Computational Fluid Dynamics Using Finite Volume Method
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Lecture - 29
Finite Volume Method for Convection and Diffusion: Discretization of steady convection equation

Hello everyone, welcome to another lecture as part of our Computational Heat and Fluid Flow, ME 6151 course. Let us get started.

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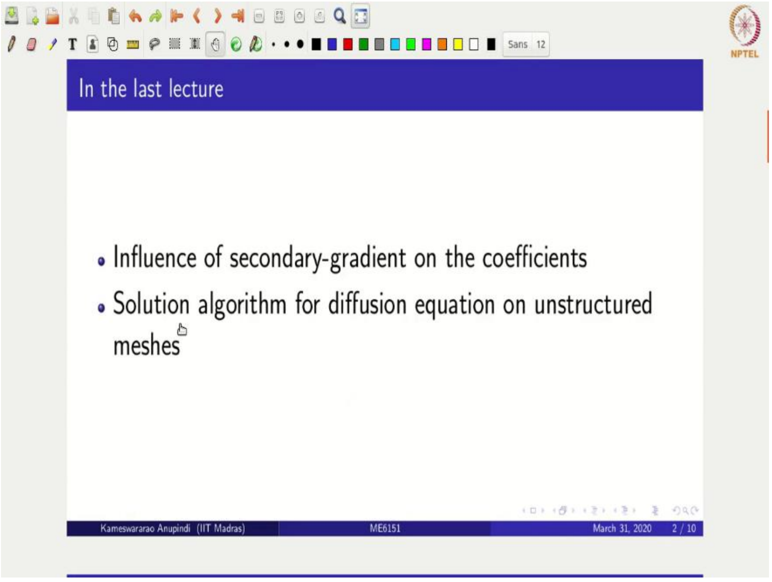

So, today, in the last lecture, we looked at the influence of secondary-gradients on the coefficients right, we kind of reasoned out that secondary-gradients create negative coefficients which may lead to oscillations as a result it is kind of a good practice to have a good quality mesh right which is kind of as orthogonal as possible. And then we have also looked at the solution algorithm for implementing a diffusion equation on unstructured meshes right.

We said instead of going through a cell centered approach, if we traverse the mesh through a face based approach, then access the cells and fill in the coefficients, and then go to the cells and calculate the central coefficients, so ap coefficients and so on, then that is a better way of solving the unstructured problem right that is what we have kind of discussed in the last class. So, we have also finished the chapter 3 which is on the diffusion equation.

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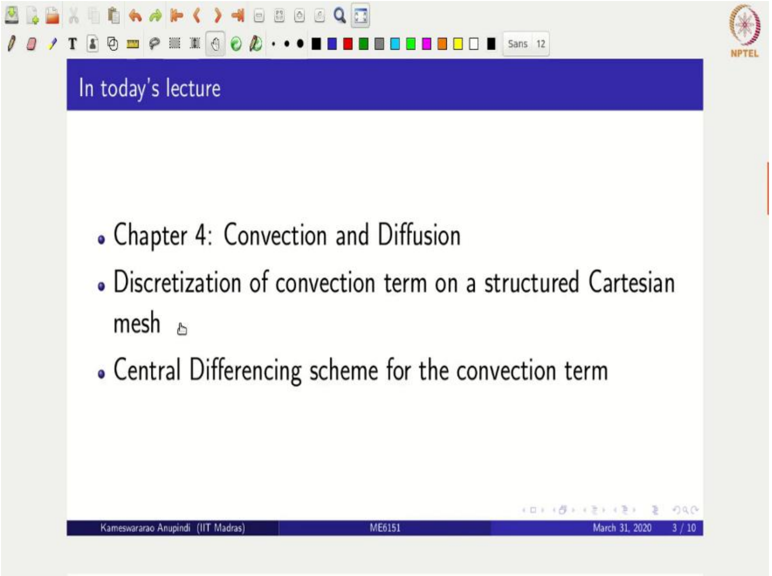

So, today we are going to look at the next chapter that is the chapter on convection. Essentially, we are going to look at the convection and diffusion as part of chapter 4. So, the two specific things that we will discuss today is the first is the discretization of the convection term right which we have not seen till now on a structured Cartesian mesh ok.

So, we will kind of go back to the structured mesh to start with, then we will learn how to discretize the convection term on a structured mesh, thereafter we will introduce the unstructured or a non-orthogonal mesh ok, so that is kind of the steps we will take. And we will also look at a particular scheme for discretizing the convection term that is the central differencing scheme ok. So, these are the two things we are going to see in today's lecture alright. Let us get started.

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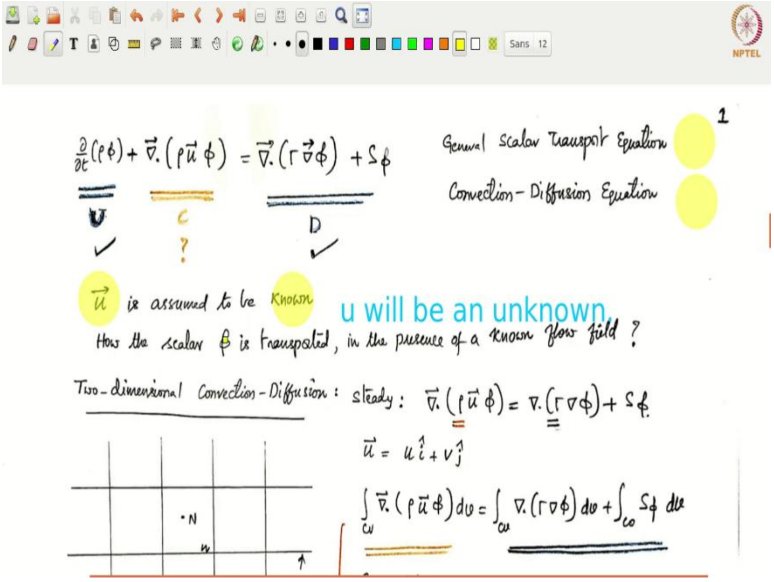

So, we will start with our general scalar transport equation ok. So, general scalar transport equation is written here right. So, basically the general scalar transport equation or the convection diffusion equation as it is known as right is written here in which the first term is the uncertainty term that is $\frac{\partial}{\partial t}(\rho\phi)$ plus the second term is the convection term that is $\nabla \cdot (\rho\vec{u}\phi)$. And on the right hand side, we have the diffusion term that is $\nabla \cdot (\Gamma\nabla\phi)$ plus S_ϕ right.

