Computational Fluid Dynamics Using Finite Volume Method
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Lecture – 24
Finite Volume Method for Diffusion Equation: Steady diffusion in unstructured meshes
Part I

Good morning. Let us get started. So, in the last lecture we discussed about discretizing an unstructured mesh that is orthogonal right and also we made some comments. Today we are going to look at discretizing of a of steady diffusion equation on an unstructured and non-orthogonal mesh ok.

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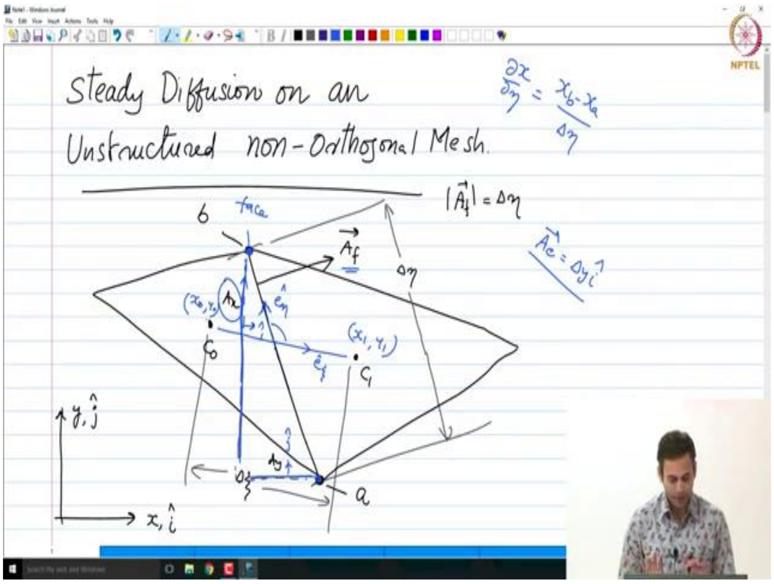

So, that is essentially steady diffusion on an unstructured non-orthogonal mesh ok.

So, if we were to draw a typical cell arrangement for an non-orthogonal mesh how let us kind of see how it looks like. So, we have a face and then we have then we have the cells ok. The cells are let us say more like this ok. So, essentially we have this is the face f in between this is my face and this one is the cell centroid; let us say this is the C_0 cell and this is the C_1 cell ok.

So, we have a 0 cell and we have one cell. Of course, we have two more cells that are sharing a face with C_0 cell, but we are only concentrating on one of the neighbors right.

Once you we generate the equation for one neighbor we can extend it to all other neighbors ok. So, we have a face f here whose area normally is in this direction. So, this is my this is $\overrightarrow{A_f}$ right.

And, the two directions that we have here one is the one connecting the cell centroids that is C_0 to C_1; this we call it as $\hat{e_\xi}$ right that is one particular direction and the other one is along the face. So, this is the other direction which is $\widehat{e_\eta}$ ok. Now, as you can see these two do they have a angle of 90 degrees between them?

Student: No.

They do not; that means, $\overrightarrow{A_f}$ is not parallel to e_ξ right $\overrightarrow{A_f}$ is not parallel to e_ξ, as a result the angle between e_ξ e_η is not 90 degrees. Then we can of course, also define the lengths that are between the cell centroid. Let us say this length we would like to define it as $\Delta\xi$. This is $\Delta\xi$ and we would also like to define this distance between t 2 vertices as $\Delta\eta$ ok.

So, we will like to call this as $\Delta\eta$; that means, $\Delta\eta$ is nothing but the magnitude of the area vector right. $\overrightarrow{A_f}$ magnitude is nothing, but $\Delta\eta$. We would also like to name these two vertices this one we would like to call it as vertex a, this one we would like to call it as vertex b that basically defines the direction for e_η unit normal. We of course, also have the global coordinate system that is x, $\hat{\imath}$ and y, $\hat{\jmath}$ ok.

So, that defines a typical cell with it is neighbor in a in an unstructured non-orthogonal mesh alright. So, of course, the magnitude A_f is equals to $\Delta\eta$ magnitude of $\overrightarrow{A_f}$ equals $\Delta\eta$ and the distance is $\Delta\xi$ between C_0 and C_1, fine? Alright.

Let us move on then. Let us try to integrate our diffusion equation steady diffusion equation on this particular cell ok.

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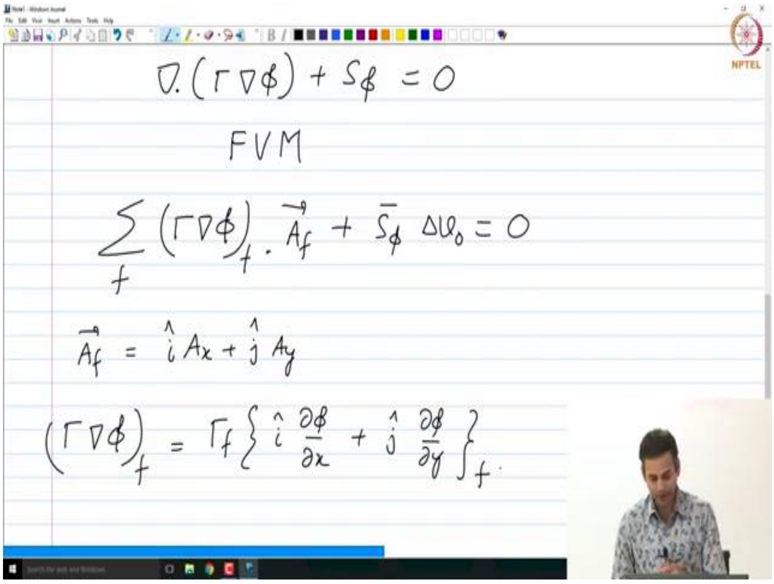

So, that is if I have to write it here this is basically $\nabla \cdot (\Gamma\nabla\phi) + S_\phi = 0$ that is our steady diffusion equation. And, we if we apply finite volume method we are going to go through all the steps that is basically integrate this on a C_0 cell then invoke Gauss divergence theorem converting volume integral into surface integral and then make the surface integral into a summation and so on.

And also approximate the source term with the cell centroid value and so on and then we finally reach this particular equation that is summation on the faces for $(\rho\vec{u}\phi)_f \cdot \overrightarrow{A_f}$ plus $\overline{S_\phi}$ times ΔV_0 equals 0 right. That is our discrete equation right which is basically the first balance for the $\nabla\phi$.

Then, of course, what would be our $\overrightarrow{A_f}$ here. $\overrightarrow{A_f}$ has two components right one is along $\hat{\imath}$ the other one is along j cap ok. So, $\overrightarrow{A_f}$ can be written as some A_x i cap and A_y $\hat{\jmath}$, where A_x is actually the vertical distance right this should be your A_x and this should be your A_y ok. So, remember A_x is not the projection on x-axis. A_x A_x is the component of $\overrightarrow{A_f}$ in the direction of i ok.

So, that means, we can write the $\overrightarrow{A_f}$ as $\overrightarrow{A_f}$ is $\hat{\imath}$ A_x plus $\hat{\jmath}$ A_y right that is what we have for $\overrightarrow{A_f}$. And, of course, the $(\Gamma\nabla\phi)_f$ can be written as Γ_f times $\nabla\phi$ and $\nabla\phi$ can be written as $\left(\hat{\imath}\frac{\partial\phi}{\partial x} + \hat{\jmath}\frac{\partial\phi}{\partial y}\right)$ on the face f right which is shared between C_0 C_1 cells.

