Computational Fluid Dynamics Using Finite Volume Method

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Lecture – 04

Review of governing equations: Navier-Stokes equations and energy equation

We looked at derivation of conservation of mass, momentum and energy right. Today we are going to see if we can solve the fluid flow and heat transfer using these equations with the required number of unknowns that we have ok.

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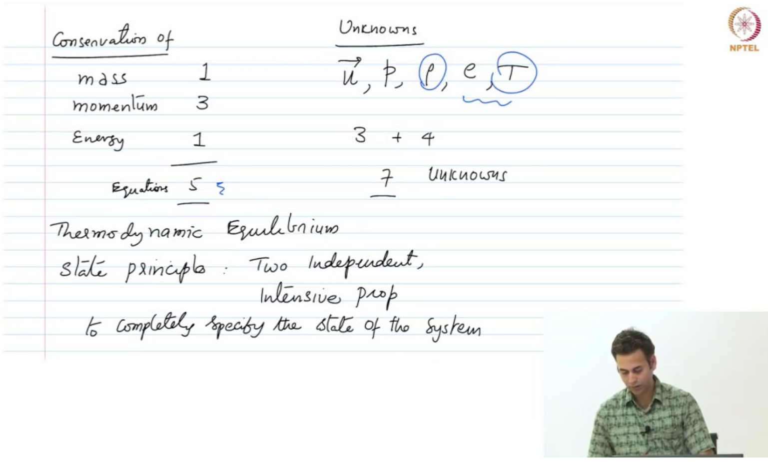

So, we have looked at conservation of mass, momentum and energy right. So, essentially how many equations we have? We have 1 equation for mass right and momentum equation is how many? 3 of them right, we have one in each direction and then energy equation is 1 equation. So, essentially we have 5 equations at our disposal and how many unknowns we have? So, how many unknowns do we have? We have what are the unknowns we have, for a compressible fluid right general compressible fluid that we have derived u right.

So, essentially u mean u bar right essentially u v w. So, that is 3 unknowns right and then we have pressure is an unknown right; pressure is an unknown and then density rho is an unknown, then we have an equation for internal energy e right. So, little e is an unknown and then temperature T is also an unknown right, unless we have a special relation which relates internal energy to the density we do not know the relation between these two right.

So, as a result e is an unknown and T is an unknown. So, how many of how many unknowns do we have in total? We have 3 plus 4 right we have total of 7 unknowns whereas, the number of equations we have are only 5 right. So, can we solve for this 7 unknowns with these equations? We cannot of course, solve for this. So, we need to invoke further assumptions right.

So, we need to invoke something known as thermodynamic equilibrium we need to invoke something known as thermodynamic equilibrium right. What does thermodynamic equilibrium mean? It means that if you have a simple compressible system right assuming that is in thermodynamic equilibrium then we just need two independent intensive properties to completely specify the state of the system ok.

So, essentially from, essentially the state principle what we have is we need two independent intensive properties to completely specify the state of the system right if we have a simple compressible system right; that means, there are no other external forces acting on it on this particular system ok. So, we just need two independent intensive properties and all other properties can be obtained from these independent properties that we have specified ok.

So, if we choose as our properties as let say density and temperature as our independent properties, then what we can do is we can go about and write and relate the other properties.

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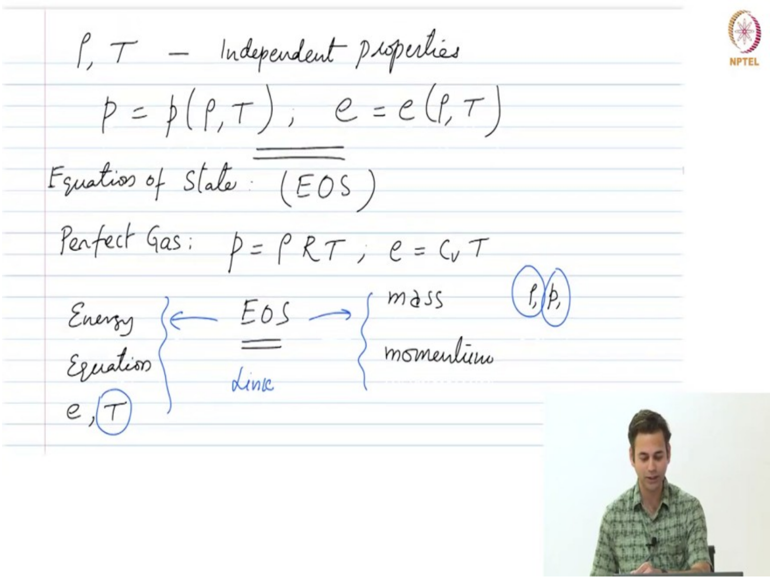

So, for example, we choose let say density and temperature as the independent properties. Then the other two quantities that is the other two thermodynamic variables that is pressure and internal energy; so, pressure can be obtained as a function of density and temperature, similarly the internal energy e as a function of density and temperature right.

So, essentially we have now two more equations right. So, this is the 6th equation and then we have the 7th equation right. So, essentially we have 7 equations and 7 unknowns as we go here right. So, we have plus 2 here which gives us 7 equations and 7 unknown. So, theoretically in principle we can solve for all the set of variables ok.

So, what does the so, what is the principle that gives us these two? This is known as the equation of state right essentially equation of state for a particular; for a particular fluid relates the pressure and internal energy to the density and temperature variations right. So, essentially equation of state gives us these extra equations which I probably write it as sometimes as EOS as a short form. Now, if you have a perfect gas then we know that the pressure is related to the density and temperature using what?

Student: Ideal gas.

Ideal gas equation that is$P=\rho RT$ p and the internal energy for a fluid right for a perfect gas is given as$e=cVT$ right so, the absolute temperature. So, we have these two equations which we use to relate density and temperature to pressure and the internal energy alright. So, that is good.

