Computational Fluid Dynamics Using Finite Volume Method

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Lecture – 41
Finite Volume Method for Fluid Flow Calculations: SIMPLE algorithm for Colocated mesh

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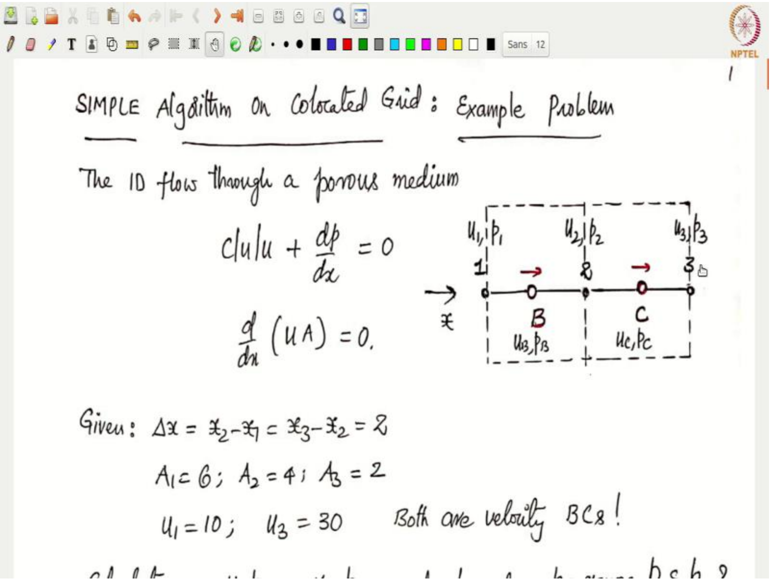

Hello everyone let us get started. So, in the last lecture essentially we looked at simple algorithm right on a colocated grid, essentially how to solve it the complete algorithm we have a kind of listed it. Now, in today's lecture we are going to solve a problem using simple algorithm on a colocated grid, ok.

So, essentially we will first set up the problem and then we will see how to quote this and try to run it, ok. So, that is the agenda for today. So, we will take the same problem that is there in the Patankar's book which was kind of we have already done this for a staggered grid, ok.

So, we are now reformulating the problem statement such that, we can apply it for a co-located grid, alright. So, let us consider the 1 dimensional flow through a porous medium governed by $C|u|u + \frac{\partial P}{\partial x} = 0$, where C is a constant this is the constant that its kind of represents the porosity of the medium and the continuity equation is given by $\frac{d(uA)}{dx} = 0$.

So, we have given both equations and then now we have unlike the staggered grid we have the colocated grid where we have 2 cells that is the cell B and cell C, where the velocities and the pressures are stored here ok. So, u_B P_B and u_C P_C are all stored at the cell centroids of the cells ok. Now, in addition we have 3 faces that is 1, 2, 3 here which are basically west and east faces for B cell and 2 and 3 form the west and east face for the C cell.

Now, in it, so, on these faces we also would need the velocities that is u_1, u_2 and u_3 which would be utilized in the in writing the continuity equation for cell C and cell B ok. So, we would also need to keep track of these velocities and we would also need pressures on the faces that is P_1, P_2, P_3 which would be required when we write the adjacent pressures for these equations ok. So, I am kind of denoting the faces using 1, 2, 3 and the respective variables are u_1 P_1 u_2 P_2 and u_3 P_3 ok.

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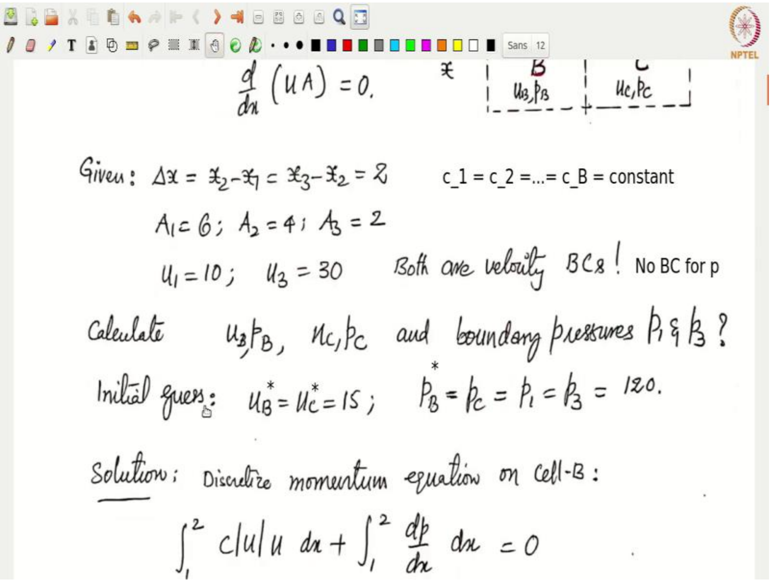

Now, the other problems are given. Other variables are given in the problem are the grid spacing that is x_2 minus x_1 equals x_3 minus x_2 equals Δx is 2 that is given and the cross sectional areas at the faces that is A_1 is 6, A_2 is 4 and A_3 is 2 that is what is given.

Of course, the value of C at all the location C_1 equals C sub 1 equals c sub 2 and so on. All the C's can be taken to be also constant and also it can be even at some C_B and so on ok. So, this can be taken to be constant. Then the boundary conditions are given that is the velocity at the face 1 is given as 10 units per second and on 3 is given as u_3 equals 30 meters per second ok.

Now, what we see is that both the boundary conditions that as given are for velocities ok. So, essentially there is no boundary condition for pressure that is given in this problem. Now the question says you calculate the cell velocities and the cell pressure; that means, u_B P_B and u_C P_C and as well as you we were asked to calculate what is the boundary pressures P_1 and P_3 because these were not given only the velocities are given as boundary conditions so calculate what will be the boundary pressures ok.

Now, it also tells us take an initial guess of u_B^* equal u_C^* equals 15. So, essentially these are the star values; u_B^* equals u_C^* equals 15 and then it also gives you the pressure guess these are also the star values. P_B^* P_C^* star P_1^* P_3^* can be taken as 120 ok.

So, that is given in the problem statement ok, the initial guess is given alright. Let us look at the now the solution using simple algorithm on a colocated grid ok. So, essentially the first starting point is basically we have to first integrate the momentum equations on cells B and C ok, so; that means, discretize the momentum equation on cell B ok. So, that is the 1st step. So; that means, we have $C|u|u + \frac{\partial P}{\partial x} = 0$, cell B has faces as 1 and 2 right if you look at here cell B has 1 to 2.

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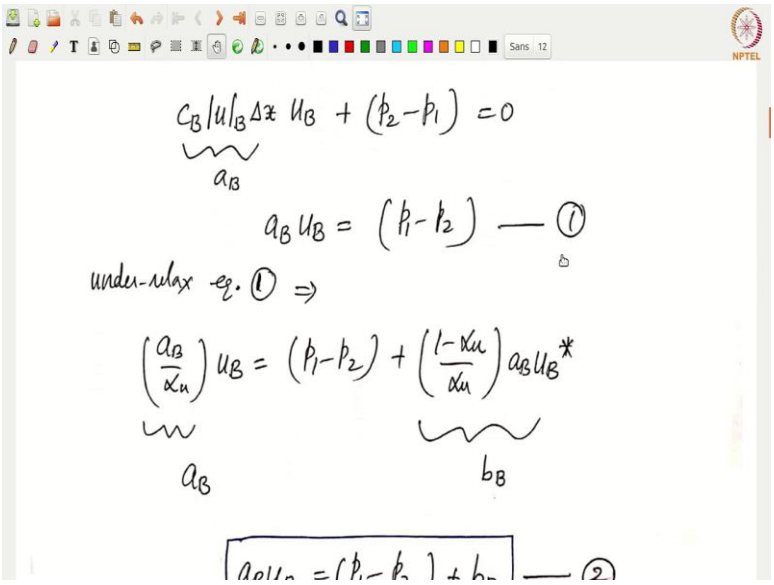

So; that means, 1 to 2 integral 1 to 2, $C|u|u$dx plus integral 1 to 2, $\frac{\partial P}{\partial x} dx$ equals 0 ok. So, this as you know we are treating it as a this basically is a source down right. It does not belong anywhere else. So; that means, C mod u Δx would be evaluate the cell centroid that is B that is C_B mod u_B Δx we would consider this as a B ok.

So, a_B times u_B plus integration of this quantity would give you pressure; that means, P_2 minus P_1 equals 0 ok. So, if you write this equation with considering a_B as C_B mod u_B Δx then the momentum equation the discrete momentum equation for cell B is $a_B u_B$ equals P_1 minus P_2 ok.

