**Computational Fluid Dynamics Using Finite Volume Method**
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**Lecture - 36**
**Finite Volume Method for Fluid Flow Calculations: SIMPLE algorithm- Part I**

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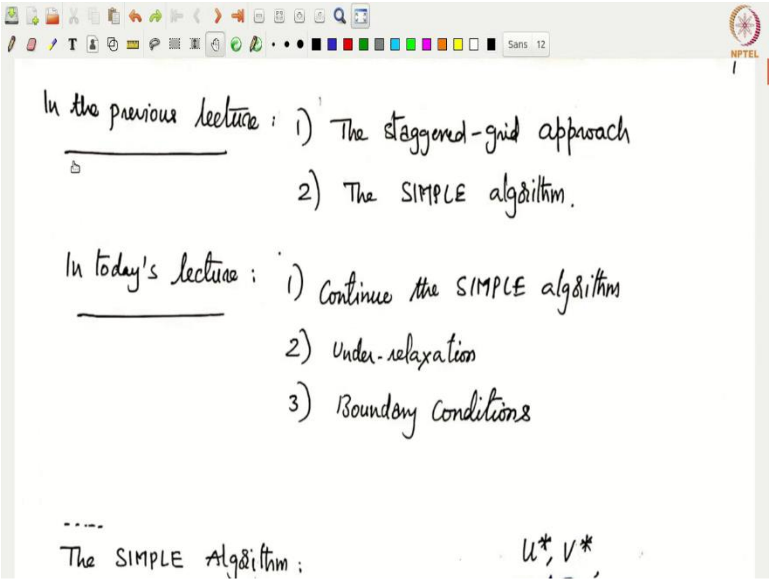


Hello everyone, let us get started. So, welcome to another lecture as part of our ME6151 computational heat and fluid flow course. So, in the last lecture, we looked at the staggered grid approach essentially to account for the pressure velocity coupling. Then, we decided it should be stored as, the velocity should be stored staggered to the pressure and the x component of velocity is that is $u_e$ and $u_w$ was stored on the east and west faces.

And, the v component y component of velocity that is $v_n$ and $v_s$ were stored on the north and south faces and the pressure was stored at the cell centroids right the main control volumes, alright. Then, we also looked at the simple algorithm the semi-implicit method for pressure linked equations in which we noted or we kind of created an equation for pressure correction from the continuity equation and the momentum equations right.

So, we have kind of went step by step through the algorithm and then, in today's lecture, we are going to continue with this algorithm, we will write the overall simple algorithm. Then, we will look at under relaxation for pressure, correction as well as for velocity and also we are going to discuss the several boundary conditions if you want to solve for a fluid flow problem alright. Then, let us move on.

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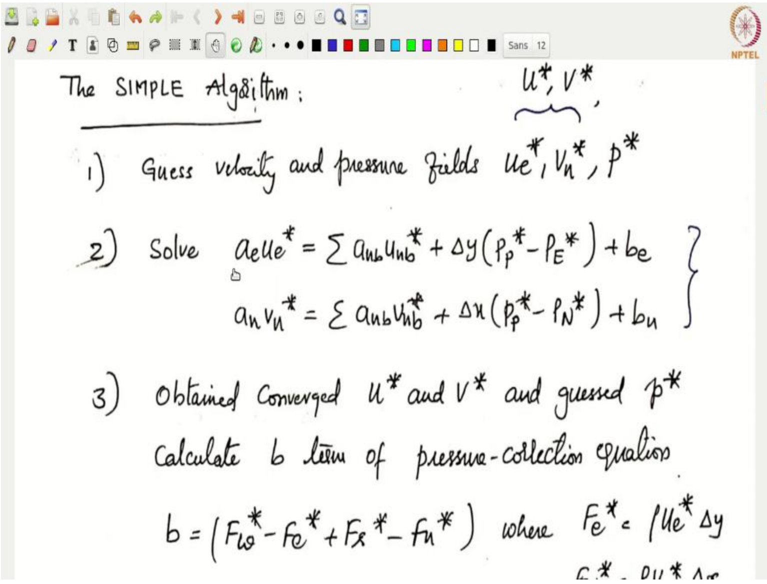


So, coming to the simple algorithm; the first step was to guess the pressure field right. Essentially, you have to guess the velocity as well as the pressure field. So, guess u star, v star or $u_e^*$, $v_n$ as well as the $P^*$ ok. So, once you guess the pressure velocity fields, you have to solve the discrete momentum equations, the discretized momentum equations which are basically for the control volumes for u and the control volumes for v right.

These are written on control volumes that are centered around the east face, west face, north face and the south face right for the cell center p ok. So, then, the discrete equation is given by $a_e u_e^*$ equals $\sum a_{nb} u_{nb}^*$ plus $\Delta y(P_P * - P_E^*)$ plus $b_e$ ok. Similarly, the y momentum equation is given by $a_n v_n^*$ equals $\sum a_{nb} v_{nb}^*$ plus $\Delta x(P_P^* - P_N^*)$ plus $b_n$ ok.

Now, in doing this, we realize that of course, we have guessed the pressure everywhere and we have also guessed the u star values because these are required when we try to solve this equation. Now, how do we solve this equation? You have to use either a Gauss-Seidel or a line by line TDMA and then, converge this equation to some tolerance right.

And now in doing so, essentially, what we realize is that the initial guess that we had for velocities would be gone right by the time this equation converges, the new $u^*$ would be something that is very different from what we have guessed, but that will be the converged value.

So, we are not using a different variable for the converged value of $u^*$ rather we are using the same notation. So, basically once you solve this equation, the $u^*$, $v^*$ you get from these two equations is something that is converged by solving these two with a guessed pressure value of $P^*$ alright.

Then, we also know that the $a_{nb}$'s that are occurring in both these equations are different because the cell control volumes themselves are different. And further, we also realize because we have not solved two equations together till now, we what we realize is that basically these two equations are now coupled right.

Essentially, in some sense because essentially, you have $a_{nb}$'s which may contain u and v whereas, $a_{nb}$'s here also would contain $u_{nb}$ because each of these $a_{nb}$'s have D term and an F term write the diffusion term and the convection term.

So, the convection term would actually have both u and v right depending on the face we are looking at. But because we have linearized system, these values going into D and F and eventually into $a_{nb}$'s are known at this point ok. So, they are basically calculated based on these guessed values ok. So, as a result, these are not actually although they are coupled, you can solve, you can converge for the first equation and after that you can converge with the second equation alright.

So, essentially what we realized that the once we converge these two equations, we got, we obtained a converged $u^*$ and $v^*$ fields with the guessed pressure ok. So, these are obtained with the guessed pressure value, pressure field ok.

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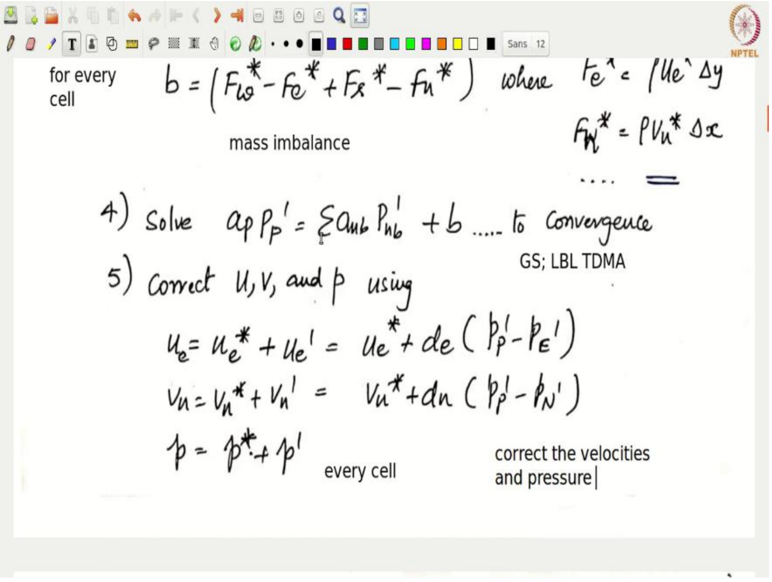


Now, the next step is basically is to calculate the source term coming in the pressure correction equation that is the b term. You remember, the b term on the in the pressure correction equation that is basically a that is basically a the amount by which the mass conservation is not satisfied right.

So, that is basically the mass imbalance that we have in the flow field that means, depending on how far these $u^*$ and $v^*$ values are from the continuity equation this mass balance, mass imbalance will drive the pressure correction equation to give the corresponding pressures ok, the corresponding pressure corrections alright.

So, the next term is to basically calculate, next thing is to calculate the b term which contains F, the mass flow rates on the west, east, south and the north faces. These are now calculated as $\mathrm{F}_e^* = \rho u_e \Delta y$ similarly, $\mathrm{F}_w^*$ would be $\rho u_w \Delta y$ and $\mathrm{F}_n^*$ would be $\rho v_n \Delta x$ and $\mathrm{F}_s^*$ would be $\rho v_s \Delta x$.

Now, these $u_e^*$, $v_n^*$ are basically the ones which you have got them as converged right. Now, these are the values that will be used in calculating the star value of the flow mass flow rates on the faces ok. Now, that means, essentially you will calculate this b term for every cell that you have in the entire domain right so, that is what is done. After that what we do is basically you we can now discretize the pressure correction equation that is basically $a_P P_P' = \sum \mathrm{a_{nb}} \mathrm{P}_{nb}' + \mathrm{b}$.

