Computational Fluid Dynamics Using Finite Volume Method
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Lecture - 02
Review of governing equations: Conservation of momentum

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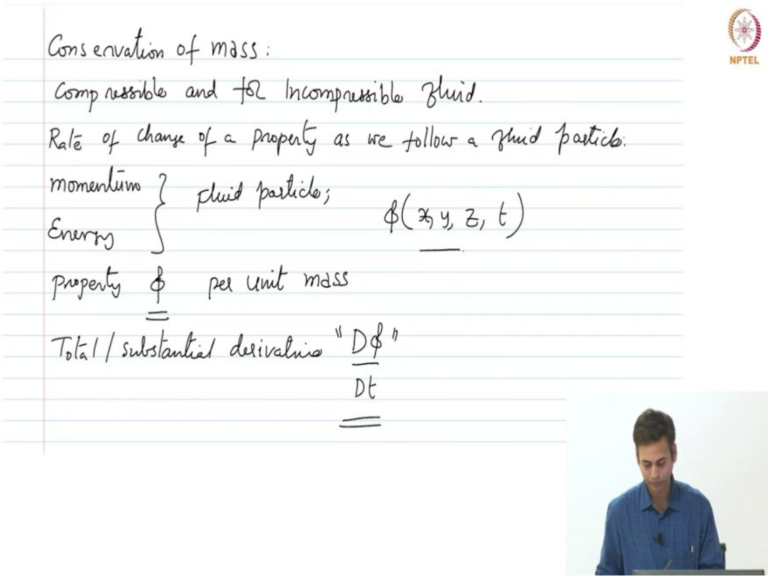

Alright, good morning everyone.

Student: Good morning.

Welcome to the 2nd lecture as part of this course. So, yesterday we looked at conservation of mass, we derived a an expression right, for both for a compressible and for an incompressible fluid.

Today, we are going to look at rate of change of a property as we follow a fluid particle ok. So, this is kind of in preparation to have essentially the momentum equations and the energy equation ok. So, we are going to look at rate of change of a property of fluid as we follow a fluid particle ok.

So, the statements of conservation of momentum and energy are applied for a fluid particle or for a finite fluid element. For what are they applied, for they applied for a fluid particle right? These are all the Newton's second law and the first law of thermodynamics these are all applicable for a fluid particle or for a fluid of certain mass right.

So, essentially these are applicable for a fluid particle right. So, we said the rate of change of momentum of a fluid particle equals the resultant forces acting on the fluid particle right. So, essentially we have to derive for a fluid particle.

Now, let us think of a particular property of a fluid, which we call it as represent with phi ok. And, this property is per unit mass ok. Now, this property phi is of course, a function of both the particle position that is x, y, z as well as time ok.

So, we want to know how this property changes with time as we follow the fluid particle. So, we are kind of we want look at what is the total or substantial derivative of this property.

That is D phi D t ok. We are going to represent with D phi D t. So, we want to calculate what is this quantity? Ok.

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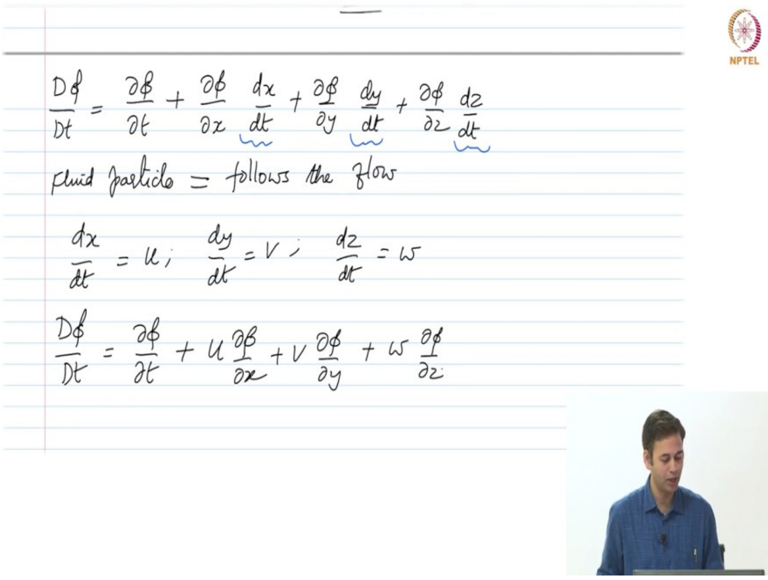

So, because it is a function of both space and time, what would it be? It would be what will be $\frac{D\phi}{Dt}$

The substantial derivative it will be rate of change with respect to time $\frac{\partial \phi}{\partial t}+$¿ , if you apply the chain rule what do we get? We have $\frac{\partial \phi}{\partial x}$ partial phi partial x right and then the position of the particle itself is changing with respect to time.

So, this will be $\frac{\partial\phi}{\partial x}\frac{dx}{dt}+\frac{\partial\phi}{\partial y}\frac{dy}{dt}+\frac{\partial\phi}{\partial z}\frac{dz}{dt}$ This is what we have for the substantial derivative? Now, we know that this fluid particle follows the flow right.

So, what would be the instantaneous derivatives of the position vector that we have here. What will be $\frac{dx}{dt}=u$ d for this fluid particle, that will be the x component of velocity of the flow.

So, this will be u similarly $\frac{dy}{dt}=v$ and $\frac{dz}{dt}=w$ right. That is what we have they essentially the time derivatives of these spatial coordinates of this fluid particles position would be nothing, but the instantaneous velocity vector ok.

So, we are going to substitute back these three quantities into the substantial derivative definition. So, we are going to get $\frac{D\phi}{Dt}=\frac{\partial\phi}{\partial t}+u\frac{\partial\phi}{\partial x}+v\frac{\partial\phi}{\partial y}+w\frac{\partial\phi}{\partial z}$ ok.So, we kind of got an expression here, which we can write in a compact notation using Nabla.