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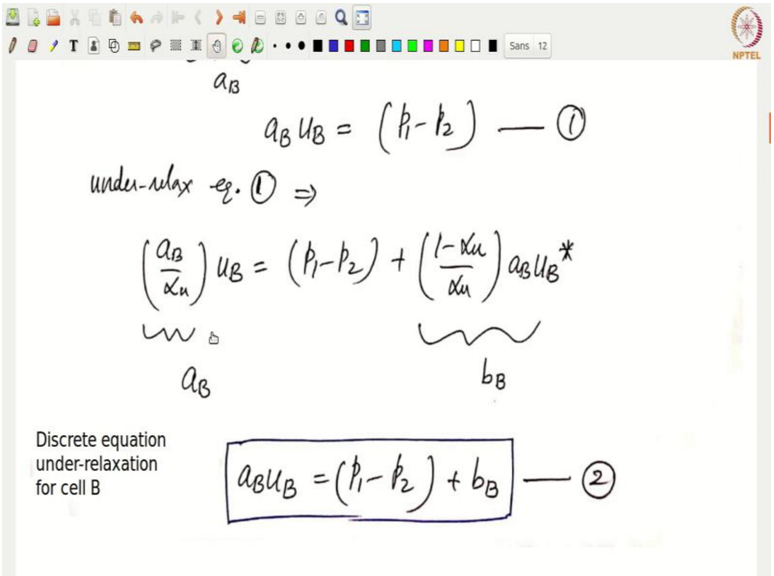

Let us call this equation number 1. Now, we have to of course, under relax this equation. So, if you under relax this equation what you get is basically divide by α_u here and add the respective quantity on the right hand side that would be a_B by α_u times u_B equals P_1 minus P_2 plus you have to add this extra quantity on the right hand side that is $(1 - \alpha_u)/\alpha_u$ times $a_B u_B^*$ ok.

So; that means, if you redefine your a new a_B as a_B/α_u then I can still write this as $a_B u_B$, where I would just say I divide by α_u for my a_B and again update its value ok; that means, that is my new a_B. So, $a_B u_B$ equals P_1 minus P_2 plus we would this term basically is known quantity it would be called as b_B ok.

So, this is $a_B u_B$ equals P_1 minus P_2 plus b_B and that is your equation 2 ok, which is basically the under relaxed form of equation 1 that is equation 2 alright. So, this is the discrete equation; this is the discrete equation in after under relaxation for cell B ok, alright.

Of course, the problem statement is such that we do not have any other dependents on the neighboring cells right. The only dependencies on the cell itself and on the pressures there is no neighboring cell that is coming into play here ok. Now, let us see let us go and discretize the momentum equation for the other cell that is for the B C cell ok.

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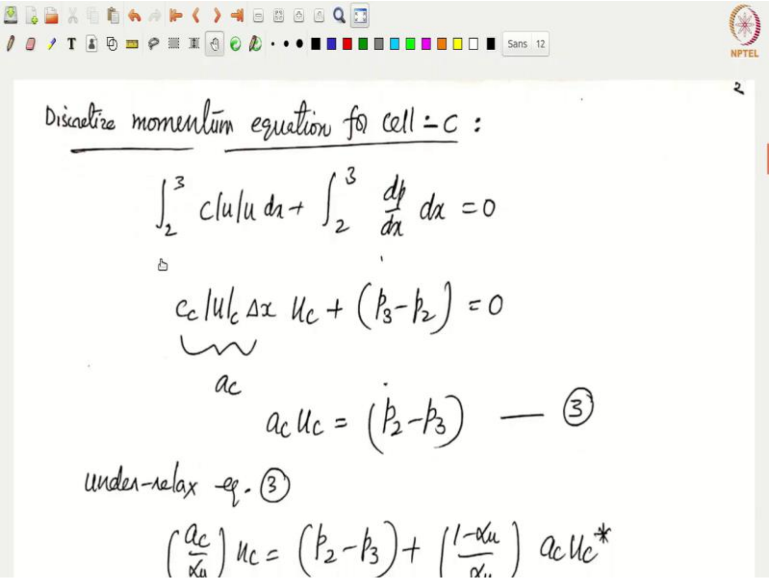

So, discretize momentum equation for cell C that is your faces are now 2 to 3, C mod u u dx plus integral 2 to 3, $\frac{\partial P}{\partial x}$ equals 0. So, again the cell centroid value of this would be C that is C_C mod u_C Δx times u_C plus integration of this is pressure this will be P_3 minus P_2 equals 0 ok. So, if you denote again this coefficient C_C mod C_C Δx as a_C then what we have is $a_C u_C$ equals P_2 minus P_3 on the right hand side ok.

So, this is let us call this as equation 3 which is the discrete a momentum equation for cell C. Of course, because of the non-linearity in the source term we have to also under relax this thing.

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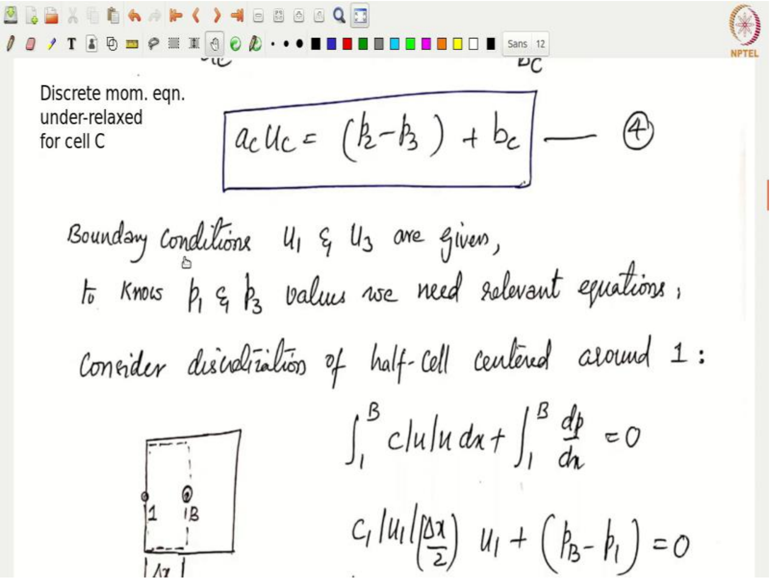

So, the under relaxed equation for the equation 3 is basically a_C/α_u times u_C equals P_2 minus P_3 plus $(1-\alpha_u)/\alpha_u$ times $a_C u_C^*$ ok. So, again this quantity can be considered as some b term that is b_C and this is your pressure term that is P_2 minus P_3. On the left hand side if you denote a_C/α_u as again as an a_C then we can write this as $a_C u_C$ equals P_2 minus P_3 plus b_C ok. So, that is what we have alright that means, this is the discrete momentum equation right.

That is also under relaxed under relaxed right for cell C right. So, we have these equations equation 2 and 4 for cell B and cell C. Now, one more thing that we have to do different from what we have done in the staggered mesh case is basically is that the boundary conditions u_1 and u_3 are given right.

But we do not know the pressures for these locations, P_1 and P_3 are not known, but in order to know them we need relevant equations right. So, the equations are basically the same governing equations. It is just at the same governing equations control how pressure and velocity are related.

So, in order to do this, in order to know the value of the pressures instead of making an approximation, what we do is we take an approach where we discretize the governing equation on this kind of an half cell ok.

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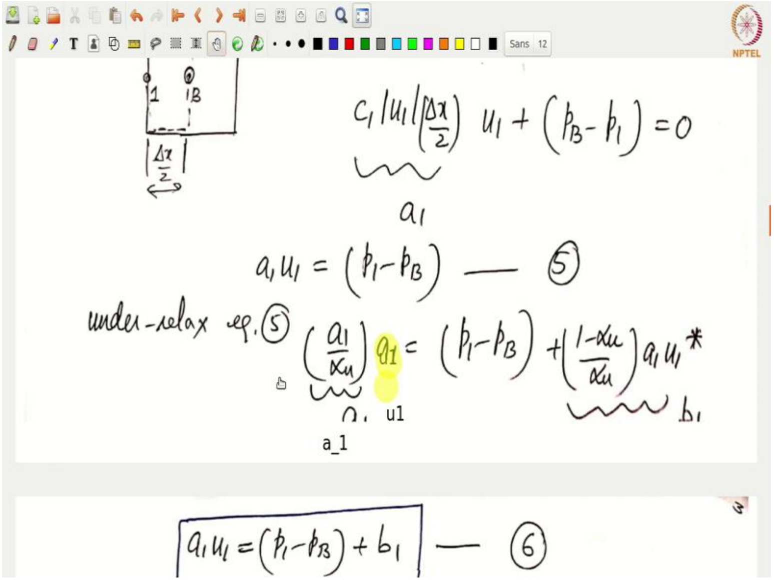

So, let me take a half cell with centroid assumed to be at 1. So, we have a half cell that is kind of like this; we you can see here that I am not taking this centroid to get the at this cell center ok.

So, essentially I have taking the cell is like this and 1 happens to be its representative value in order to; in order to derive an equation for pressures in velocities on the boundaries ok, which would be needed to update the pressures. So, if I consider a control volume that is like this. So, then I can go from 1 to B, C mod u u dx plus integral 1 to B, $\frac{\partial P}{\partial x}$dx equals 0.

This is basically your integration of the momentum equation for some kind of an half cell with 1 as its representative value ok. So that means, then we can write this as C_1 mod u_1 times integral dx would give you Δx by 2 because nor this is only over a range of Δx by 2 times u_1 plus we have P_B minus P_1 equals 0 ok.

So, this is basically going to give you if you call this as some coefficient a_1 what you get is $a_1 u_1$ equals P_1 minus P_B right. Now, P_1 and P_B of course, are not equal right because essentially you have c mod u u dx that is the this porosity is going to have a pressure drop from P_1 to P_B that is why we are now deriving this equation right, how the velocities and the pressure are related within this half control volume such that this pressure can be computed accordingly right.

