Computational Fluid Dynamics Using Finite Volume Method
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Lecture – 19
Finite Volume Method for Diffusion Equation: Discretization of unsteady diffusion equation

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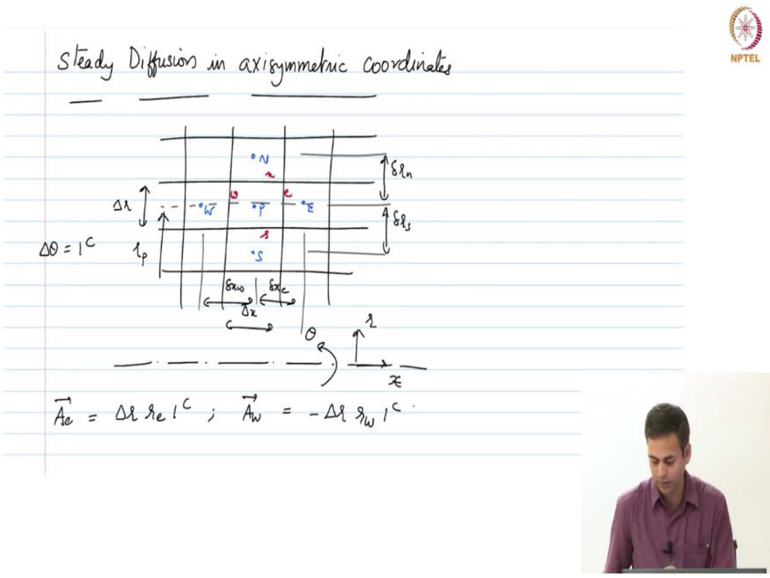

So, we were discussing Diffusion in other coordinate systems. So, we looked at we were halfway through steady diffusion in axisymmetric coordinates. So, this is a steady diffusion in axisymmetric coordinates. I will kind of quickly draw the figure here, the domain and the discretization so that it can help us formulate the problem. So, unlike the two dimensions, we have now an axis about which this will be revolved right. Essentially, there will be this is an axis of revolution and then, we are talking about Δθ equals 1 radian right.

So, all these cells that we have are swept in the θ direction by 1 radian ok, that is what we have and we also, have the cells that is the primary cell P, east cell, west cell, north and the south and the distances, we have are Δx in the x direction, Δr in the r direction right. We also have the distances between the cell centroids. Those are how much? δr_n and δr_s and right and the distances in the x direction between the cell centroids are how much? They are δx_e and δx_w, correct.

So, those are the metrics we have and then, we also know the radii right. The radii r_p or r_e right this is r_p and so on to r_n, r_s and so on right. So, we have the faces which are between the cell centroids, those are east, west, north and south right and then, what about the area vectors now? For these cases, we have we said $\overrightarrow{A_e}$ bar is how much?.

So, Δr has to be swept by one radian in the along the theta direction. So, that should be how much? Δr times r_e times 1 radian right; r_e would be same as r_p right. So, essentially, I just put it there because its east right, that is $\overrightarrow{A_e}$, bar and what about $\overrightarrow{A_w}$? Minus Δr times r_w times 1 radian ok.

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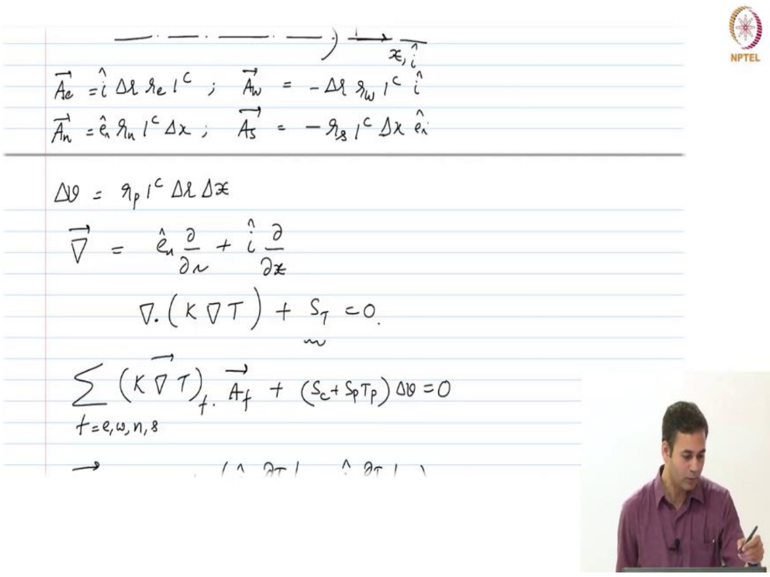

What about other area vectors? So, what about $\overrightarrow{A_n}$? r_n times 1 radian times Δx and $\overrightarrow{A_s}$ would be minus r_s 1 radian Δx ok. So, and what about the volume, ΔV for this cell? r_p ok. Times.