Now, what we can see from here is that the equation of state relates the energy equation on one hand and the mass and the momentum equations on the other hand ok. So, essentially equation of state is the kind of connecting link between these two ok. So, why do we say that? We say that because the energy equation contains internal energy right e whereas, the mass and momentum contain the density, pressure and of course, the energy equation also contains the temperature right; so, we have these things.

Now, the changes in density; the changes in density and the changes in pressure cause a changes in temperature right now that is only possible if you have a compressible fluid or a compressible flow right only under if you have a compressible flow or a compressible fluid you have density changes which are caused by changes in pressure as well as temperature ok.

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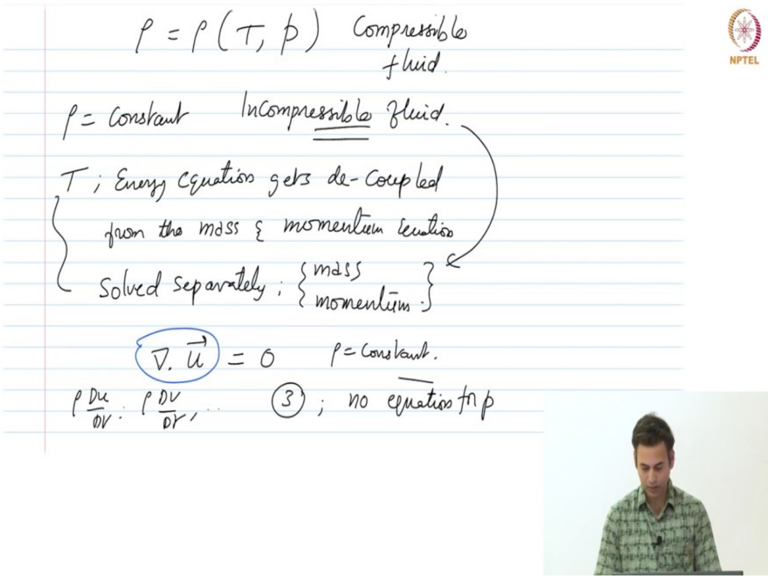

So, essentially what we mean is that ok. So, the changes in density are caused by temperature as well as pressure if you have a compressible fluid or a compressible flow. Now, this is what relates the energy equation kind of links the energy equation to the mass and momentum equations ok.

Now, if you have a an incompressible flow or a fluid for which density is constant right. So, if we have an incompressible fluid then what happens? Then what happens is basically you do not have any density changes. As a result your energy equation right, the equation for temperature or internal energy gets decoupled from your mass and momentum equations ok.

So, essentially this link is will not be there between mass and momentum and energy as a result temperature right the changes in temperature are not brought about because of the changes in density or in pressure right if you have an incompressible fluid ok. So; that means, for incompressible fluid the temperature or the energy equation gets decoupled from the mass and momentum equations ok.

So, what is a consequence of that? The consequence of that is the energy equation can be solved as a separate passive scalar transport equation ok. So, this can be solved separately because it is no more coupled to the mass and momentum. As a result in many incompressible fluid flow problems we could just get away by solving only the mass and momentum equations if we have an isothermal flow.

If we are solving for a non-isothermal flow, then we have to solve for temperature as well as a separate scalar with the with different boundary conditions that it has ok. So, that is kind of the take away message now as far as the complexity of the solution procedures is concerned; if you have a compressible flow system, then you have a equal number of equations and unknowns right you have 7 equations 7 unknowns you can solve for them one by one ok.

Now, we will see that if you have an incompressible fluid or incompressible flow your continuity equation becomes just del dot u bar equals 0 right. As a result you do not have an equation for density right your density is again now constant. And, what we will see is that this will bring about another significant change which is basically your you have 3 equations of the momentum which is $\rho \frac{Du}{Dt}.\rho \frac{Dv}{Dt}$ and so on right you have these 3 equations and you end up with no equation for pressure ok.

So, as a result the equation for pressure is only again another equation for in terms of velocities ok. So, as a result the solution procedures for incompressible flows are quite different from the solution procedures that you have to adopt for solving for a compressible flow ok. So, that is what we are going to see in the rest of this course when we come to the solution of fluid flow equations for incompressible flows. Questions till now? Ok. So, let us move on ok. Then let us look at Navier - Stokes equations for a Newtonian fluid.

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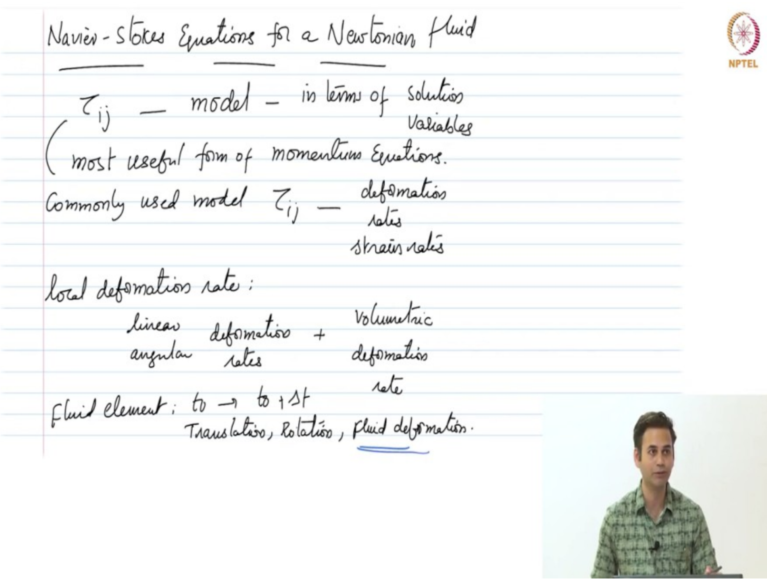

Now, one thing we did not of course, consider was the shear stresses right, we talked about tau i j, but when I have listed down the unknowns I have comfortably not listed down the tau i js neither you told me that tau i js are still unknown right we all assume that tau i js are known right that is kind of good, but we know that these are still unknown at this point of time right.