Now, you solve these two convergence because now b is computed everywhere and we know what are these $a_{nb}$'s and $a_P$'s because these again depend on the $\Delta y/a_E$, $\Delta y/a_N$ and so on. So, those are all already known and those $a_E$, $a_N$ are actually the same as what we had here so, they are not updated between here and here ok. So, they are the same $a_E$, $a_N$ that we have used in the momentum equations before ok.

So, and then, we can solve this two convergence. Now, how do to solve this two convergence? Again, you have to use either a Gauss-Seidel or you have to use line by line TDMA and then, converge this to some tolerance value ok. So, once you finished with once you are done with step 4, essentially what you get is you get a field for your pressure correction p prime would be known everywhere in the domain alright.

Then, once you know the pressure corrections, of course, we can now go ahead and correct the velocities and the pressures with the obtained pressure correction. So, correcting the pressures is trivial $P = P^* + P'$ that is straightforward so, you do this basically for again for every cell and then for the velocities, we have already the $u^*$ value which is the converged value after step 2 right.

Now, you add $u_e^*$ to $u_e'$. Now, $u_e'$ is not directly known, but $u_e'$ is related to $P'$ through $d_e(P_P' - P_E')$ right basically through the simple algorithm we have related this thing. Similarly, $v_n = v_n^* + v_n'$ where $v_n'$ is equal to $d_n(P_P' - P_N')$ ok. So, basically, using the pressure corrections, correct the velocities and pressures everywhere in the domain ok. So, that is basically that step is done.

Now, what we know is that because of this correction of $u_e'$ and $v_n'$ to $u_e$ and $v_n$, $u_e^*$ and $v_n^*$, we get this $u_e$ and $v_n$ field. Now, this field will satisfy continuity equation because we have just solved continuity equation in a reformulated state right essentially, the solution for pressure correction is nothing, but the solution for continuity right. So, then this field will now satisfy continuity equation.

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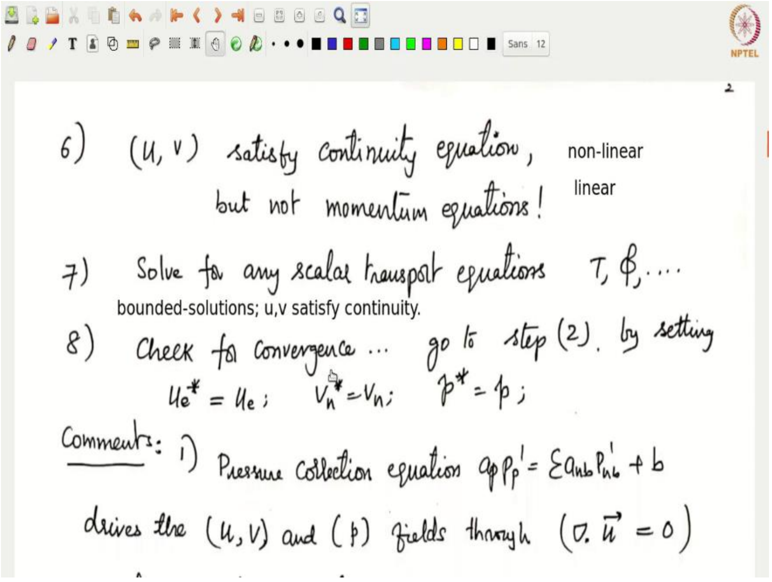


So, $u_e$, $v_n$, P the corrected velocities and pressures will satisfy continuity equation, but they will not satisfy momentum equations ok, they will not satisfy momentum equations. So, this kind of requires little bit of explanation, what do we mean by they do not satisfy momentum equation? What we mean by not satisfy momentum equation is basically that the momentum equation itself is non-linear right.

So, essentially, we have already linearized it so, that means, they will essentially, this field will satisfy the linearized equation with the original $a_{nb}$'s that we already have because that is what we have done in arriving at these pressure correction equation and so on, but the u and v and P that you got would not satisfies the original momentum equation right, the continuous momentum equation is still not satisfied by this u and v right.

So, if the momentum equation were not if it was if it were linear, let us say if the momentum equation were linear, then there is no problem because if it were linear, then these coefficients $a_{nb}$ would not have had any non-linear component which were linearized sorry which were linearized here. So, as a result, this equation would be the same as the linear momentum equation only this would be the discrete form.

As a result, then this obtained u and v would also satisfy not only continuity, but also momentum, the continuous momentum equation alright. But because we have non-linear terms, this obtained u and v and pressure fields will not satisfy the momentum equation after one iteration; however, it satisfies the continuity equation alright.

Now, because we have obtained a velocity satisfying velocity continuity satisfying velocity field, we can now solve for any scalars that we have. For example, if you are solving for transport of let us say some scalar ϕ or if you want to solve for temperature in your equation and so on, then you can solve for all those scalar transport equations after you have obtained the continuity satisfying velocity field.

Now, why do we do it after this, why cannot we do it before and what does the resemblance it, what does the what is the significance it has if u and v were not to satisfy continuity? The thing is if you do not fit in a continuity satisfying field to the scalar transport equation, then essentially you will get, you do not get bounded solutions.

So, you will only get bounded solutions, if your u and v satisfy continuity right. Essentially, that means, if you do not have a continuity satisfying velocity field even if you use upwind difference schemes, you will not get bounded solution for your ϕ and temperatures for temperatures ok. So, that so, it is very important that all the scalar transport equations are solved with a continuity satisfying velocity field ok.

So, as an intermediate step, this is what we have and we do not want to have unboundedness here because they are they cannot grow more than what value that is coming in right. So, they cannot grow more than that unless there is a source or something so; that means, this is important. So, once we solve for all the scalars, then we know that of course, we have not reached convergence yet, then we check for convergence.

Now, what do we mean by checking for convergence at this stage? You check this with the original u star and v star that you started off with right that was basically whatever values you had guessed here, you compare that with whatever u, v you have obtained here right.

Now, these will most likely be different from what you have guessed right and then as a result, we will not converge so, that is why basically you go to step 2 by setting you again use the new guess values as whatever you have just obtained excuse me; that means, your $u_e^*$, $v_n^*$; $u_e^*$, $v_n^*$ would be whatever $u_e$ and $v_n$ that you have got it here after correction step ok.

So, you would use that and then go back to step 2 and then again, you with the new pressure field, you update now $\mathrm{a_{nb}}$'s because $\mathrm{a_{nb}}$'s now contain u, v, $u^*$, $v^*$ values which are now updated right with that you would again solve for converge x momentum equation, converge y momentum equation and then, fill the right-hand side, then converge the pressure correction equation and then, correct velocities and pressure and then again, solve for transport equations and so on ok.

So, how many let us say if we do not have any scalar transport equations to solve so, how many Gauss-Seidel loops do we need here? We need essentially, we need one to converge the x momentum, one to converge the y momentum and one to converge the pressure correction right.

Either we need essentially three Gauss-Seidels or three line by line TDMA that is massive, isn't it because if we let us say take a cell, if we take a domain with let us say a 100 by 100 cells; that means, we have 10 power 4 cells; that means, we have to solve Gauss-Seidel to convergence for three such three times on this 10 power 4 cells right.

Which if it is not fast might take, might consume a good amount of time alright. Of course, if we have another scalar transport equation, then we would need a need to solve it here as well right ok. So, that is the overall algorithm.

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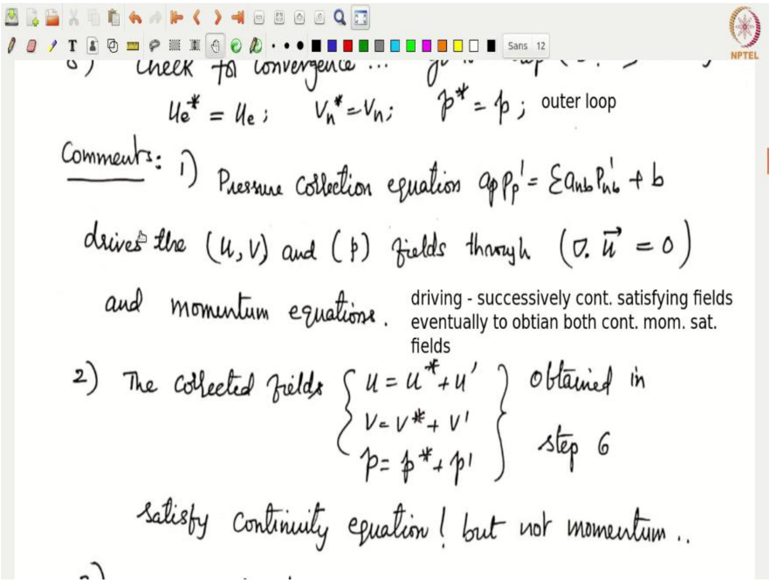


Let us make some comments on what we have learned so far ok. So, coming to the comments, the pressure correction equation that we have $a_P P_P' = \sum a_{nb} P_{nb}' + b$ so, what this is doing is basically it is driving the velocity field that is u, v and the pressure fields through successively continuity satisfying fields and eventually to arrive at a velocity and a pressure field that will also satisfy momentum equations ok.

So, essentially, the pressure correction equation is driving the velocities and pressures through essentially successively continuity satisfying fields right and eventually to obtain both continuity and momentum satisfying fields so that is what we are doing. So, essentially, we are eventually reaching the velocity and a pressure field that satisfies both continuity and momentum, but we are reaching this goal on this path through successively continuity satisfying fields.