Student: (Refer Time: 03:22).

Times 1 radian times.

Student: Δr.

Δr.

Student: Then, Δx.

Δx ok. So, that is your volume and now, we are talking about the unit vectors. Unit vectors are $\widehat{e_r}$ in the rdirection and $\hat{\imath}$ in the x direction. We had also of course defined what is our ∇ right, that is our ∇ operator. ∇ operator for axisymmetric coordinates is $\widehat{e_r}\,\frac{\partial}{\partial r}$ plus $\hat{\imath}\,\frac{\partial}{\partial x}$ that is our ∇ operators. Then, we would go ahead with the governing equation. The governing equation is what? $\nabla \cdot (K\nabla T) + S_T = 0$right ok.

So, we used temperature here instead of ϕ to not get confused with the angle theta right. We also have θ there which is for the azimuthal coordinate ok. Now, if you apply gauss divergence theorem and then, integrate this on a finite volume, apply gauss divergence theorem and then, write the discrete form of this equation that would be what?

That would be $\sum_{f=e,w,n,s}(\mathrm{K}\nabla\mathrm{T})_{\mathrm{f}} \cdot \overrightarrow{\mathrm{A_f}}$ right plus if you also say that there is an average value for S_T which we are modelling using a linear source term, then this would be $S_c$plus $S_P T_P$times ΔV equals 0 ok, that is fine? Then, what about the faces? The faces are the east, west, north and south.

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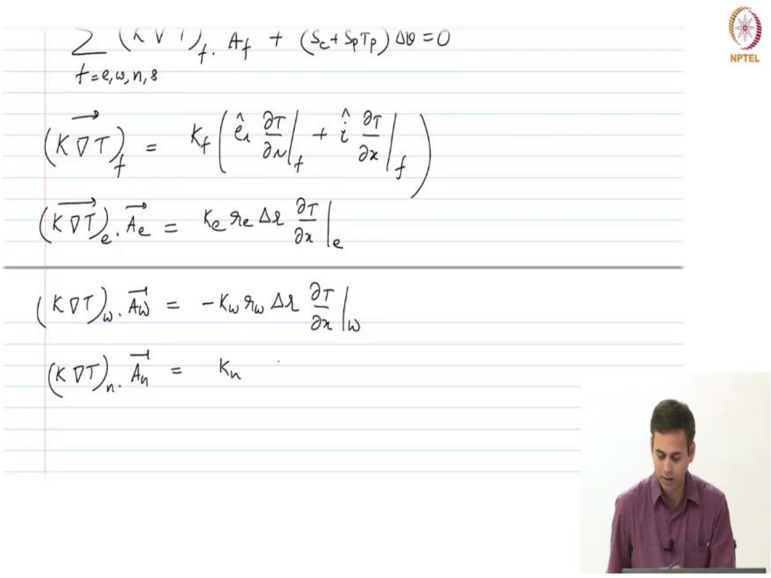

So, if I calculate what is $(\mathrm{K}\nabla\mathrm{T})_{\mathrm{f}}$this would be how much?

$K_f\left(\widehat{e_r}\frac{\partial \mathrm{T}}{\partial \mathrm{r}}\Big|_{\mathrm{f}} + \hat{\imath}\frac{\partial \mathrm{T}}{\partial \mathrm{x}}\Big|_{\mathrm{f}}\right)$ right, that is our $(\mathrm{K}\nabla\mathrm{T})_{\mathrm{f}}$vector. Now, what will be then our $(\mathrm{K}\nabla\mathrm{T})_{\mathrm{e}} \cdot \overrightarrow{\mathrm{A_e}}$

? A east is oh sorry, I think I missed the unit vectors here; is not it? $\Delta r\ r_e 1$ radian times which direction is it pointing in; $\hat{\imath}$ north is in $\widehat{e_r}$, this is $\hat{\imath}$ and this is $\widehat{e_r}$ right. Those are the unit vectors. Now, what about the evaluation of this quantity on the east face? $(K\nabla T)_e \cdot \overrightarrow{A_e}$; how much would this be? So, which term survives only the?

Student: i term.

i term right because it is only in the x direction. So, only the i terms are waves and or how much would that be? K_e times?

Student: r_p.

Times r_p that is I would still write it as $r_e$ok; K_e r_e. What else? Δr.

Student: Partial T (Refer Time: 06:17).

Partial T.

Student: (Refer Time: 06:19).