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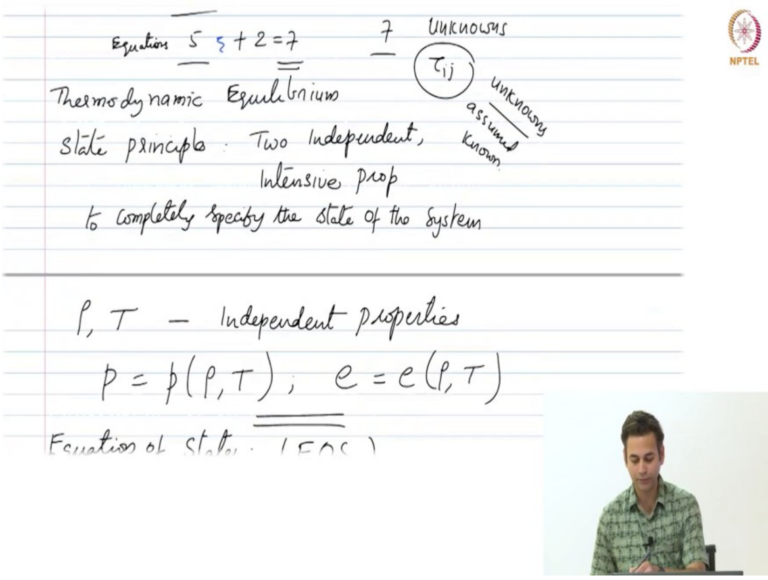

The shear stresses that we have in the momentum equations are still unknowns nobody told us how to evaluate these terms right, these are still unknowns at this point of time which we kind of assumed while deriving this unknown and the equations balance that these are somehow known ok.

So, we assume that these are known which is what we are going to now discuss how do we introduce or how do we model the shear stresses. So, the tau i j the 9 terms that we have out of which the 6 are the independent ones for an isotropic fluid are the ones which needs to be somehow kind of need a model right to specify them in terms of the other solution variables that we have ok. So, we need a model to specify these in terms of solution variables ok.

Now, by introducing a model for this we are going to derive the most useful form of momentum equations alright. So, a common commonly used model is to relate the shear stresses to the deformation rates are the strain rates in a fluid. So, that is what we are going to use as a model which was proposed by Navier and Stokes in the 19th century and then that is what we are going to use and kind of substitute these back into the momentum equations and derive the Navier - Stokes equations.

So, the local deformation rate is composed of what, is composed of linear deformations right shear deformations as well as volumetric deformations right. So, the strain rate contains the linear or the angular right deformations deformation rates as well as volumetric deformation rate ok. So, this is a deformation rate ok.

So, essentially if you take a fluid element in general you can model the change of a fluid element at a time t naught to another time t naught plus delta t by a series of elementary super positions right. These super positions would be you would recall from a fluid mechanics course that we will be composed of translation, a pure rigid body translation, a pure rigid body rotation and fluid deformation. And this fluid deformation would be again composed of two components, one is angular deformation right, another one is a volumetric deformation ok.

So, essentially you have all these four we are only looking at only the later part which is the fluid deformation the angular and the volumetric deformations because the other two do not come into picture in modeling the shear stresses that we have for a fluid ok, alright.

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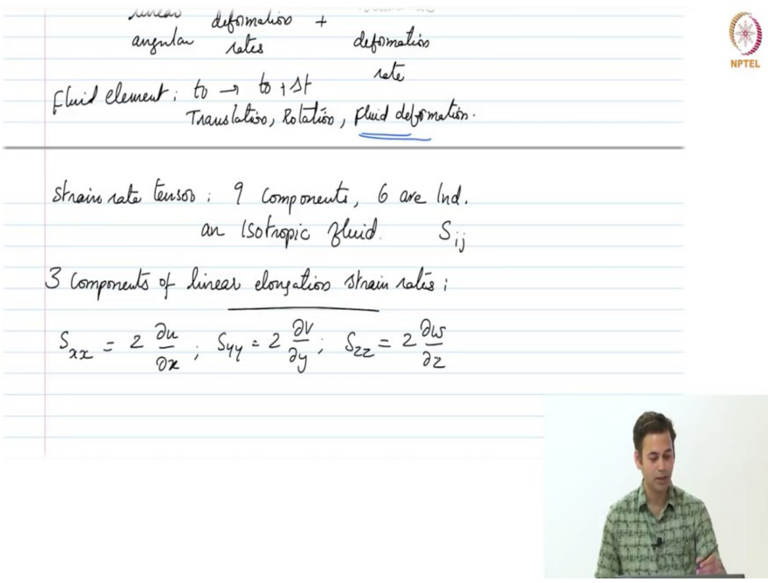

So, the we call these as the strain rates or the strain rate tensor; which has again 9 components out of which 6 are independent if we have an isotropic fluid. So, how do we represent these deformation rates? We have 3 components of we have 3 components of linear elongation strain rates these are given as. So, we use the symbol S sub i j ok. So, we have just like we have tau i j we use this S sub i j to represent the strain rates ok.

So, again i j go from x y z each of them. So, the linear elongation strain rates would constitute $S_{xx}=2\frac{\partial u}{\partial x}, S_{yy}=2\frac{\partial v}{\partial y}, S_{zz}=2\frac{\partial w}{\partial z}$ could there be a 2 here? No, 2 right it is just; this is just, this is just partial u partial x partial v partial y partial w partial z that is for the linear elongation strain rates.