So, that is what that is why we have done this thing alright. So that means, we got a equation this is basically discrete momentum equation that is written for the boundary half cell that is for centroid 1 ok. Let us call this equation 5. So, similar to what we have done for B cell and C cell here also we would do the under relaxation.

So, if you under relax cell 5 what you basically equation 5 what you get is a 1 by alpha u times this is incorrect this supposed to be; this supposed to be u_1 is not it, is supposed to be u_1 equals.

So, this is basically u_1 ok. This is not correct. Supposed to be u_1 equals P_1 minus P_B plus $(1-\alpha_u)/\alpha_u$ times $a_1 u_1^*$ is what we have. Let us call this again as some a sub 1, this co efficient a 1 alpha u then let us this with basically become P_1 right.

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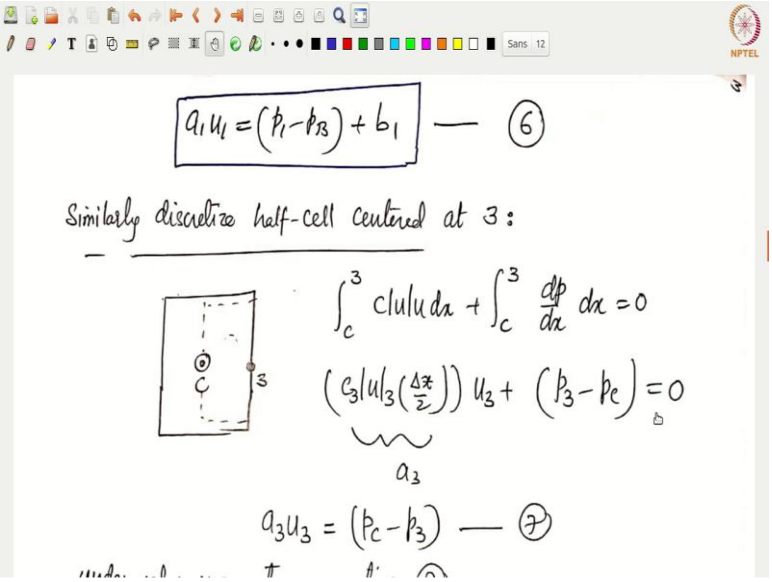

Then we have an under relaxed equation that is basically $a_1 u_1$ equals P_1 minus P_B plus b_1 ok. So, equation 6 is basically under relaxed discrete equation momentum equation for some kind of a half boundary cell which we want to have because in order to apply boundary conditions and in order to get the value of the pressure P_1 on the boundary we would need this equation ok.

Now, similarly we will also discretize the half cell that we would get on the other side that is at the vertex 3 right on face 3. So, basically this would be if you consider a half cell that is from extending from C to 3 right, I will we will again integrate the momentum equation in order to get the pressure relation between P_3 and P_C. So, this will be integral C to 3, C mod u u dx plus integral C to 3, $\frac{\partial P}{\partial x} dx$ equals 0 ok.

So, what we have is basically C_1 again if you consider that the for this cell 3 is kind of representative of the centroid then what we have is C_3 mod u_3 Δx by 2 times u_3 plus we have P_3 minus P_C equal 0 ok.

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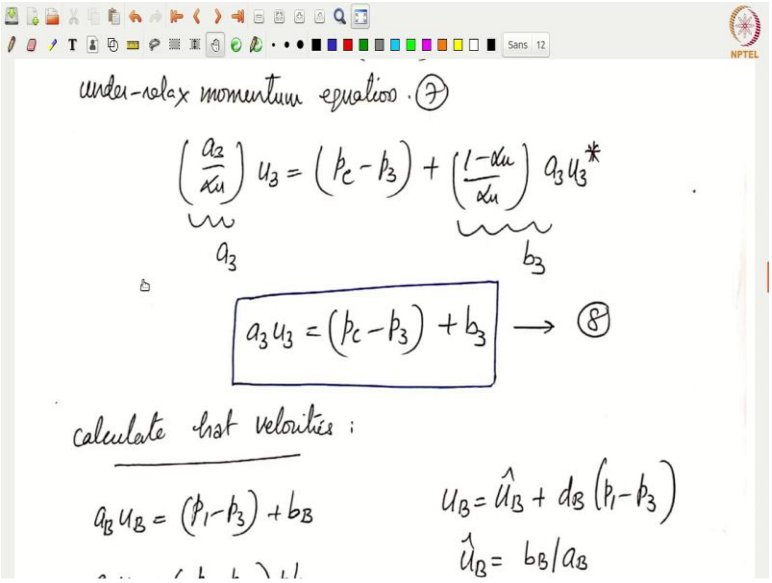

So, again if you call this coefficient as some a_3 what we have is $a_3 u_3$ equals p C minus P_3 ok. So, equation 7 is basically your discrete momentum equation for the half cell that is about list written about face 3 right ok. Now, we will also under relax this equation this is basically gives you a_3/α_u times u_3 equals p C minus P_3 plus we have on the right hand side $(1-\alpha_u)/\alpha_u$ times $a_3 u_3^*$ ok.

So, let us again call this a_3/α_u as a_3 itself essentially you divide and then store it in the same variable then we have $a_3 u_3$ equals P_C minus P_3 plus this entire term would be called b_3 ok. So, now, equation 8 is your discrete momentum equation written for the half cell that is basically centered or about face 3 right ok.

So, now equations 8 and 6 that we have derived before these 2 equations now relate the boundary pressures to the cell pressures with the boundary velocities ok. That means, we got relation between the boundary pressure and the cell pressure essential pressure in relation to the boundary velocity. These are required in order to update the face pressures from the velocity space velocities that are given and the cell pressure that we would calculate ok.

So, that is the significance of writing this kind of a half cell thing which was of course, not there in the staggered mesh case right because you already had because the pressures in the velocities are anyway staggered. So, you never had to do such a thing before alright, now ok. So, let us now move on with the algorithm. So, essentially once you have written the discrete momentum equations you can solve for them you can also calculate the hat velocities ok.

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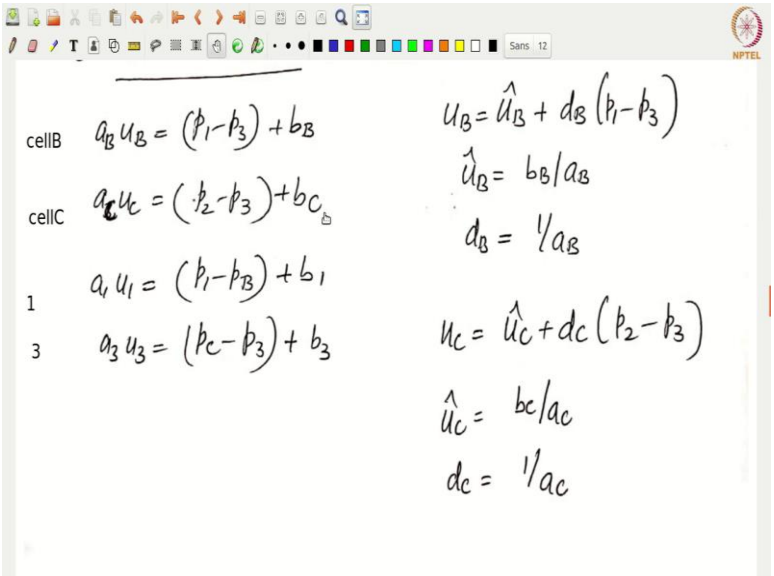

So, we go back and list out all the 4 equations that we have for the cells. This is for essentially for momentum equation for cellB and this is for cellC right and this is for node 1 and this is for node 3 right. What we had was $a_B u_B$ equals P_1 minus P_3 plus b_B and $a_C u_C$ equals P_2 minus P_3 plus b_C.

Similarly, we had for nodes 1 and 3 as $a_1 u_1$ equals P_1 minus P_B plus b_1 and $a_3 u_3$ equals P_C minus P_3 plus b_3 right. So, this is what we have these are the 4 equations that we have derived. Now, we can of course, write these in $\hat{u}_B$ notation by dividing with a_B right on the right hand side that means I can write this as u_B equals $\hat{u}_B$ plus some d_B times P_1 minus P_3; that means, where $\hat{u}_B$ is your bb_B/a_B right.

b_B upon a_B that is your $\hat{u}_B$ plus your d_B would be $1/a_B$ that is multiplying P_1 minus P_3. So, d_B would be $1/a_B$ right. Now similarly we can write a equation for cell velocity u_C in terms of hats as divide everything with a_C then what you have is u_C equals $\hat{u}_C$ plus some d_C times P_2 minus P_3 that is your u_C equals $\hat{u}_C$ plus d_C times P_2 minus P_3, where your $\hat{u}_C$ is your b_C/a_C right and your d_C value is your $1/a_C$ right that is what we have.

We have basically just divided with the central coefficient on the both sides such that is how we got these hat velocities and the coefficients B that multiply the pressure differences that is clear. Now we of course, would also like to write these 2 also in hat in terms of hat velocities.