$\frac{\partial T}{\partial x}$ on the east face right, that is what we have and then similarly, $(K\nabla T)_w \cdot \overrightarrow{A_w}$ would be how much? Would be a minus right; would be a $-K_w r_w \Delta r \left.\frac{\partial T}{\partial x}\right|_w$ and then, $(K\nabla T)_n \cdot \overrightarrow{A_n}$ would be how much? Would be K_n times only the first terms survives now, the $\widehat{e_r}$ terms survives. How much is $\overrightarrow{A_n}$?

Student: (Refer Time: 06:58).

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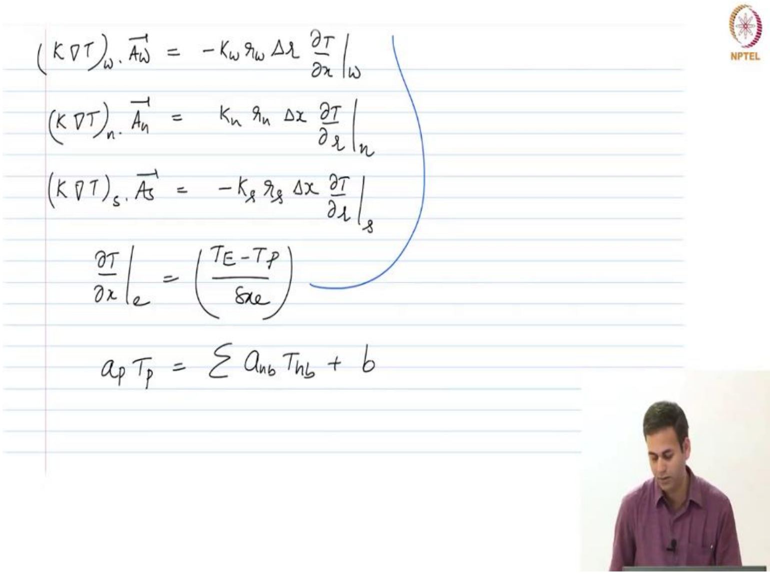

$r_n \Delta x$. So, this would be $K_n r_n \Delta x \frac{\partial \mathrm{T}}{\partial \mathrm{r}}\Big|_{\mathrm{n}}$ on the north face right and then, we have $\widehat{e_r}$ dot $\widehat{e_r}$ inner product ok, then $(\mathrm{K}\nabla\mathrm{T})_{\mathrm{s}} \cdot \overrightarrow{\mathrm{A_s}}$ would be $-K_s r_s \Delta x \frac{\partial \mathrm{T}}{\partial \mathrm{r}}\Big|_{\mathrm{s}}$ on the south face. Of course, now we can apply a linear profile assumption for the variation of temperature between the cell centroids and evaluate these gradients on the faces right.

For example, $\frac{\partial \mathrm{T}}{\partial \mathrm{x}}$ on the east face, we can write it as $\frac{T_E - T_P}{\delta x_e}$ and so on ok. And then, you we will use those relations to substitute back in here right, I will use this substitute back into this and then rearrange the terms right and then, we get our favourite equation that is $\mathrm{a_P T_P} = \sum \mathrm{a_{nb} T_{nb}} + b$ ok.

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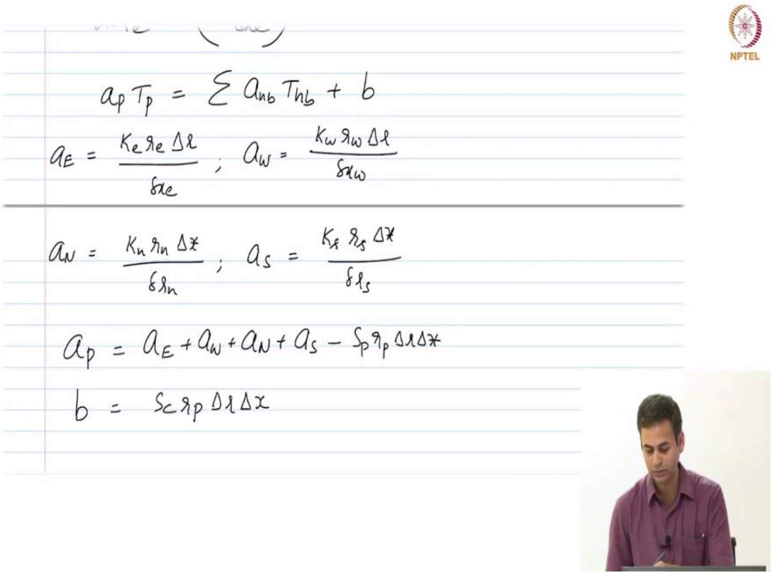

Out of which what is a_E now? a_E is $\frac{K_e r_e \Delta r}{\delta x_e}$ fine ok. a_W, $\frac{K_w r_w \Delta r}{\delta x_w}$ right and a_N, $\frac{K_n r_n \Delta x}{\delta r_n}$ north and then a_S would be $\frac{K_s r_w \Delta x}{\delta r_w}$. Of course, what is a_P? a_P would be $a_E + a_W + a_N + a_S$ and what else? a_P term minus.