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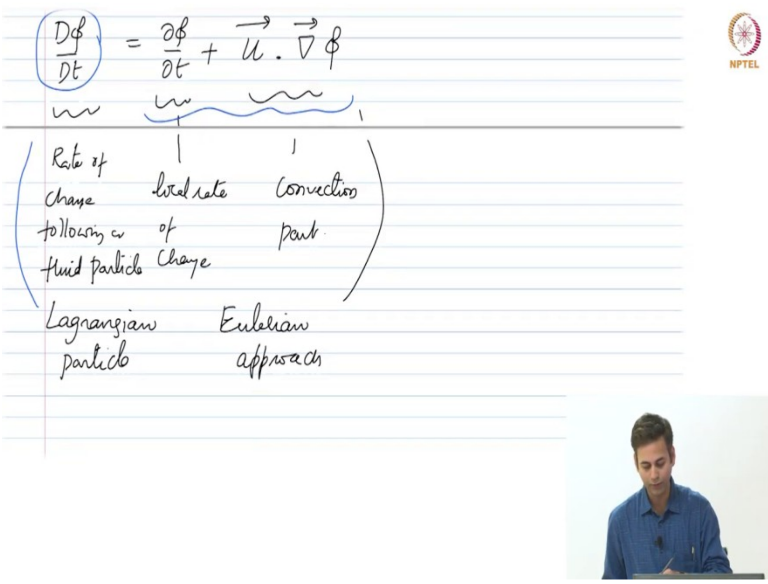

So, this will be $\partial\phi/\partial t$ plus, how can we write $u\frac{\partial\phi}{\partial x}+v\frac{\partial\phi}{\partial y}+w\frac{\partial\phi}{\partial z}$u? . We could write it as the velocity vector that is u bar right, dotted with gradient of the scalar field phi ok. So, this will be $\bar{u}.\nabla\phi$.

So, that is your $\frac{D\phi}{dt}$ ok, ok. Now, we have in the process developed an expression, which is this part is the rate of change following a fluid particle right, which we said is nothing, but the rate of change ok. So, this would be the local rate of change right and this would be the convection part of the change in phi ok. So, this is the convection part.

So, we have now two expressions for the same quantity. On the left hand side what we have we call it as what in what approach is this, we call this derivative that we have here. What do we call where do we use this derivative in? It is in a Lagrangian approach right. Essentially you are following the particles.

So, this is a Lagrangian description right, wherein you follow the particle on the right hand side what we have here, these two quantities, these describe the flow from a an Eulerian approach right. So, this is a an Eulerian approach where you are focusing on a certain quantity or a certain domain right a region of interest in the flow domain.

And, that is the Eulerian approach. Now, certainly all the governing equations that we have the conservation of momentum and the conservation of energy these are all defined in a Lagrangian approach right. These are for a fluid particle.

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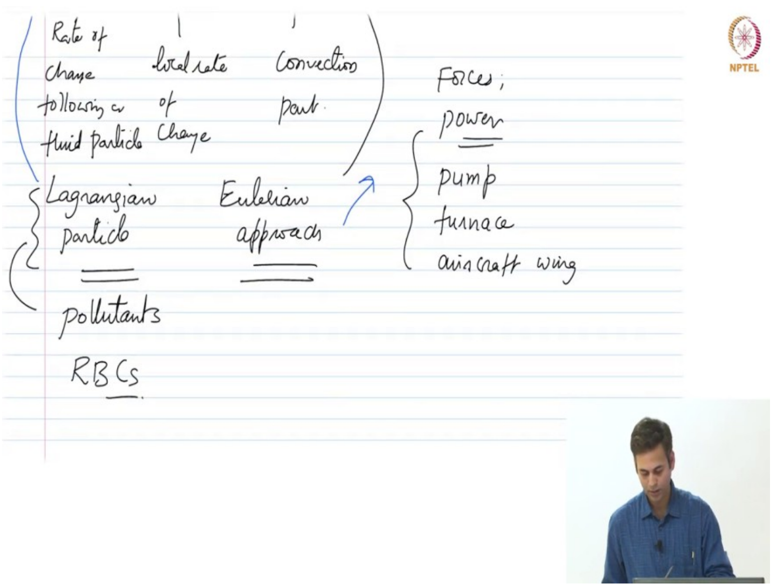

Now of course, we can write and derive equations in a Lagrangian approach. However, the most common approach is the Eulerian approach when it comes to fluid mechanics, that is because our interest is in calculating the forces and the power developed by a certain region.

For example, let us say you are interested in what will be the power required by a pump, or you are interested in what will be the power consumed by a furnace, or maybe you are interested in a lift that is produced by a an aircraft wing and so on. In which case, your domain of interest is fixed ok.

So, in that sense Eulerian approach is very much used and that is what we are going to use in this part of the course. And, of course, Lagrangian approach is also useful in certain calculations. For example, if you want to track the pollutants that are coming out of a some plant right, or if you want to kind of track let us say red blood cells in your body.

So, for all these things you would use a Lagrangian approach, which would be much more convenient and straightforward ok, alright. Questions till here, you probably are already you know aware of all these things ok.

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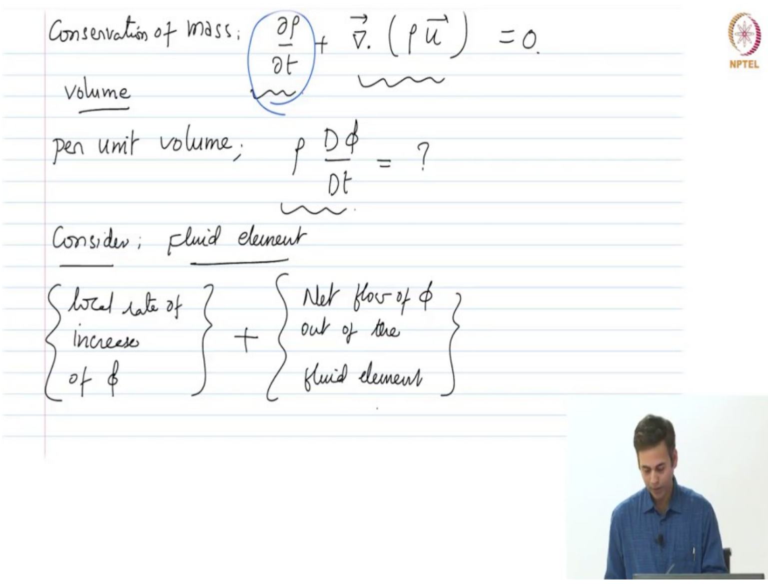

Let us move on ok. Now, the just like the conservation of mass right, where we said we derived the conservation of mass as $\frac{\partial p}{\partial t}+\vec{\nabla}.(\rho \vec{u})=0$partial rho partial t plus what was the expression plus we had $\vec{\nabla}.(\rho \vec{u})$ right. Now, $\vec{\nabla}.(\rho \vec{u})=0$, we said there is a local change of density right.

And, then we have net flow out of the fluid element that we have considered. These two together constitute 0 right, because the mass is conserved in this flow. Now, remember if this equation is developed per a unit mass or per a unit volume.

Student: (Refer Time: 09:19).