So, at every step, at every iteration, we have at every step in one loop of the simple algorithm, we have a velocity field that satisfies continuity and then, this will be driven to satisfy momentum through successive iterations.

That means, we have this outer big iteration, like this outer loop which is basically check for convergence go to step 2, this is basically the outer loop. So, once you are done with this outer loop, then you have a velocity and a pressure field that satisfies not only continuity, but also momentum ok. I hope that part is clear alright.

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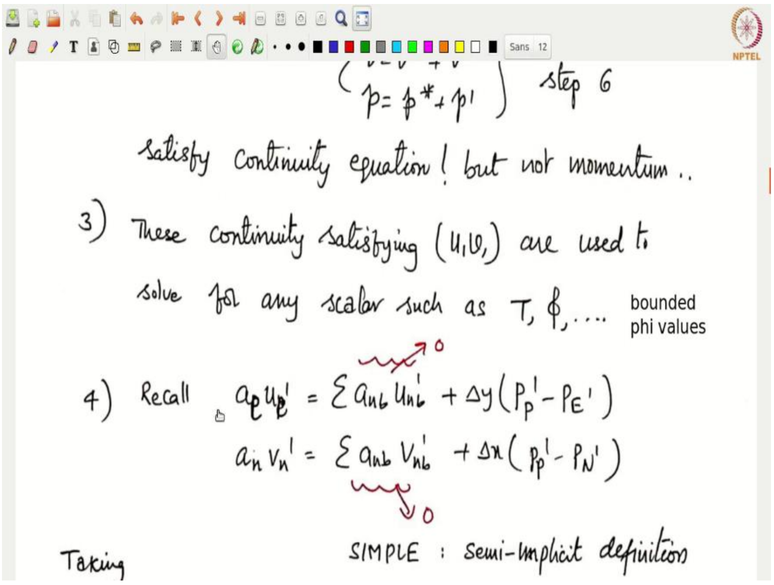


Now, let us look at the kind of approximations we have made. So, the corrected fields u and v, p obtained in step 6 basically after the correction step, they satisfy continuity, but not momentum right, they do not satisfies the original momentum equations. And these continuity satisfying fields are used to solve for any scalar such as temperature and phi.

Of course, if you do not do that, then you do not, you will not get basically bounded phi values even if you use bounded schemes such as upwind difference scheme, even with that it is it will not come because essentially your velocity itself is not continuity satisfying right. Then you have the right-hand side where remember in we had $F_e$ minus $F_w$ plus $F_n$ minus $F_s$ that would not go to 0 and as a result, you will not get boundedness for your scalars alright.

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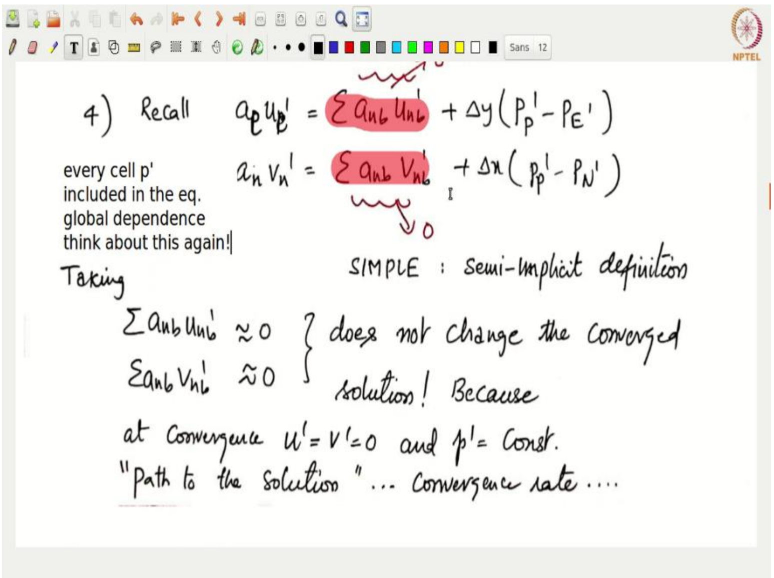


Then, one of the major approximations that we made in this simple algorithm is basically after we arrived at the velocity correction equation in terms of pressure corrections that means, we wrote two such equations, one for $u_e'$ and one for $v_n'$ as $a_e u_e' = \sum \mathrm{a_{nb}} \mathrm{u}_{nb}' + \Delta y(P_P' - P_E')$. And, the other one was $a_n v_n' = \sum \mathrm{a_{nb}} \mathrm{v}_{nb}' + \Delta x(P_P' - P_N')$ right.

In doing this, what we have; what we have done is essentially, we have assumed because of the simple algorithm, we have assumed this term and this term to be 0 right; that means, we have assumed these two neighboring dependency on the neighboring corrections is said to 0; essentially what that means is that will not change the converged solution.

So, one question would be like is it ok, can you arbitrarily take this to be 0, what if what will happen if we do not take it to be 0 ok? We will see what will happen, but if you take it to 0, it only says that essentially you are putting the entire burden of correcting velocity corrections on pressure corrections because the neighboring velocities are not contributing ok.

But we realize that this kind of an approximation will not change the final converged answer. Now, why? Why is it so? Because essentially, your $u'$, $v'$ would be 0 at convergence right because $P'$ would reach a constant value as a result, $u'$, $v'$ would be 0. So, once you have $u'$, $v'$ 0, your u equals u star. As a result, u primes are 0 everywhere so, $u_{nb}$'s are all those $u'_{nb}$ are also 0, $v'_{nb}$ primes are also 0.

So, as a result, this kind of an approximation would not change the convergence solution, it only changes the path to the solution and more precisely it changes the convergence rate. So, but we are with it because the trouble of including these terms would be much bigger than actually change in the convergence rate because so that means, this kind of an approximation only changes the convergence rate.

Now, what will happen let us say if you have not neglected these guys, if you have let them there, let them be there, then what you happen? So, what was the algorithm? The algorithm was to basically substitute for velocity corrections in terms of pressure corrections. Now, because we could make such an approximation, we could nicely substitute for east prime as in terms of P and capital E, v n as p and N and similarly for west and south and get a nice diffusion like discrete equation.

But if you had let us say not neglected these things, what would have happened? What would have happened is basically $u'_e$ requires not only $P'$ when you substitute in the continuity equations, it also requires $u'_{nb}$, then $u'_{nb}$ also would be would have a similar equation which will require on, which will depend on its own neighbors right that means, here it will be like $a_e u'_e$ this is for the neighboring cell would depend on its neighbors and some pressure corrections and so on.

And then, this keeps going until essentially every cell the pressure correction for every cell will kind of b included in the equation right that means, we are essentially talking about a global dependence right of solving the entire p primes in one place which is what we do not want to do because that would basically make it a very dense matrix and then which we do not want to go in that direction right.

So, if this is not clear, you have to kind of think about this again ok; that means, if you had basically not taken this to be 0, then just like the equation we have written here for east prime, you will write an equation for $u'_{nb}$ right when you go to the next cell, then $a_{nb}u'_{nb}$ would depend on sigma its own neighbors, its own $a_{nb}u'_{nb}$ and then, the pressure gradients.

So that means, when you go back and substitute these in the velocity correction equation in the continuity equation, then you would get essentially not only $P'_E$, you will get $P'_E$, $P'_{EE}$ and so on all the way to the boundary right that means it is just a big massive equation which will make it basically unmanageable to solve ok.

So, that is why we are avoiding that is why, simple algorithm makes that a this is these terms are 0. Now, that is where the naming for simple has come. Basically, it says that semi-implicit method if you that is because this neighboring coefficients are basically made to 0 as a result, we are only solving for a semi-implicit ok, this is only implicit in pressure primes, but not a fully implicit right.

If it were fully implicit, then u prime would also depend on $u'_{nb}$ ok. So, in order to save computational effort, we are doing that and that is where the naming for the algorithm as semi-implicit comes into play here and the remaining part the pressure linked equations is basically we have the momentum equations are linked to the pressure that is what the naming comes from alright.

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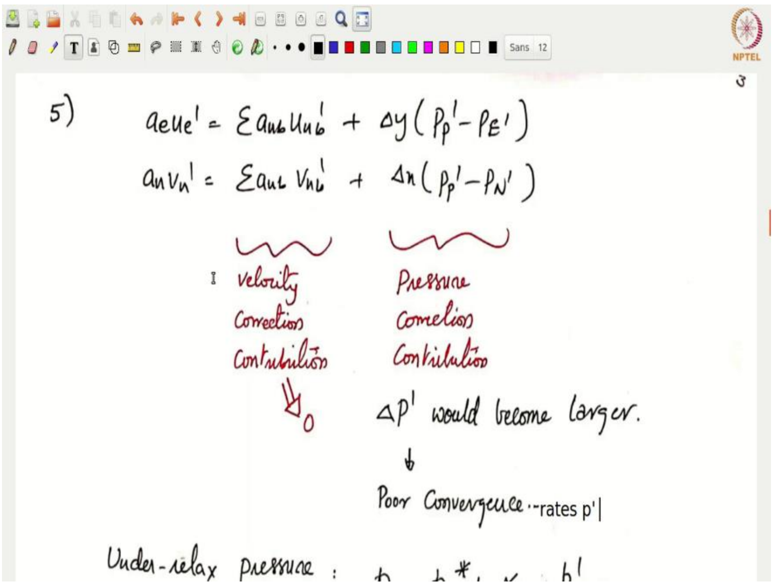

Then, essentially, if we continue to look at this neglecting the neighboring coefficients, then basically your $a_e u_e'$ has two components, one is $\sum \mathrm{a_{nb}} \mathrm{u}_{nb}'$ plus $\Delta y(P_P' - P_E')$similarly, $a_n v_n'$ equals $\sum \mathrm{a_{nb}} \mathrm{v}_{nb}'$ plus $\Delta x(P_P' - P_N')$that means there are two components, one is the velocity correction here depends on neighboring velocity corrections and on the pressure corrections.