Now, so far so good, but what is the difficulty here we have this derivatives that is $\frac{\partial \phi}{\partial x}$ and $\frac{\partial \phi}{\partial y}$ which we cannot directly calculate, right? Why is it so? because?

Student: Because phi values are not.

Because the phi value are not stored along x-direction or y-direction other they are stored in a unstructured way right all over the place. So, that means, ϕ values are only stored at the cell centroids and these cell centroids do not align themselves along these directions of x and y, ok.

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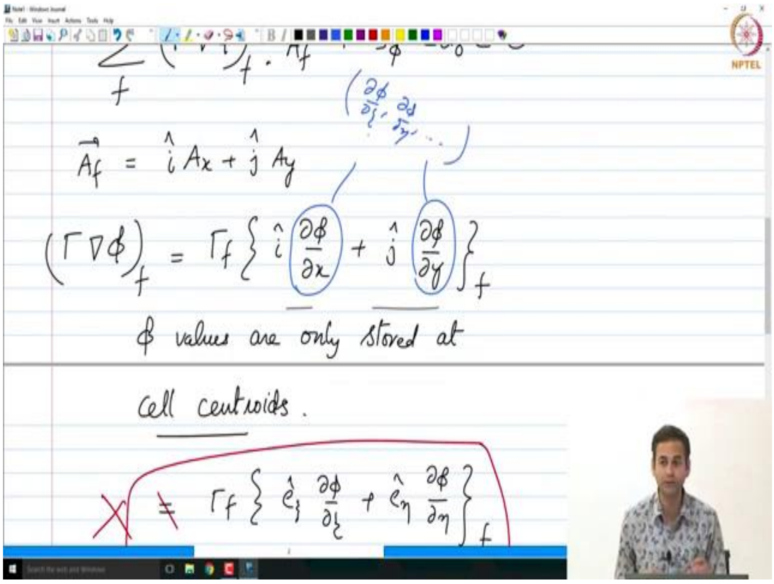

So, the difficulty is we cannot work with $\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}$ because ϕ values are only stored at cell centroids as a result what would be a better way to work with?

If we can calculate gradient in the ξ direction right that is a better way to work with because ξ direction kind of connects C_0 and C_1, where we have the values, right? But, of course, because the mesh is not orthogonal can I right can I evaluate the gradient as $\left(\widehat{e_\xi} \frac{\partial \phi}{\partial \xi} + \widehat{e_\eta} \frac{\partial \phi}{\partial \eta}\right)$ can I write like this?

Student: No.

I cannot write like this because this is not a orthogonal coordinate system right. So, this is not equal to this. So, we cannot work with this. So, this is because now the ξ, η is not an orthogonal coordinate system right. So, as a result we cannot write it there decompose into two components of e_ξ and e_η.

Rather we have to see how the partial derivatives $\frac{\partial \phi}{\partial x}$ and $\frac{\partial \phi}{\partial y}$ these two can be related to the other gradients ok. That means, we want to look at the relation between $\frac{\partial \phi}{\partial x}$ and these two derivatives that is $\frac{\partial \phi}{\partial \xi}$ and $\frac{\partial \phi}{\partial \eta}$ ok.

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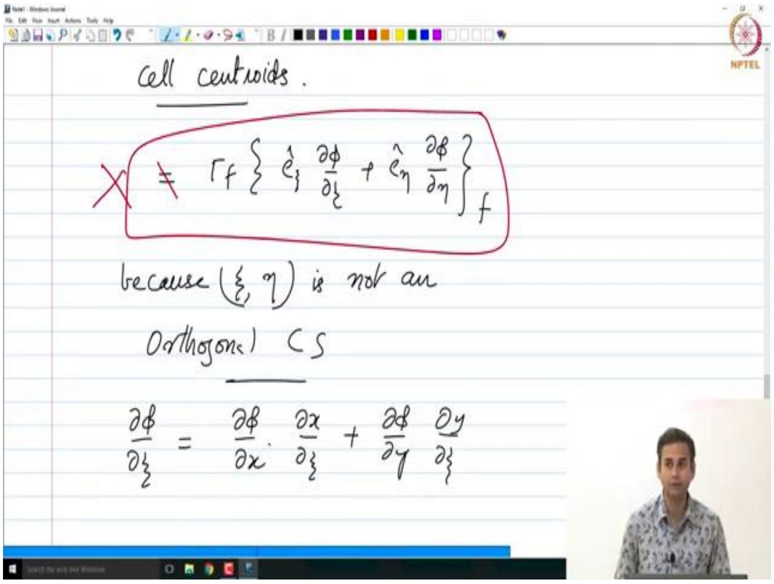

In order to do that we recall that phi as a if I want to calculate what is $\frac{\partial \phi}{\partial \xi}$, where ϕ is a function of both x and y right. I can write this as $\frac{\partial \phi}{\partial \xi} = \frac{\partial \phi}{\partial x}\frac{\partial x}{\partial \xi} + \frac{\partial \phi}{\partial y}\frac{\partial y}{\partial \xi}$, can I write like this? ok.

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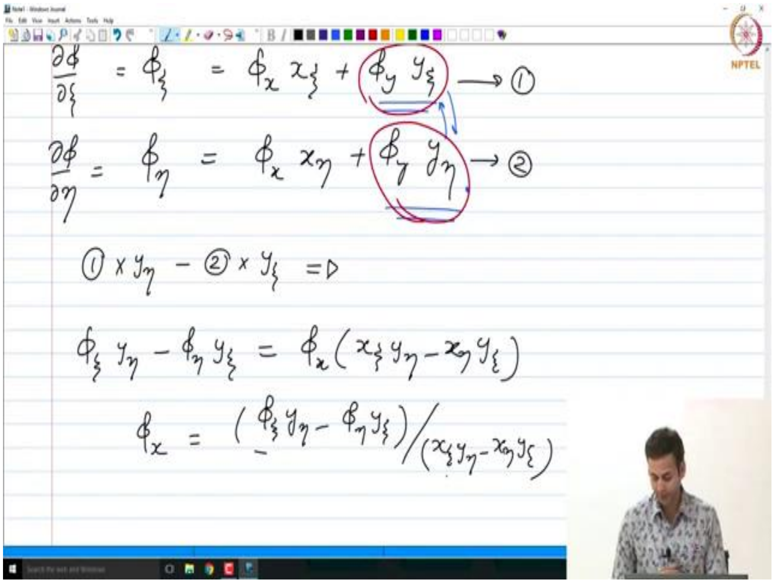

That means if I use a short hand notation here little bit, we can write this as ϕ_ξ which indicates $\frac{\partial \phi}{\partial \xi}$ ok. So, the partial derivative is indicated represented using a subscript then can be written as $\phi_x x_\xi + \phi_y y_\xi$ right; I can write like this. Similarly, if I want to relate $\frac{\partial \phi}{\partial \eta}$ this is ϕ_η would be what? Phi is again a function of x and y. So, I can write this as $\phi_x x_\eta + \phi_y y_\eta$, can I write like this? Ok.

So, I am relating the gradients in the local coordinate system that is ξ and η to the gradients in the global coordinate system that is ϕ_x and ϕ_y, right. And of course, together with the grid matrix those are how does x change with respect to ξ and how does x change with respect to eta and so on right. Let us call these equations as 1 and 2; let us call this as 1, this as equation 2.

Now, our motivation is now to calculate what is ϕ_x in terms of ϕ_ξ and ϕ_η, ok. So, we want to get expressions for ϕ_x, ϕ_y because we want to kind of replace this term and this term with what?

Student: (Refer Time: 11:25).