This is developed per unit volume right. So, we would be developing the momentum and the energy equations as well for a per unit volume basis ok. Now; that means, if I know that the rate of change of a fluid particle is as I follow it is D phi D t right. This is the substantial derivative this is per unit volume or per unit mass? this is per the unit mass right. Because, phi we said that this is a property per unit mass now I want to calculate it per unit volume, how do I do that?.

I just have to multiply with the density of the of the particle right. Essentially, I multiply with density. So, this will be the rate of change of the property phi right per unit volume of this particle that we have considered ok.

Now, in order to develop this we would consider. Let me consider a similar to the earlier case I want to consider a fluid element ok. Inside this fluid element I am interested in the local increase of phi ok, local rate of increase of phi, which is similar to the partial rho partial t that we have ok.

And, we want to consider what will be the, I want to know what will be the net flow, essentially net flow of phi per unit volume out of the fluid element ok. So, I want to consider these two quantities; one of them is the local rate of increase of the property phi per unit volume and also the net flow essentially out of flow of phi out of the fluid element ok.

So, you want to consider these two ok. So, this is certainly an Eulerian approach right. As, we discuss now what will be this local rate of increase of phi per unit volume, that will partial partial t times rho phi right.

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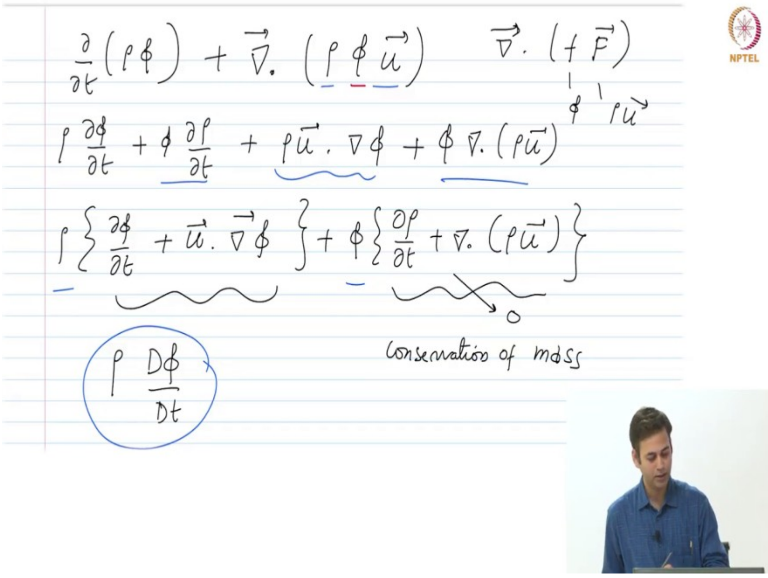

Because, the instantaneous value of phi would be per unit volume will be rho phi right inside the fluid element. And, then the local rate of increase is this much, plus what would be the net flow out of the fluid element of phi, what would that quantity be? We said, you remember what was this quantity? Del dot rho u bar was the net flow rate right.

This was net flow rate out of the fluid element right. We said if it is on the right hand side it was net flow into the fluid element, you brought into the left hand side you, see this is the net flow out of the fluid element right. So, you have the divergence.

So, what will be the net flow of the flow out of the fluid element. This would be del dot rho phi u bar right. This would be the net flow out of the fluid element for quantity phi a power unit volume basis ok. So, we have this quantity. Now, I would like to expand this, by considering rho and u bar together and phi to be together ok. I have two quantities inside this divergence I want to expand this.

So, what will be partial partial t of rho phi? This will be rho partial phi partial t right, plus phi partial rho partial t ok, fine plus. I have del dot rho u bar phi ok. I would write like this as rho u bar dot. So, we are essentially expanding the divergence operator on this product of the scalar and a vector right.

So, essentially we have a del dot f times F bar ok. So, we have this quantity, which is where f little F is a scalar F bar is a vector. So, my F bar is nothing, but in this case rho u bar. And, little f that I have is my phi ok. We have these two and then I would I am just expanding the identity, that is divergence of a operating on a product of vector and a scalar.

So, this will be rho u bar dot grad phi right plus, what else it would be? Phi times divergence of rho u bar right, that will be the identity ok. Now, if I collect the terms that are multiplied with the density and with phi, I have rho times partial phi partial t, plus I have this term here, which is u bar dot grad phi right, plus we have the remaining two terms are this guy and the last one here ok.

So, these are nothing, but phi times partial rho partial t plus del dot rho u bar ok. So, I kind of got two expressions; one multiplying the density, the other on multiplying the phi ok. Now, what would be the value of this quantity that we have here? Partial rho partial t plus del dot rho u bar.

Student: 0.

That is 0 by virtue of the.

Student: Conservation.

Conservation of mass, ok. So, this is 0 owing to the conservation of mass. Now, what is this quantity that we have here? Partial phi partial t plus u dot grad phi.

Student: (Refer Time: 14:48).

This is the substantial derivative of phi right. So, essentially this is rho times, D phi D t right. That is what we have ok. That means, we have just proved, that the rate of change phi per unit volume in a Lagrangian approach that is given by, this quantity right is nothing but, is nothing but this right.

So, this is essentially the local increase of phi per unit volume, and the net flow out of the fluid element of the phi per unit volume ok. So, what we have just derived is? Ok.

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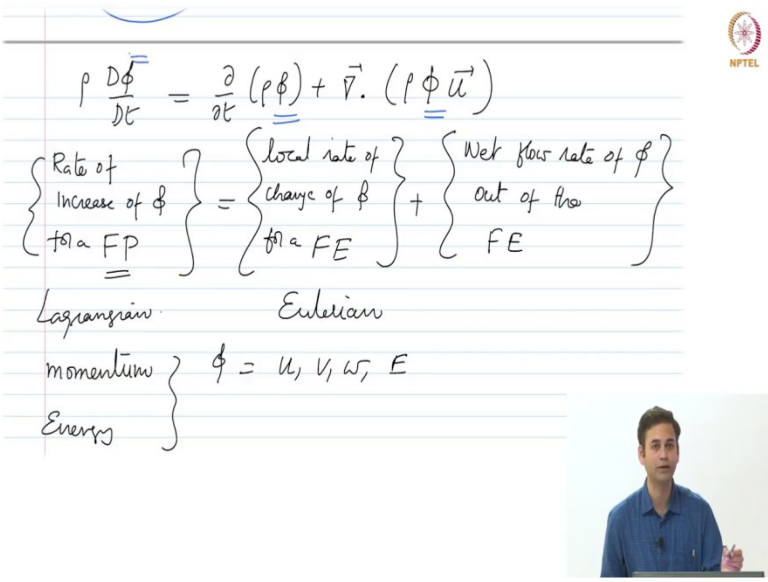

That is so we said rho D phi D t equals, partial partial t of rho phi, plus divergence of rho phi u bar ok. So, this is what we started off with and we arrived at this expression right. So, this is rate of increase of phi for a fluid particle, I am writing F P ok. That is as we follow the fluid particle equals, the local increase or local rate of change of phi for a I would right fluid element ok, which is F E plus we say the net flow rate of phi, out of the fluid element ok.