So, we are taking the neighboring velocity corrections to be 0 that means, what we are doing is by doing so, the velocity correction at a particular phase has to be completely done by the pressure corrections alone right because the neighbors do not contribute. As a result, the delta $P'$ values that you would good get would be quite large that means, having larger values for; larger values for this $P'$ would lead to poor convergence rates; this will lead to poor convergence rates.

As a result, the $P'$ equation that we have the pressure correction equation will not converge very quickly ok, it will kind of be very sluggish. So, as a result, we have to kind of under relax the pressure that we get otherwise this will become like very large values.

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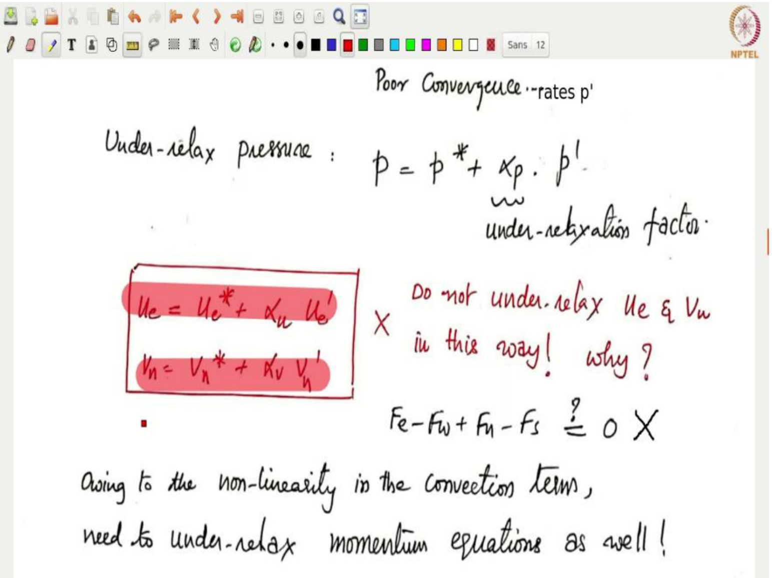


So, one way to under relax is basically the pressure equation is P equals when you correct the pressure, you add only a portion of whatever $P'$ that you got out of the pressure correction equation. That means you write $P = P^* + \alpha P$ this is some under relaxation factor it could be 0.2, 0.3 depending on situation or 0.8 times p prime is what you do.

And the non-linearity of the momentum equations also calls for under relaxing the velocity corrections because of the non-linearity, but we will not under relax it in this way ok, in the way we have done the pressure under relaxation because, let us say you will not write $u_e = u_e^* + \alpha_u u_e'$ and $v_n = v_n^* + \alpha_v v_n'$.

You will not do this, because if you do it this way, essentially, the velocities you are getting after correction will not satisfy continuity so essentially this will not be satisfied ok, the continuity this will not be satisfied so, do not under-relax basically this way so, do not under-relax them this way.

We will see how to under relax the velocities using our original way that means, using the momentum equations itself because if we do this, if you modify, then you are $u_e, u_w, v_n, v_s$ that you get would not make it to 0 right, you are again you will get a; you will get a velocity field that will not satisfy the continuity because F east minus $(F_w - F_e + F_s - F_n)$ would not be equal to 0 right so, entire purpose would be gone.

So, as a result, the non-linearity in the convection term of the momentum under momentum equations also calls for under relaxation and we under relaxing momentum equations in the following way essentially in the original way that we have discussed.

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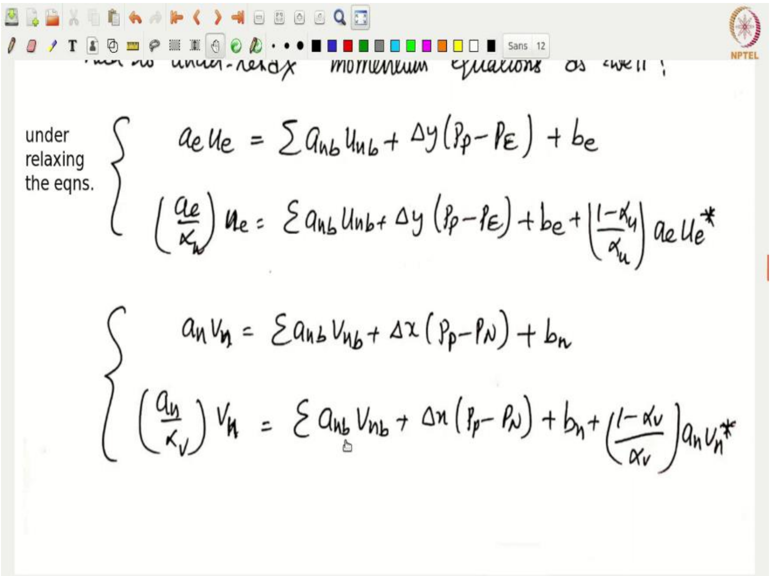


So, if you have the discrete momentum equation as $a_e u_e = \sum a_{nb} u_{nb} + \Delta y(P_P - P_E) + b_e$, then of course, how do you under relax this thing? Basically, you add and subtract $a_e u_e^*$.

And then, you multiply the component with some $\alpha_u$ ok, then if you rearrange, then you what you get is you get this coefficient $a_e$ going up by divided by $\alpha_u$ times $u_e$ equals $\sum a_{nb} u_{nb} + \Delta y(P_P - P_E) + b_e$ plus you have this extra term right which is $(1-\alpha_u)/\alpha_u$ times $a_e u_e^*$.

Now, again at convergence $u_e^*$ equals $u_e$ as a result, this $(1-\alpha_u)$ times $a_e u_e^*$ would go to would get cancel with this term and then your $\alpha_u$ and $\alpha_u$ get cancel here and what you get is $a_e u_e^*$ with a minus that can be taken back to the left hand side and you get the original equation here ok. Now this we already have discussed in the context of under-relaxing the equations right, this part we have already discussed.

So, we can do a similar thing for the y momentum equation that is$a_e v_n = \sum a_{nb} v_{nb} + \Delta x(P_P - P_N) + b_n$ and if you want to under-relax this equation, you write basically $a_e/\alpha_v$ times $v_n$ equals $\sum a_{nb} v_{nb} + \Delta x(P_P - P_N) + b_n$ plus you have $(1-\alpha_u)/\alpha_u$ times $a_n v_n^*$.

So, one question could be again what will be the values of $\alpha_u$ and $\alpha_v$, again they are depending on the case you could probably take them around like 0.8 or 0.7 and they need not be also they need not be the same they can be different as well and they need not be same as the pressure under-relaxation as well alright.

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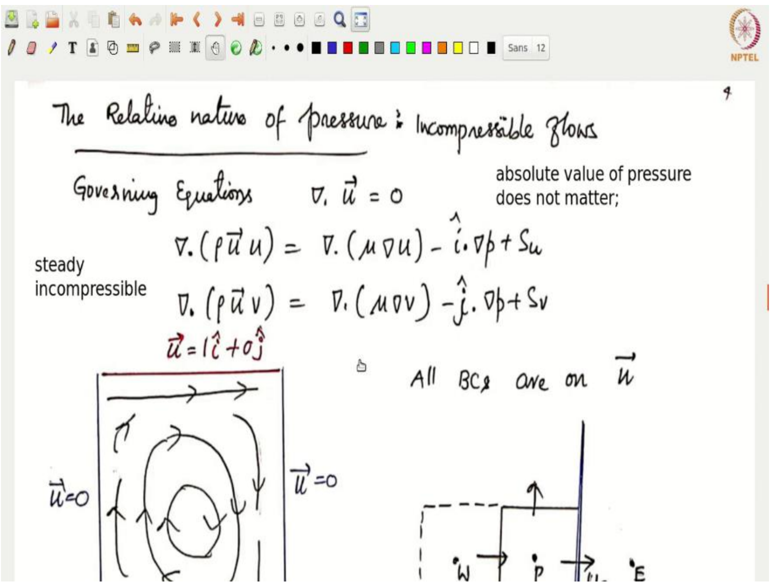


Now, let us look at couple of things that is basically before we go on to the boundary conditions, we will look at the relative nature of the pressure in incompressible flows. In this also, we will kind of invoke little bit about the boundary condition and thereafter we look at the complete set of boundary conditions that we can apply in the solution of fluid flow equations ok.

So, in the context of incompressible flows, we say that the pressure has got a relative nature; that means, what we say is that the absolute value of pressure does not matter because it does not feature in the equations right, it only needs the gradient of pressure that is featured in the equations in the governing equations which are basically $\nabla \cdot \vec{u} = 0$.

Essentially, if you are talking about a steady incompressible flow, your continuity equation is $\nabla \cdot \vec{u} = 0$ and your momentum equations are $\nabla \cdot (\rho \vec{u} u) = \nabla \cdot (\mu \nabla u) - \hat{\imath} \cdot \nabla P + S_u$.

And the y momentum equation is $\nabla \cdot (\rho \vec{u} v) = \nabla \cdot (\mu \nabla v) - \hat{\jmath} \cdot \nabla P + S_v$ right essentially, it is only the gradient of pressure that matters so, because you do not have a an equation of state or something that can control the absolute that can introduce the absolute value of pressure into the system. So, as a result, it is only the gradient that matters, but that is fine.