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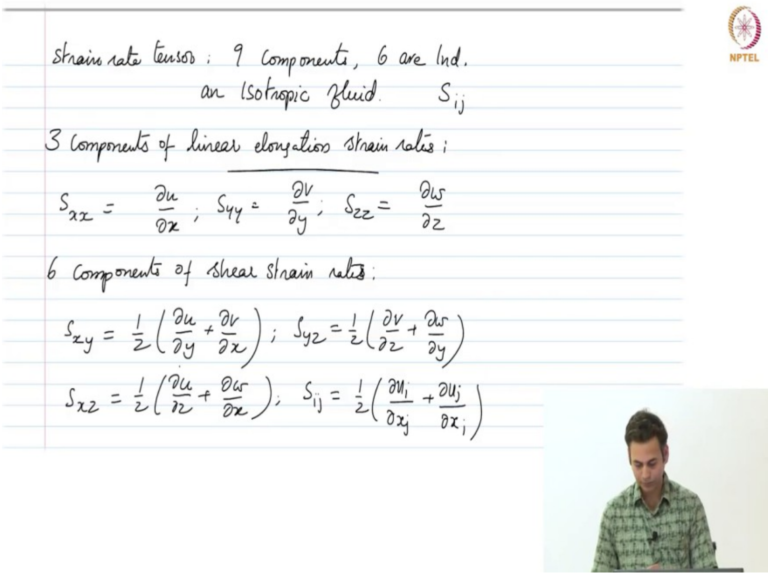

And when it comes to the 6 components of shear elongation rates, elongation strain rates; so, these are the angular deformation rate. So, that is S sub x y equals half of partial u partial y plus partial v partial x ok. Now, you see that how if you have $S_{xy}=\frac{1}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) S_{yz}=\left(\frac{1}{2}\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right) S_{xz}=\left(\frac{1}{2}\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)$ S sub x x how do you get partial u partial x where you have two of them summing up to one ok. So, there is no two in there as I wrote before similarly S y z would be equal to half of partial v partial z plus partial w partial y ok, similarly S x z would be equal to half of partial u partial z plus partial w partial x ok.

Now, this looks like we can write in a convenient summation notation a very simple formula that would be nothing, but S sub i j would be equal to what; half of right partial u i by partial x j plus partial u j by partial x i right that is all where we plug in i equals i and j as x y z each of them and get these 3 components of the linear deformation rates. And of course, I said 6 components of shear elongation strain shear rates right I am sorry I think I wrote elongation here there is no elongation it is just shear strain rates all right.

So, what about the other 3? They are the same as this right because of the isotropy fine. So, we have all these strain rates that we get.

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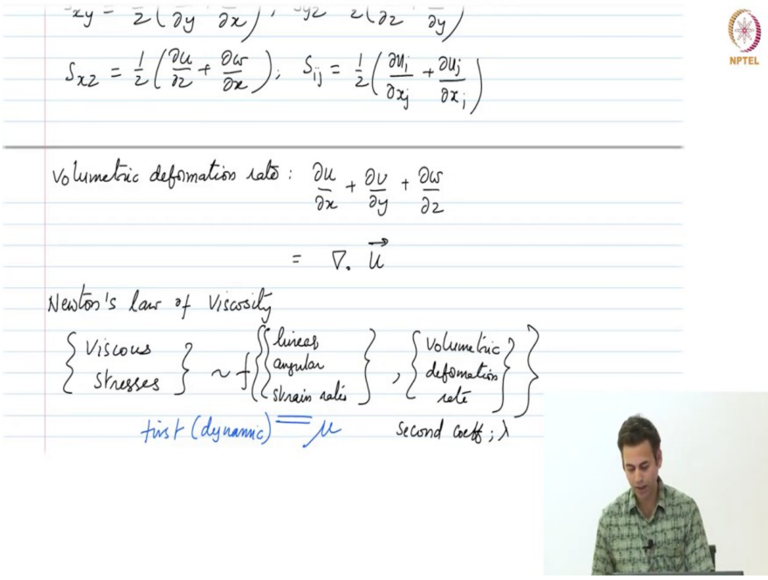

And of course, we have this additional volumetric deformation rate which is given by what, which is the sum of $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}$. So, in a convenient form we can write this as $\nabla . \vec{u}$ which would be 0 for if you have an incompressible fluid or incompressible flow which is not 0 if you have a compressible fluid right; we are still talking about a general compressible fluid, alright.

Now, we have introduced this model now we need to know how do we relate the shear stresses that we got the viscous stresses to the these strain rates right. So, if we have if we consider if we consider Newton's law of viscosity right, then the Newton's law of viscosity relates the viscous stresses that we have to the strain rates and in the strain rates are again we have these linear or angular strain rates and we also have the volumetric deformation rate right.

So, essentially the Newton’s law of viscosity relates the viscous stresses we have to each of these strain rates as a result we end up with two coefficients of viscosity one relating to the linear or angular strain rates that is known as the first or the dynamic coefficient of viscosity which is denoted with the symbol mu and the other one which is known as the second coefficient of viscosity which is usually denoted with the symbol lambda ok.

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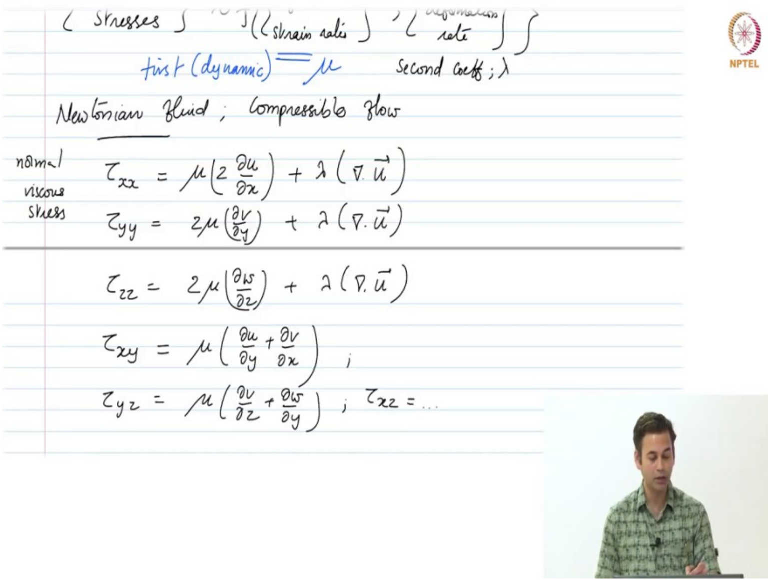