So, this is the equation we have kind of started off with. We already know how to solve the or taken by account the unsteady part and the diffusion terms right. These two things we already know. The only thing we did not know is how do I now include the convection term in there which will allow us to solve the entire general scalar transport equation right or the convection diffusion equation ok, so that is the thing.

So, essentially we already know how to discretize the this part right ok. Now, here when we solve this problem, you see that we have a velocity field that is introduced. So, as far as the convection diffusion is concerned, we will assume that u bar is a known quantity ok, so that is what we will assume. We will assume that u bar is known ok, that means, the underlying flow field is known. However, let say in a real flow field, in a real flow field, u will be an unknown right. This will be an unknown which we have to calculate ok.

And the question we are interested in basically is how is this phi that we have right how is this phi getting transported in the presence of a known flow field. So, essentially u bar is known and how is my phi getting transported that is the real question we are asking. And what we do is initially we will assume that we will not we do not have the unsteady term in here ok. So, we will only solve for a steady convection diffusion equation ok.

Later on of course, we can introduce the unsteady part as we have done for our pure diffusion equation ok, so that is what we would be we would be doing. So, let us start off with the two-dimensional structured mesh as shown here right.

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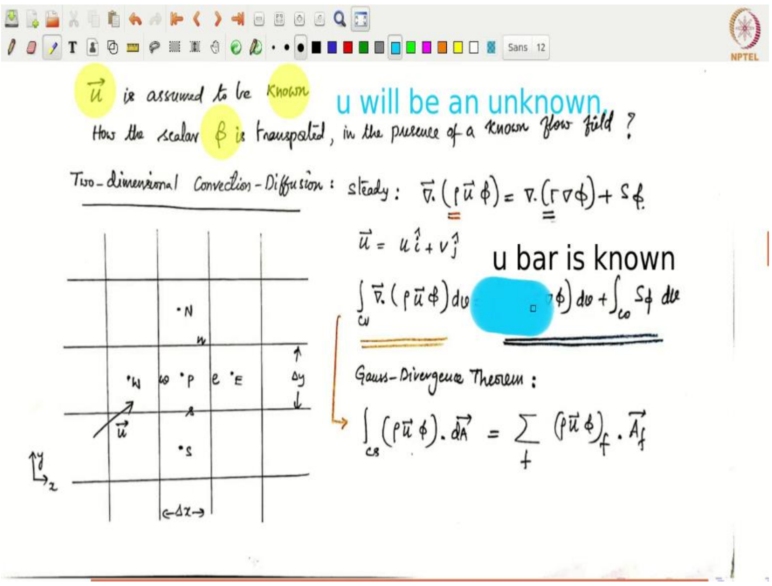

This is the same mesh we had before. Essentially we have a Cartesian mesh with either uniform cells or non-uniform cells. And we have the primary cell P on which we are focusing our discretization on and then we have an east cell, west cell, north and the south cells, and the corresponding faces are the little e, little w, little north, and little south ok. And the width of the cell in the x-direction is Δx, and the width of the cell in the y-direction is Δy.

And of course, the other terminology which is the distance between P and the east cell in the x direction would be equal to δx_e, and this would be δx_w, this would be δy_n, and this would be δy_s, all those terminology would be valid here as well I am not repeating those terms here ok.

So, assuming that we are not interested in right now in the unsteady part, we can write the steady diffusion convection diffusion equation like this. So, basically $\nabla \cdot (\rho\vec{u}\phi) = \nabla \cdot (\Gamma\nabla\phi) + S_\phi$ that is basically the same equation that we have here. However, we notice that we have now introduced a new variable right that is your that is your rho right.

So, this is a new variable that we have introduced, up till now we only had gamma that is the diffusion coefficient of the flow right now with convection we are addressing the density of the of the flow field ok. So, we also saw the density gets introduced even if you have an unsteady term in the past in the past lectures right, so that is what we have already seen.

So, it is good to keep in mind that the rho is introduced ok and then the velocity vector that we have that is $\vec{u}$. I can of course, write it as $\hat{\imath}u + \hat{\jmath}v$ ok. So, we have basically two scalar components u and v, and u bar would be the velocity vector that is in this particular case as far as the as far as the solution is concerned. This is basically is known right. So, basically u is known right, $\vec{u}$ is known right, for this for the solution of general scalar transport equation alright.

Then what is the first step in finite volume method? The first step is basically to integrate the convection diffusion equation on a control volume that is basically this control volume that is if I integrate this you have integral CV $\nabla \cdot (\rho\vec{u}\phi)$ dV equals on the right hand side control volume $\nabla \cdot (\Gamma\nabla\phi)$ dV plus integral on the control volume S_ϕ dV right ok.

Again what we what we know is we already know how to solve for the diffusion part right, that is $\nabla \cdot (\Gamma\nabla\phi)$ dV plus S_ϕ dV equals 0 is what we already know right. So, we already know how to solve for solve for this term right. We already know solving for this term ok. So, we will not worry about that part, rather we will worry about the convection term that is $\nabla \cdot (\rho\vec{u}\phi)$ dV.

So, we will invoke the Gauss divergence theorem just the way we have done it for the diffusion term here, we will do it for the convection term. So, invoking the Gauss divergence theorem the convection term here can be rewritten as instead of a volume integral, we can convert it to a surface integral of $\rho\vec{u}\phi$ dot $\overrightarrow{dA}$ right. So, $\overrightarrow{dA}$ is now your area vector.

And again assuming that the this quantity that we have is a constant on the on the faces that these are control surface is made up of. And that constant can be represented using the face centroid value. We can transform this continuous integral into a discrete summation of sigma over f, $(\rho\vec{u}\phi)_f \cdot \overrightarrow{A_f}$ right. $\overrightarrow{A_f}$ is basically the area vectors for each of the faces that is east, west, north and south which we already know right ok.

So, for example, what would be $\overrightarrow{A_e}$, $\overrightarrow{A_e}$ would be $\Delta y\ \hat{\imath}$ right similarly $\overrightarrow{A_n}$ would be $\Delta x\ \hat{\jmath}$ right. All these things are already known ok. So, now, we have to focus on basically evaluating this quantity right that is basically $(\rho\vec{u}\phi)_f \cdot \overrightarrow{A_f}$ on all the faces. So, once we once we know how to solve this, then we can actually solve for the convection diffusion equation ok. So, let us move on with our this quantity.

