

So, the only thing remaining now is the multi grid method in chapter 5 ok. So, we will be taking maybe 2 to 3 lectures and then kind of motivate for why we need to go for a multi grid kind of method and what is the need for it and how does the multi grid methods work ok. So, that is probably another 2 to 3 lectures maybe and then we should be able to wrap up the syllabus alright. So, I am going to stop here if you have any questions do let me know through E-mail ok.

Thank you. Talk to you in the next lecture.