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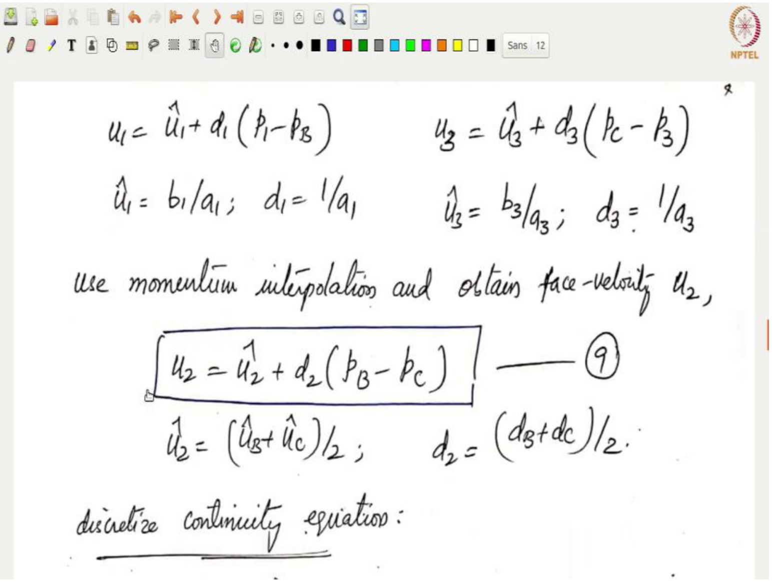

Then you divide this entire equation with a 1 then what you would get is you get an equation u_1 equals $\hat{u}_1$ plus d_1 times P_1 minus P_B where $\hat{u}_1$ is b_1/a_1 that is b_1 by a_1 and d_1 would be $1/a_1$ right ok. Similarly, we would also like to write a corresponding equation in terms of hat for node 3 as well. So, this will basically give you if you divide everything with a_3 you will getting u_3 equals $\hat{u}_3$ plus d_3 times P_C minus P_3.

So, where you are $\hat{u}_3$ is b_3/a_3 and d_3 would be$1/a_3$ right. So, again we basically we just converted the 4 discrete momentum equations, excuse me into these kind of hat form ok.

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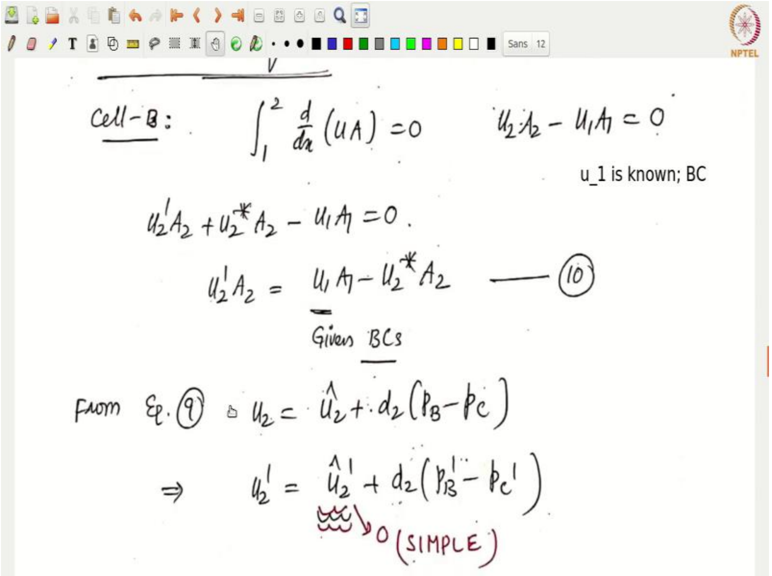

Now, what is the main step? The main step in simple algorithm on colocated grids is basically use momentum interpretation and calculate the face velocities right. So, we have 2 cells B and C, the only face that we have is 2 right, the other 2 faces that are on the boundaries 1 and 3 are not very much interesting as such because the velocities there are already given, u_1 and u_3 are given.

So, we do not have to use as such momentum interpolation there to obtain the velocity there. So, we will use momentum interpolation and obtain the face velocity u_2. So, what do you do using momentum interpolation you can write the face velocity u_2 that is u sub 2 equals $\hat{u}_2$ plus d_2 times the adjacent pressures.

For a face 2 the adjacent pressures are B and C right, so; that means, u_2 equals $\hat{u}_2$ plus d_2 times P_B minus P_C right that is let us call this equation 9, where $\hat{u}_2$ would be you would obtain it as average of arithmetic average of $\hat{u}_B$ and $\hat{u}_C$ and d_2 you would have obtain as a arithmetic average of d_B and d_C, right.

This is basically the simple algorithm on co-located grid right. So, this is the main assumption that we have made. Obtain the face velocities by through momentum interpolation that is by through the hat velocity and the addition pressure alright.

So, this is clear. Now once you have the momentum interpolated face velocity we can go and discretize the continuity equation. Now where do you discretize the continuity equation? In the context of staggered meshes we were discretizing the continuity equation or the pressure cell alright. Now in the context of colocated meshes you will discretize both the momentum equations and the continuity equations always on the single cell that is basically the primary cell which are basically cell B and cell C.

Because you do not have any other cell to discretize right ok. So, discretize continuity equation on cell B that is basically gives you integral 1 to 2, $\frac{d(uA)}{dx}$ equal 0. So, this is going to give you u times A evaluate 2 minus 1 that is u_2A_2 minus u_1A_1 equals 0. So, if I write this as if I split this again into prime and star values what you get is $u_2'A_2$plus $u_2^*A_2$ minus u_1 is already is u sub 1 is known right.

This is basically given boundary condition. So, you cannot split u_1 into star and u prime, you would leave it as it is this is u_1A_1 equals 0 ok. So, from here we are interested in of course, obtaining a pressure correction equation eventually. So, what we do is we leave $u_2'A_2$ on the left hand side and send everything else to the right hand side that is known right. So, what is known here? u_1 is known u_2^* is known of course, the cross section area A_1 A_2 are known.

So, what you have is you get an equation $u_2'A_2$ equals u_1A_1 minus $u_2^*A_2$ right. Of course, u_1 is a given boundary condition ok. So, let us call this equation number 10.

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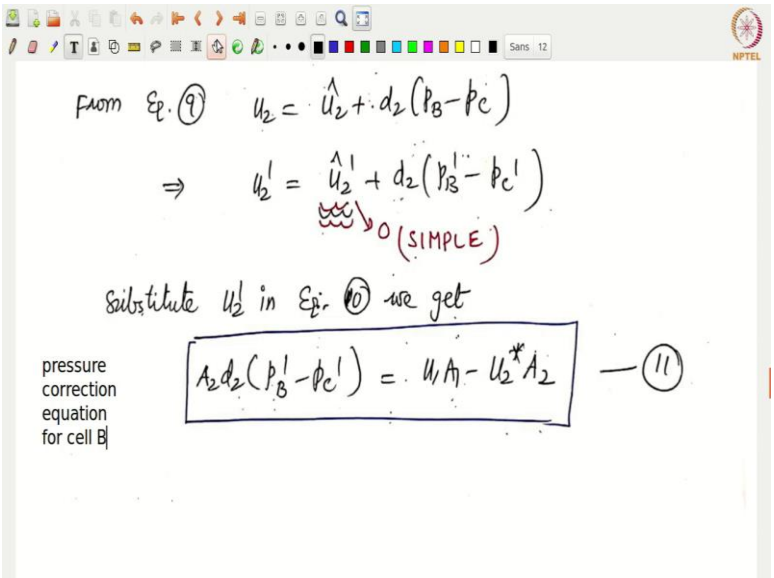

Now what we can do is we can of course, take the momentum interpolated face value in order to substitute for u_2' we have to go back and look at this equation 9 that is your momentum interpolated face value and obtain the prime equation in terms of pressure correction.

So, this is basically from momentum interpolated face value what we have is u_2 equals $\hat{u}_2$ plus d_2 times P_B minus P_C right. So, from here if I subtract off this from or if I subtract a star a corresponding equation for stars from this equation what you get is an equation for primes right. So, this minus u_2^* would give me u_2'. Similarly, on the right hand side.

That is basically give you basically going to give u_2' equals $\hat{u}_2'$ plus d_2 times P_B' minus P_C'. Again $\hat{u}_2'$ would contain all the neighboring coefficient and everything which will be neglected as part of the simple algorithm right; that means, u_2' hat is 0. So, what you have is u_2' equals d_2 times P_B' minus P_C'.

So, this is what we are going to substitute for u_2' in equation 10 ok. So, that is going to give you essentially d_2 times P_B' minus P_C' if we substitute for u_2' in equation 10. On the right hand side we have u_1 A_1 minus u_2^* A_2 ok. So, essentially we obtained a pressure correction equation right for cell B right.

So, this is a pressure correction equation in terms of P_B' and P_C' equals u_1 A_1 minus u_2^* A_2. So, that is equation 11 ok. So, we have 2 pressures 2 pressure corrections that are unknown. We have obtained one equation. How do you get the other equation? Basically write the continuity equation for cell C right because we have now written equation for this is equation for cell B. So, now, write an equation for cell C ok.