Student: S_P.

S_P.

Student: ΔV.

ΔV that is nothing but $r_P \Delta r \Delta x$ right and then, what is b? $S_C r_p \Delta r \Delta x$ right that is all fine ok. So, essentially everything else remains the same. And of course, apply boundary conditions as special cases on the on the boundary faces, whenever it is given and we can go ahead and solve solutions you know for these axisymmetric problems. Questions till now?

So, given let us say any other 2 D geometry, you will be able to do it this way right. The steps are basically the integration and the gauss divergence theorem, then calculate the areas and then find out what will be the resulting equations right using a linear profile assumption and so on right.

So, that is easy. Questions? No ok, clear; very good all right. So, that kind of finishes the diffusion equation steady diffusion equation other coordinate systems. We will kind of move on to now to unsteady diffusion ok, then we will come back to steady diffusion for unstructured meshes ok, after we finish unsteady diffusion and also, we look at the stability criteria, a truncation error all those things and then, we will come back to unstructured meshes after we finish all these discussions fine ok. Then, let us move on to the discussion on unsteady diffusion.

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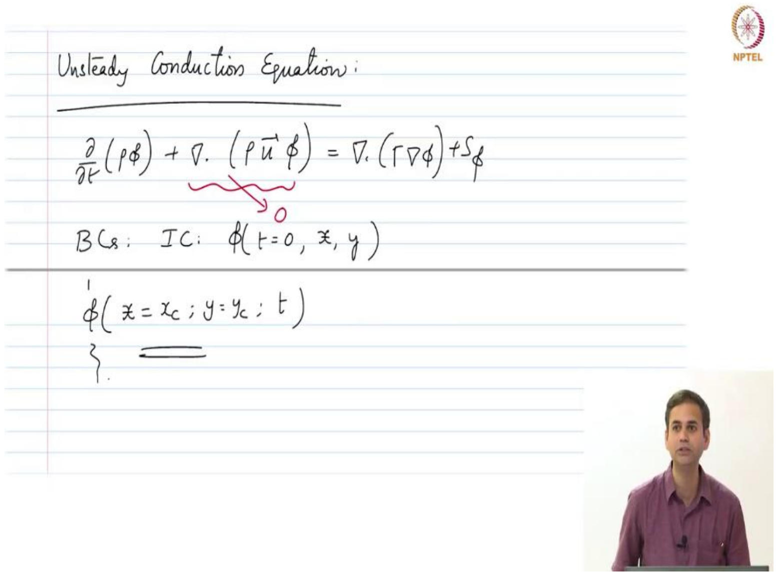

So, this is unsteady conduction equation, alright. So, how do we obtain unsteady conduction equation? Essentially, take the general scalar transport equation and knock out the which term? Remove the convection term right that is all.

Essentially, what was the general scalar transport equation? That was, $\frac{\partial}{\partial t}(\rho\phi) + \nabla \cdot (\rho\vec{u}\phi) = \nabla \cdot (\Gamma\nabla\phi) + S_\phi$ right. Up till now we have set everything on the left hand side as 0, right. We knocked out the unsteady term and the conviction term and we were working only with the $\nabla \cdot (\Gamma\nabla\phi)$ plus S_ϕ.

Now, we will only set this term to 0 right. As a result, we have unsteady diffusion equation ok. Through this unsteady diffusion equation, we are bringing in one more property of the fluid. What is that or the whatever the material we are considering?

Student: Density.

Density right. Up till now, we have only Γ, now we are done bringing in density ρ is coming into play through the unsteady term ok. Then, what all do you need to solve this problem? You would need of course the boundary conditions right, you would need the boundary conditions; what else do you need to solve this problem?

Student: Initial condition.

You need an initial condition because you have a time derivative in there right. So, essentially, you would need an initial condition; initial condition will be specification of the dependent variable that is ϕ. Do you need it specified at $t = 0$ right that is your initial condition and where do you want it to be specified? Everywhere in the domain or only on the boundaries?

Student: Everywhere.