So, essentially we have considered a fluid element on the right hand side, which is a an Eulerian approach. And, on the left hand side, what we have is a rate of change of phi, as we follow the fluid particle. So, this is an this is Lagrangian approach ok. Questions till here, no this is all probably you have already learnt in fluid mechanics.

Now, what about the sizes of this fluid element and the fluid particle, I just keep interchanging these two right. What would what is that is in your mind in about fluid element and a fluid particle? Yesterday, we discussed that you know, we are up, I mean applying continuum approach and then this fluid particle we have taken is the smallest possible element right in the fluid, such that the molecular motions and the molecular structure, can be ignored right that is what we discussed?

Now, today we have this fluid element and fluid particle being interchanged. What would be the you know the size difference between these two? Like, we have I am conveniently kind of drawing this kind of a rectangular geometry right, which I did not draw today, for a fluid element right. For a fluid particle I never drew anything right.

So, what would be these sizes in terms of the comparison? They are of the same order of magnitude right. So, whether we call it a fluid particle or a fluid element they are of the same order of magnitude ok. We are still looking at a very infinitesimally small cube right, in for both of them. Only the thing is in the first case we are riding with the particle, in the other case we are fixed in our reference frame right. And, far as the fluid element is concerned ok, alright.

Now, that we have derived this expression, it is very easy for us to write the three component of momentum equation and the energy equation ok. By simply changing phi to assume u, v, w, and E ok. So, where u v w are the three components of the velocity and E is the total energy, that is stored in fluid ok.

So, because the momentum equation says that, the rate of change of momentum as we follow the fluid particle right. That is nothing, but the three components would be like rate of change of momentum in the x direction as we follow the fluid particle.

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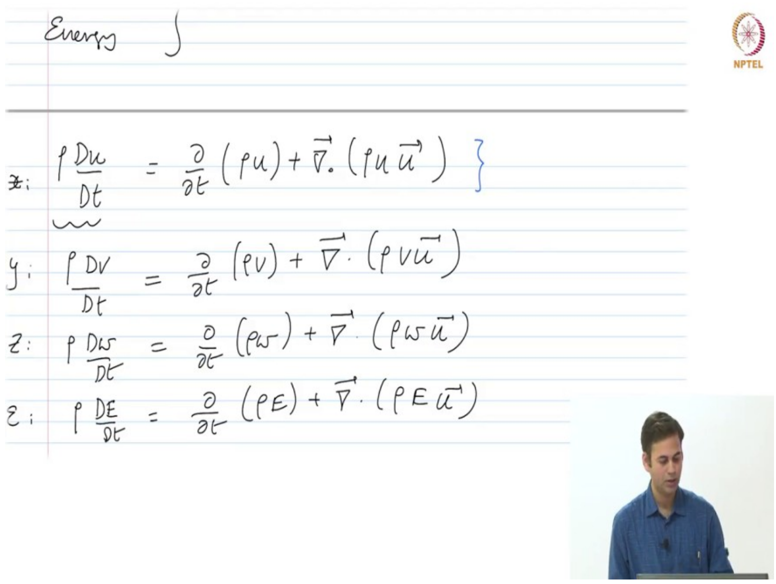

So, that would give you rho D u D t right, we have essentially replacing c with u, that would give you what in the Lagrangian approach? This will be partial t of we just have to plug in instead of phi, we have to plug in u, in both the terms on the right hand side as well as on the left hand side right.

So, essentially this gives us rho u right plus del dot what do we have here? We have rho phi u bar. So, replace phi with u. So, this would be rho u u bar ok. So, that would be your rate of change of momentum, in the x direction for a fluid particle, in the Lagrangian approach right. As dictated by Newton's second law and on the right hand side what we have is the same quantity in Eulerian reference frame ok. Now, similarly the y component would be rho D v D t right.

So, this is nothing, but partial partial t, rho v, plus del dot rho v u bar ok. That is your y component and the z component is rho times, t w d t equals partial partial t rho w plus del dot rho w u bar ok. That is what we have and for the energy equation, we have rho D E by D t equals partial partial t rho E plus del dot rho E u bar ok.

So, we can simply obtain the rate of change term just by changing the or replacing phi with either u v w ok, alright ok. So, we will move on to the momentum equation ok.

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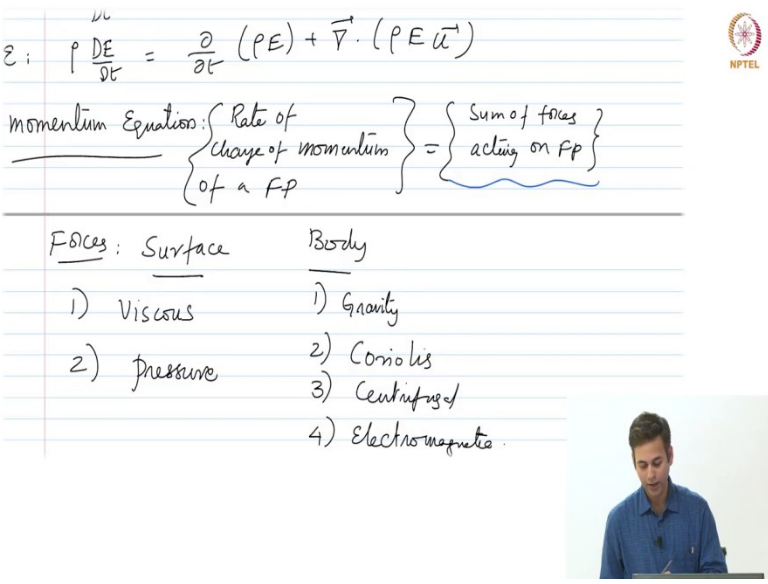

So, the momentum equation is I would write it here rate of change of momentum of a fluid particle equals, the sum of forces or resultant of forces acting on the particle ok, acting on fluid particle. So, that is what we have, we have already derived what is on the left hand side right. The rate of change of momentum we have already just derived.

Now, we have to look at what kind of forces that act on the fluid particle? What are the forces that act on a fluid element? We can kind of categorize them into surface forces and body forces right. What are the surface forces that you can think of?