Now, we are assuming a Newtonian fluid. What is a Newtonian fluid? The viscous stresses are proportional to the linearly proportional to the strain rates ok. So, essentially that is how we get these things. So, we have a Newtonian fluid, but the fluid is still compressible right we are still considering a compressible fluid, but a Newtonian fluid. Would that be possible or should a Newtonian fluid be always incompressible? Need not be right, you can still have a compressible fluid and it could be still a Newtonian fluid ok. So, we are still looking at a compressible flow with as a and the fluid behaving as a Newtonian fluid ok.

So, the viscous stresses are proportional to the linear deformation rates and as well as the volumetric deformation rates ok. Now, if this is the case we can again list down the viscous stresses that we have as using Newton’s law of viscosity as $\tau_{xx} = \mu(2\frac{\partial u}{\partial x})$ tau. So, that is the coefficient which is making the proportionality constant go away which is making the proportionality symbol go away you have this first coefficient of viscosity plus you have lambda times del dot u bar ok. So, that is your shear stress right the normal stress we have this is the normal viscous stress ok.

Similarly, we can write down $\tau_{yy}=2\mu\left(\frac{\partial v}{\partial y}\right)+\lambda(\nabla.\vec{u})$ and we also have $\tau_{zz}=2\mu\left(\frac{\partial w}{\partial z}\right)+\lambda(\nabla.\vec{u})$

tau z z. Of course, we have the shear components right that is the $\tau_{xy}=\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)$ right that

is what we have. Similarly, $\tau_{yz}=\mu\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)$ and of course, you can also write down what is tau x z similarly ok.

Now, of course, the volumetric deformation rate only shows up in the normal stresses right because that is a linear part of the deformation rate. Now, not a whole lot of stuff is known about this second coefficient of viscosity ok.

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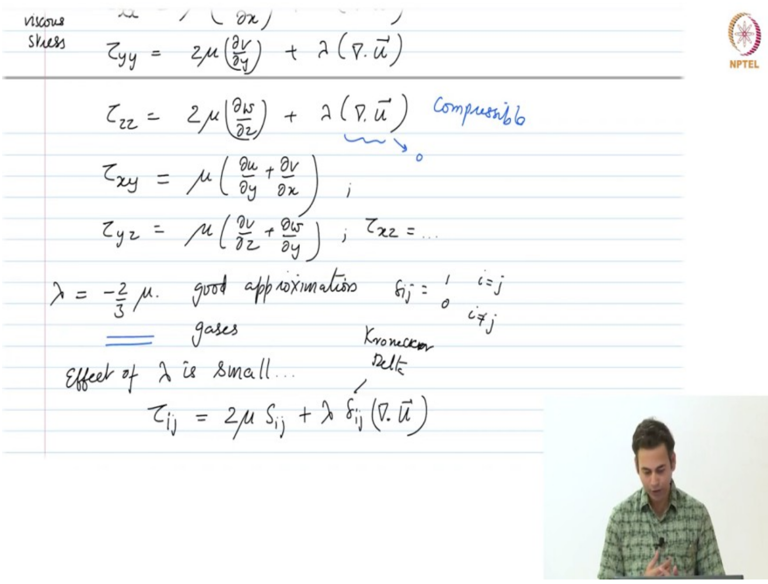

So, second coefficient of viscosity is usually taken to be from experiments usually taken to be minus two-thirds mu this is a good approximation for gases for flow of gases. Now, of course, we know that if you have an incompressible flow what will happen to del dot u bar? This goes to 0, so it does not matter what will be the second coefficient of viscosity anyway ok. So, partially the success of the originally proposed minus two-thirds mu is related to the observations right that delta u bar was even 0 in most of the compressible flows that were there ok.

So, as a result the proposed lambda equals minus two thirds mu was a huge success for a for over few decades ok. Now, we kind of tend to realize that that is because of this term going to 0 even for compressible flows ok. Nonetheless the effect of the; effect of the second coefficient of viscosity is small even in you know practical flows ok. So, the effect of lambda is small as a result it is not going to make a whole lot of change or difference in the results even if you have a compressible flow, but anyway we are not worried about it because we are looking at simulation of incompressible fluid flows.

Now, can we also write down a simple expression in terms of the index notation for the shear stresses from whatever we have proposed here? Right. We can write down one what would that be that would be$\tau_{ij}=2\mu s_{ij}$ very good ok. So, essentially what we have is 2 mu S i j because S i j is again half of partial u i partial x j plus partial u j partial x i plus now how do I bring in this extra component in here which is non-zero for normal which is 0 for the shear. I use a delta function right that is Kronecker's delta.

So, that will be what lambda right times delta i j times del dot u bar right can I write this where delta i j is the Kronecker delta function which is equals 1 if i equals j which is 0 if I naught equals j right. So, we can write down this one. So, this is a simple formula that relates the shear stresses to the strain rates that we have alright.

Now, we have brought in Newton's law of viscosity and related the shear stresses to the strain rates; now what do we have to do? We have to plug back these shear stresses into the which equations? Into the momentum equations right and of course, into the energy equation as well ok. So, we will plug these back into the momentum equation and see if we can get an I C equation that we can use to that we can use to further integrate ok.