With the local coordinate derivatives; that means, we now replace it with $\frac{\partial \phi}{\partial \xi}$, $\frac{\partial \phi}{\partial \eta}$ and so on right and these all grid matrix and all. So, that is the motivation; because $\frac{\partial \phi}{\partial \xi}$ can be directly calculated ok.

Now, if it is if we have these two equations can be calculate what is ϕ_x from these two equations? Ok, of course, we can calculate. So, let us say we want to get rid of the second term here, we want get rid of this guy right. So, I can multiply the first equation with y_η right and the sorry, second equation with y_η and the first equation with sorry, first equation with y_η and second equation with y_ξ right and then subtract that essentially get rid of the second term here.

Can we do that? So, that is basically first equation times y_η minus second equation times y_ξ and calculate what would be ϕ_x that it will be ϕ_ξ y_η minus ϕ_η y_ξ equals ϕ_x times x_ξ y_η minus x_η y_ξ and the second term I am multiplied with y_η and y_ξ, it canceled right? Can we do that? Is that correct? Yeah? Ok it is correct.

Now, what about so, this is basically gives us what is ϕ_x alright this is basically gives us what is ϕ_x? ϕ_x is ϕ_ξ y_η minus ϕ_η y_ξ upon x_ξ y_η minus x_η y_ξ right that is what we have for ϕ_x. Now, similarly can we find an expression for ϕ_y in terms of the local derivatives? To calculate ϕ_y what should I do? I should multiply the first equation with?

Student: (Refer Time: 13:49).

x_η and the second equation with?

Student: x_ξ.

x_ξ and then subtract ok.

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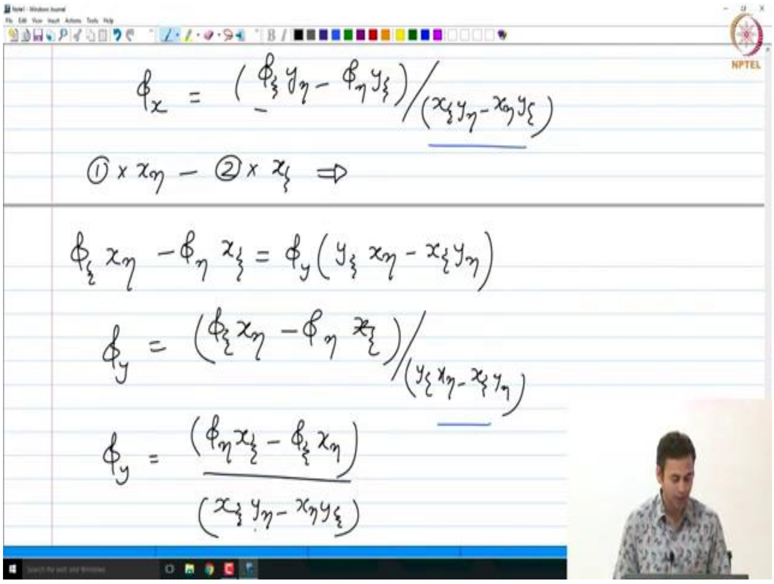

So, that means, I would write first equation times x_η minus second equation times x_ξ. This would give me alright ϕ_ξ x_η minus ϕ_η x_ξ equals the first terms get canceled in the 1 and 2 equations. And, the second terms remind that is ϕ_y times y_ξ x_η minus x_ξ y_η right; that means, ϕ_y is ϕ_ξ x_η minus ϕ_η x_ξ upon y_ξ x_η minus x_ξ y_η. Is it correct? y_η yeah it is correct? Ok.

So, now we will rearrange this little bit. I want to get the minus from the denominator to the top; because we have in the first equation we have x_ξ y_η whereas, this is kind of reverse. So, I would rearrange this and write the numerator as ϕ_η x_ξ minus ϕ_ξ x_η upon I would rewrite in denominator as x_ξ y_η minus x_η y_ξ, right. I just took a minus from the top.

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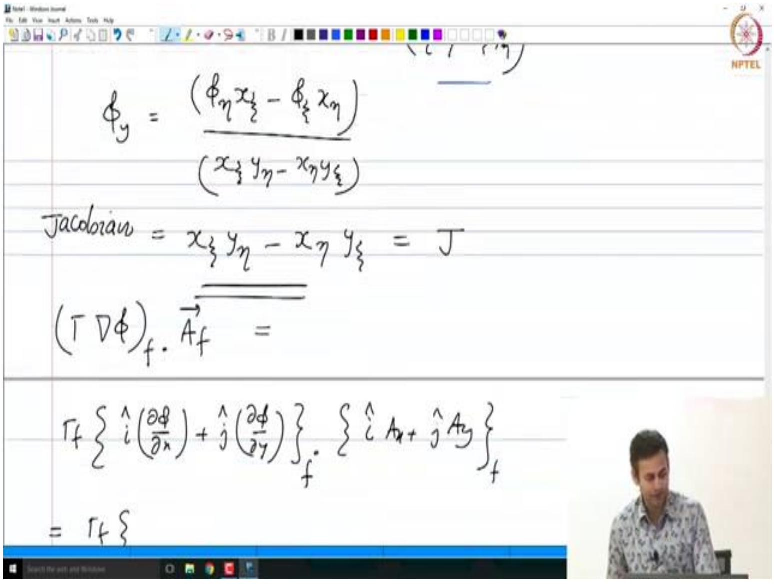

Then the denominator that we have here which is basically this expression x_ξ y_η minus x_η y_ξ which is the denominator in both of the expressions, this is known as Jacobian this expression. So, we would represent this using J which relates essentially which is a combination of the derivatives of x and y with respect to ξ and η some combination of that, fine. So far so good, any mistakes till now? No? They are all fine, ok.

So, that means, we have now somehow related all of these guys ϕ_x and ϕ_y in terms of the ϕ_ξ and ϕ_η ok. So, we have kind of related them. Now, I can go back and substitute these guys into the gamma grad phi right this is the original expression right this is the expression we want to evaluate. So, I am going to substitute for in this $(\Gamma\nabla\phi)_f \cdot \mathrm{A_f}$ right that is what we want to evaluate. So, I am going to substitute in there these terms.

So, I think I would need your help to tell me what would be the expression. So, we want to calculate $(\Gamma\nabla\phi)_f$ dot $\overrightarrow{\mathrm{A_f}}$; $\overrightarrow{\mathrm{A_f}}$ is î A_x plus ĵ A_y ok. So, how much this would be? This would be Γ_f times $\left(\hat{\imath}\frac{\partial\phi}{\partial x}+\hat{\jmath}\frac{\partial\phi}{\partial y}\right)$ dot î A_x plus ĵ A_y, right. This is evaluate the face and this is also on the face right that is what we have. So, that means, equals Γ_f times we will again get two terms right $\frac{\partial\phi}{\partial x}$ and $\frac{\partial\phi}{\partial y}$ multiplying A_x and A_y.

So, that would be how much is $\frac{\partial\phi}{\partial x}$? Basically, that is this quantity, right? This quantity is $\frac{\partial\phi}{\partial x}$ ok.