When we stated theoretically, let us see how, why we say that the gradient is the one that matters not the absolute value by using couple of examples and also through some analyzing some of the boundary conditions ok.

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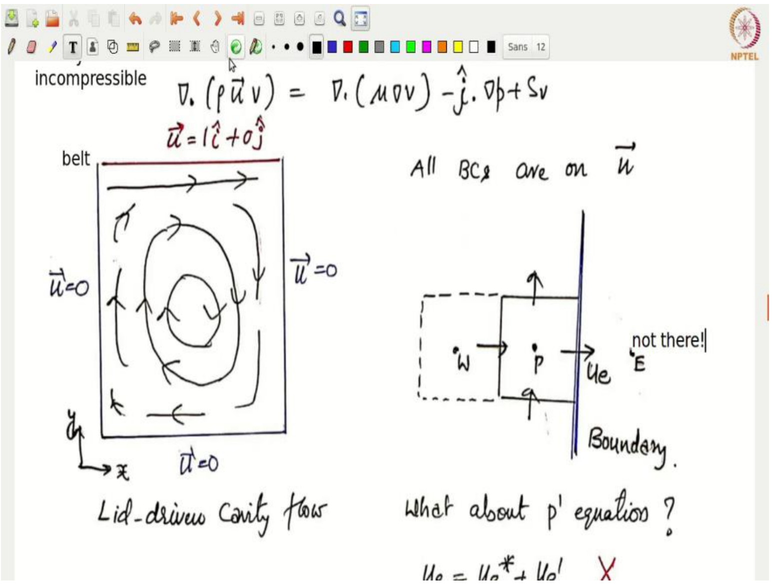


So, let us take basically an example. Here, this on the left-hand side, you see basically a cavity let us call this as a square cavity. In the literature, this is known as lid-driven cavity flow. Essentially, let us say we have a box like this on which the top of this box is you could think of this as a belt or something that continuously moves in the positive x direction.

So, there is a belt that is moving and then, it kind of comes back and essentially it has a lid on the top which kind of moves continuously in x direction with a velocity of let us say 1 meter per second. So, $\vec{u}$ for the lid is $1\hat{\imath} + 0\hat{\jmath}$ whereas, the all three other boundaries of this box are stationary so, they are not moving so, their velocity is 0 on all the three sides and this box is filled with let us say water so, it is filled with water.

And then, if the belt is moving at a certain speed, then after let us say if particular steady state is reached, then you will see that the water inside, the fluid inside the cavity also starts rotating with because of the belt and these are some of the stream lines.

Of course, these are only a representative because they are not drawn continuously and we know that streamlines cannot end like this somewhere in the middle of the flow abruptly ok, these are only kind of indicating the direction. So, we have some kind of flow that is set up.

Now, what we see here is that what are the boundary conditions that we have specified in solving this problem? The only boundary conditions we have specified are basically for velocity right. Essentially, velocity is 0 here and velocity is 0 here as well and 0 and it is $1\hat{\imath}$ on this top lid where we have not specified pressure anywhere ok. So, we have not specified pressure anywhere. So, what consequences does it have in the solution of fluid flow equations or even in the solution that is obtained here? Ok.

So, to do that, let us look at; let us look at one of the cells that is sitting right next to the right-hand side boundary here. So, we are talking about this is the right-hand side wall and if we have let us say a staggered velocity representation basically u east on the faces, u east u west on the west face, v north, v south and P cell and we do not have essentially this east cell at all right this is not there, this is only shown for representation. So, you have P and W cells, and this is the primary cell alright.

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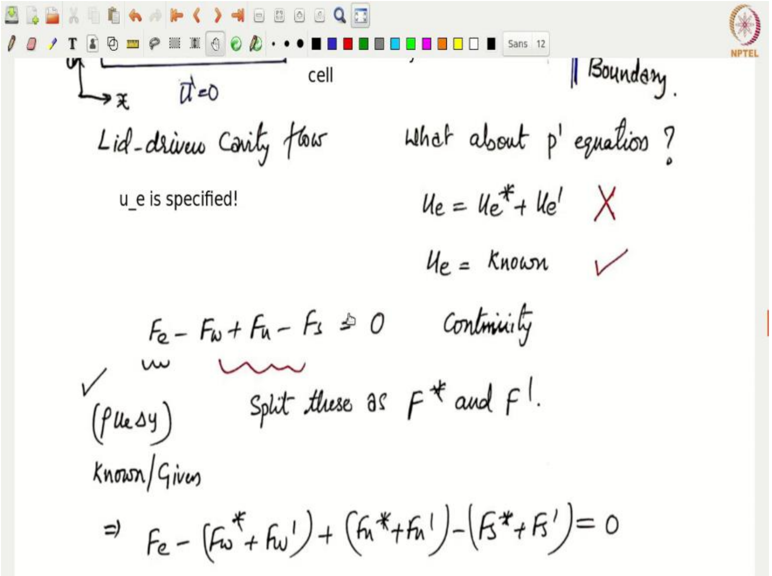


That means, what happens well now what will happen to the p prime equation? When you try to solve for this near boundary cell essentially for the near boundary cell, what will happen to the p prime equation? That means, your $u_e$ is $u_e^*$ plus $u_e'$ right that is what we would use in obtaining the $P'$ equation, but we know that $u_e$ is already specified right.

So, $u_e$ is $u_e$ value is specified so, we cannot correct it so, we cannot write $u_e$ equals $u_e^*$ plus $u_e'$. So, leave $u_e$ as it is. So, this is a known value; that means, once you know; that means, velocity correction on this for this particular face would be 0, $u_e'$ is 0 on this particular face in fact, for all the faces that share this boundary.

In fact, if you look at this problem, $u_e'$ would be 0 for all the cells here that share near this boundary and here, all the $v_s'$ would be 0, similarly $u_w'$ prime would be 0 here and $v_n'$ prime would be 0 for all the cells that share (Refer time: 33:11) addition to this top boundary that is what we are saying; that means, we already know what is the value, then we cannot, we should not write it in terms of star and the correction values alright.

Then, if you look at the starting point for the pressure correction equation, we are starting off with the conservation of mass right essentially, $F_e - F_w + F_n - F_s$ equal to 0 this is the continuity equation and because these are now used together with $u_e$, $u_w$ and so on, these will satisfy continuity. If there were stars on this, these would not satisfy continuity right. If there were stars here, this will be not equal to 0 because there are no stars here, this is equal to 0.

Now, this $F_e$ value is already known. Of course, in this particular context, if $u_e$ is 0, $F_e$ is 0, but in general, let us say $F_e$ equals $\rho u_e \Delta y$ this is already known so, let us substitute for this guy. Once that this is known, where will this term go? This term will go into the b term right that means, we can write this as and the remaining terms that is $F_n$, $F_w$, $F_n$ and $F_s$ can be written can be decomposed into star values and the prime values leaving $F_e$ as it is.

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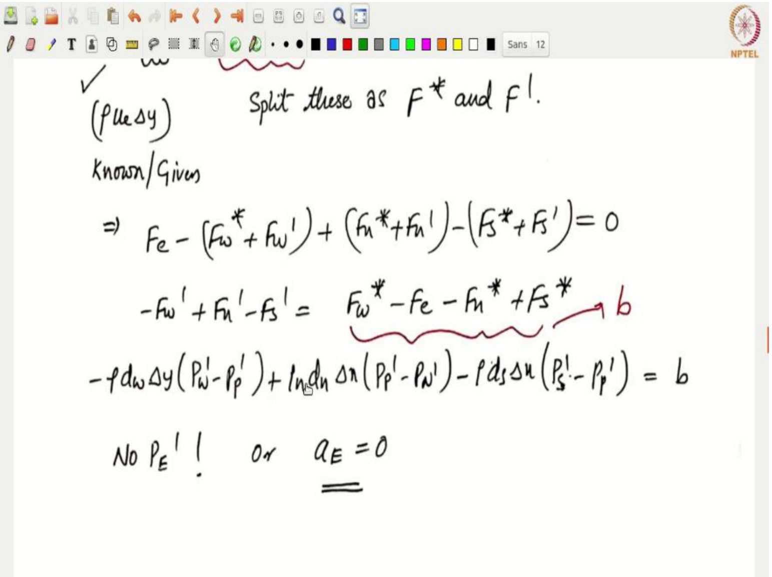


So, $F_e - (F_W^* + F_W') + (F_n^* + F_n') - (F_s^* + F_s') = 0$ ok. Then, we can rearrange this by sending the star values to the right-hand side and also the $F_e$ to the right hand side that means, the b term now contains $F_W^* - F_e - F_n^* + F_s'$.

Again, if $F_e$ happens to be 0 because of something let it be 0, but otherwise this is the formulation right and on the left-hand side, now, we are left with only three terms that correspond to the primes of the flow rates those are $-F_W' + F_n' - F_s'$.

And for which we can now substitute in terms of; in terms of the velocity corrections, in terms of $u_w'$ and $u_w'$ can be again written in terms of $P_W'$ and $P_P'$ and we have this particular equations right basically $-\rho d_w \Delta y (P_W' - P_P')$ plus its basically $\rho d_n \Delta x (P_P' - P_N')$ minus $\rho d_s \Delta x (P_S' - P_P')$ equals b alright.

That means, we see that there is no $P_E'$; that means, there is no connection to the $P_E'$ right of course, that makes sense it should not be there. What about; that means, what about $a_E$?