Everywhere. Essentially you want it for all x and all y right. You want an condition to start off with that is your initial condition and then of course, you have boundary conditions which are on the specific boundaries from x equal to x constant and y equal to y constant and you want it for all time right. You want so many of these boundary conditions also to work with for all the combinations ok; for all x, for all y equal to y constant, for all y for all x equal x constant and so on ok.

So, you would need all the boundary conditions and the initial condition. Now, what kind of an equation? This is still an elliptic equation. Let us say we say Γ is some finite value ok. So, there is a lot of diffusion in the problem; what kind of what type of equation is this? What type of PDE is this? Still elliptic PDE?

Student: No.

No, this is now what is the?

Student: Parabolic.

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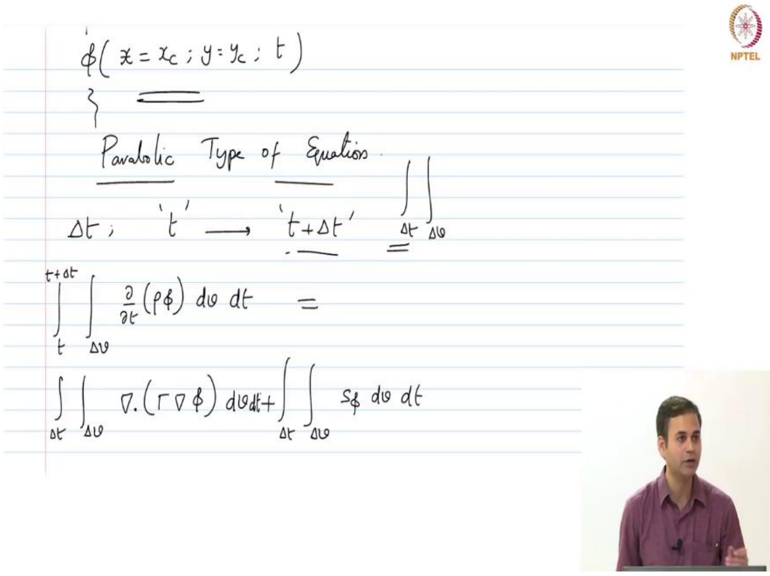

It might most likely, it will be either a parabolic or hyperbolic depending on the amount of diffusion we have. Let us say it is a parabolic say that there is a lot of diffusion happening because its unsteady conduction.

So, this is a parabolic type of equation fine. Then, what we need to do is up till now, we have been only integrating the equations on a particular control volume right, that is the first step in finite volume method; now, we have this time derivative ok. So, in addition to integration on the volume, we have to also integrate along time right. We have to go in steps of some Δt right. We start off with let us say some time t or sometime 0 and then, we move on to $t + \Delta t$ ok.

So, essentially, we have to integrate on time and on space right. We have to do both these steps. So that means, if I take the governing equation which is this equation and integrate both on its time as well as space, then what we have is integral t to $t + \Delta t$ and integral on the control volume. The first term is $\frac{\partial}{\partial t}(\rho\phi)\, dV\, dt$ right that is our unsteady term. I am integrating both on the control volume also in time right t to $t + \Delta t$.

So, I can also shorten this and write instead of integral t to $t + \Delta t$, I can also write it as integral over Δt integral over a ΔV right. We understand Δt is basically we are going from some time t to the next time $t + \Delta t$ ok. What else is there on the left hand side; any other term? No other term, we have to move to the right hand side. What is there on the right hand side? Integral Δt integral ΔV, we have $\nabla \cdot (\Gamma \nabla \phi)\, dV\, dt$ right plus integral Δt integral

ΔV $S_{\phi} dV$ dt. Now, let us kind of look at each and every term one by one. Let us look at term by term ok. So, let us take the unsteady term that is the first one.

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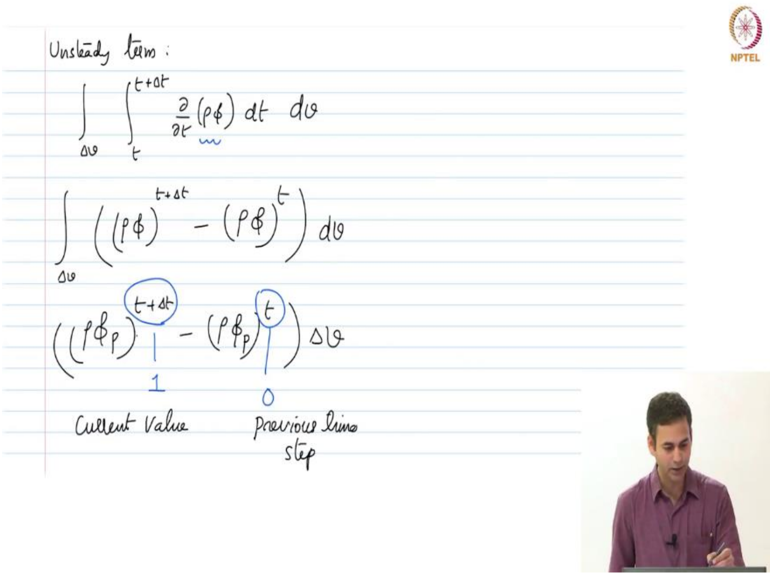