Student: (Refer Time: 22:34).

The stresses right, this is shear stresses or the viscous stresses. So, you have the viscous forces that are arising out of the viscous stresses and then what else.

Student: (Refer Time: 22:44).

Pressure? Ok. So, you have the pressure forces. And, the viscous forces right coming out of the pressure and the shear stresses that we have in the flow, what about the body forces?

Student: Gravity.

Sorry.

Student: Gravity.

Gravity, ok. What else? If you have a rotating domain or something you would probably get a coriolis force right, or centrifugal forces right and so on right you can probably also have electromagnetic forces and so on. Right anything that acts on the volume you would call it as a body force ok.

Now, we need to kind of understand these forces such that we can develop the right hand side that we have in the Newton's second law of motion ok. So, we look at the surface forces first ok.

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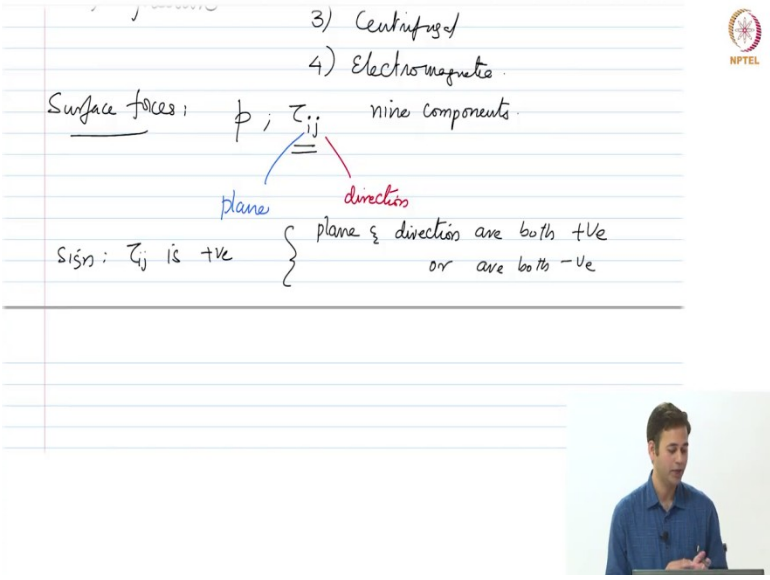

So, the surface forces arise out of the pressure and the viscous forces that we have right. So, this in order to know what kind of you know the forces that act on it we have to know, what is the stress system that is a fluid element is subjected to right?

So, we have pressure that is being acting on a fluid element. And, we have shear stresses right or the viscous stresses, those are the represented by tau i j right. These are this is a stress tensor this has 9 components ok. Now, what is the notation here? We say tau sub i j what does i and j denote.

Student: (Refer Time: 24:28).

i denotes the.

Student: Plane.

Plane on which the stress is acting ok. So, this is the plane on which the stress is acting and j denotes the.

Student: Direction.

Direction ok. So, the direction is denoted by j and what about the sign convention? We consider a shear stress to be positive when so, the sign convention is tau i j is positive right. Both, the plane and direction are both positive or are both negative right.

So, essentially both the plane in which on which the stress is acting and the direction which acting is both of them should be either positive or both of them should be negative. Then, we can call tau i j as a positive quantity ok. If, they have opposite signs then tau i j would be negative ok.

So, for the purpose of this demonstration this class we would be only using will be kind of representing only the positive shear stresses on the fluid element ok. Because, the equations that we developed would not change even if you consider the other shear stress components ok, it is just a easier thing to do ok. And, what about the pressure in which direction does the pressure act on the fluid element. Pressure is always a compressive force compressive or is it tensile.

Student: (Refer Time: 26:03).

Its always compressive right, it always acts on the fluid element trying to compress the fluid ok. So, let me consider a fluid element ok. So, we consider a fluid element here.

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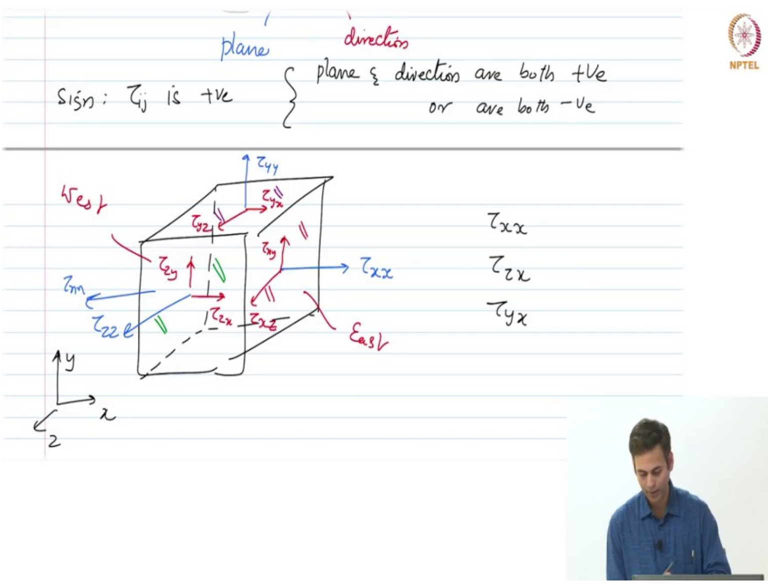

Now, where we would first represent the shear stresses that we have. So, we have x y z ok. Then, let me first represent the normal stresses. So, the this is τ_{xx} right, it is acting on a let us say on the x plane right. This plane has a normal in the x direction and this is also acting in the positive x direction ok.

So, this is τ_{xx}. And, similarly tau y y would be this one, where this plane is a y plane right. Whose normal is in the y direction, positive y direction and this is also acting in the positive y direction? And, similarly we have the normal stress the other normal stress is τ_{zz} right, this is also acting on the z plane and acting in the positive z direction ok.

Then, we can go and look at the shear stresses right. So, what about the other two shear stresses that act on this thing, those are τ_{xy} right. That is this one τ_{xy}tau x y and this would be τ_{xz} ok. And, then here similarly we have τ_{yx} acting on the y plane in the x direction similarly we have τ_{yz}, which is on the y plane acting in the z direction.

Similarly, we have τ_{zx} and τ_{zy} ok. So, here all the normal stresses are represented in the blue and the shear stresses are in all in red ok. Now, these are all positive right ok. Now, if I were to draw a tau x x on the let us say this face on the west face. Let us say if we call this as east face and this as west face, if I were to draw a tau x x as a positive shear as a positive stress in which direction should I draw it. Is it in the negative x direction or in the positive x direction?

Student: Negative.