That means, $a_E$ equals 0 because there is no contribution to of that term to either to P W or either to P east or to P p right so, $a_E$ equal to 0; that means, we already have, only have a three neighbors and the contribution to $a_P$ will only come from west, north and south, it will only come from north, west and south.

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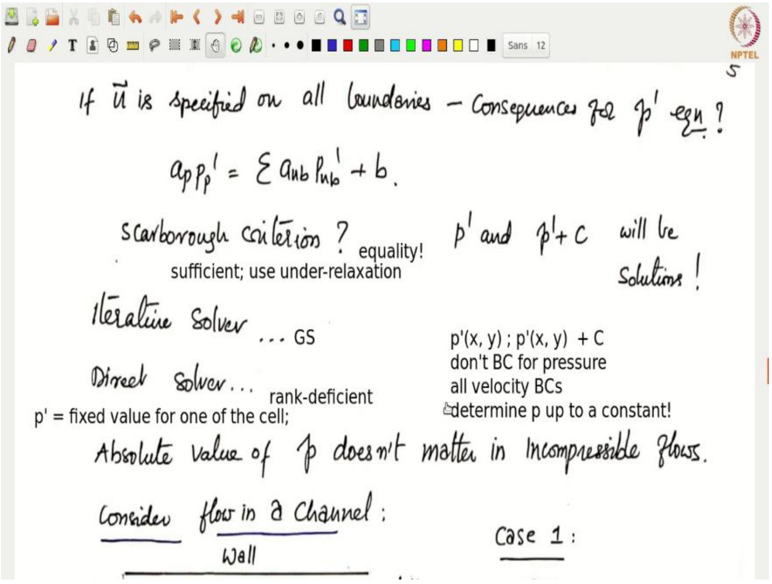


Let us say if you have this kind of boundary condition that is specified on all the boundaries right, on all the boundaries velocities only specified, then what will be the consequence of p prime equation? Let us say similar to this, what will happen then $a_P P_P' = \sum a_{nb} P_{nb}' + b$ and would it satisfies Scarborough criteria?

Because $a_E$ equal to 0 that means $a_P P_P'$ would be only a summation of $a_W$, $a_N$ and $a_S$ right. So, it would satisfy, it will satisfy Scarborough criteria, but it only satisfy in equality because for this particular problem, every cell behaves like this that means, everywhere all the boundaries it will only satisfy in inequality right because the corresponding coefficients will be different, but; that means, all the cells including the boundary cells will only satisfy inequality that means, it will never satisfy in inequality.

But then ok, then how do I solve this problem? Will it work? Yes, it will still work because Scarborough is only a sufficient condition right. So, you will still be able to solve for this in case if you cannot solve for this, then use under relaxation here right essentially bump of b a p value by dividing it by some under-relaxation and then it will kind of converge.

But what we see is that we see that because of this Scarborough satisfied inequality, we see that both $P'$ as well as $P'$ C will be solutions right because now $a_p$ equals $\sum \mathrm{a_{nb}}$ so, whatever p prime you get if you add. So, whatever $P'$ equation that you got in terms of your x, y distribution, this will satisfy the equation that you get here, not only that your $P'$ x, y if you add a constant value of something that will also satisfy your pressure correction equation ok.

So, that means, when you have, when you do not have a boundary condition for pressure right, when you specify all velocity boundary conditions, then you can only determine pressure up to a constant right, right you cannot specify; you cannot specify the you cannot get pressure to absolute value right, you can only specify you will only get pressure up to a constant right alright.

Then, how does the iterative solvers and direct solvers behave? So, if you are using an iterative solver like Gauss-Seidel, then this will converge otherwise you have to use some kind of an under-relaxation and what will be consequence for a direct solver? If we have a direct solver, what it means is basically when you have all velocity boundary conditions, you have essentially what we are talking about is we are talking about a rank deficient system right.

Because; you have these continuity equations and one of the cells, the last cell that you have let us say if you keep writing the continuity equation, the last cell would not be contributing anything new. As a result, one of the cells the equation for one of the cells can be written as a linear combination of the equations for using all other cells ok.

So, as a result, you will direct solver will not work in this context when you have all velocity boundary conditions. So, one fix for this is basically you have to set pressure or pressure correction equal to some fixed value for one of the cells and then do not solve for the cell and then, solve for everybody else. That means, what we talking about is that if you have all velocity specified, the absolute value of P does not matter in incompressible flows.

It is only the pressure gradient because whatever pressure you get, if you add another constant to that will also satisfy the equation, a pressure correction equation also as the momentum equations as well ok. As a result, your p does not matter in incompressible flows ok, it is only the pressure gradient that matters alright.

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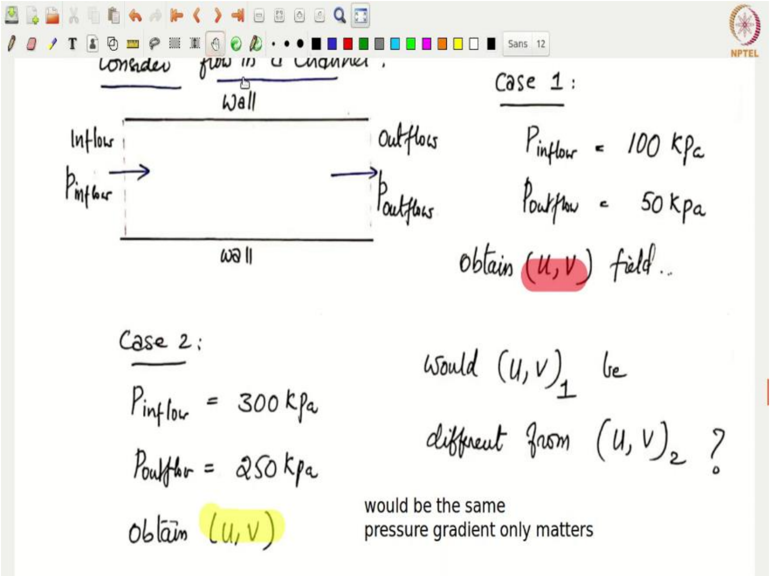


Now, let us take some examples and see why we say that the pressure gradient only matters. If you have let us say consider a channel flow so basically, we have two walls, one on the top, one on the bottom, the flow is coming from the left, we have inflow and we have an outflow and what we specify is we specify pressure here.

Let us say we are now talking about a different problem and we are not talking about velocity boundary condition, we are talking about pressure boundary conditions both at the inflow and at the outflow.

Now, if this pressure at the inflow is higher than the pressure at out flow of course, the flow will go from left to right and in case 1, we maintain a uniform pressure of 100 kilo Pascal's at the inlet and at the outlet we maintain a uniform pressure of 50 kilo Pascal's that means, and then, we can solve for this system and obtain let us say using simple algorithm and then, obtain what is the velocity and the velocity field for this.

Now, in the 2nd case, we change the inflow pressure 300 kilo Pascal's and the outflow pressure to 250 kilo Pascal's and again solve for the velocity field here and they obtain another solution u and v for the entire domain here. Now, would this velocity field that you have obtained be different from this velocity field that is obtained? Would u, v obtained through the case 1 be different from u, v obtained from 2 or not? What would be the 3rd process here? Would they be the same?

The pressures are different. It basically, in the 1st case, we have 100 and 50 and here, we have 300 and 250, but still the velocity field that you calculate using the solution or in an experiment would be the same right the u, v that you get from both systems would be the same because it is only the pressure gradient that matters right, it is only the pressure gradient that matters because you have 50 kilo Pascal’s from inlet to outlet over certain length l here and same pressure loss here as well and that is what is matters.

So, the pressure gradient only matters as a result, the absolute value of the pressure at the inlet or outlet does not matter as long as you have the same pressure difference over the same length, then you will get the same velocity field that is something to know about which is also same as what we have discussed before in the previous problem.

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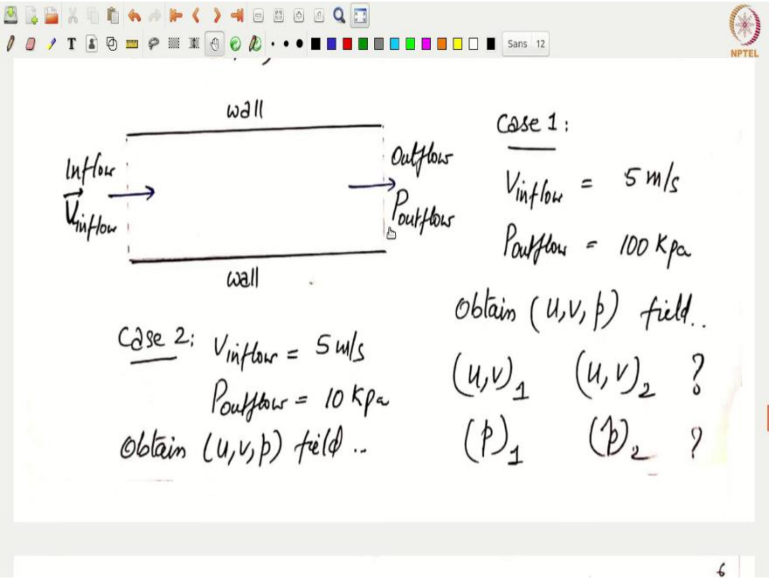


Now, what about if you take another case where instead of having both pressure inlet and pressure outlet, we maintain pressure outlet whereas, we maintain a velocity inlet. So, we provide $\vec{v}$ or $\vec{u}$ at in flow and we provide a pressure at the outflow. So, in the case 1, we provide 5 meters per second as velocity at the inlet and we maintain a pressure of 100 kilo Pascal’s at the outlet.