So, that is the unsteady term and if I assume that these integration commutes ok, I can rearrange this integration as in the integration over the ΔV and integration over t to $t + \Delta t$, $\frac{\partial}{\partial t}(\rho\phi)$ dt dV ok. I have rewritten it, assuming that the integration can commute here. Now, what is the integration of the inner term here? $\frac{\partial}{\partial t}(\rho\phi)$ dt integral of this thing, what would be this quantity?

Student: ρϕ.

ρϕ right. This is essentially ρϕ. Do you have limits for integration?

Student: Yes.

Yes, you have. So, until it will be ρϕ at $t + \Delta t$ minus t right into ΔV. Of course, that is there you have to sum it for all the small volumes you would get. So, this is definitely ρϕ right at $t + \Delta t$ minus ρϕ at t integration over the each of the control volumes that you would consider ok. Now again, I make an assumption which is ρϕ is for a for a small control volume right.

So, I will say that this the particular control volume, the cell centroid value of ϕ_P prevails over the entire cell ok. Essentially that means, I can rewrite this as and I have an integration over the over the over the particular cell right, all the particular control volume. So, I can rewrite this as $\rho\phi_P$ at $t + \Delta t$ minus $\rho\phi_P$ at t ok. So, this is basically the cell centroid value of ϕ_P ok. And I am integrating on the control volume ΔV. So, this would be times ΔV. Can I write this?

Student: Yes.

Yeah ok. Similar to what? We have done for the source term right, we are calculating the average value which is the cell centroid value and then, we say that that prevails over the entire cell right. The small control volume instead of integrating, you have a constant value over the entire cell some average value and then, we are multiplying with the volume. Is that correct? Is there a dt multiplying also? No, there is no dt, but there is ΔV multiplying it right. Is that correct fine? Ok.

Now, I will kind of switch to a simpler notation, I would like to represent $t + \Delta t$ instead of $t + \Delta t$, I would like to write it as 1, superscript 1 and t with superscript 0 ok, indicating that 1 is the current value, 0 is the previous time step value ok. So, essentially, this is the current value and this is the previous time step value.

Student: Sir?

Yes?

Student: (Refer Time: 18:54) rho (Refer Time: 18:56).

Yeah, for ρ as well. Essentially, $\rho\phi_P$ as such, I am assuming that the density is not changing Let us say if you have an incompressible flow or something ok. I am assuming the density is constant; otherwise, it will be ρ_P as well ok. It will be $\rho_P\phi_P$ fine ok. So, fine and let us move on to the next term, that is what was the next term?

Student: Diffusion.

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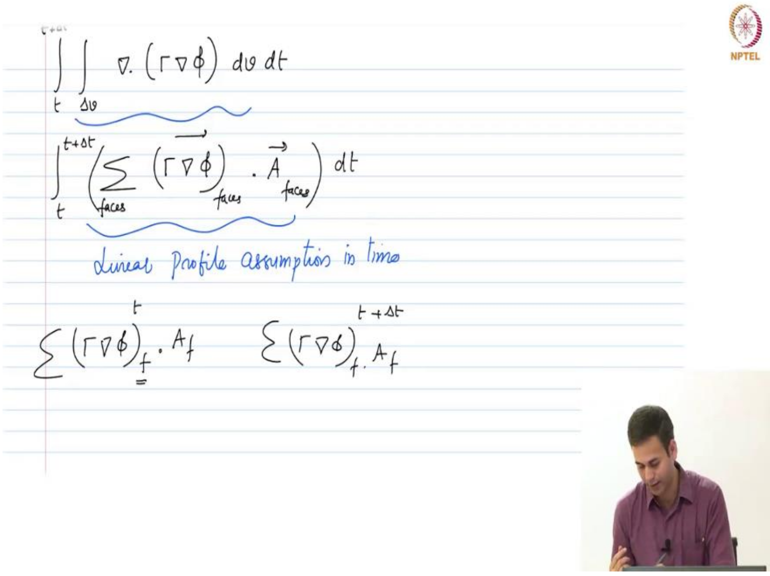

Diffusion right, that is this term ok. So, that is if I write it down, it is integral t to $t + \Delta t$ integral over the control volume $\nabla \cdot (\Gamma\nabla\phi)$ right. Is it correct? Well, it $\nabla \cdot (\Gamma\nabla\phi)\ dV\ dt$, but we realize that this term is on the right hand side of the equation ok. We are done with the left hand side, there is only unsteady term. We have come to the right hand side, this is important.