In the negative x direction, because it is acting on a negative x plane right. So, it should be. So, this τ_{xx}tau x x would be a positive quantity right acting on a negative x plane in the negative x direction ok. So, we would consider that for the derivation here. All right now let me go and consider the forces ok, in the x direction.

That are arising because of the all the stresses that are acting in the x direction ok. So, here for example, what are the stresses that are acting in the x direction? Tau x x is acting in x direction, which would give you a force in the x direction right. What are the other stresses that are acting in the x direction on this fluid element?

Student: τ_{zx}.

τ_{zx} .

Student: τ_{yx}Tau y x.

τ_{yx} right essentially it is the second subscript should be x right, all of them having a second subscript x will be acting in x direction and y in y direction and so on ok. Then, because a momentum equation has three components, I am going to write the first component. The x component first and then we would kind of try to guess the other two components instead of deriving it all through again alright.

So, then let me redraw this same fluid element with only these three stresses ok, τ_{xx} , τ_{zx} and τ_{yx}.

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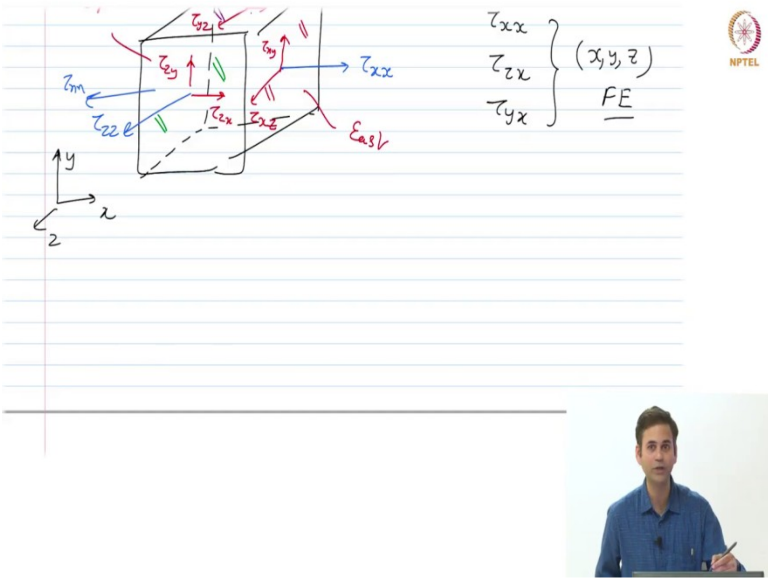

Now, all these three quantities are acting at the centroid of this fluid element right. These are all acting at x y z, which is the centroid of the fluid element ok. Now, we have to again use Taylor series expansion ok, if we were to kind of draw them on the fluid element faces ok. Here only to represent this state of stress I have drawn it on the surfaces, but I will go back to my original fluid element right, that we have used in the definition of the conservation of mass ok.

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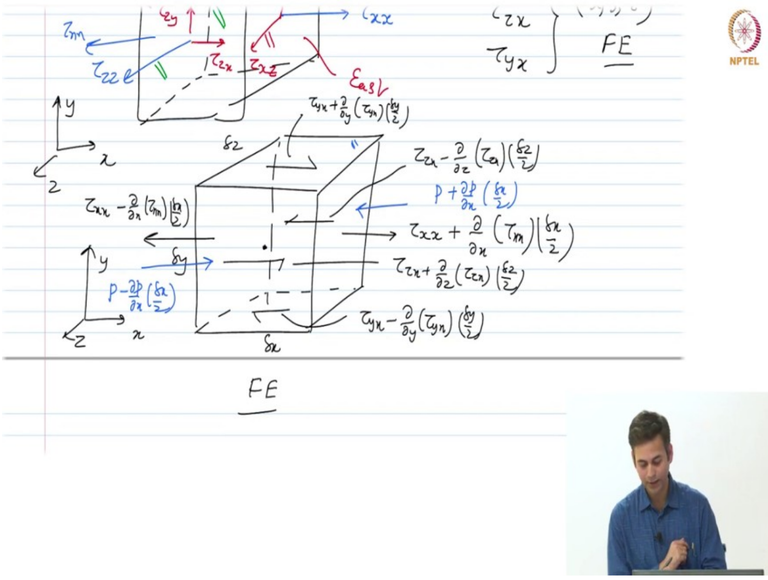

So, I am kind of redrawing this figure with our new or with our original way, that was where the shear stresses or the properties are all defined at the centroid ok. So, this is the centroid which is x y z, and I have τ_{xx}tau x x defined at the centroid of this fluid element ok.

So, this is the fluid element. Then, what would be it is value on the east face? What would be it is value on the east face? This would be τ_{xx}plus right plus you are coming in the positive x direction plus partial partial x, tau x x and we say that this fluid element has delta x delta y delta z in the x y z directions as it is length.

So, this would be the partial partial x tau x x times delta x by 2 right, because we define tau x x at the centroid of this fluid element ok. Then, in which direction would the tau x x on the west plane will act it will act in this direction right. This would be what this would be tau x x minus partial partial x tau x x delta x by 2? Ok.

So, that is what we have here? What about the other quantities that we have tau y x right? So, tau y x would be acting in which direction on this north plane, on the north plane here. If, it has to be positive, it has to act in the positive x direction right, because this is a positive y plane. So, this would be again tau y x defined in the centroid. So, this would be this would be tau y x plus partial partial y tau y x delta y by 2 right, that is what we have here.

Then, I would draw a positive shear stress on the negative y plane here in the opposite direction right, because this is a negative y, it is positive it is acting in the negative x direction right. So, this is tau y x minus partial partial y, tau y x delta y by 2 right.

So, that is the tau y x stresses that we have on the north and south faces. What about tau z x? It will be acting on the front and the back faces. So, on the front face it will be acting in the positive x direction right. So, this is tau z x plus partial partial z tau z x delta z by 2 right, that is what we have and on the negative side. So, we have we have it here, which is tau c x minus partial partial z tau z x delta z by 2 right is that clear is that correct? ok.

So, these are the three viscous stress components that we have drawn. We have left one more surface force pressure right, we have left pressure. So, in which direction does pressure act. So, again pressure is defined as p at the centroid x y z right. What would be it is value on this face? Again you have to use Taylor series expansion and in which direction would act it would act in the negative x direction on this face ok.

So, this would be p plus partial p partial x delta x by 2 and on the negative x face on the west face it would be acting in the trying to compress it right. It will act in the positive x direction. This would be p minus partial p partial x delta x by 2 right. That is what we have essentially we have now 8 stress components right, that we have kind of drawn two per face right.