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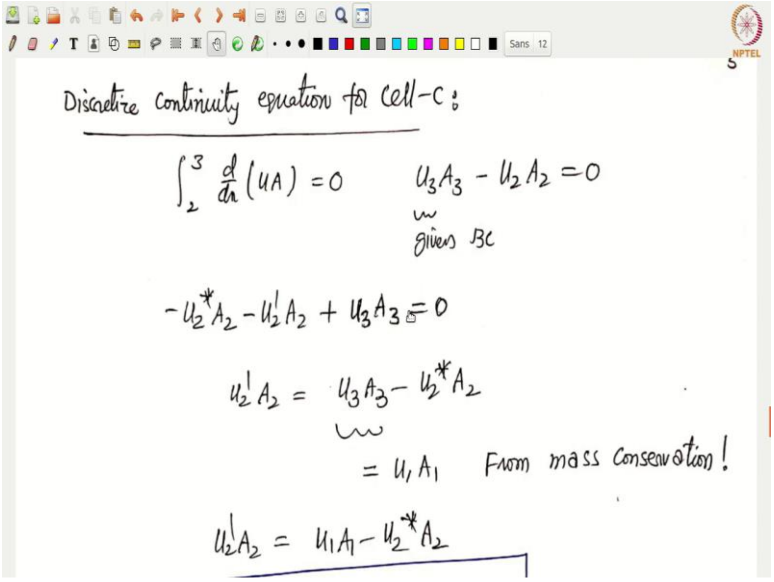

So, that we basically discretize the continuity equation for cell C. So, that is going to give you integral 2 to 3, $\frac{d(uA)}{dx}$ equals 0, this will give you u_3A_3 minus u_2A_2 equals 0. Again u_3 is given as a boundary condition. So, this would be this cannot be split into star and prime quantities whereas, u_2 can be split.

So, this is basically minus $u_2'A_2$ minus $u_2^*A_2$ minus $u_2'A_2$ plus u_3A_3 equals 0. Again you leave out $u_2'A_2$ to the right hand side then $u_2'A_2$ equals basically u_3A_3 minus $u_2^*A_2$ ok.

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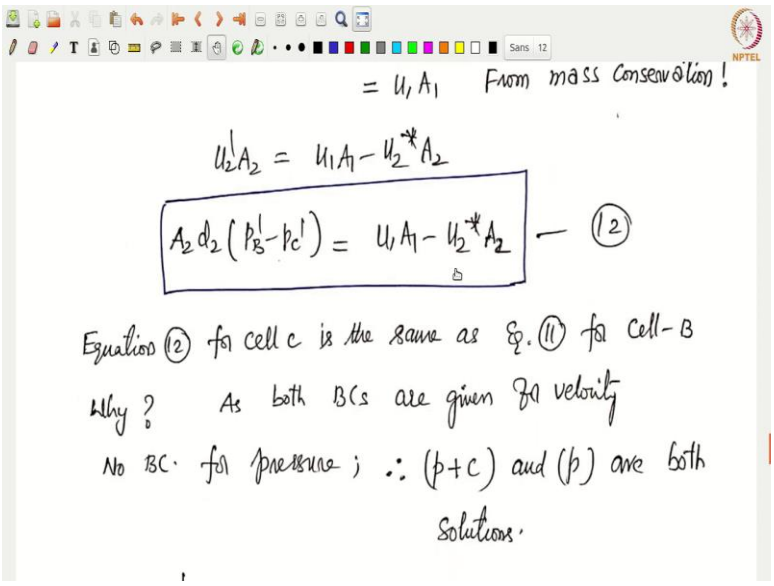

Again of course, from mass conservation we know that u_3A_3 is nothing but u_1A_1 ok. So, I can write this equation as $u_2'A_2$ equals u_1A_1 minus $u_2^*A_2$. This is exactly the equation same equation we have got before here in the context of cell B as well right. $u_2'A_2$ equals u_1A_1 minus $u_2^*A_2$ ok.

So, that is the pins we got the same equation for both the cells. Again if you plug in for u_2' in terms of pressure primes what you get is A_2u_2 2 times P_B' minus P_C' equals u_1A_1 minus $u_2^*A_2$.

So, equation 12 for cell C is the same as equation 11 that means, what we got here for cell B right. Now why is that why is the 2 equations the same? Why are the both equations are same? That is that means, that we do not have any additional equation; that means, that the 2 cells together is not giving is basically giving you only one equation. That is because basically the both boundary conditions that are given for or for velocity right.

As a result there is no boundary condition given for pressure. Therefore, the pressure that you obtain from this calculations p as well as if you add any constant to that pressure, so, both p and p plus c will be solutions for these equations ok. As a result you cannot fix pressure level rather you can only find it up to a constant right.

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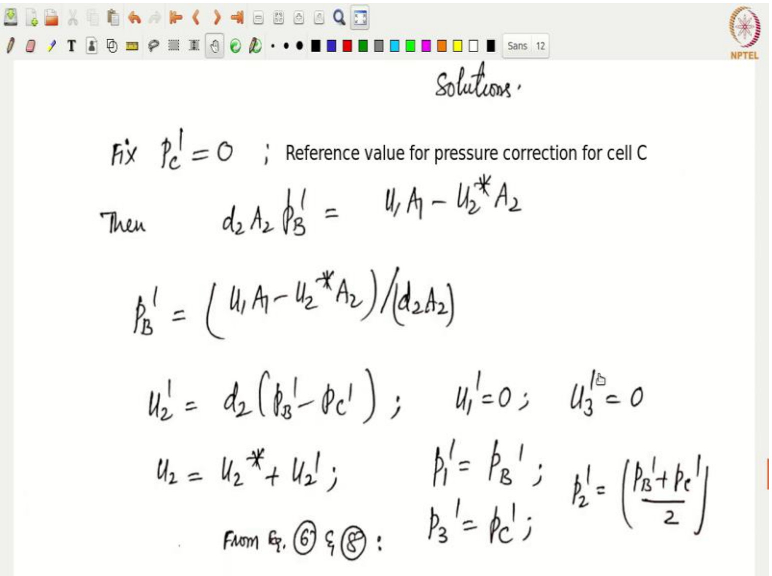

So, pressure level cannot be fixed rather you can find every pressure with in relation or with reference to one of the pressures here. So, you have to kind of we can only find pressure up to a constant because no boundary condition for pressure is given. But that is not a problem that is not going to make any issues because we know that in compressible flows its only the pressure gradient that matters it is not the absolute pressure that rise to flow ok.

As a result we would like to fix one of the pressure. So, either you can take B p prime equals 0 or any other value 10 40 whatever or P_C' equals some constant. I would like to take P_C' as 0 that is basically some kind of a reference value for pressure correction for cell C ok.

We are taking. So, everything will come in terms of P_C' alright. Now, if you once you get the solution if you take if you change P_C' to something else everything will get added up by that value ok; that means, if P_C' is 0 what you have is $A_2 d_2 P_B'$ equals $u_1 A_1$ minus $u_2^* A_2$ ok.

That means $A_2 d_2 P_B'$ prime equals $u_1 A_1$ minus $u_2^* A_2$ that is what we have. That means, we can calculate what is the pressure correction required at the cell B as P_B' equals basically this term divided by $A_2 d_2$ that is $u_1 A_1$ minus $u_2^* A_2$ upon $A_2 d_2$. So, this is the pressure correction for cell B. Once you know the velocity corrections you can calculate this value ok.

But we also know that u_2' once you know the P_B' you can update what is the velocity correction at face 2 as u_2' equals d_2 times P_B' minus P_C' right. And once you know u_2' you can add it u_2^* and update your value on the face that can be done.

And we also know that our boundary conditions are given for or given for faces 1 and 3 right. u_1 is given as some value and u_3 is given as some value as a result there is no correction for u_1 and u_3.

So, u_1' equals 0 and u_2 u_3' equal 0. So, if you go to equations 6 and 8, where u_1's are related that is basically. So, essentially we are talking about this one. $u_1 A_1$ equals P_1 minus P_B plus b 1. So, u_1' equals right u_1' equals we would neglect this guy right because if you write if you go back to maybe I should look at this one ok.

This one the corresponding equation is here ok. So, u_1 equals so, u_1' equation for here will be u_1' equals this will be neglected. So, as a result what you get is P_1' minus P_B' right because this is a neglected and simple. So, u_1' is 0. If this is 0, P_1' has to be equal to P_B' right, similarly P_C' equals P_3' when u_3' is 0 ok.

So, those are the 2 equations we have; that means, if we go back to this thing; that means, p from these equations we can write that because u_1' and u_3' are 0, the pressure correction on the face 1 is the same as the pressure correction on the cell neighboring cell centroid ok. So, P_1' equals P_B' ok, remember this is the pressure correction is the same not the pressures ok.

The pressures could be different. Now, P_3' equals P_C' because this is what is coming out from applying the momentum equations to the boundary cells and using that discretization we are able to comment on the pressure corrections on the faces ok. So, P_1' equals P_B' and P_3' equals P_B'.

And as usual if you have a central difference approximation arithmetic average essentially you update the pressure on the face 2 as P_B' plus P_B' by 2. Essentially we are doing all these once you solve for the pressure correction equation ok.

So, solve for pressure correction then update your velocity corrections also update your pressures on the faces ok. How do you update the pressure on the faces? You have to go back to the corresponding momentum equations that we have derived from the half cells and such alright.