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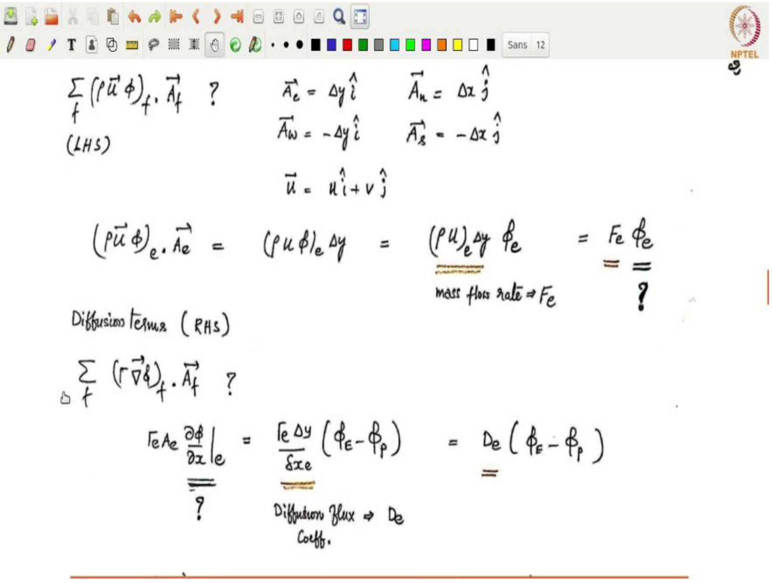

So, we can rewrite this quantity as basically we say sigma f, $(\rho\vec{u}\phi)_f \cdot \overrightarrow{A_f}$ right. And we also have to realize that this term this particular term is on the left hand side of the original equation ok. So, if you go back to the equation, the entire diffusion and the source terms are on the right hand side, whereas this convection term is on the left hand side ok.

So, this is important because when we try to assemble the coefficients later on, then we have to know on which side of the equals equation is this particular term is right because then only we can get the coefficient signs correctly ok, so ok.

So, we have the faces the faces are basically east, west, north, south, and then the area vectors are all known $\overrightarrow{\mathrm{A_e}}$ equals $\Delta y\ \hat{\imath}$; $\overrightarrow{\mathrm{A_w}}$ equals $-\Delta y\ \hat{\imath}$; $\overrightarrow{\mathrm{A_n}}$ equals $\Delta x\ \hat{\jmath}$; and $\overrightarrow{\mathrm{A_s}}$ equals $-\Delta x\ \hat{\jmath}$ right, so that we already know. And $\vec{u}$ is also known; $\vec{u}$ is basically $\hat{\imath}u + \hat{\jmath}v$. This is also already known.

So, can we write this one particular term of this summation? Out of the four terms, we have we will write first for the east face. So, for the east face, this, this will read as $(\rho\vec{u}\phi)_e \cdot \overrightarrow{\mathrm{A_e}}$ bar right. So, this is basically we know this thing as a bar is $\Delta y\ \hat{\imath}$, $\vec{u}$ is $\hat{\imath}u + \hat{\jmath}v$. So, the only term that survives would be î dot î. So, u times Δy is the only term that survives v times 0, so that would not survive. So, we can write this as $(\rho\vec{u}\phi)_e$, where u is this scalar component of $\vec{u}$ in the i direction times Δy right.

So, now you know that this have to be has to be evaluated on the east face right ok. Of course, I can rewrite this as ρu_e u evaluated on the east face times Δy times ϕ_e right. So, phi value on the face e, ϕ_e right ok. So, now, this quantity if you look at this is $\rho u_e \Delta y$ this is basically the area, this is density. So, density times velocity times area what would this quantity be?

This quantity is nothing but your density times velocity times area would be your mass flow rate right, so that is your mass flow rate. We would like to represent it using a quantity F capital f sub e, F_e that is the mass flow rate on the east face is F_e ok.

So, if I use F_e instead of $\rho u_e \Delta y$, then I can write this entire quantity as F_e times ϕ_e is what I can write this as. So, that means, one of the terms out of this four terms we have comes out to be $\mathrm{F}_e\phi_e$. Where F_e is the mass flow rate times phi is the scalar so far so good. But what about ϕ_e?

Do we know the value of ϕ_e? We do not know, because phi is only stored where it is only stored at the cell centroids right, it is not stored at the faces. Of course, we can somehow interpolate the value of ϕ_e that is what we would do. So, that is what needs to be done ok. So, how do we interpolate for ϕ_e kind of determines the kind of convection scheme we are talking about ok? So, as far as the convection term is concerned, we get we got one term that is F_e times ϕ_e ok.

Now, let us now look at the diffusion term as well because when we write the equations we want to introduce a slightly different notation than what we have used for the pure diffusion equation. So, if you look at the diffusion equation on the right hand side, what we have is $(\Gamma\nabla\phi)_f \cdot \overrightarrow{\mathrm{A_f}}$ right so for all the faces. Again if I consider the east face, this would come out to be $\Delta y\hat{\imath}$.

This has two components, so the only component that survives is the $\left.\frac{\partial\phi}{\partial \mathrm{x}}\right|_e$ right. So, this will be $\Gamma_e \mathrm{A}_e \left.\frac{\partial\phi}{\partial \mathrm{x}}\right|_e$ on the east face. Now, how do we calculate $\left.\frac{\partial\phi}{\partial \mathrm{x}}\right|_e$ on the east face? Using linear profile assumption right.

So, now what you can see is that for convection, we need to use a model to calculate this dependent variable itself right. So, we for ϕ_e itself, you would need a you need an assumption, whereas for the diffusion terms you would need an assumption for the gradient of the phi right, so that is the difference between the convection and the diffusion terms. For the dependent variable itself, here you would need a model; here for the gradient of the dependent variable, you would need a model right.

So, we have used a linear profile assumption, and we can write this as $\frac{(\phi_E - \phi_P)}{\delta x_e}$. And A_e is nothing but your Δy right $\overrightarrow{\mathrm{A_e}}$ is your $\Delta y\hat{\imath}$, but A_e is your scalar value. So, we have this value.