Now, only these stresses can give rise to a force in the x direction right ok. So, now, let us then calculate the force that would be arising because of this right. So, what will be that? If, I consider the east face first, then we have or the west face, first then we have p minus so if I consider this guy.

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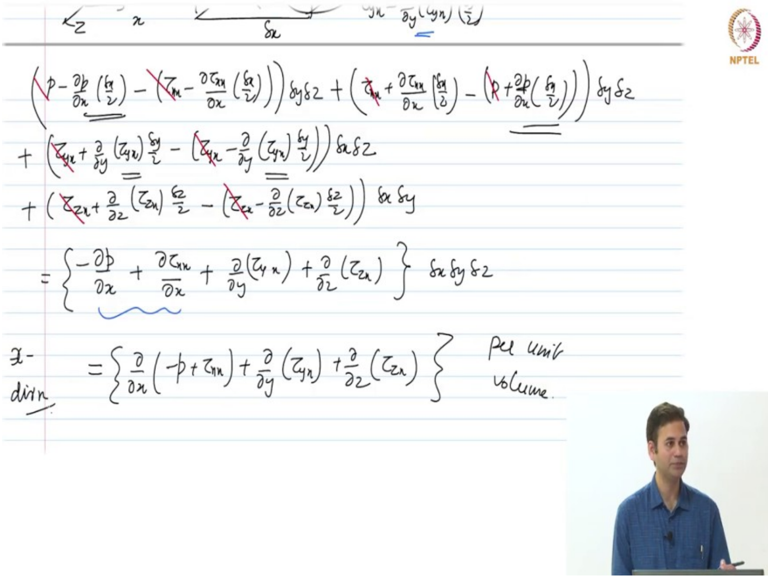

First this would be p minus partial p, partial x, delta x by 2 acting in the positive x direction, minus I have the tau x x. So, this is tau x x minus dou tau x x by dou x, delta x by 2 right. Both of these are acting on the west face right.

So, this is the stress this has to be multiplied with the area in order to obtain the force. And, what would be the area on which they are acting?

Student: (Refer Time: 35:16).

That is this one this is $\delta y\, \delta z$delta y times delta z ok, that is delta y times delta z. So, if I multiply this with $\delta y\, \delta z$delta y delta z ok. Then, what else I have, I have on the east face we have plus ok, tau x x plus partial tau x x by partial x delta x by 2 right, acting in the positive. So, I have a positive here minus we have the pressure acting in the negative x direction that will be p plus partial p partial x delta x by 2. Again this is also multiplied with delta y delta z right is that all.

Student: (Refer Time: 34:04).

No we have more terms right, we have the other two terms, which are nothing, but on the y plane and z planes ok. So, on the y plane what is other quantity we have this is tau y x plus partial partial y, tau y x times delta y by 2 ok.

Minus we have on the south face, which is this one what we have is tau y x minus partial partial y, tau y x delta y by 2 ok. Multiplied with what is area? Delta x delta z right, and we have one more quantity ok. I am going to kind of write it here plus we have tau z x plus partial partial z tau z x delta z by 2 minus alright.

So, that is this quantity. And, then and then we have the tau z x and so on it is actually on the back face right acting in the negative x direction ok. So, this would be minus tau c x minus dou by dou z tau z x delta z by 2 multiplied with area being delta x delta y.

Alright. Good. So, these are the forces in the x direction, that act on all the faces on the east, west, north, south and front and back faces, this is forces arising out of the surface forces alright. So, what terms remain and what terms get cancelled here? Does p retain or p gets cancelled.

Student: (Refer Time: 37:52) cancelled.

P gets cancelled minus what about τ_{xx}?

Student: Gets cancelled.

Gets cancelled ok, you have a plus here a minus here, what about tau y x gets cancelled tau z x also goes away right. So, we have two halves of partial p partial x terms. Here and here right they sum up to one. And, similarly all other terms are also come in pair's right and they all sum up to one ok.

So, what we have is as a result minus partial p, partial x right. Multiplied by delta x delta y delta z, which I am not writing at the moment, then we have plus ok. This is a plus here minus and minus. This is again partial tau x x by dou x right, partial tau x x by partial x again multiplied by delta x delta y delta z.

And, then we have plus another half here and a half here that would be partial partial y, tau y x ok. Plus we have partial partial z tau z x all these quantities are multiplied with delta x, delta y, delta z right that is what we have ok. Is that correct? do you all see that all right.

So, I can rewrite this as partial partial x of I can club these two terms in the derivative terms of the x which is minus minus p plus tau x x plus partial partial y tau y x plus partial partial z tau z x. So, if I write per unit volume. Then, I don't have to multiply with my volume of the fluid element anymore ok. So, this is the force acting on the fluid element per unit volume, in the x direction ok.

Now, will you be able to guess the corresponding terms for forces in the y and z directions, if I were to write let us say y direction, what would that be partial partial x.

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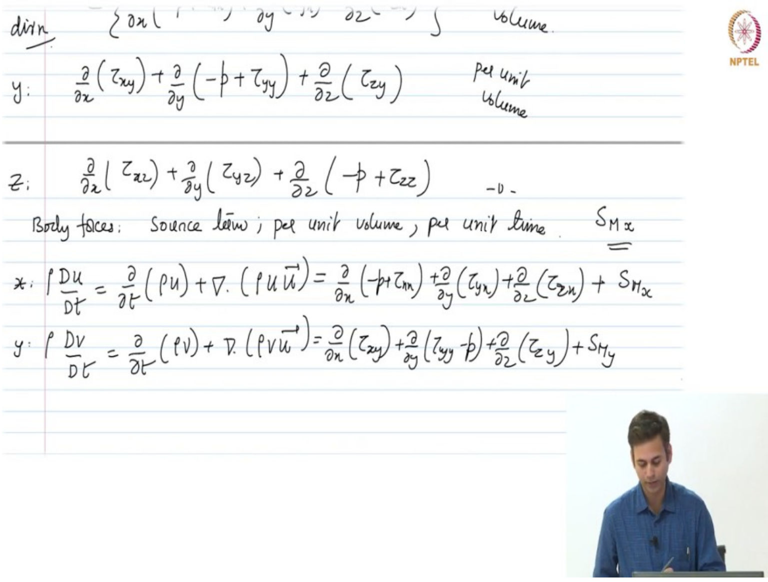

So, because we are writing in the y direction, the second subscript has to be y right. Second subscript has to be y. So, this will be partial partial x tau x y plus partial partial y, you would get the minus p right. Similar to the previous 1 plus tau y y plus partial partial z tau z y right, this is again per unit volume, this is the force that is acting on the fluid element in the y direction ok.