And in the case 2, we keep the same velocity that is 5 meters per second, but we change the pressure to a very small value we just make it 10 kilo Pascal’s. Then you obtain let us say velocity and pressure field using simple algorithm and then, here also you solve using velocity and pressure field everywhere and compare this field with this field.

Now, would u and v obtained through case 1 be different from u and v obtained through for case 2? Would they be the same or different? They should be the same because we have maintained the same in flow. Although, the pressure is different, the pressure at the inflow would come out to be this pressure plus this gradient times this length so, that pressure gradient will still be there will come out through your algorithm and essentially your p in flow will be different.

That means, p 1 and p 2 would give a different pressure field here ok, but the pressure gradient would be the same for both cases and the velocity fields that you get from the case 1 and case 2 would come out to be the same. But your algorithm will cure itself to get a accordingly a pressure here which will be this outflow pressure plus the gradients times this length ok.

So, as a result, you will get the corresponding pressure field here which will be of course, different from here, but the pressure gradients will be the same and the velocities will be the same, the velocity vectors, magnitudes everything will be the same here between the two cases alright. So, that kind of emphasizes the role of pressure only as a relative nature essentially because the gradient is only matters the absolute value of pressure does not matter alright.

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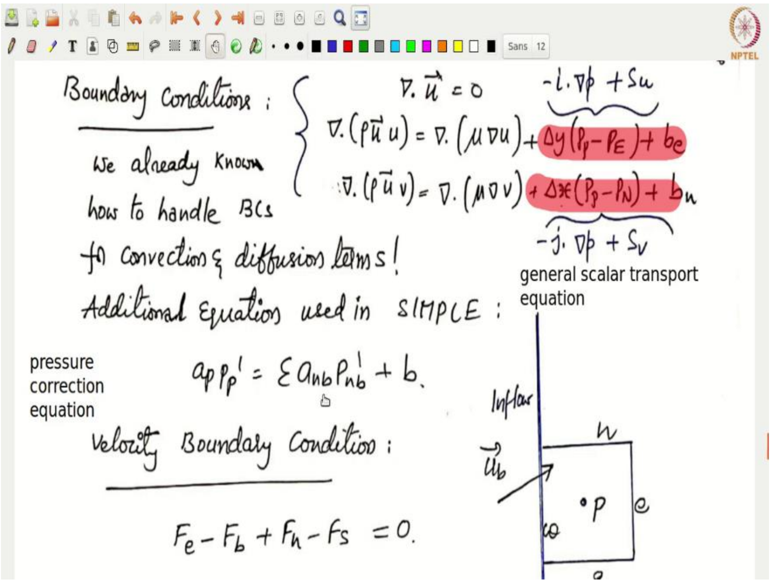


Now, let us look at the final component of this today's lecture that is basically the boundary conditions out of which we have already seen how to tackle the velocity boundary condition. So, we will look at other boundary condition that is the boundary condition pressure as well.

So, if you look at the equation, we have the continuity equation that is $\nabla \cdot \vec{u} = 0$ and then, the x and the y momentum equation that is basically $\nabla \cdot (\rho \vec{u} u) = \nabla \cdot (\mu \nabla u) - \hat{\imath} \cdot \nabla P + S_u$.

This is the discrete part, so this do not worry about these two basically. And then, we have for the y momentum equation, what we have is $(\rho \vec{u} v) = \nabla \cdot (\mu \nabla v) - \hat{\jmath} \cdot \nabla P + S_v$ ok. These are both are basically similar to the general scalar transport equation.

So, as such we know how to handle the boundary conditions for these convection and diffusion terms ok. It is only basically a matter of changing $\phi$ to either u or v and solving them right for Cartesian meshes or for orthogonal meshes. In fact, we also know how to do this for non-orthogonal meshes ok.

So, there is only one extra equation that was introduced in the simple algorithm and that is basically your pressure correction equation right, this is the only one that was introduced in the simple algorithm that is the pressure correction equation. So, we need to see now, how do we handle the pressure correction boundary conditions in the pressure correction equation is something that we have to look at because everything else is already known to us ok.

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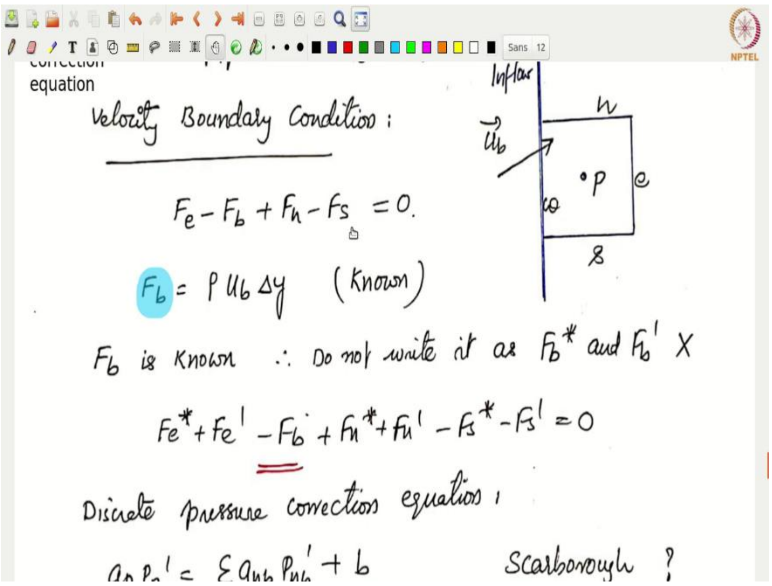

How does the pressure correction equation behave if there is a velocity boundary condition? This thing we have just already seen. So, that means let us say we will kind of look at it again. So, that means, if we have an inflow and this is a cell that is adjacent to the inflow where u b bar is specified let us say this west face is nothing, but b and this P cell has east face, north face and south face and of course, there is an east cell, here north cell, here and south cell here which are not shown here.

And if you write the conservation of mass for this particular cell, that means, what we get is $F_e$, the mass flow for the east face minus $F_b$ that is the for the west face plus the mass flow rate for the north face and the south face equal to 0 right. And, we know that $F_b$ is basically your $\rho u_b \Delta y$ that is already known right essentially this value is already known ok.

This is already known because $u_b$ is known and everything else is known ok; that means, what we do is we will not substitute $F_b$ in terms of $F_b^*$ and $F_b'$ we will just leave $F_b$ as it is and we will write other components F e star F e, F n, F south in terms of star and the prime values. So, the equation we get is $F_e^* + F_e' - F_b + F_n^* + F_n' - F_s^* - F_s' = 0$.

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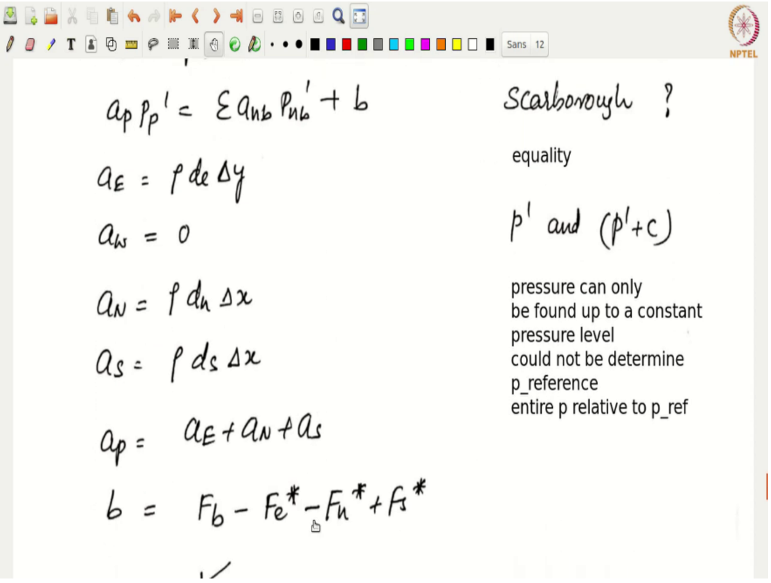


So, the discrete pressure correction equation now becomes $a_P P_P' = \sum a_{nb} P_{nb}' + b$. So, your $a_E$ which is $\rho d_e \Delta y$ and there is no $a_E$ because we do not have any terms coming from there, $a_N$ equals $\rho d_n \Delta x$, $a_S$ equals $\rho d_n \Delta x$. And your $a_P$ would be summation of only east, north and south because there is no west coming into play and b equals your $F_b - F_e^* - F_n^* + F_s^*$.

That means, what we have got is basically an equation which is in terms of north, south and east and the b cell, b values have gone into the right-hand side ok. Now, does it satisfy Scarborough criteria?

Yes it does, but it will only satisfies in equality because $a_P$ equals a $\sum \mathrm{a_{nb}}$ only so that means, in the absence of source terms also, it only enclose this thing; that means, both because of this property $a_P$ equals $\sum \mathrm{a_{nb}}$ at convergence both $P'$ at convergence b goes to 0 that means, both $P'$ and $P'$ plus constant are both functions.

As a result, the pressure can only be found up to a constant right essentially the pressure level could not be; could not be determined only up to a constant so, we cannot determine the pressure level, but the pressure gradient can be found. But if there is a; if there is $a_P$ reference, pressure reference, then the entire pressure can be found or expressed relative to P reference that can be done in case if you have all velocity boundary conditions like in the case of the lid driven cavity problem ok. This we have already seen.