Now, just like we have used a notation for the mass flow rate that is multiplying ϕ_e as F, let us introduce another term for this coefficient that is multiplying the ϕ_E and ϕ_P as the diffusion flux coefficient that we call it as D_e ok.

So, this is the diffusion flux coefficient on east face, basically $\frac{\Gamma_e \Delta y}{\delta x_e}$ right. So, if I plug in this, we can rewrite this equation as D_e times $(\phi_E - \phi_P)$, so far so good. So, we have now looked at discretization of the convection term and the diffusion term on one particular face that is the east face alright.

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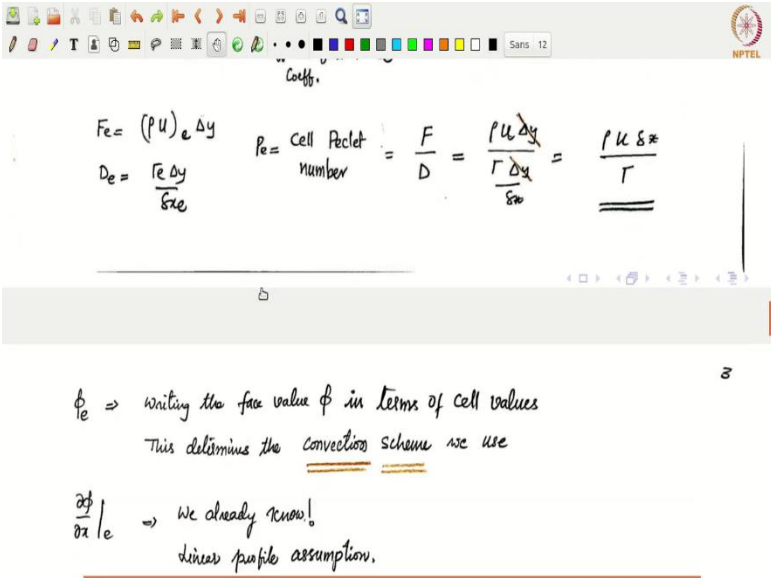

Let us also kind of define a non-dimensional number that would be that would be an indication of the relative strengths of the convection and the diffusion terms which we would call it as a Peclet number which is based on the cell dimension. So, we will call it as a cell Peclet number ok.

So, going by definition the Peclet number, we would like to define it as the ratio of the convection to the diffusion coefficients that is F to the D right. Whereas, we know what is the mass flow rate, the mass flow rate is basically F_e equals $\rho u_e \Delta y$, and the coefficients. D_e for the diffusion flux is $\frac{\Gamma_e \Delta y}{\delta x_e}$ right.

So, we can calculate now what would be the cell Peclet number. So, this is $\rho u \Delta y$ upon $\frac{\Gamma \Delta y}{\delta x}$. So, delta y gets cancelled. So, what you get is $\frac{\rho u \delta x}{\Gamma}$ ok. So, this is your cell Peclet number.

We need this in order to know whether it whether the problem is a convection dominated problem or if it is a diffusion dominated problem depending on the value of the cell Peclet number ok. So, that is going to define the relative strength of these two these two physical quantities alright. So, we define this.

Now, let us get back to our discussion. Basically we have now discretized on only for the east face. Now, what do we have to do? We have to do the same thing. Essentially, we have to introduce a model for ϕ_e right. And then once we introduce a model for calculating phi on the east face, we have to write equations for all the faces that is for west, south and north, and then assemble all of these together, and then put them in a form that is $a_P \phi_P = \sum a_{nb} \phi_{nb} + b$ plus b ok. So, that is what we are going to do next ok.

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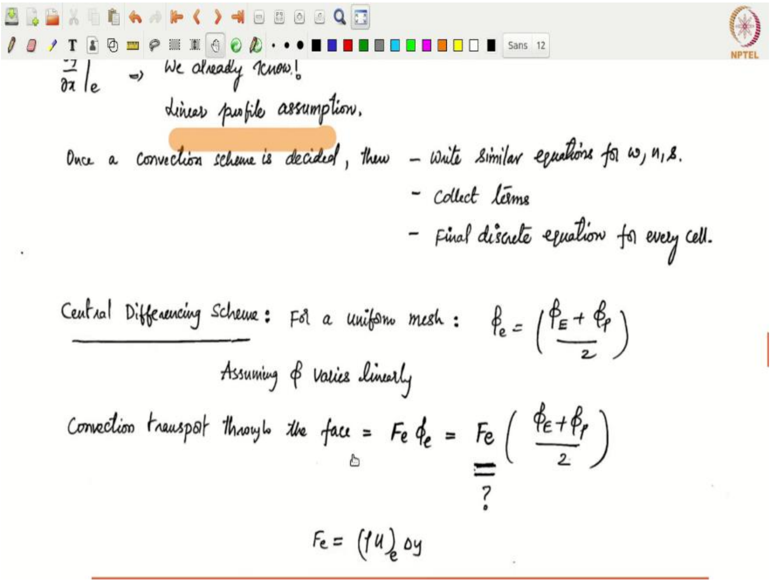

Now, ϕ_e is basically we need to write the face value in terms of the cell centroid values that is basically where we have ϕ_e stored right. Now, how do we write this? Actually determines the convection scheme ok. So, how we are going to write this is going to determine the convection scheme ok. So, so, the convection scheme we would use would be based on how the face phi value will be written in terms of the cell values ok.

But for the face gradient $\frac{\partial \phi}{\partial x}$, we have already introduced a model that is the linear profile assumption right. The linear profile assumption is already introduced. What is, what is the linear profile assumption we have used can be called as what kind of scheme can we call the linear profile assumption as maybe you have to kind of think and come back ok.

Now, essentially once we introduce a convection scheme, once we decide on how we calculate ϕ_e in terms of the cell values, what we need to do is, we need to do similar right similar equations for the west, north, and the south faces. Collect all the terms and then formulate the final discrete equation for every cell that is what we would do.

Now, that means, we would introduce one particular scheme which we call it as a central differencing scheme ok. Now, assume that we have a uniform mesh ok. If we have a uniform mesh, then ϕ_e can be written as ϕ_E plus ϕ_P by 2. Now, this we can write assuming that phi varies linearly between the cell centroids which is also the same assumption we have used in evaluating the face gradients ok.