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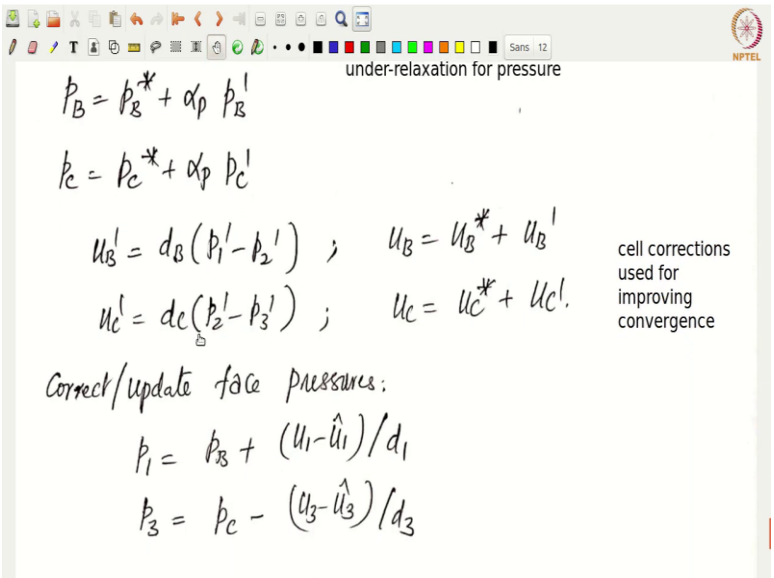

Then we have all these pressure correction and everything ready then we what we have do is we have to kind of correct the pressures. So, cell pressures are now P_B equal P_B^* plus α_P times P_B'. So, essentially have some kind of a under relaxation for pressure and also the pressure at cell C is basically P_C equals P_C^* plus α_P times P_C'.

Then what you do? You can correct the velocities at the cells cell centroid velocity that is u B prime equals d B times P_1' minus P_2'. u_C' equals d_C times P_2' minus P_3'. How did these equations combine? These are again coming from the corresponding cell equations right. Now, we are doing this only to increase the convergence, improve the convergence ok.

Then you can correct the velocity corrections to the starred value at the cells. So, these are basically cell correction use for improving convergence otherwise these are not required ok.

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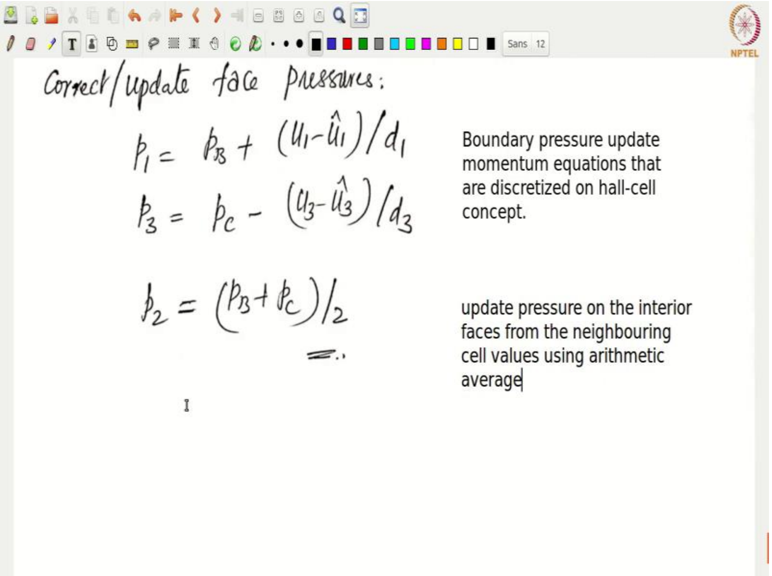

Then one more thing that is still remaining to be done is basically you need to update the boundary pressures ok. We have calculated what is the pressure correction on the faces but we have not updated the pressures on the boundaries. So, how do you update the pressures on the boundaries?.

Again from the momentum equation if you remember we had u_1 equals $\hat{u}_1$ plus d_1 times P_1 minus P_B. So, that equation I am recasting it as an equation for pressure on the boundary that is P_1 equals P_B plus u_1 minus $\hat{u}_1$ by d_1 ok. So, everything is known on the right hand side the boundary pressure can be updated ok.

So, basically these are equations for boundary pressure update ok. These are again coming from the momentum equations that are discretized on half cell concept ok. Then similarly we had an equation for u_3 as on the right hand side boundary face. u_3 equals $\hat{u}_3$ plus d_3 times P_3 minus P_C or something. So, from there I can write it as P_3 equals P_C minus u_3 minus $\hat{u}_3$ by d_3 ok.

So, once you know these cell pressures you know the hat velocity. So, you can update now the boundary pressures. Now, you can see that P_1' equals P_B' prime, but not P_1 is not equal to P_B ok,

P_1 and P_B are cos by u_1 and $\hat{u}_1$ hat ok. Similarly, once you know the once you correct the pressures on the boundary faces, you can also correct the pressure on the interior faces. This is basically arithmetic average of P_B and P_C ok.

So, essentially update pressure on the interior faces from the neighboring cell values using arithmetic average alright. That finishes the algorithm then we can go back with the new pressures and new velocities and then continue doing this alright. So, that is about the simple algorithm on a collocated grid. Now, let us look at the corresponding code. Again I have written this in Fortran ok. I will go back to the front of this thing ok.

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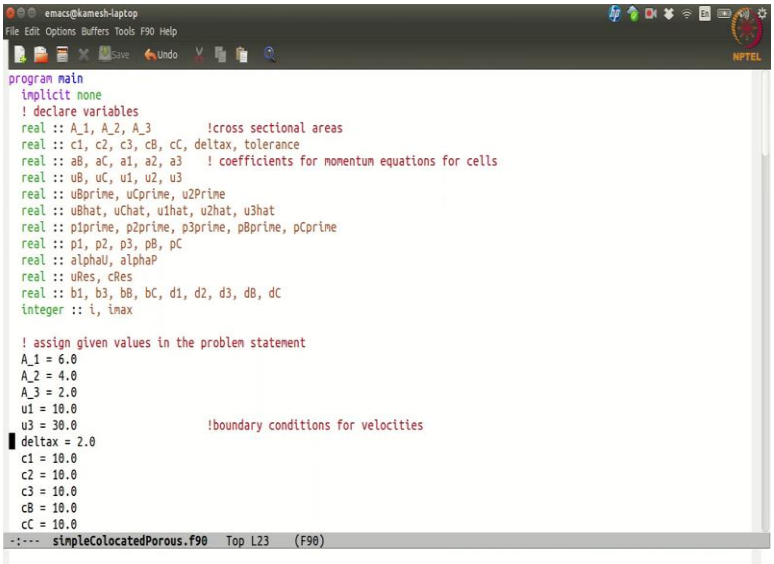

So, I will go back to the code we have ok. This code is now again also written in Fortran. We will kind of try to understand, draw correspondence between what we have derived and this code.

So, essentially this part is basically declaration of variables. So, what we have is A_1, A_2, A_3 these are basically your areas. Then c1 c2 all these things are deltax tolerance and these are the porosity constants and the deltax and tolerance. Then we have a B, a C, A 1, A 2, A 3. So, remember here I have capital A underscore 1, 2, 3 to define basically the cross sectional areas ok, whereas, these ones are aB aC I mean A 2, these are basically the coefficients for momentum equations for cells ok.

So, and then we have u B, uC u, u_1, u2, u3 these are u B, uC are the velocities for the cells u_1, u2, u3 are the velocities on the faces. And then we have uB prime, uC prime, u2 prime and then we have uB hat, uC hat and then u1 u1 hat, u2 hat, u3 hat these are all basically the variable declarations.

Then assign given values in the problem statements. So, the cross section area is given as 6, 4, 2 for at location faces 1, 2, 3 and u1 is given and as 10, u3 is given as 30. These are basically the boundary conditions for velocities then deltax equals 2.

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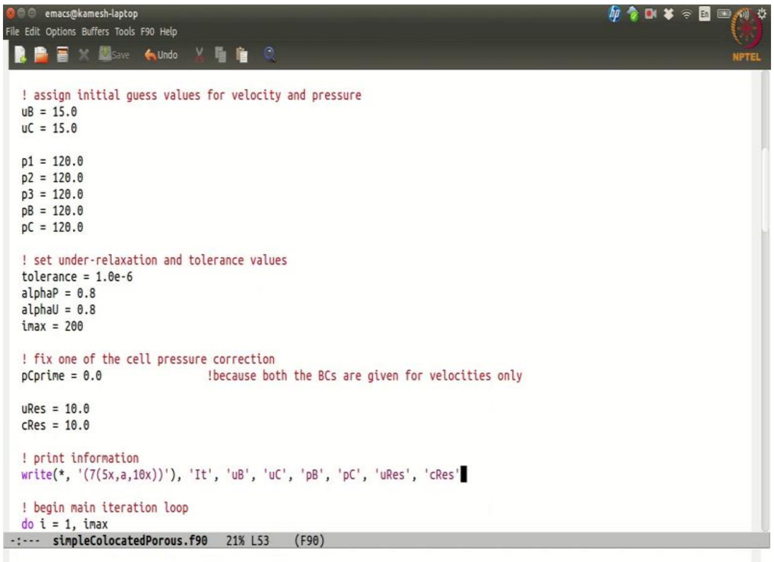

Then of course, the porosity constant c1 everything I am taking as 10, you are welcome to take any other values here in a particular problem then we need to star of with an initial guess. The initial guess for this cell centroids we are taking it as 15 for both uB and uC and the pressure we are taking it as a initial guess for pressure at all the cell centers as well as on the faces we are taking it as 120.

And of course, let us set the tolerance value that is come out of this loop if you are sum of your u residual and momentum residual and the continuity residual is falls below 1 e minus 6. So, that is what it is and then we also set some under relaxation parameters; alphaP and alphaU these are under relaxation for pressure and velocity and let us hope that we do not hit 200 iterations in this loop and we try to converge much before that.