Because when we calculate the coefficients, where we should be putting them with correct plus minus ok, all right. Now, what do we do here? Essentially, we can again invoke gauss divergence theorem, convert this volume integral into surface integral and with the discrete summation ok.

If I do that integral t to $t + \Delta t$, I can replace this entire thing of here with the already known value that is that is what? Summation on all the faces right. So, this is on all the faces let me write it as faces ok. And what would be this value? $\Gamma\nabla\phi$ on the faces dot A right. A for the faces ok. $\vec{A}$ fine, this is all the faces that make up the control volume fine, times what do we have?

Student: dt.

dt right fine. Any questions till now? No questions ok. Now, what we have done is we have to; so, of course, this $\nabla\phi$, we will assume that its again a linear profile assumption between cell centroids. Now, we have to do something similar here. Now, so, essentially, we have to carry out this integration. We have this term in the parenthesis that has to be integrated over this time step of Δt right.

So, what we say is that we again make use of a linear profile assumption in time for this quantity ok. So, I am making a linear profile assumption in time for this particular quantity in parenthesis that means, similar to the interpolation of Γ. You remember, interpolation of the diffusion coefficient on the faces, we had some factor right. We had some $f_e$times Γ_P plus something right.

Similarly, I have two quantities here; one is one is this $(\Gamma\nabla\phi)_f \cdot A_f$, where the sub f stands for the face ok. We have this value at $t + \Delta t$ right and we have another value which is $(\Gamma\nabla\phi)_f \cdot A_f$ at t right. I have these two values. So, let me write it this way. Let me write it t on the left hand side and t plus already. So, I have two values right. Of course, this is again a summation right, on all the faces that is fine right. We have these two values at different time steps at t and $t + \Delta t$.

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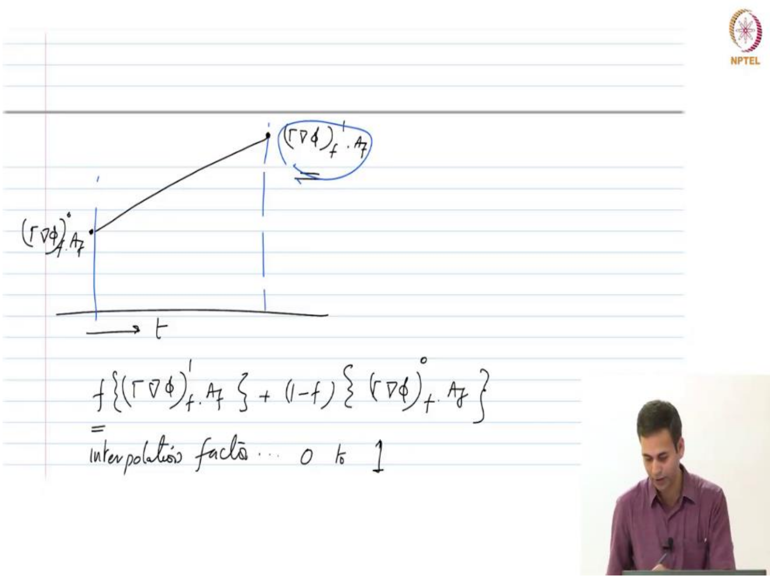

So, if I were to draw it, essentially I have this is my $(\Gamma\nabla\phi)^1{}_f \cdot A_f$ and then this is my $(\Gamma\nabla\phi)^0{}_f \cdot A_f$ right. I have two values and I am considering this to be some kind of a linear variation between two these two values. We still have the summation right; we are talking about time in this direction right. I have this; that means, I can write this with some factor ok; that factor, I would like to call it as f and that f is an interpolation factor which is different from this f, we have here ok. So, if we have an interpolation factor f, I can calculate this entire quantity as an interpolated value right. That would be how much?

So, whatever is there in the parentheses that would be f times this quantity that we have that is $(\Gamma\nabla\phi)^{1}{}_{f} \cdot A_f$right plus (1-f) times $(\Gamma\nabla\phi)^{0}{}_{f} \cdot A_f$. Is this? Ok. Of course, we have it for all the faces summation is there on the faces right, where this f here is my interpolation factor which I have not defined yet ok; it is some linear interpolation factor. It could be anywhere between 0 to 1 right fine, that is this value, I have introduced some kind of a linear interpolation between 0 value and 1 value right.