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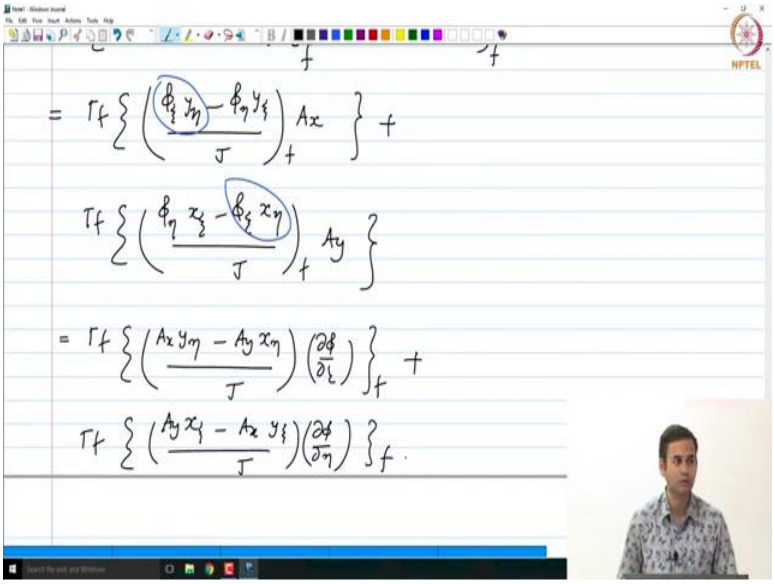

That means $\phi_\xi\ y_\eta$. So, this is $\phi_\xi\ y_\eta$ minus $\phi_\eta\ y_\xi$ upon Jacobian times A_x right this is you write it on the face plus Γ_f times, what would be partial phi partial y? That is this quantity right that is $\phi_\eta\ x_\xi\ \phi_\eta\ x_\xi$ minus $\phi_\xi\ x_\eta$ upon Jacobian times A_y is what we have, right. Is it correct? Any mistakes in here? Ok, yeah, expression settled ok.

Now, we want to kind of rearrange this little bit such that we get the two derivatives separated ok. We have these two derivatives ϕ_ξ and ϕ_η. Now, I want to collect terms and then I write as a multiplication of something times ϕ_ξ plus something times ϕ_η, right. Just like what we do for phi partial phi partial x and $\frac{\partial \phi}{\partial y}$ ok. If you want to do that what would this be? This would like Γ_f times if I collect all the coefficients for ϕ_ξ what would that be?

Student: (Refer Time: 19:18).

A_x.

Student: y.

y_η minus A_y.

Student: x_η.

x_η upon Jacobian times $\frac{\partial \phi}{\partial \xi}$ right all on the face plus what else will be there? So, that means, we essentially collected this term and this term right because both of them have ϕ_ξ as a multiplication factor. What is the next term plus Γ_f times?

Student: A_y x_ξ.

A_y.

Student: ξ.

x_ξ minus A_x y_ξ upon Jacobian times.

Student: ϕ.

$\frac{\partial \phi}{\partial \eta}$ on the face f, is this correct? A_y x_η minus A_x y_ξ right this is correct A_y x_ξ minus A_x y_ξ, fine ok. So far so good so, essentially we got now our two derivatives $\frac{\partial \phi}{\partial \xi}$ and $\frac{\partial \phi}{\partial \eta}$ with some multiplications in there ok, alright.

Now, we will go back to the schematic that we have drawn about the mesh and see what actually these things are ok.

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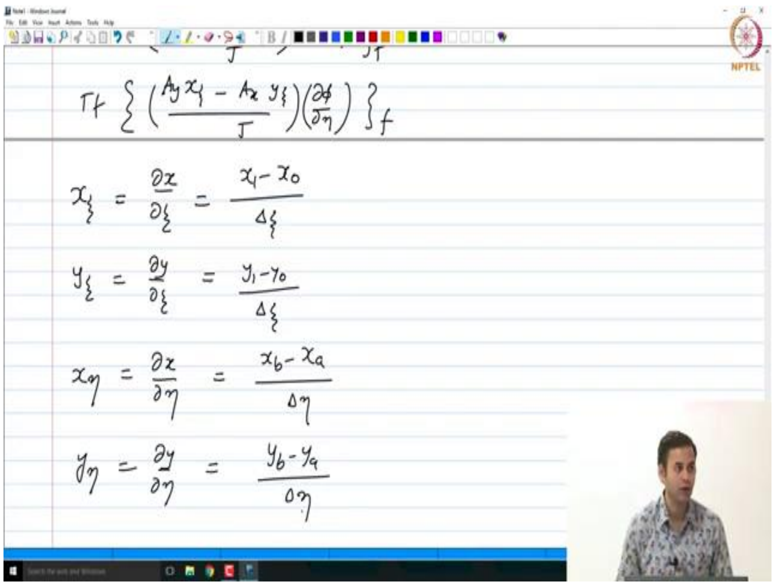

So, we have Jacobian has all these matrix right x_ξ y_η and so on. So, what is what is x_ξ? x_ξ is nothing but $\frac{\partial x}{\partial \xi}$. So, what would be $\frac{\partial x}{\partial \xi}$ if you were to calculate $\frac{\partial x}{\partial \xi}$ would be what? So, I am calculating what is x value at 1, how do I numerically calculate $\frac{\partial x}{\partial \xi}$? How do you calculate $\frac{\partial x}{\partial \xi}$?

Student: (Refer Time: 21:23).

So, essentially what would be $\frac{\partial x}{\partial \xi}$ here?

Student: (Refer Time: 21:30).

x_1 minus x_0.

Student: (Refer Time: 21:32).

Divided by $\Delta\xi$ right essentially that would be x_1 here, x_0 here divided by $\Delta\xi$, right. We are we are essentially relating the derivative of x with respect to xi that is alright change of x with respect to xi. So, that would be how much? x_1 minus x_0 upon $\Delta\xi$ right what would be, let me do this one. What would be y_ξ, $\frac{\partial y}{\partial \xi}$ if you want to calculate numerically what would this be? So, we have this cell centroid is C_1 or 1, the other cell centroid is C_0 or 0, right. The coordinates are x_0, y_0 and x_1, y_1.

We can always calculate what is the derivatives of these coordinates with respect to xi and eta, right that is all we are doing. Everybody follows? If I have to draw it here essentially we have these two right. So, the coordinates are x_1, y_1 and x_0, y_0 right and we are going in the direction of e_ξ and calculating the partial derivatives right, fine. So, what would be $\frac{\partial y}{\partial \xi}$ now?

Student: (Refer Time: 22:50).

y_1 minus y_0 upon.

Student: Delta y.

Delta y. So, that would be y_1 minus y_0 upon $\Delta\xi$. Now, we have another direction which is the eta, right. So, what will be x_η? That will be $\frac{\partial x}{\partial \eta}$ we have also identified a name for it, we have these vertices named as b and a, right. So, what will be and eta is along a to b right. So, what will be $\frac{\partial y}{\partial \eta}$ or $\frac{\partial x}{\partial \eta}$?

Student: (Refer Time: 23:24).

x_b minus x_a upon?

Student: Delta.

$\Delta\eta$.

Student: Eta.

Right, ok. Similarly, what will be y_η?

Student: (Refer Time: 23:35).

y_b minus y_a upon $\Delta\eta$, fine. So, we have kind of setup all of these things these are the matrix for all the coordinates. It is quite simple 0 and 1, a and b right. So, we have these four coordinates from which we are calculating the matrix ok, alright. So, ok.