Now, what about z direction? Partial partial x tau, anybody tau x x z ok, x z partial partial y y z.

Student: y z.

Partial partial z minus p plus tau z z ok, that is what we have for the force per unit volume in the z direction ok, very good. Then, we are now ready to write the momentum equation ok. Usually, it is customary to kind of group all the body forces as a source term on the right hand side ok.

So, essentially all the body forces we are going to represent and represent them as a source term ok, which is basically per unit volume right it has to have the same units as the other term.

So, whenever you have a source term you have to represent as per unit volume, per unit time right because this is a rate equation right. So, you have to have so, if I represent a source term with some S and if I write S sub m for representing source in the momentum equation. And, we would use different subscript x y z to represent the source of the momentum equation in the x y z directions then, which is per unit volume per unit time

And, now, I can collate all of these things and write the final momentum equation in the x direction as what now we are writing it for a fluid element? Ok. So, what will be the rate of change of momentum for a fluid element?

That we have we have already derived that that is nothing, but the $\rho \frac{Du}{Dt}$, which we wrote it as partial partial t rho u plus del dot.

Student: Rho u.

$\frac{\partial}{\partial t}(\rho u)+\nabla.(\rho u\bar{u})$. That is the rate of change of momentum equals partial partial x minus p plus tau x x plus partial partial y tau y x plus partial partial z tau z x plus in addition we have this source, which is momentum source which constitutes all the body forces that are acting in the x direction ok.

Similarly, in the y direction we have $\rho \frac{Dv}{Dt}$ equals $\frac{\partial}{\partial t}(\rho v)+\nabla.(\rho v\bar{u})$ equals

$$\frac{\partial}{\partial x}(\tau_{xy})+\frac{\partial}{\partial y}(\tau_{yy}-p)+\frac{\partial}{\partial z}(\tau_{zy})+S_{My}$$

Ok.

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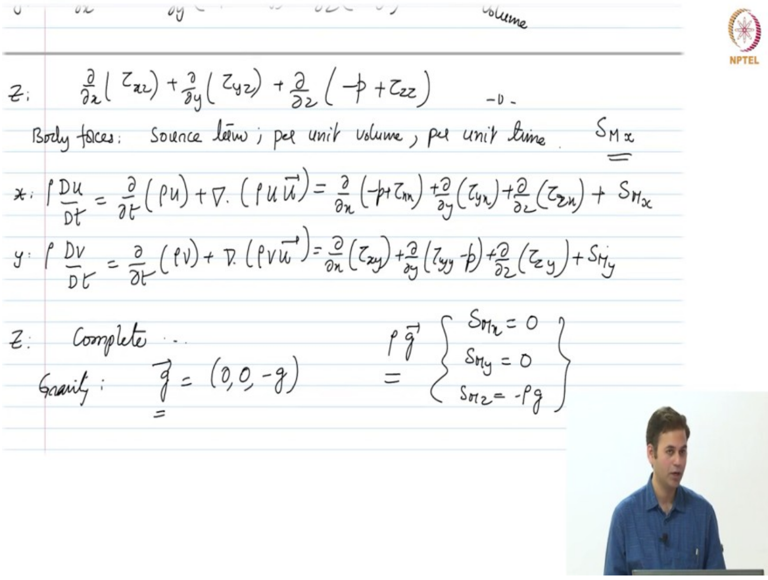

Of course, you can also write the c momentum equation ok. I would leave for you to complete later ok. Questions till now anything on the moment equation derivation? on the stresses?

Student: (Refer Time: 44:05).

So, we have this body force let us say we have a gravity force ok, gravity force acting on the fluid element ok. For example, you have let us say acceleration due to gravity g bar is acting in the negative z direction ok. This would be $(0, 0, -g)$0 0 minus g ok, then what would be your body force out of gravity? It would be rho g bar right.

So, in this context you would write your source term in the x direction as 0, source term in the y direction as 0, and the source term in the z direction as rho g with a minus right, that is your body force. If it is arising out of a gravitational force ok. If, it is not something else, then you have to kind of write these terms. The source terms in a per unit volume per unit time basis ok.

So, essentially this kind of completes the derivation for momentum equation ok. Then what we have next is the energy equation?

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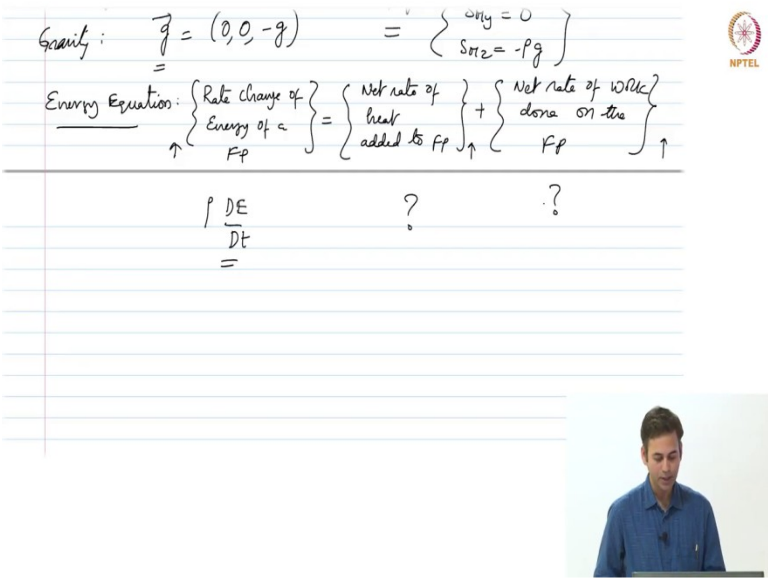

So, what does the, what is the energy equation state? It says, again the rate of change of energy of a fluid particle right equals. Net rate of heat added to the fluid particle plus net rate of work done on the fluid particle right.

So, that is how the rate of change of energy of a fluid particle is going to change? Right. This is going to increase depending on, if work is done on the fluid particle or if heat is added to the fluid particle, then it is rate of then it is energy is going to increase right.

So, this is what we have now of course, we have already derived the left hand side, which is the rate of change of energy of a fluid particle that is nothing, but $\rho \frac{DE}{Dt}$ right. Now, we have to look at these two terms the rate of a net rate of heat added and net rate of work done on the fluid particles.

So, the work done part the rate of work done is can be calculated again from the surface forces that are acting on the fluid particles. And, the net rate of heat added that can be calculated from the temperature field right, the gradients in temperature that we have in the fluid domain ok. So, we are going to take up these two in the next class.