So, again we have to fix p C prime because both the boundary conditions are given for velocities only right. So, as a result we fix p C prime and then let us initialize the residuals value of u residuals and c residuals is some arbitrary values.

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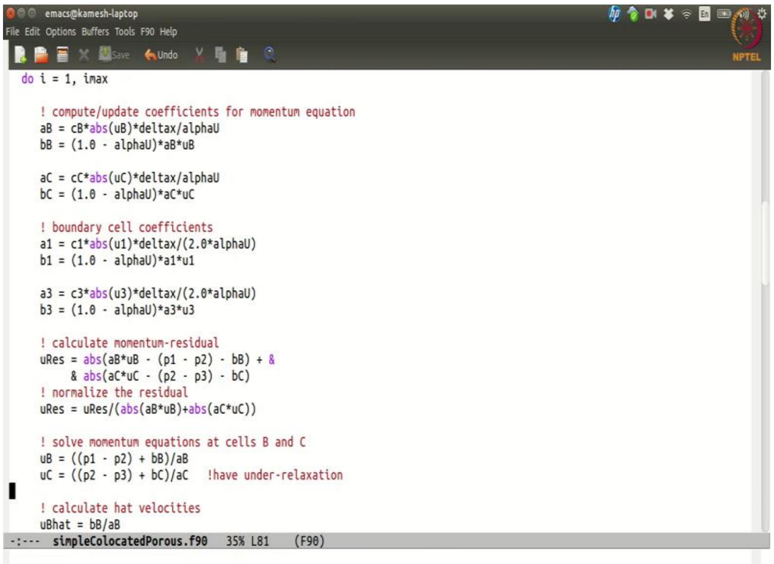

Then, we have some print statement to print all the as we progress, we want to print the iteration, uB uC the velocities and the pressure at the cells and we also want to print the momentum residual and the continuity residual ok. So, this is the main iteration loop; do i equals to 1 to imax this is like your for loop.

Then compute and update coefficients for momentum equation. So, what is your a B? aB equals cB mod uB cB mod uB deltax times divided by alphaU right, remember this thing. So, this is basically aB right Ab cB mod uB deltax that is your cB mod uB deltax divided by alphaU this is basically divided by alpha u. So, that is your aB then your bB would be 1 minus alphaU by alphaU that is y alphaU is already in here in aB.

So, I do not have to write that this is basically 1 minus alphaU times aB times uB star that is your bB value. Similarly aC equals cC mod uC deltax by alphaU and bC would equal to 1 minus alphaU by alphaU times a C, but aC is already divided with alpha U. So, we do not have to write that and we have aC uC star ok. These are your aC and bC values and aB and bB values that we have here ok.

So, this is your bC and aC these are populated. Then the similar to what we have for the cells we also have these boundary coefficient that is basically a1 equals c1 mod u1 deltax by 2, remember we had 2 in here right. So, we had a 2 in here. Similarly, b1 would be equal to 1 minus alphaU times a1 times u1 star right.

1 minus alphaU by alphaU that is here ok, similarly a3 would be equal to c3 mod u3 deltax by 2 and then divide by alphaU and your b3 would be equal to 1 minus alphaU by alphaU times a3 u3 star ok, but a3 already got the division. So, we do not have to be again write it. So, these are basically you want this equation and this equation basically these equations are written here.

So, this basically completes the momentum equations for the cells B and C as well as the boundary cells also right ok. Now, we calculate what is the momentum residual. Momentum residual is nothing, but take everything to the left hand side that is basically aB uB minus p1 minus P_2 minus bB that is basically if we go here or we do not have the equation here ok.

So, this is your momentum equation taken to the left hand side ok. Similarly this your momentum equation taken to the left hand side for cell C that is aC uC minus p2 minus p3 minus bC alright then I want to normalize this thing. So, basically this is your residual calculation from the momentum equations ok, these are the momentum equations they are right here.

So, from here you can construct what these qualities are right, what this quantity is basically this equation everything taken to left hand side ok. Then you solve for the momentum equations at cells B and C. So, uB equals p1 minus p2 plus bB upon aB similarly uC equals p2 minus p3 plus bC upon aC. These equations already have under relaxation in them right because we have already divided with alphaU ups here ok.

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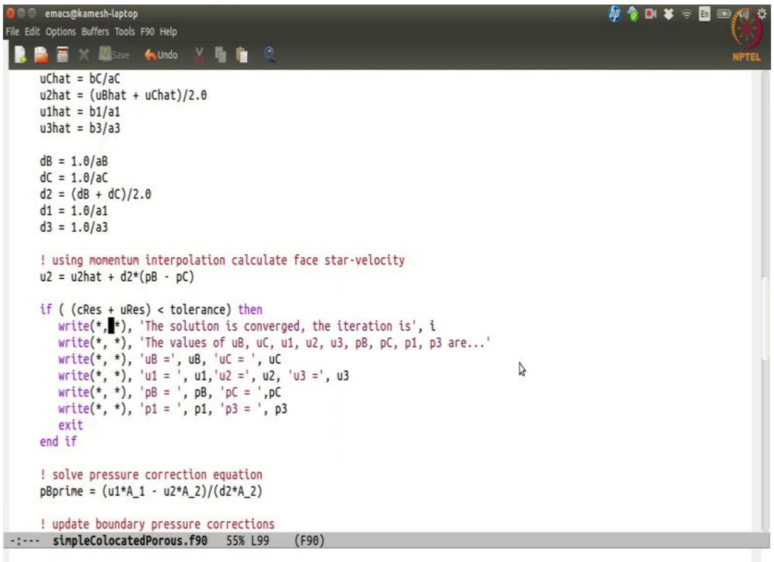

So, these equations already have incorporated under relaxation in them because aB aC bC already contain those things right. So, once you have uB and uC now it is time to update the hat velocities. How do you calculate uB hat? uB hat is your bB upon aB right, essentially that is this guy divided by aB right and similarly uC hat is bC by aC. This is what we have written here right. uB hat is bB by a B, dB is 1 by a B, similarly, uC hat is bC by a C.

So, I update what is uBhat uChat then u2 hat you will find it as arithmetic average of uB and uChats then you are u1hat would be b1 by a1, u3hat would be p3 by a3 right. So, that is what we have here right. Basically u3 by right you have b3 by a3 is u3 hat ok. Then we can also write the coefficients d that multiply the pressure difference or dB equals 1 by aB dC equals 1 by aC d2 equals db plus dC by 2 excuse me.

And then this is basically coming from momentum interpolation. Then d1 equals 1 by a 1, d3 equals 1 by a3 alright. Then use momentum interpolation and calculate the star velocity right face velocity face star velocity. So, u2 equals u2 hat plus d 2 times p B minus pC right that is your momentum interpolated value u2 equals u2 hat plus d 2 times pB minus p C because we have already calculated what is u2 hat.

What is d2 we can do this thing right. Use momentum interpolation. Then here I am checking for convergence. If my continuity and momentum residual together are less than the tolerance then I would print off a several values alright. And once you have the momentum interpolated face values we can go and solve for the pressure correction equation.

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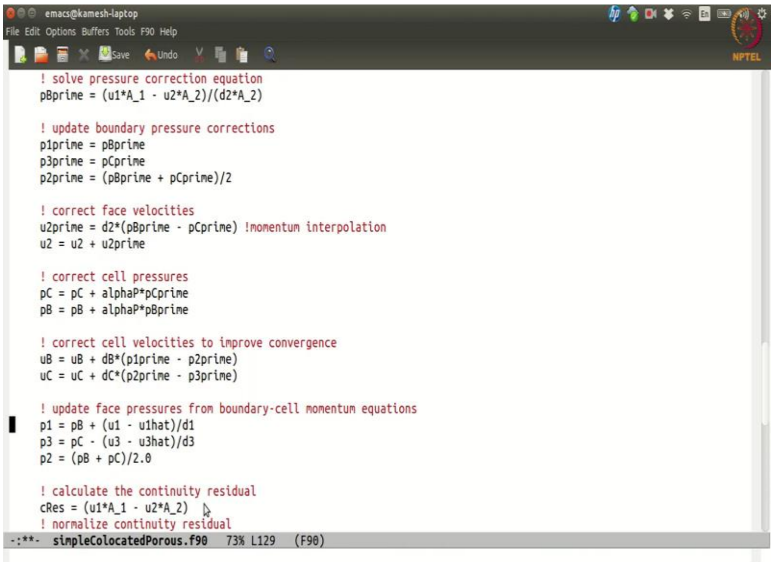

So, this is the pressure correction equation that is pBprime equals u1 a1 minus u2 star a2 divided by d 2 a2 right that is your pressure correction equation. If you remember we set pC prime equal to already 0 right which gave us this equation right. This is p B prime equals u1 a1 minus u2 u2 star a2 by d 2 a2 ok. So, that is the equation here then once you solve for pressure at the cells pressure correction this cells you can update the boundary pressure corrections.

So, what we have is p1prime equals pBprime that is coming from here and p3prime equals pCprime that is coming from here right and then p2prime equals arithmetic average of the neighboring cell values that is written here. So, we have these are the pressure correction. First 2 are written from the boundary cell momentum equations right which relate the pressure to the boundary velocities alright.