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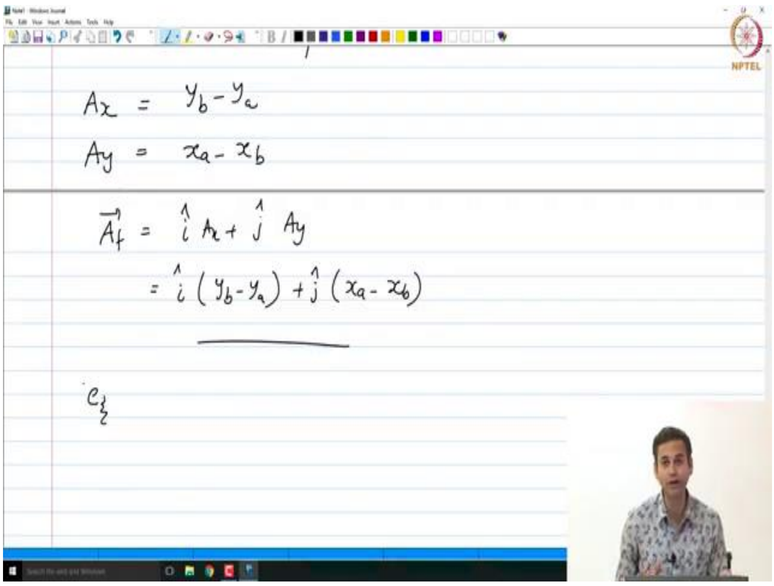

Now, what about what about A_x? A_x would be how much? So, A_x we have identified it as the length in the y direction right; that means, the projection of the face area vector onto the y-axis. So, what will be A_x in terms of b and a values? What will be this height? A_x is this one, right.

Student: (Refer Time: 24:45).

x or y?

Student: y.

y_b minus.

Student: y_a.

y_a right ok. So, y_b minus y_a is my A_x. This is y_b minus y_a. What about A_y?

Student: x_a minus x_b.

x_a minus that is this length right that will be how much?

Student: x_a minus.

x_a minus.

Student: x_b.

x_b. So, this is x_a minus x_b, alright? That is what we have. So, that means, my $\overrightarrow{A_f}$ is nothing but î A_x plus ĵ A_y this I can rewrite it as î times y_b minus y_a plus ĵ times x_a minus x_b, ok. Everybody agrees to this? Is it is the area vector is fine? The notation we have used for the A_x and A_y. For the A_x it is y_b minus y_a for the A_y it is x_a minus x_b it is not the same. It is b minus a and a minus b we see that ok.

Now, this will work even if the face vector is not inclined like this even if it goes like this way it will work, right. Accordingly x_a becomes smaller than x_b and A_x becomes negative all that works fine.

Yes?

Student: Sir, (Refer Time: 26:09).

Yeah.

Student: (Refer Time: 26:12).

Hold on. So, the question is what is the partial x partial eta, right? So, this is partial x partial eta. So, we are talking about the eta direction that is this guy right what are the coordinates we have. So, this is the direction arrow indicates the direction it would be you are talking about x. So, it will be x_b minus x_a upon $\Delta\eta$, right x_b minus x_a upon $\Delta\eta$, is that correct? Because it is it is always this guy minus this guy divided by the distance right that is what I have written right ok. Other questions? Area vectors derivatives, is fine? Ok fine.

So, we have all of these guys then area vector is also done, then what about the unit vectors. We do not know, what are these unit vectors, right. We only know what is x and y, i and j. The unit vectors will be specific to?

Student: Cells.

Cells; that means, it is specific to faces right each and every face will have a local coordinate system right because it is a unstructured, right. You may recall we have a let us say I find a difference kind of mesh, then you would probably use one particular transformation which will relate entire your x, y into some curvilinear geometry. But here we are talking about an unstructured mesh where each and every cell can be in its own directions right. So, as a result you will have a local coordinate system for each and every face right.

So, these e_ξ, e_η are specific to the face in question right and this is have to be computed for every cell for all the phases ok. So, that is why is $\widehat{e_\xi}$; $\widehat{e_\xi}$ is a unit vector right. But, if you look at the; that means, it is a unit vector that is connecting C_0 and C_1 we do not know unit vector, but we know a vector that is connecting C_0 and C_1. We can certainly normalize that and calculate the unit vector right. So, what is the vector that is connecting C_0 and C_1 with coordinates x_0, y_0, x_1, y_1?

Student: (Refer Time: 28:27).

So, we have two coordinates x_0, y_0, x_1, y_1. What is the vector that is connecting these two points?

Student: (Refer Time: 28:38).

x_1 minus x_0 î plus y_1 minus y_0 ĵ that is the vector right that gives you the total vector; if I want to normalize it what will be the distance?

Student: (Refer Time: 28:46).

Divided by?

Student: (Refer Time: 28:49).

$\Delta\xi$. $\Delta\xi$ is nothing, but again square root of x_1 minus x_0 which we do not have to write because we have indicated as $\Delta\xi$ right of course, when you code it up you have to use square root of x_1 minus x_0 square and so on, ok. So, that is the unit vector; so, that means, this is probably we will do it slow ok.

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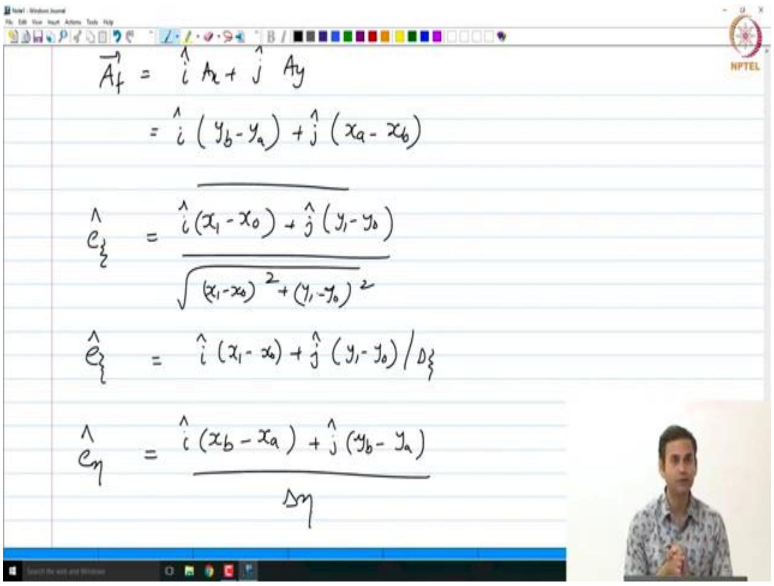

So, this is basically x_1 minus x_0 î plus ĵ times y_1 y_0 this is a vector that is connecting C_0 to C_1 centroids right. Of course, the distance would be that divided by if you want to normalize it and find some unit vector it will be square root of x_1 minus x_0 and so on which you would calculate when you write a code, but for now I would denote this as simply $\Delta\xi$ to kind of reduce the notation.

So, this is î times x_1 minus x_0 plus ĵ times y_1 minus y_0 divided by $\Delta\xi$, fine. That is your $\hat{e}_{\xi}$,. Everybody agrees with it? We have essentially two points. How do you calculate a vector between two points? Difference of the x values times i ok, fine. What about similarly can you tell me what will be e_η hat?

Student: (Refer Time: 30:15).

x_b.

Student: Minus.

Minus x_a.

Student: î.

î.

Student: Plus.

Plus.

Student: (Refer Time: 30:24).

y_b minus y_a ĵ.

Student: (Refer Time: 30:29).

Upon $\Delta\eta$ right. You have chosen b first because you are vector is pointing from a to b that is alright, that is we know the thing right. Similarly, just like one first because it is pointing from 0 to 1, ok. These are all known fine, everybody with the unit vectors? Similarly, $\Delta\eta$ you would calculate from the magnitude of the face $\overrightarrow{A_f}$ magnitude would give you $\Delta\eta$ right when you actually code it up.

Student: Sir.

Yes.

Student: (Refer Time: 31:04).

i A_x plus j A_y.

Student: (Refer Time: 31:11).

But, the notation is A_x is not the x component of the face; A_x is the component of the face vector that is in the x-direction, in the i-direction.

Student: (Refer Time: 31:24).