So, if you have a uniform mesh and if you assume that phi varies linearly between the cell centroids; we can write the face value ϕ_e as ϕ_E plus ϕ_P by 2 ok, very good. Well, we want to do this because our eventual equation is in terms of the cell centroid values for ϕ_P, ϕ_E, ϕ_N, and so on right, only then this can go into the matrix right alright.

So, so this particular scheme of taking it as the linear average is known as central differencing scheme if you have a uniform mesh; otherwise this will be basically in terms of the linear interpolation right ok. Now, if you introduce this, then what will happen to the convection transport through the particular face e through the face east? In the face east, the east face has basically the convection term is F_e times ϕ_e right, so that is what we have from we have reduced rho u bar phi dot e dot A e bar as F e times phi east right.

Now, if I introduce ϕ_e equals ϕ_E plus ϕ_P by 2, then I can write this $F_e\phi_e$ as F_e times ϕ_E plus ϕ_P by 2 right, that is my I am just substituting for phi east as ϕ_E plus ϕ_P by 2. Now, what about this coefficient F_e, is this known or unknown? This particular F_e equals $\rho u_e \Delta y$, is this known or unknown? This is known, because density is known, area is known for the purpose of the convection diffusion equation, we know that the velocity vector is also known ok.

So, essentially this coefficient F_e is known and so is the diffusion coefficient D_e ok. So, these two are known fine. So, we have now introduced a model for the east face. Can the same model be extended for other faces, for example, for the west face? Yes, this can be only is only thing is that for west face ϕ_w would be half ofϕ_W plus ϕ_P by 2 right ok.

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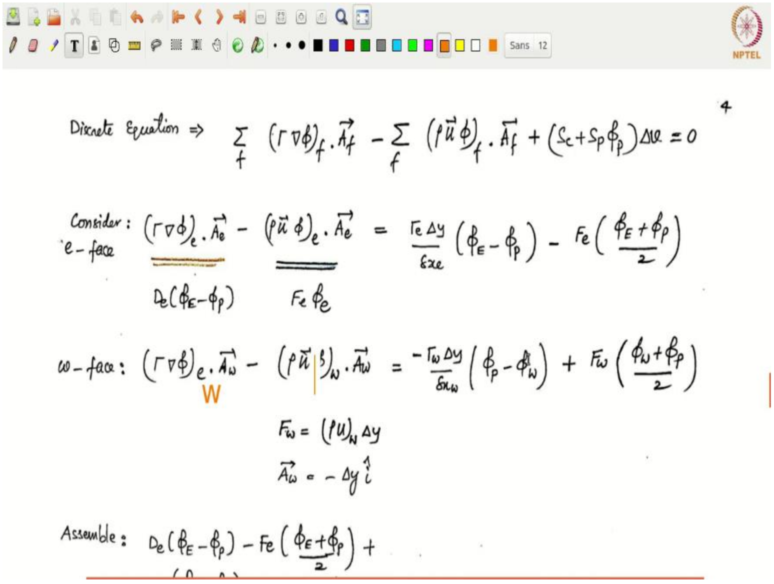

So, if we do the same thing, we can now formulate the total problem ok. So, coming to the discrete equation, so the total discrete equation would read as bringing the convection term to the right hand side ok. So, earlier this was on the left hand side with equals these two right now I brought this to the right hand side with a minus. So, what we have is sigma f, $(\Gamma\nabla\phi)_f \cdot \overrightarrow{\mathrm{A_f}}$ minus sigma f, $(\rho\vec{u}\phi)_f \cdot \overrightarrow{\mathrm{A_f}}$ plus $(S_{\mathrm{C}} + \mathrm{S_P}\phi_{\mathrm{P}})\Delta V$ ok.

Now, you already know this last term and the first term right this is what we have been doing in the diffusion equation. Now, the only extra term is this one which is rho u bar phi times dotted with $\overrightarrow{\mathrm{A_f}}$ that is the only extra term alright.

Now, let us consider only the east face ok. Let us consider the only the east face, and also only consider these two terms – the diffusion and the convection. So, these two would read it as $(\Gamma\nabla\phi)_e \cdot \overrightarrow{\mathrm{A_e}}$ minus $(\rho\vec{u}\phi)_e \cdot \overrightarrow{\mathrm{A_e}}$ right that is what we have. And we have written this diffusion as using linear profile assumption D_e times ϕ_E minus ϕ_P, and this as F_e times ϕ_e ok.

So, if you, if you plug in these two, what we get is for D_e we have $\frac{\Gamma_e \Delta y}{\delta x_e}$ times ϕ_E minus ϕ_P minus F_e times ϕ_e is ϕ_E plus ϕ_P by 2 right, this is what we have for these two terms alright. Now, can we write a similar expression for the west face? Yes, we can. So, this has this is of course, a mistake; this has to be supposed to be w, this should be w ok, ok.

Now, this would be $(\Gamma\nabla\phi)_w \cdot \overrightarrow{A_w}$ minus $(\rho\vec{u}\phi)_w \cdot \overrightarrow{A_w}$ ok. So, that would be equal to what would this quantity? This would be minus why minus? Because $\overrightarrow{A_w}$ would be $-\Delta y\hat{\imath}$. So, this would be $-\frac{\Gamma_e \Delta y}{\delta x_e}$ times $\left.\frac{\partial\phi}{\partial x}\right|_w$ on the west face would be ϕ_P minus ϕ_W by δx_w. And then here also A_w would be a $-\Delta y\hat{\imath}$, so that minus and this minus would make it a plus and this we would call it as F_w, F_w times ϕ_w would be what ϕ_w would be ϕ_W plus ϕ_P by 2 right that is what we have.

Now, notice here, here we got a minus, here we have got a plus. Similarly, here we have got a plus and we have got a minus here ok. So, there is some small difference between the east and west ok. Now, what is the definition F_w is $\rho u_w \Delta y$; and of course, $\overrightarrow{A_w}$ equals $-\Delta y\hat{\imath}$ ok.

So, with these two things, we now kind of wrote for the west face as well. Can you write for the north and south faces similarly? Yes, you can write, but we have to keep in mind the corresponding plus minus sign. So, maybe you should try writing it out as well as a verification, as of now I am going to assemble all of these things ok.