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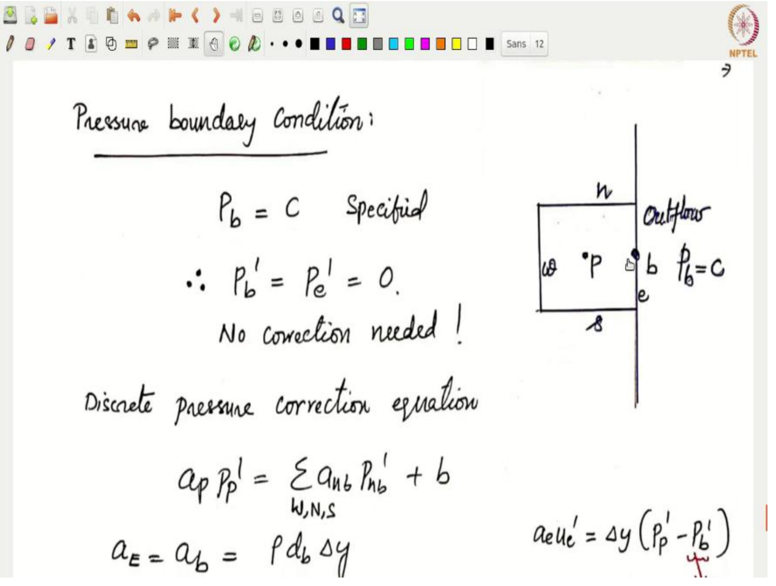


What about the pressure boundary condition? So, the other boundary condition you can get is a pressure boundary condition, then you can specify some value for the pressure that means, P let us say we are talking about an outflow, this is a cell that is adjacent to the outflow. So, we have the P cell we have east, west, north, south and b is the now the boundary face and the pressure on the boundary face e is constant this is specified.

That means, we are let us say the channel is flowing out into some atmospheric pressure because $P_b$ basically specified the pressure is fixed so there is no correction; that means, $P_b'$ which is also equal to $P_e'$ equal to 0 so, there is no correction required for the face value of east for pressure ok; that means, $P_b'$ equal to 0, then what about, what happens to discrete pressure correction equation?

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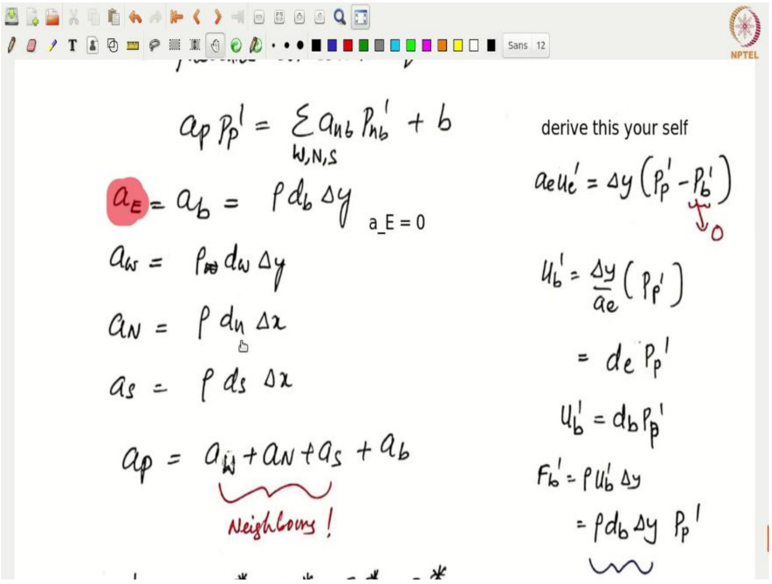


We have $a_P P_P' = \sum \mathrm{a_{nb}} \mathrm{P}_{nb}' + b$. Now, if you look at the equation, what happens is basically you will get in the equation for velocity correction for the face east ok, what will happen is $a_e u_e' = \Delta y(P_P' - P_b')$ is what you will get for $u_e'$ ok.

But we know that the $P_b'$ is 0 right this value is 0 that means, $u_e'$ also equal to $u_b'$ is $\Delta y / a_e$ times $P_P'$ that means this contribution of $d_e$ is only going into the $a_P$, but not into $a_e$ ok.

As a result, $a_e$ is 0 right, $a_e$ is not there, it is only going into $a_P$ that means, but we know that what is that means, if you write the entire system, what you get is essentially there is no $a_e$ so, this $a_e$ is basically $a_b$. $a_b$ equals $\rho d_b \Delta y$ which we got it from here basically once you know what is $u_b'$, your $F_b'$ is $\rho u_b' \Delta y$ so, $\rho d_b \Delta y$ this becomes $a_b$ right which is basically written here $\rho d_b \Delta y$times $P_P'$.

So, there will be a contribution going into $P_P'$ coefficient that is $a_P$, but not to $a_E$ that means $a_P$ would again be summation of $a_W$, $a_N$ and $a_S$ and $a_b$ right $a_b$, but $a_E$ itself is 0 so, we can say $a_E$ is 0, but $a_b$ term will go into $a_P$ for pressure boundary condition. So, you may need to; you need to derive this yourself once again.

So, one thing that you would see which is different here is because you have only pressure gradient, you do not get that half $\Delta x$ terms here because, this $\Delta y$ corresponds to area, as a result, you do not have that half you remember, for the diffusion. We had the $\Delta x/2$, $\Delta y/2$ coming that would not be there because now we are talking about gradient theorem applying to grad P, you will only get P east minus P something minus P something here that would be $P_P$ minus $P_b$.

And $P_b$ itself is 0, $P_b'$ is 0 because of the pressure boundary condition only this term survives; that means in the original equation, in the pressure correction equation for this particular cell what we get is $a_P P_P'$ equals $\sum a_{nb} P_{nb}'$ plus b where the neighbors are only the capital W, capital north and capital south west, north and south.

And $a_E$, $a_N$, $a_S$ are the same as before that is $\rho d_w \Delta y$, $\rho d_n \Delta x$ and $\rho d_s \Delta x$ and $a_P$ would be equal to $a_W$ plus $a_N$ plus $a_S$ plus $a_b$ and your b term now has $-F_b^* + F_w^* - F_n^* + F_s^*$ basically this is nothing, but my row minus F e star.

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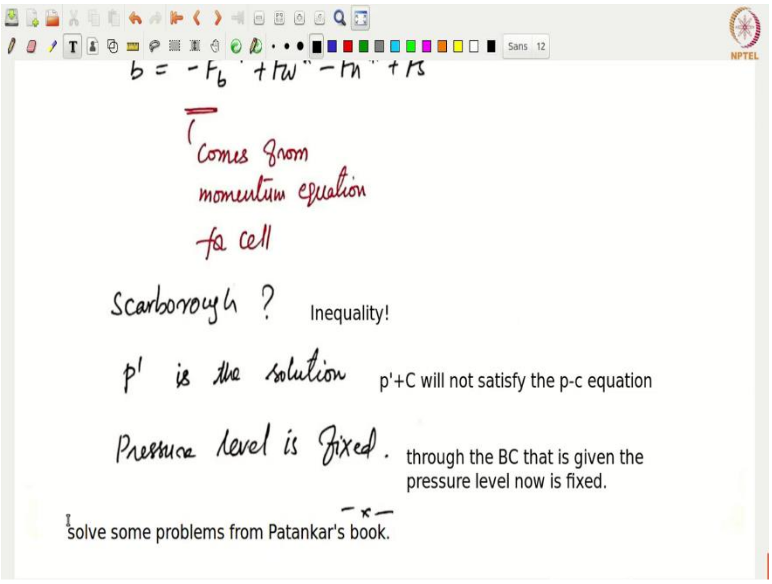


And how do you get a $F_e$ minus $F_e^*$? This comes from the momentum equation for the cell right. We know the how to discretize the momentum equation and from there, you will get what is $F_e$ star here that we will substitute. So, what about Scarborough criteria now for this particular problem if you have a pressure boundary condition?

So, Scarborough is it satisfied? Yes, it is satisfied. Is it satisfied in equality or inequality? Inequality because, $a_P$ is now greater than its neighbors by $a_b$ ok. So, essentially Scarborough is satisfied in inequality that is a good news because then, our iterative solvers will have no problem and what about convergence? At convergence would both $P'$ and $P'$ plus C would be solutions? No, because $a_P$ is not equal to $\sum \mathrm{a_{nb}}$, this will always have this $a_b$ effect right.

As a result, only $P'$ is the solution right. So, basically when you have pressure; pressure boundary conditions $P'$ plus C will not satisfy the pressure correction equation ok. It is only the p prime that you get as a result now the pressure level is fixed because through the boundary condition; through the boundary condition that is given the pressure level is now fixed right.

That means you cannot have any arbitrary pressure values inside the domain, they are all expressed relative to the pressure boundary condition that is given so, pressure, but, still it is only the relative pressure that matters that does not, that is not any different ok. So, that is how essentially we apply boundary conditions for pressure and velocity when it comes to the pressure correction equation in the solution of the incompressible flow equations using simple algorithm ok.

So, what I am going to do is I am going to stop here and in the next lecture, we are going to see, we are going to solve some problems, exercise problems from Patankar's book. So, I will try to post these problems and then, we can first solve them by hand, setup the algorithm, then we can go back and look at the code and see how to run them and get the solution and stuff like that ok. I am going to stop here. If you have any questions, write back to me through email, we will, I will respond back.

Thank you, talk to you in the next lecture.