So, once you have all the pressures the cell pressures and the face pressures then you can correct the face velocities by computing the face velocity corrections ok. So, u2prime equals this is again d2 times pBprime minus pCprime. This is coming from where? This is coming from momentum interpolation right then u2 equals u2 plus u2 star plus u2 prime.

Then your pC, the cell pressure equals pC plus alphaP times pC prime that is basically pressure correction. So, that is these equations, p C and p B equals p B plus alphaP times p B prime then you can correct the velocities cell velocities as uB equals uB plus dB times P_1' minus P_2' because P_1' P_2' are already corrected updated here.

So, that can be used and correct the cell values. This we are doing essentially primarily this is basically to improve convergence right that is what we have written here. We do in this thing ok, you calculate uB prime and add to the uB star values alright. Then update the face pressures that is basically define a step here right. Update the face pressures because only the face velocities are given. So, update the face pressures from the u1 and u3 equations right.

The face values on the pressure p1 equals pB plus u1 minus u1 hat by d 1 p3 equals p C minus u3 minus u3 hat by d 3 that is written here and then p2 would be arithmetic average of the cell values that is written here alright. That means, again; that means, we are done with all the corrections right these are the face pressures updates ok.

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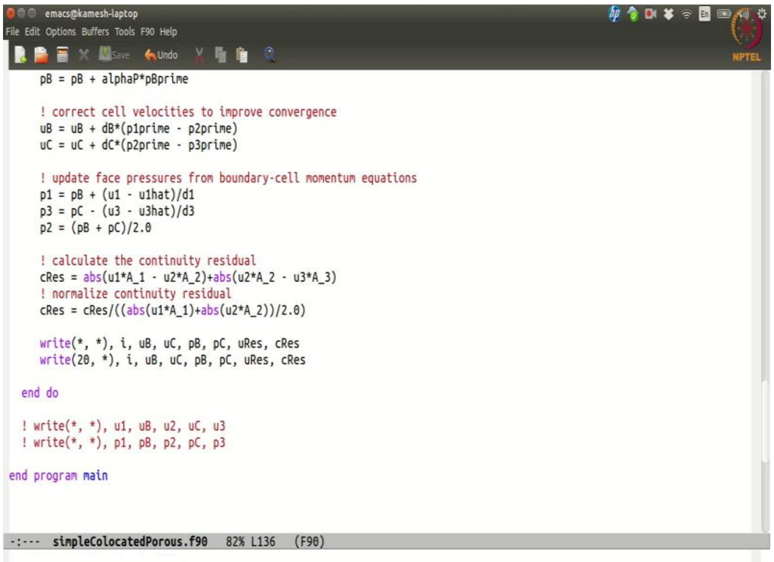

Then calculate the continuity residual. c residual equals essentially u1 a1 minus u2 a2. Of course, even you can even add the other component that is for cell B, you can also add the other component that is coming for cell C as well and then normalize with the continuity value.

And essentially write down all these information fine. We can also do the continuity residual absolute was absolute of u2 star a2 minus u3 star a3 right that be the total value for both the cells and of course, it can be also divided ok.

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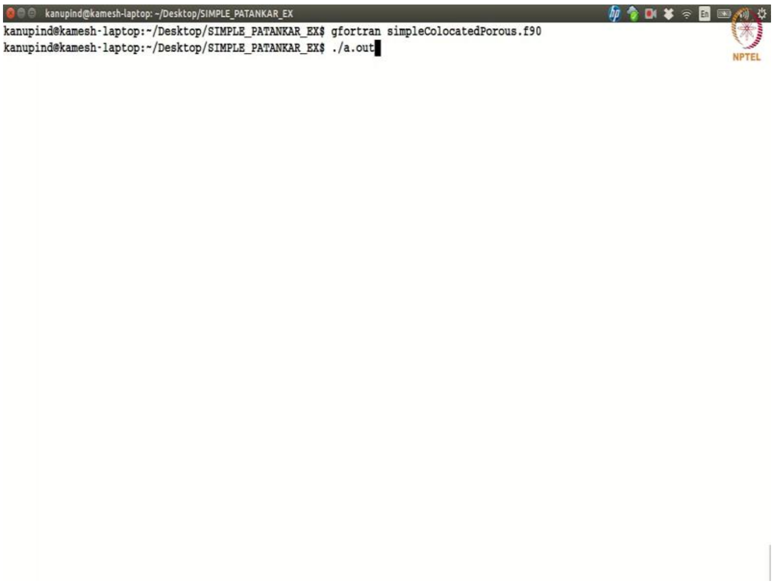

So, this is the main this is the complete program. So, let us try to compile and run and this is in 3 4 runs simple porous colocated porous. So, dot slash a dot out ok.

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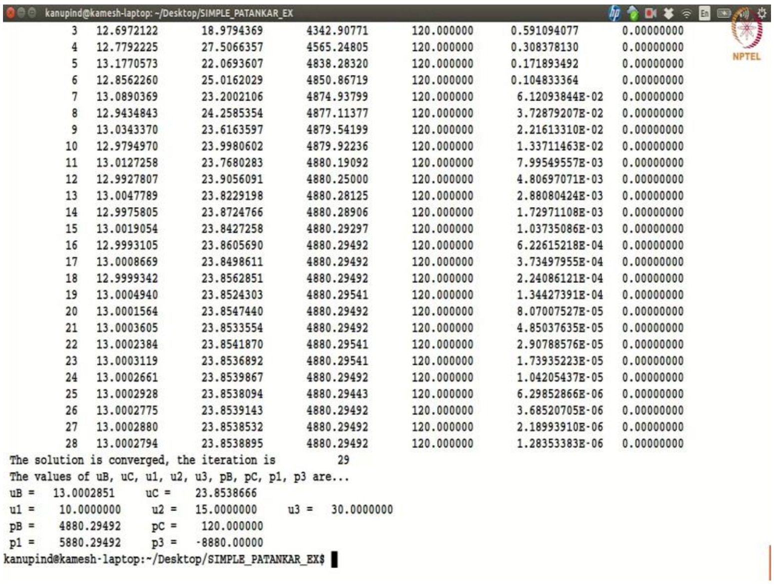

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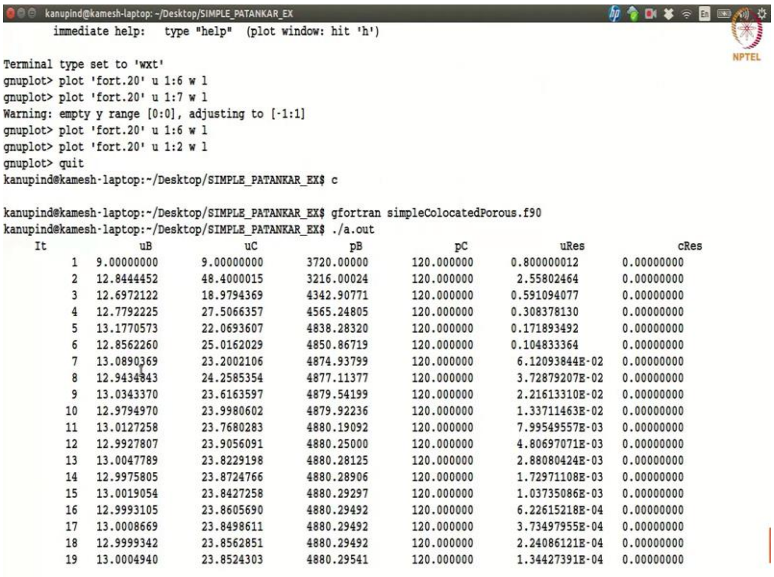

That is good. So, the program ran. Now, what we are looking at here is basically the iteration count uB value, uC value pressure at B and C and u residual and the continuing residual. As you can see continuing residual is always 0 as it should be because simple algorithm make sure that the velocity is satisfy continuity after the correction ok.

Whereas the momentum residual you can see that it slightly increases and then it stops it starts to decrease to all the way to 1 u minus 6 and because pC prime was set equal to 0. So, pC reminded it s fixed value of 120 that was the initial guess and pB of course, changed and the solution converge into 29 iteration and u B and u C values converge to 13 and 23, ok.

So, the converged values of u B is 13, u C is 23 and the cell pressure are 4880 that is 5000, cell pressure p C was set to 120 and $p1$ is basically the face pressure that is around 5900. So, you can see there is a difference between $p1$ and p B right because this is because of the porosity right this pressure difference should be there so that the flow can accelerate while it is going through the porous media right.

Similarly, we have p3 equals minus 8880. So, basically this can be thought of as this is coming out to be minus because we have fixed is this to be 120 right. Otherwise if you make this as 0 everything will get increased get added up correspondingly ok.

So, that is the converged pressure and velocity fields. This of course, satisfy the momentum equation and the continuity equations to this precision ok. I mean we can also see how the you can also change any of these coefficients and play with them as well. You can change the essential in the porosity constants or the cross sectional areas and things like that as long as the problem is well post fine.

So, that is an example on how would you go about solving simple algorithm on a colocated grid ok. So, this is this kind of co-located algorithms are only used in the regular software when you try to solve incompressible flows ok. So, that is kind of good to know ok.

So, I will share this program. So, you can kind of go through it and if you have any questions you can get back to me. So, I am going to stop here for today. We will catch up in the next class. If you have any questions do send me an email ok.

Thank you.