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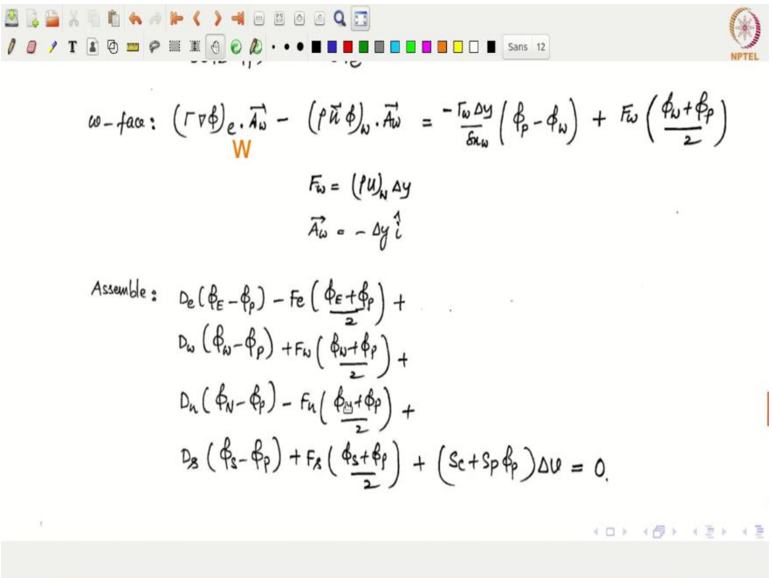

So, this is basically D_e right, this is D_e times ϕ_E minus ϕ_P minus F_e times this thing ok. And what about this thing this will be minus D_w time's ϕ_P minus ϕ_W plus F_w times ϕ_W plus ϕ_P by 2 ok. So, if I assemble all of these things what do we have? We have $D_e\phi_e$

minus ϕ_P minus F_e times ϕ_E plus ϕ_P by 2 plus D_w here I have changed the ϕ_W and ϕ_P the order of ϕ_W and ϕ_P that is why I could get a plus here instead of minus ok.

And we have plus F_w time ϕ_w plus ϕ_P by 2 plus the north would be similar to the east face ok, this is what we have to verify ok. The north would be D_n ϕ_N minus ϕ_P minus F_n times ϕ_N plus ϕ_P by 2 plus what would be the quantity for south, south would be it would this would actually south also would come to be $-\frac{\Gamma_s \Delta x}{\delta y_s}$, but I would write with a plus with these two flipped ok.

So, this would be D_s times ϕ_S minus ϕ_P plus F_s times ϕ_S plus ϕ_P by 2. So, this is basically coming from your diffusion terms and the convection terms. And we have of course, the $(S_C + S_P\phi_P)\Delta V$ equal to 0 ok. So, this is the final equation alright.

But what do we want our equation to be written as? We want it to be written as $a_P\phi_P = \sum a_{nb}\phi_{nb} + b$ right. So, what should I do now? We have to send all the ϕ_P coefficients and ϕ_P to the right hand side, and leave all remaining things on the left hand side ok. So, if we do that, what will be the coefficient for ϕ_E? ϕ_E will get D_e minus phi east will get F_e upon 2 right that is what phi east will get ok.

What about the coefficient for ϕ_P? When you send it to the right hand side, this becomes a plus, and this becomes a plus as well. For ϕ_P that will be D_e plus F_e by 2 would be the coefficient for ϕ_P. And the coefficient for phi east is D_e minus F_e by 2. You see there is a difference between the coefficients now ok.

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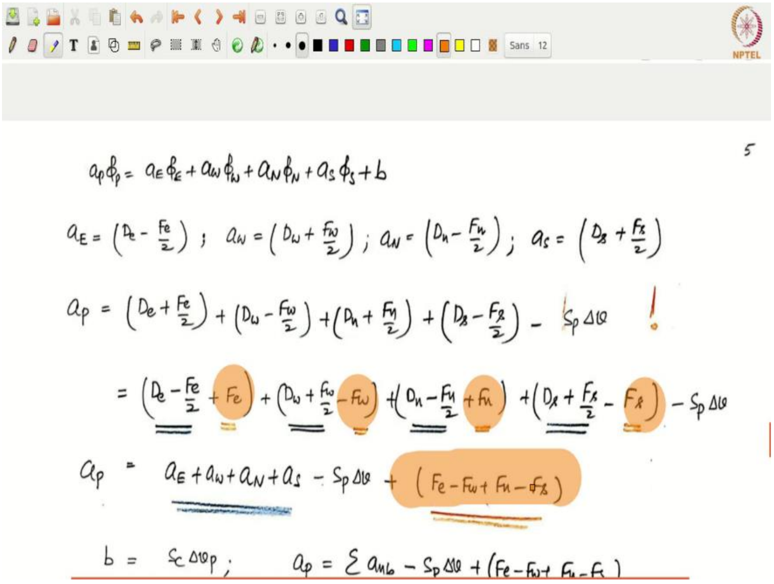

So, if I were to write it as $a_P \phi_P = \sum a_{nb} \phi_{nb} + b$, the coefficient for a_E would be this east phi east remains on the left hand side. So, this will be D_e minus F_e by 2 ok, and a west would be D_w plus F_w by 2 ok; a north would be D_n minus F_n by 2; and a south would be D_s plus F_s by 2 very good. Now, what about a_P? a_P basically has a similar contribution, but these a plus minus for F_w getting flipped, because consistently for all the diffusion terms ϕ_P has a minus right, so that takes care of the same term for same coefficient as here.

But here we see that ϕ_P has a plus, so as a result this part of the convection that is going into the contribution of the neighbouring coefficients would change its sign when it goes to the coefficient of a_P right ok. So, what is the coefficient for a a_P? a_P would be D_e plus F_e by 2, because this goes to the right hand side, this becomes plus, this becomes plus. And then D_w minus F_w by 2 plus D_n plus F_n by 2 plus D_s minus F_s by 2 minus of course we have our $S_P \Delta V$ as well right this becomes minus ok.

Now, what do you see here? What you see here is basically a_P is not now, not the summation of the neighbours, is not it? Because you cannot write this as a summation because this is minus, whereas you have a plus, here you got a plus, you got a minus. So, when you had only the diffusion term in there, then this could have been this was written as a_P equals $\sum a_{nb}$ right, but now it is not the case. So, but we have only different values here. Of course, we can now make it actually a summation of a and b. So, by adding and subtracting certain quantity that is nothing but this plus half F_w, F_w by 2 can be written as F_e minus F_e by 2 right. I can write this.