Yeah, A_x is not the component along x axis. A_x is the component along y axis because it will point in the x direction, right. $\hat{\imath}$ a A_x is chosen on the y axis as that it points in the i cap direction, right. If you want I can write it as A_y x it will be confusing, right. A_x is so, go back to the figure. We have $\overrightarrow{A_f}$ here, right. This is $\overrightarrow{A_f}$. I am choosing projecting this on to two axis.

But, the axis are not corresponding to the subscripts they are interchange. So, the projection on the y axis I am calling it as A_x, the projection on the x axis I am calling it as A_y because A_x would be along $\hat{\imath}$ right and A_y would be along $\hat{\jmath}$, is that ok? Then A_x actually is the length in the y direction right and A_y is actually length in the x direction, ok. Fine? No? Alright?

Student: Magnitude of A_x is lesser than a 1.

Magnitude of A_x can be less a 1, we do not we do not really bother. Sorry?

Student: Sir, according this diagram (Refer Time: 32:44).

According to this diagram length of A_x is greater than length of A_y.

Student: (Refer Time: 32:54).

Hold on. So, the question is I think there is some confusion between A_x and A_y. Let us. So, A_x is what is A_x by definition? What I have drawn according to what I have drawn?

Student: (Refer Time: 33:06).

No. On the y axis.

Student: (Refer Time: 33:10).

It is on the y axis right this is y axis right. It is not along x axis. It is along y axis such that it points in the i cap direction right. So, what was if you go back to our diffusion equation what was the $\overrightarrow{A_e}$? $\overrightarrow{A_e}$ was how much? Was it Δy times $\hat{\imath}$ or Δx times $\hat{\imath}$?

Student: Delta y.

Delta y times $\hat{\imath}$ right, you remember? So, essentially I am using the same notation here A_x is nothing, but A_x is pointing in the $\hat{\imath}$ direction and that is projection along the y axis, right. You see that? No? ok. Otherwise we will get back to this after the class, for now A_x and A_y are have to be interchanged from what you are thinking alright. So, fine?

So, we have now have all these matrix that are set up ok. So, if we go back, and evaluate each of these things that we have in terms of the Jacobian and all the coordinates so, let us look at the Jacobian ok.

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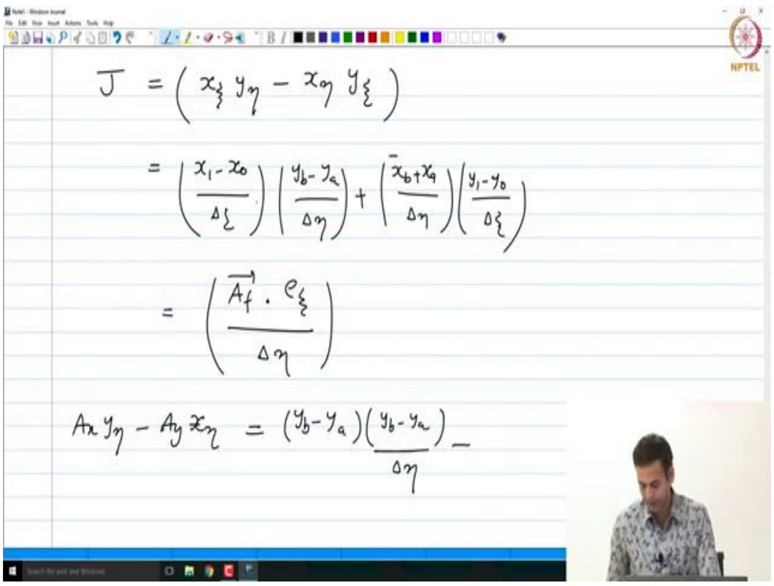

So, what was the Jacobian given as? x_ξ y_η minus x_η y_ξ, is this correct? Jacobian x_ξ y_η minus x_η y_ξ, is this correct? Yeah, this is correct ok. Now what is x_ξ?

Student: (Refer Time: 34:44).

Yeah, $\frac{\partial x}{\partial \xi}$ we have written it as something right x_1 minus x_1 minus x_0 upon?

Student: (Refer Time: 34:54).

$\Delta\xi$ and what is y_η?

Student: (Refer Time: 34:57).

y_b minus y_a upon.

Student: $\Delta\eta$.

$\Delta\eta$ minus x_η will be how much?

Student: x_a x_b.

x_b minus x_a by.

Student: Delta.

$\Delta\eta$ times y_ξ would be?

Student: y_1

y_1 minus y_0 by.

Student: Delta.

$\Delta\xi$ ok, fine. I just substituted whatever we have calculated for this.

Now, we have a multiplication of $\Delta\xi$ $\Delta\eta$ in the denominator right and what is the numerator? We have x_1 minus x_0 y_b minus y_a that is, if I go back here x_1 minus x_0 that is this quantity multiplying y_b minus y_a that is this quantity and what is the other one? x_b minus x_a and y_1 minus y_0 right. So, that is y_1 minus y_0 instead of x_b minus x_a I have x_a minus x_b right, but I can of course, change this minus to plus and make this minus and plus right.

Then, can I rewrite this as a dot product of these two quantities, this guy and $\vec{A}_f$? Do you see that or no? No?

Student: Yes.

Followed or shall I repeat it again? Ok, let us see it again. So, we have what is the first quantity we have? x_1 minus x_0 times y_b minus y_a, right. So, that can be thought of as multiplying y_b minus y_a from here multiplying x_1 minus x_0, right and the second one is x_a minus x_b times y_1 minus y_0, right. Of course, there is a $\Delta\xi$ in the denominator right. So, that also can absorbed into this.

So, I can write this essentially as $\overrightarrow{A_f}$ dot e_ξ by $\Delta\eta$. Can I do that? Can you check whether this comes back here? alright, we are kind of reducing it to the matrix that we have alright can we write like this?

Student: Yes.

A f dot e_ξ by $\Delta\eta$ because there is a $\Delta\xi$ in the e_ξ definition, right. So, there is this oh, I am sorry yeah, there is a $\Delta\xi$ in the e_ξ definition right this also gets absorbed right. So, x_1 minus x_0 divided by $\Delta\xi$ times y_b minus y_a would give you A f dot e_ξ the i th component divided by $\Delta\eta$ would remain. Right. Can you verify this? Tell me whether it is correct or not? Fine. Yes, any mistakes or is it ok? fine, this is fine ok.

So, Jacobian is done, then what else we have? We have again the other two quantities which are basically this one A_x y_η minus A_y x_η upon Jacobian we will come to Jacobian little later ok. I will evaluate the numerator here which is A_x y_η A_y x_η ok. So, that is how much? A_x y_η minus A_y x_η, is that correct what I have written A_x y_η? Yeah. So, how much would this be A_x is how much? A_x is how much?

y_b minus.

Student: y_a.

y_a times how much is y_η? y_η?

Student: (Refer Time: 39:10).

y_b minus y_a divided by $\Delta\eta$ minus what we have, A_y would be how much?