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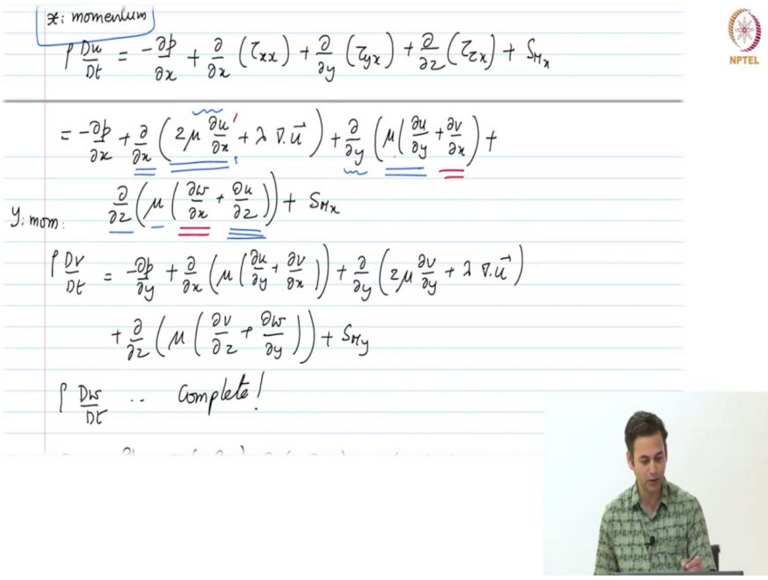

So, what was our x momentum equation? If you go back our x momentum equation was $\rho \frac{Du}{Dt} = ¿$rho D u D t right equals; what was there on the right hand side? We had a pressure term right that was minus partial p partial x I will first write down the equation in terms of the shear stresses. So, $\rho \frac{Du}{Dt} = \frac{-\partial p}{\partial x} + \frac{\partial}{\partial x}(\tau_{xx}) + \frac{\partial}{\partial y}(\tau_{yx}) + \frac{\partial}{\partial z}(\tau_{zx}) SMx$ which is a source for the momentum equation in the x direction very good.

Now, can we plug in these shear stresses that we have obtained here into the into this equations and see what happens ok. So, how does these equations look? So, if I plug in these we have minus partial p partial x plus partial partial x of how much was tau x x? This was $(2\mu \frac{\partial u}{\partial x} + \lambda \nabla . \vec{u})$ as well is not it ok, which I have missed out.

So, p plus$\frac{\partial}{\partial y} ¿ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} ¿$+partial partial y of what we had? Mu times partial u. So, this was tau y x this would be partial u partial y plus partial v partial x$\frac{\partial}{\partial z}\left(\mu\left(\frac{\partial w}{\partial x} + \frac{\partial w}{\partial z}\right)\right) + SMx$

Very good I will write down the v momentum equation also for the sake of completeness. So, that would be rho D v D t. So, this is the y momentum equation

$\rho \frac{Dv}{Dt} = \frac{-\partial p}{\partial y} + \frac{\partial}{\partial x}\left(\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right) + \frac{\partial}{\partial y}\left(2\mu \frac{\partial v}{\partial y} + \nabla . \vec{u}\right) + \frac{\partial}{\partial z}\left(\mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)\right) + SMy$ I will directly write in terms of the shear stresses in terms of the velocity gradients that would be what? Partial partial x of this should be tau x y right.

I would not write down the rho D w D t which you can complete later very good.

Now, we have obtained all these equations which are only in terms of velocities right. On the right hand side now we do not have this shear stress vector anymore as a result our unknown velocity and its gradients appear on the right hand side which could be obtained somehow if we know the velocity field at a particular time. Now, we are going to do some kind of a rearrangement here, so that these equations look somewhat nicer and also clean and also they look in a more nicer way for to work with incompressible fluids ok. So, for that we are going to work kind of rearrange some of these terms.

So, I would go back to the x momentum equation and what we what I would like to do is I am going to split this 2 mu partial u partial x term into 2 terms ok. So, mu dou u dou x plus mu dou u dou x ok. And then I am going to collect one of that partial u partial x term from here, from here and then the other the term I am going to take it is this one this is first term of the this term which is mu times dou u dou y and then I think there is some mistake in here is it no ok.

So, I would take the second term here I were not consistently so, this is dou u dou z ok. So, I would take down write down these 3 terms together and I will write down the remaining terms separately ok.

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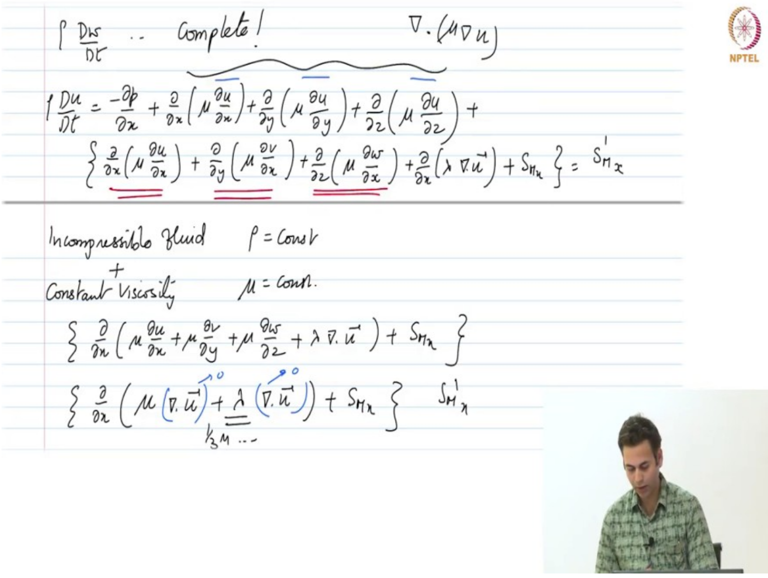

So, can you help me write this equation by looking at this? So,

$$\rho\frac{Du}{Dt}=\frac{-\partial p}{\partial x}+\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)+$$

$$\frac{\partial}{\partial z}\left(\mu\frac{\partial u}{\partial z}\right)+\{\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial z}\left(\mu\frac{\partial w}{\partial x}\right)+\frac{\partial}{\partial x}\left(\lambda\nabla.\vec{u}\right)+SMx\}$$

Is that correct, yeah?