Similarly, this minus F_w by 2, I can write it as minus F_w plus F_w by 2 right. I can basically add and subtract these quantities ok. So, I can write like this and in which case D_e plus F_e by 2 is written as D_e minus F_e by 2 plus F_e right. But then this quantity is now what? This quantity is nothing but your a east right. This is nothing but your a east.

Similarly, this quantity is now nothing but your a west, and this is your this is your a north here, and this quantity is now your a south ok. Of course, now we have introduced this extra terms which have to be taken out. So, this is F_e minus F_w plus F_n minus F_s is the extra term that we have introduced in the process, but nonetheless we can write now a_P as a_E, a_W, a_N, a_S summation minus $S_P \Delta V$ plus this extra terms right that we have introduced.

For example, what are the extra times that we have introduced, we have introduced this quantity, this quantity, this quantity and this quantity ok. These are the four quantities that is F_e minus F_w plus F_n minus F_s ok. This entire thing is extra which we have introduced. But barring this a_P is now equals $\sum a_{nb}$ minus S_P. So, this particular quantity has some significance which we will see in little while ok.

What is this quantity F_e is the flow rate, mass flow rate through east face. F_w is also mass flow rate through the west face. And F_n mass flow rate to the north face, and F_s is the mass flow into the south face ok. So, that means, mass flow rate through east minus west plus north minus south that is the quantity we are talking about ok, very good alright.

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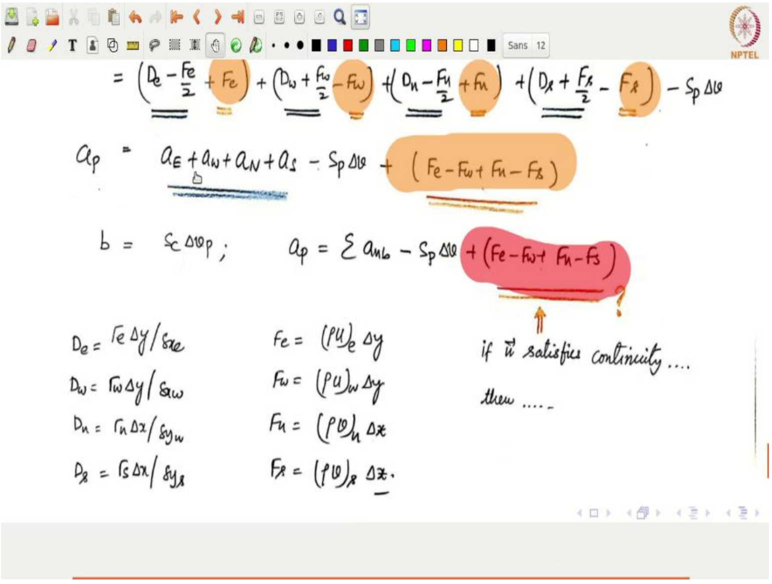

Then what is our b? b of course, is your S_C times ΔV. And a_P is your now $\sum$ a_{nb} minus S_P ΔV plus this quantity ok. Now, because the c, these are all the mass flow rates through the faces right, so these are basically the mass flow rates that are going out of the domain.

Let us say if your u is positive u and v are positive, then this is the total mass for it that is going out of the out of the control volume right, because $\rho u \Delta y$ is u is positive, then it is basically $\rho\vec{u}$ dot $\overrightarrow{A_e}$ right which will give rise to $\rho u \Delta y$ that will be a positive quantity. So, that means, this entire thing is basically the net mass that is entering and leaving through the control volume through all the faces.

Now, if the given velocity field $\vec{u} = \hat{\imath}u + \hat{\jmath}v$ satisfies continuity let us say if satisfies mass conservation, then what will this quantity be? The amount of mass leaving through the east face minus the amount of mass essentially this is the amount of mass entering through the west face plus the amount of mass leaving through the north face and the amount of mass entering through the south face, this is basically the conservation of mass right. This has to go to 0, if the given velocity field $\vec{u}$ satisfies continuity ok.

So, that means, this would be 0 if you have a velocity satisfying flow field that is given to you for which you have to calculate the transport of the scalar ok. So, that means, that means, this quantity would go to 0 if you have a continuity satisfying velocity field ok, that you have to verify once again alright now ok.

So, now, in all these quantities we have just literally listed out what is D_e and F_e, but the definitions are we already know right. D_e would be $\frac{\Gamma_e \Delta y}{\delta x_e}$; D_n would be $\frac{\Gamma_n \Delta x}{\delta y_n}$. And similarly F_e is $\rho u_e \Delta y$; F_w would be $\rho u_w \Delta y$; F_n will be $\rho v_n \Delta x$ and so on right. So, these are all known values because density is known, velocity is known, diffusion coefficient is known ok, so all these are known fine. So, essentially we have now formulated the equation.

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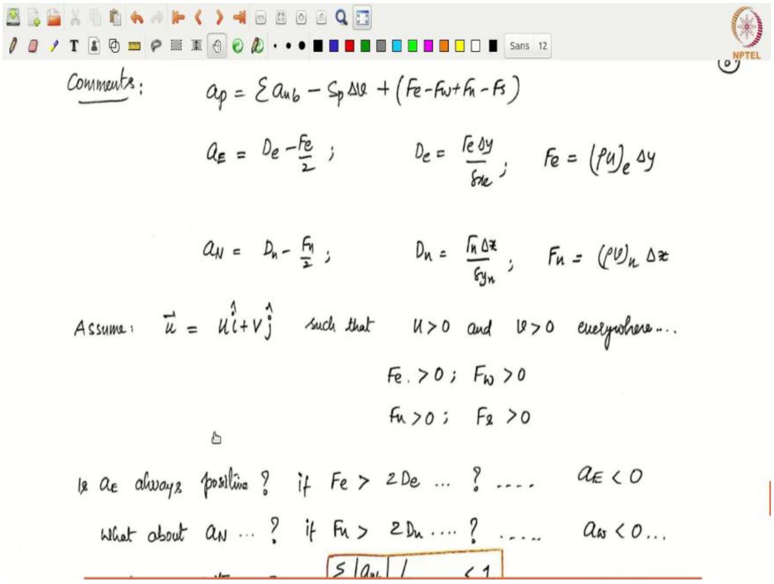

Let us make few comments ok. The comments are now a_P is not just summation $\sum a_{nb}$ minus $S_P \Delta V$ like we what we have seen before, but we have this extra quantity ok and what about the coefficients? The coefficients are a_E equals D_e minus F_e by 2, where D_e and D_e are known. Similarly, a north is D_n minus F_n by 2, where D_n and F_n are known ok.