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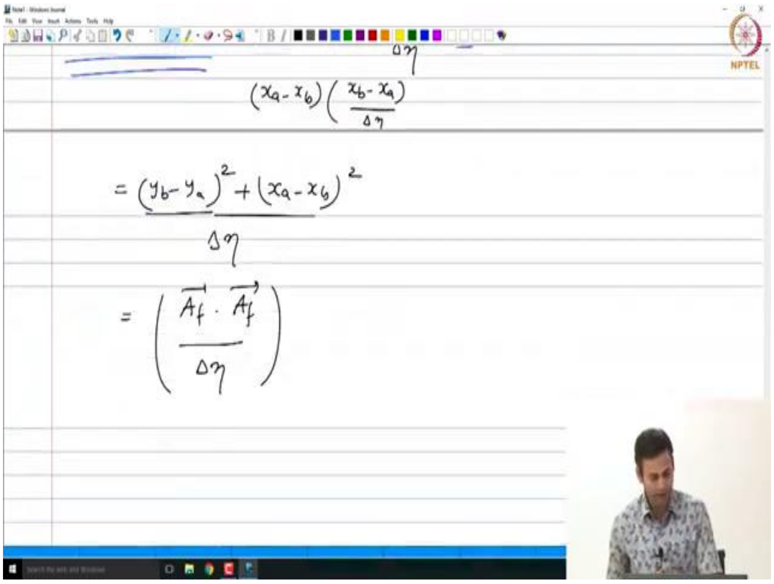


Student: x_a minus x_b.

x_a minus x_b times, what would be x_η?

Student: x_b minus x_a.

x_b minus x_a upon $\Delta\eta$, fine. You have to use your notes and see whether this is correct. A_x is the vertical distance that is y_b minus y_a, y_η of course, is between a and b so, that is y_b minus y_a upon $\Delta\eta$. Similarly, A_y is the projection along x axis that is x_a and x_b and x_η would be in the a to b direction that is x_a, x_b minus x_a by $\Delta\eta$ right.

Of course, there is some asymmetric here. What we can do is we can just rewrite this by absorbing the minus here we can rewrite this as y_b minus y_a whole square plus x_a minus x_b whole square divided by $\Delta\eta$ right. What is y_b minus y_a whole square plus x_a minus x_b whole square.

Student: (Refer Time: 40:34).

That is nothing but.

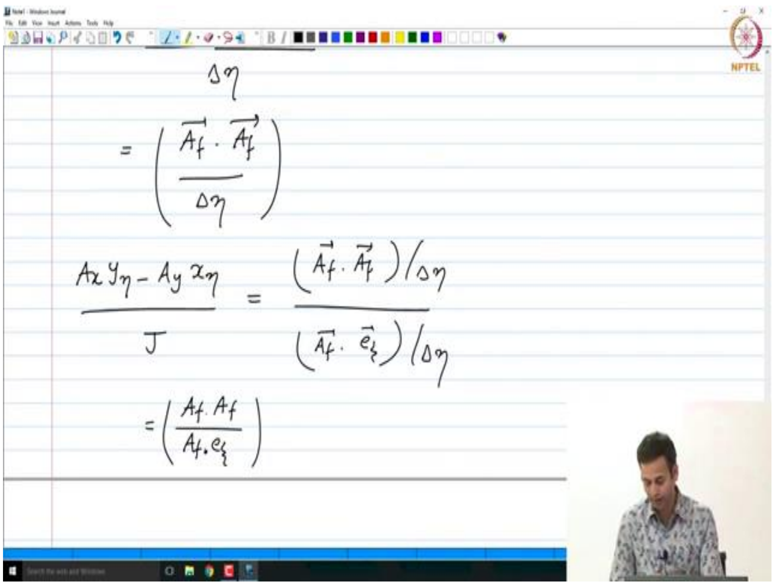
Student: A dot.

A dot a, right, y_b minus y_a whole square plus x_a minus x_b whole square $\Delta\eta$ square right the same thing. Essentially ah; that means, I can write this as $\overrightarrow{A_f}$ dot $\overrightarrow{A_f}$ which is nothing, but $\Delta\eta$ square upon $\Delta\eta$ is what we have right.

Upon sorry? Upon $\Delta\eta$ is what I have, right. This essentially nothing, but $\Delta\eta$ because $\Delta\eta$ square upon $\Delta\eta$ this is $\Delta\eta$ right, the entire term, correct? I would leave it there because in the Jacobian also there is a $\Delta\eta$ in the denominator. I want to cancel this $\Delta\eta$ with that one because this is the numerator we are talking about right this is divided by Jacobian and Jacobian we just had a $\Delta\eta$ here write that gets canceled. So, I want to leave it here as it is.

So, what is the first coefficient now? If I have to complete the first coefficient for $\frac{\partial\phi}{\partial\xi}$ is this entire term, right.

(Refer Slide Time: 41:59)



What is that? That is nothing, but A_x the first coefficient is A_x y_η minus A_y x_η right, is that correct?

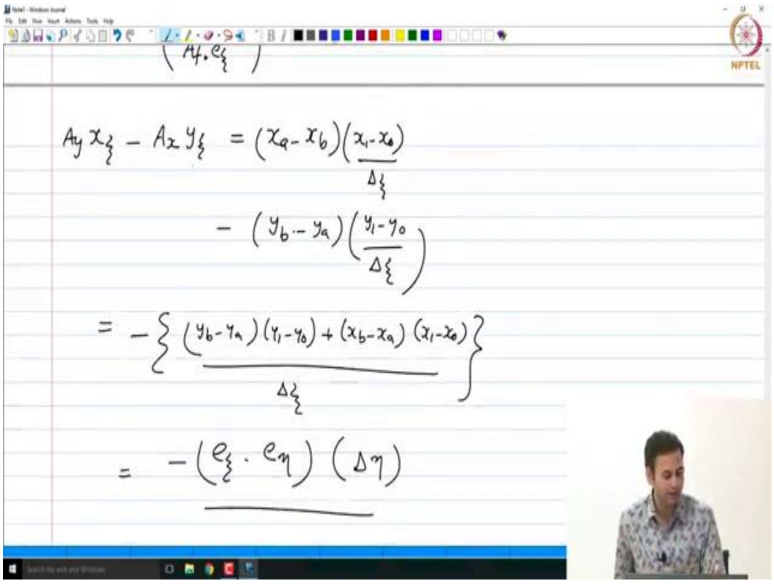
Student: Yes.

Yeah, divided by Jacobian. This would be what? This would be A f dot A f upon $\Delta\eta$ divided by how much was Jacobian?

Student: (Refer Time: 42:22).

A_f dot e_ξ upon $\Delta\eta$ right. So, this is nothing, but what? This is nothing, but A_f dot A_f by A_f dot e_ξ, fine. $\Delta\eta$ gets canceled, this is what we get. So, this is multiplied by $\frac{\partial\phi}{\partial\xi}$ right, that is all very good, fine. Everybody is following ok?

(Refer Slide Time: 43:18)



Now, we will do the second term. The second term was how much that is multiplication of $\frac{\partial\phi}{\partial\eta}$ this is A_y x_ξ A_y A_y how much can you let me write A_y x_ξ minus A_x y_ξ, is it? Yeah, divided by Jacobian we will come to that little later. So, what is A_y?

Student: (Refer Time: 43:30).

x_a minus x_b times how much is x_ξ?

Student: (Refer Time: 43:36).

x_1 minus x_0 upon.

Student: $\Delta\xi$.

$\Delta\xi$ plus how much is A_x?

Student: Minus.

Minus sorry, yeah. So, minus.

Student: y_b minus.

y_b minus y_a times, how much is y_ξ?

Student: (Refer Time: 43:53).

y_1 minus y_0 by $\Delta\xi$ ok. So, we got x_a minus x_b, x_1 minus x_0 y_b minus y_a and y_1 minus y_0. So, if you go back x_a minus x_b x_1 minus x_0, what would that be a dot product of?

Student: Unit vectors.

Unit vectors, right. e_ξ and e_η have these things, but there is plus minus references, is it not? So, I should take a minus out because the first one is x_b minus x_a, here whereas, what we got is x_a minus x_b right. So, if you take a minus out that would be y_b minus y_a times y_1 minus y_0, right and then plus x_b minus x_a times x_1 minus x_0 is that correct divided by $\Delta\xi$ right. So, we have b minus a b minus a 1 minus 0 1 minus 0 multiplying the y coordinates getting multiplied and x coordinate.