So, we have partial partial x of mu dou u dou x then partial partial y of mu dou u dou y and then partial partial z of mu dou u dou z right these 3 terms I have written down together, then what do we have? We have one more term remaining here right which is again mu dou u dou x operated by partial partial x. And then we have one term here this is partial partial y of dou v dou x then partial partial z of this guy right. In all these terms I would like to interchange the order of differentiation for example, here instead of writing partial partial y of mu dou v dou x assuming that I will come to that point.

So, let me first write down these things. So, what do what is that remains here? So, plus what is the first term I would write down in some brackets here So, that is what remains plus I think we had this del dot u which is also remaining, is not it this guy right. So, that guy would be what?

Student: (Refer Time: 35:49).

Yeah. So, that will be that is the remaining term, do we have any other terms left out?

Student: Source term.

We have the source term of course, so, that would be what? That would be plus S M x I have written all of these into curly braces ok. So, is this correct? This is correct very good. Now, let say if we have a so, this entire thing in the curly braces I am going to write it as denote it with a new source term called S M prime x fine.

Now, we will see that that S M prime x should be equal to S M x if we have an incompressible fluid that is what we are going to look at. So, let say if we have incompressible fluid or flow plus we have a constant viscosity so; that means, our mu is also constant if you have a constant viscosity as well as incompressible fluid; that means, both my what properties are constant, density is constant as well as mu is constant ok.

So, if my mu is constant I can interchange the order of differentiation here right instead of writing. Can I write; can I write these terms like this right; these terms underlined in red as can be rewritten like this by just interchanging the order of differentiation because viscosity is now constant plus we have of course, this extra term is also in here.

So, I would go back and write this guy here which is plus I have lambda times del dot u bar right plus S M x is the term in the curly braces. Now, we can of course, simplify this little bit better this would be dou by dou x of mu times what would the; what would be these 3 terms together?

Student: (Refer Time: 38:32).

Del dot u right this is del dot u bar which is actually 0 plus we have lambda times del dot u bar right plus S M x. So, as we just discussed, this term would be 0 right so, as this term right if we have incompressible flow or fluid with constant viscosity ok. So, these are 0 in which case your S M prime x is what?

Student: (Refer Time: 39:01).

Is the same as a sum x ok. So, this is what you have to keep in mind. So, we are throwing everything all these extra strain rate terms that we do not like into this curly braces we are throwing them into the source term. But you have to keep in mind that that source term would be the same as a source term that is brought about by somebody forces or any other forces, but it would not be the just the same it would have these extra terms in case you have a compressible fluid or if you have a non constant viscosity in those things your source term is not just coming out of the body forces very good.

Now, this looks good of course, you can again use if you look into some textbooks they would use this lambda equals minus two-thirds mu in which case you have one mu here and minus two-thirds mu which would give you one thirds mu and so on that is kind of a some simplification, but nonetheless that will term the term will go to 0 ok. So, can we now rewrite this in a more compact way?

Now, if you take a look at these 3 terms can we kind of gain some insight here. So, all these 3 terms are operating on only one component of velocity that is u right, they are all operating on u and then there are two derivative operators right one is operating inside one is operating outside of course, we are saying that we are looking at a constant viscosity fluid. So, this mu can be kind of taken out or it can be left inside ok.

Now, what would can we write this in a compact way/ So, this if you have dou u dou x dou u dou y dou u dou z that is nothing, but a gradient of u right which will give you a vector right. So, essentially these 3 terms can be written as gradient of u which is a vector and then it has to be multiplied with mu right. Now, eventually we have to get a scalar out of this and they have to sum together then if I take again we have dou by dou x dou by dou y dou by dou z we have another gradient right. So, that would be should be another del, but that should operate as a.

Student: (Refer Time: 41:07).

Dot product it should operate as a divergence right divergence of mu times grad u kind of aptly puts these 3 terms into one term right. So, this is del dot mu grad u ok.

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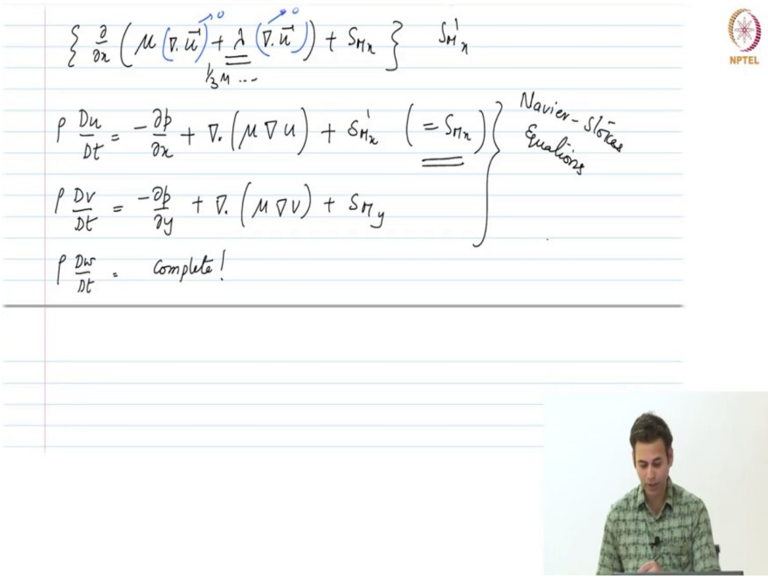

Now, we have made a tremendous simplifications. So, we are going to come down and then write this in a nice way which is$\rho \frac{Du}{Dt} = \frac{-\partial p}{\partial x} + \nabla . (\mu \nabla u) + SMx$¿ I am writing the source term which is S M x prime, but we know that this is same as S M x because we have incompressible flow with constant viscosity. So, even if I leave out the prime sometimes so, do not get confused this is same as S M x whatever is brought about by the body forces ok.