Now, what about the coefficients? Do the coefficients look ok? They do not really look ok. Because if you consider let us say your velocity field is sum 2 times i plus 3 times j where u and v are both positive quantities ok. So, basically your velocity is going in the positive quadrant direction right.

It is basically has 2, 2 i plus 3 j, it is going something like this. Then is that means, if you have this your all your flow rates are now positive right, because your u_e, u_w, v_n, v_s, so that all are positive, so these are all positive quantities. And diffusion is always a positive quantity right. Then is this coefficient a_E always positive? Need not be right.

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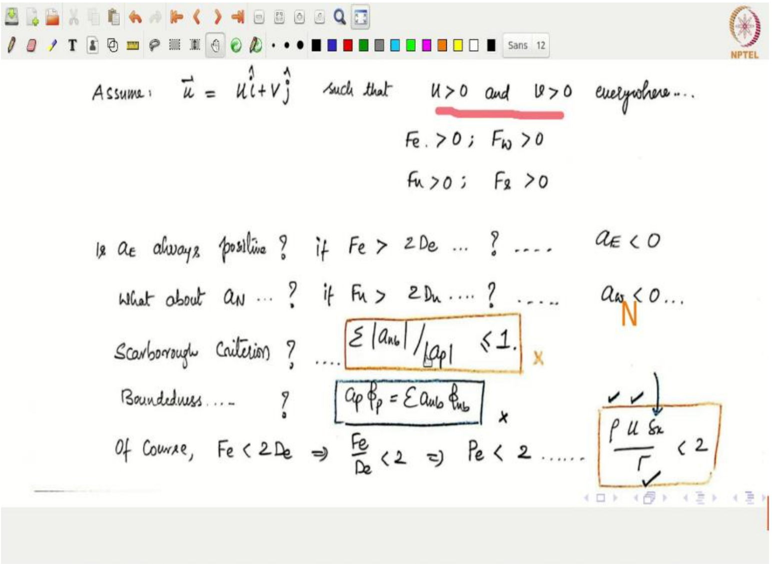

Because, if F_e is greater than D_e by twice of D_e, then this; actually can become negative, because these two are positive quantities. Now, D_e is positive, F_e is positive, but the magnitude of F_e can be more than if it is more than 2 times D_e then this coefficient becomes negative, then a becomes c less than 0. Similarly, if F_n is greater than 2 times D_n, then even a north can become this should be a north ok. So, this should be a north, a north can become negative right.

Now, of course, I have assumed, the I have assumed that the u and v are here positive quantities. If you assume u and v are both are negative quantities, then instead of a_E and a_N, you would get a south a west and a south to be the quantities that may become negative right. Because if you go back if you assume you know u to be negative, then this always becomes positive, and this always becomes positive. Whereas, this will become now negative right, because u is negative.

So, your F_w would come out to be negative, and this would come out to be negative alright, so that, that still happens. So, that means, what we have is we have the coefficients are not guaranteed to be positive. And the coefficients are not guaranteed to positive depending on the relative importance of the convection and the diffusion ok, that is what we see ok

What about this Scarborough criteria? If these becomes negative what will happen to Scarborough ok? Let us say the source is 0, source is 0 and if we have continuity satisfying flow field this is 0, sum of the coefficients a_{nb} are negative. So, a_P will be equal to $\sum \mathrm{a}_{\mathrm{nb}}$, because you have let us say some coefficients minus 2, plus 3, minus 1, plus 2 something like that you would get some value of 2 or something for a_P ok, a_P will be equal to $\sum \mathrm{a_{nb}}$.

But in the Scarborough criteria would be modulus sigma of $\sum \mathrm{a_{nb}}$ by modulus of a_P. This will not be satisfied, because now a_P will come out to be smaller than some of the modulus of the negative values right. The moment you have negative values this will this although a_P equals $\sum \mathrm{a_{nb}}$, this will not be satisfied right. This will be less than or equal to one will not be satisfied. So, with the negative coefficients, your Scarborough is not satisfied.

What about boundedness? Your boundedness is also not satisfied because now $\mathrm{a_{nb}}$ some of these are negative as a result your boundedness is also not satisfied, only if you have all positive quantities the ϕ_P value will lie between all the phi and bs right ok. So, that means, your Scarborough and boundedness are not going to be satisfied, as a result you cannot solve for this if your coefficients become negative ok.

So, in order to make sure that the coefficients do not become negative, of course, we have to do something like this. We have to choose always F_e is less than 2 times D_e, that means, F_e by D_e is less than 2. But of course, we know what is F_e by D_e, F by D, we have defined it as a cell Peclet number. So, cell Peclet number has to be always less than or equal to 2.

But what is the cell Peclet number? Definition, the definition is $\frac{\rho u \delta x}{\Gamma}$. But what is known and what is unknown in this? Velocity field is given. So, you cannot change it. You can, you cannot say I will solve for a different problem. Density is also known; gamma is known right.

So, the only control you have is basically δx. You have to choose your mesh such that your δx is such that this comes out to be less than 2 Γ by ρu ok, only then your coefficients will not become negative. And as a result, you can use central difference scheme for solving for convection diffusion equation ok.

This actually puts a very stringent restriction on the mesh because your gamma is usually very small. And depending on how large your velocities are, your δx has to be made much much finer ok. So, as a result, the central difference; central difference in scheme for convection equation comes with a very big restriction on the mesh size that you can take in order to solve for it successfully without having any divergence ok; so, alright.

Then that is all for today. So, I am going to stop here. So, in next class, we will see another discretization scheme for the convection term that is basically your upwind differencing scheme which will not come with a similar kind of which will not have these kind of restrictions.

So, we were going to look at that in the next class, alright, thank you. And if you have any questions, write back to me on my email ok, or else we will see if we can setup a Google chat or something like that, alright.

Thank you. See you, talk to you in the next class.