So, this is would be how much? This would be y_b minus y_a times y_1 minus y_0 x_b minus x_a times x_1 minus x_0 divided by $\Delta\eta$ is what we have. So, if we multiply e_ξ dot e_η right and then multiply that with $\Delta\eta$ I would get this thing right. So, that would be how much? That would be this would be I can write this as minus e_ξ dot e_η times $\Delta\eta$ right; because on $\Delta\xi$ one $\Delta\eta$ in the denominator, but this resulting expression has only $\Delta\xi$. So, I would multiply this with $\Delta\eta$ right. Fine, everybody able to follow? Ok, fine.

So, alright so, but what is the total expression we have for the second term? Basically this quantity divided by Jacobian ok.

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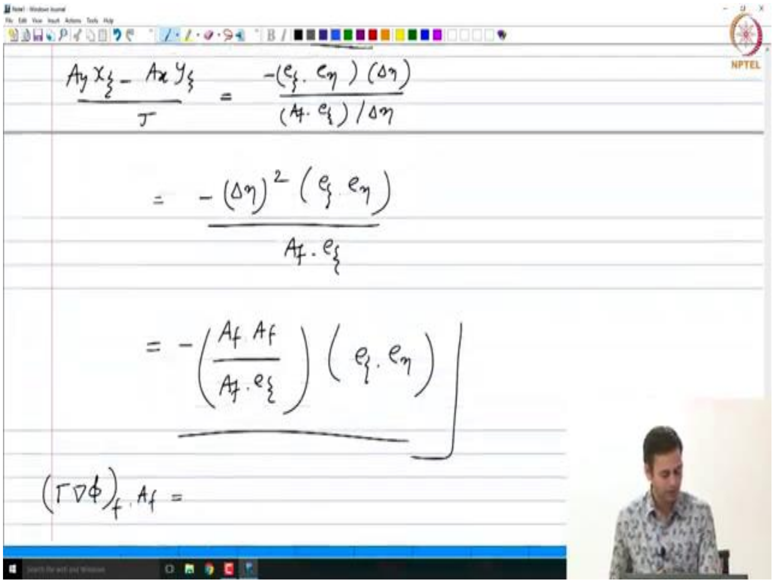


So, that would be if I put it back the second one is A_y x_ξ A_y x_ξ minus A_x y_ξ upon Jacobian is what is multiplying $\frac{\partial\phi}{\partial\eta}$. So, that would be minus e_ξ e_η times $\Delta\eta$ upon how much was Jacobian?

Student: A_f.

A_f dot.

Student: (Refer Time: 46:38).

e_ξ.

Student: Divided by.

Divided by $\Delta\eta$. So, this should how much? This will be minus $\Delta\eta$ square times e_ξ dot e_η by A_f dot e_ξ, is that correct? Yeah or would the del eta get canceled or it will become square?

Student: (Refer Time: 46:58).

Square, right it will go to the numerator it will become square. I would like to rewrite this thing $\Delta\eta$ square is nothing, but what? A_f dot A_f. So, this I would like to write as minus

A_f dot A_f upon A_f dot x_ξ times x_ξ dot e_η, ok. So, this is the another expression multiplying the $\frac{\partial\phi}{\partial\eta}$ right ok.

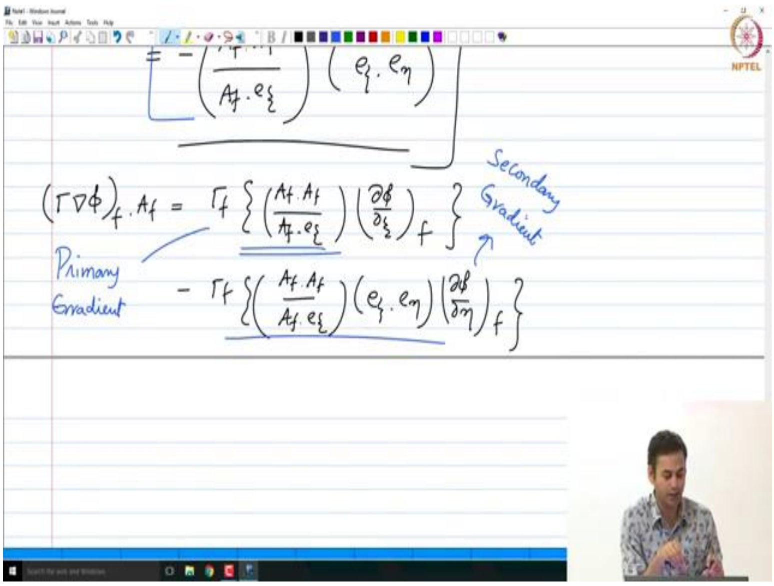
So, shall we assemble everything and write our $(\Gamma\nabla\phi)_f \cdot \mathrm{A_f}$ we are still there right we have still not written the final equation. We are trying to see if we can calculate all these matrix ok. So, what is our final expression? That would be that would be $(\Gamma\nabla\phi)_f \cdot \mathrm{A_f}$ would be would be essentially this one right. So, that is whatever we got for the first term multiplying the $\frac{\partial\phi}{\partial\xi}$ then whatever we got for the second term multiplying the $\frac{\partial\phi}{\partial\eta}$ that is what we have right.

So, what was the first term?

Student: (Refer Time: 48:19).

First one is this this guy right first is this one, right? A_x y_η minus A_y x_η by Jacobian; what is the second one? Second one is this guy right, ok.

(Refer Slide Time: 48:36)



So, can you let me write this? This is basically Γ_f times A_f dot A_f by A_f dot e_ξ right is that the first term?

Student: Yes.

Ok, multiplied by what? $\frac{\partial \phi}{\partial \xi}$ f, right and then we have minus gamma f; minus is coming from this coefficient right gamma f times A_f times A_f by A_f dot e_ξ times e_ξ dot e_η times what? $\frac{\partial \phi}{\partial \eta}$ for the face. Do you see that? ok.

Now, all this is done for one particular face f, right. This is something like a an east face right if you have a structured mesh. Now, what do you see? This is something different from what we have been doing. Till now right till now we only had one derivative coming for every face, right.

If you are talking about east face you only got $\frac{\partial \phi}{\partial x}$right or if you are talking about a north face you only got $\frac{\partial \phi}{\partial y}$. But, even in the context of unstructured orthogonal meshes we only got one quantity right that was $\frac{\partial \phi}{\partial \xi}$, but here we get $\frac{\partial \phi}{\partial \xi}$ component and a $\frac{\partial \phi}{\partial \eta}$ component that is a manifestation of the.

Student: (Refer Time: 50:06).

Non orthogonal of the mesh, because of that you got this one. So, that is why you got two terms her. Now, let us see we would like to call these two terms with different names. We call the first one which is the primary direction right which normal to face as which is which is one direction is basically called the primary gradient which is basically $\frac{\partial \phi}{\partial \xi}$. And, the second term we have here which is not the main direction for xi we call this as secondary gradient. So, we got a primary gradient and a secondary gradient um, ok.

So, we will kind of see kind of make more comments on this on how to compute these quantities and what is the consequences we get as a result of the non-orthogonality. Now, would the non orthogonal formulations switch back to an orthogonal formulation if we end up having some cells as orthogonal ok, all those things. And finally, the discrete equation all this things we will see in the in the next class, fine? I am going to stop here. We will start off with this particular equation ok.

Thank you guys. See you in the next class.