So, this is for incompressible fluid with constant viscosity now the equation looks much nicer right much more pleasing. So, we can probably work with this than with the previous gigantic expressions we had for the shear stresses ok. Now, if you tell me what would be an equation for$is\ \rho \frac{Dv}{Dt} = \frac{-\partial p}{\partial y} + \nabla . (\mu \nabla u) + SMy$¿ of course, you can complete the rho D w D t equation yourself fine.

Now, so, these are these are what these are the Navier - Stokes equations right Navier - Stokes equations which are independently derived by these two scientists by introducing this model for the shear stresses ok. So, that is important.

So, the distinction between momentum equations and the Navier - Stokes equations is clear right essentially momentum equations have the shear stresses as the unknowns still whereas, the Navier - Strokes equations do not have the shear stresses as unknowns right they have a model for it which works very well and those are the Navier - Stokes equations which we will use to solve for fluid flow equations in the incompressible fluid flow regime alright.

So, now of course, we still have if you go back to the equations we have the energy equation which also contained shear stresses tau x x tau y y all these things which we are not going to do, but rather I am going to kind of summarize the equation.

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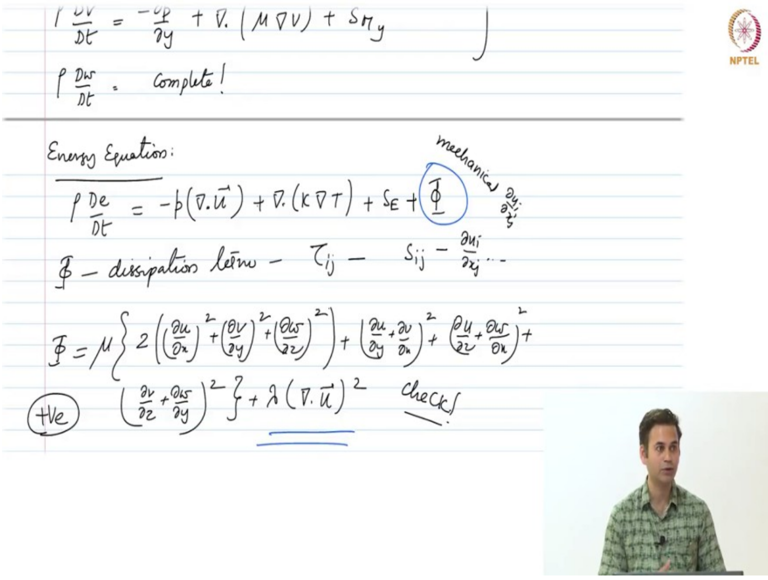

So, if you look at the energy equation which you have to solve even if you have an incompressible fluid only that this will be solved separately you do not have to kind of couple them. So, what was the energy equation, if you will remember back rho times D little e D t right; what was this?

This was some I think minus p times del dot u bar or something like that right, initially we had del dot p u bar, but then we subtracted off the kinetic energy term which gave us an equation for the internal energy $\rho \frac{De}{Dt} = -p(\nabla.\vec{u}) + \nabla.(k\nabla T) + SE$ rho . We have a lot of terms in terms of shear strains right instead of viscous stresses right instead of viscous stresses tau i j we had lot of terms about 9 or 10, 9 terms and I would like to call this all these terms as phi ok.

So, what I mean by phi is a is the dissipation term which contains the tau i j terms right we had these u times tau x x u times tau x y and so on right we had several terms. Now, what you have to do is you have to plug in the tau i js viscous stresses in terms of the strain rates right to eventually get all these terms in terms of partial u i by partial x j and so on ok.

If you do that what you would see is phi would read as $\mu\left\{2\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right)+¿$ mu

$\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)^2\}+\lambda(\nabla.\vec{u})^2$. So, this is something you have to derive or rather kind of check it ok.

So, check this by plugging in the viscous stresses in terms of strain rates and then club all these terms. So, this is the dissipation term which appears on the right hand side of the energy equation ok. Now, what is the first thing you see you notice about the dissipation term? Of course, it is composed of the strain rates ok, but what is the thing that is there that kind of catches your eye?

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Squares viscosity we have the viscosity we have the mu and del lambda that is there, but the terms are all squared now right. So, what about the values we have? So, these are there. So, essentially what will this term be if you have a if we have let say incompressible flow or a fluid this term is anyway 0, the second term is anyway 0 right now what will happen to the; so this term on phi is always a positive quantity right because of the square. So, this is always a positive quantity.

Now, what is it actually doing? It is actually what it is actually doing is you have these strain rates which are the deformation rates which describe the deformation of the fluid particles or fluid as it flows through right. And, you are getting the deformations right which is described by this phi which is acting as a source term on the right hand side of the energy equation right.

So, this is basically dissipation because it is converting the mechanical energy which is the partial u by partial x j right all these terms that is the mechanical energy right of the fluid into thermal energy right it is creating the internal energy or temperature right essentially it is increasing the internal energy of the system. So, the dissipation term always tries to increase the internal energy of the fluid by extracting the energy from the mechanical component of the fluid ok.

So, that is responsible for the change in internal energy by converting the mechanical energy into the thermal energy right ok. So, that is the only link fine. So, now we have looked at the complete set of Navier - Stokes equations including the energy equation by introducing the model for the shear stresses. So, what we are going to do next is we are going to list down all the equations you know one by one and then see if we can kind of come up with a common equation that can represent all these equations, right.

If you see there are several of these terms are common between all these equations right you have an unsteady term, you have a convection term and then you have a divergence term and so on right. So, we are going to kind of list down all these equations one by one and then see if we can find some commonalities between them and then thereafter go from there so ok.

Thank you.