And I would say this is the value of the term that is there in the parentheses here fine. Now, what will be this integration with this particular interpolated value? What will be the result of this integration that is nothing but integral t to $t + \Delta t$ and we have this big expression that is f times $(\Gamma\nabla\phi)^{1}{}_{f} \cdot A_f$ plus (1-f) times gamma $(\Gamma\nabla\phi)^{0}{}_{f} \cdot A_f$ right that is interpret value times dt right that is what I have ok, let me write it down.

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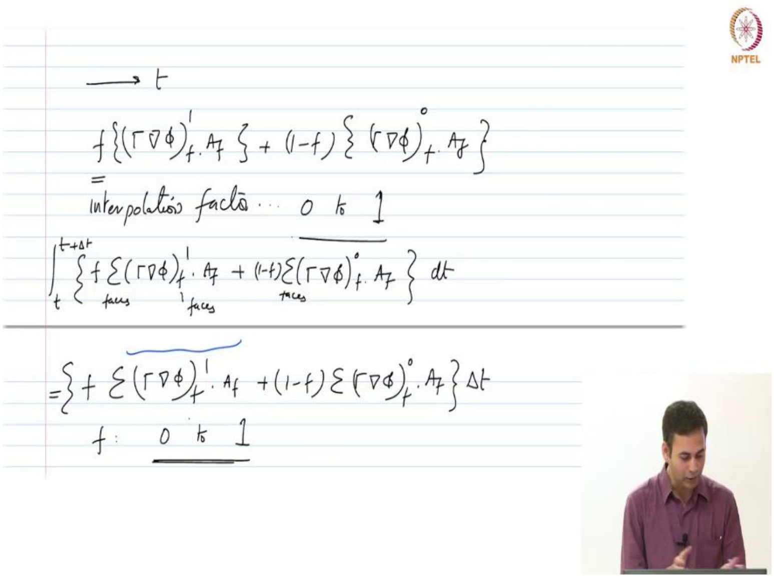

So, that so essentially the integration would be integral t to $t + \Delta t$, what I have is f times $(\Gamma\nabla\phi)^{1}{}_{f} \cdot A_f$ plus (1-f) times $(\Gamma\nabla\phi)^{0}{}_{f} \cdot A_f$. This is again summation on all the faces times dt. Is that correct? Question? Yes.

Student: Sir, (Refer Time: 25:35) for (Refer Time: 25:38).

Yeah, I would just assume that the interpolation factor is this interpolation factor is in time ok. So, maybe shall I use g or something instead of f, would that make it clear? The sub f is basically faces, this is faces.

Student: (Refer Time: 25:52) summation (Refer Time: 25:53).

Summation sign is inside. It does not matter, is not it?

Student: (Refer Time: 26:01).

You want it to be inside ok, I will put it inside what?

Student: (Refer Time: 26:02).

Inside f or something ok, I will put it there; it does not, it does not anything matter ok. I will rewrite it fine; essentially, this is summation on all faces right times f right and then, this is summation on all faces times (1-f) times dt is that correct fine. Essentially, we have some interpolation factor f which could be some value between 0 and 1; it could be 0.2, 0.5 whatever right.

So, we have this quantity gamma $(\Gamma\nabla\phi)_f \cdot A_f$ f which is being evaluated to some value at T some value at $t + \Delta t$, we are trying to calculate some interpolation between these two and then, we have some value ok. Now, that value is what we have to integrate over this dt all right ok. Now, I have interpolated this value, all right. Why do why they; I think I made a; yeah. So, essentially what will be this integration? Now, I have some value that I have already interpolated. So, I can rewrite this integration as this times the constant Δt right.

So, this is nothing but $\{f \sum(\Gamma\nabla\phi)^1{}_f \cdot A_f + (1-f) \sum(\Gamma\nabla\phi)^0{}_f \cdot A_f\}\ \Delta t$. Is that correct? This is some constant value right which I am integrating over the entire Δt.

t to $t + \Delta t$ fine. f, what is f different? f is an interpolation factor that kind of determines the kind of time stepping scheme you want to use, we will come to that little later. As of now, f is some interpolation; but remember the interpolation of Γ, where we have two particular cell values, it would go from one cell to the other cell right.

So, similarly f is something that we would decide based on the temporal scheme, we want to device ok. As of now, I would say f goes from 0 to 1 and depending on the value you choose, you will get something here ok. As of now of course, we do not know what is the value of 1. We go for example, this term I have written down, but I do not know this value right because 1 is something that I have to calculate from 0 all right. So, fine as of now, I would say f is some factor which will relate these two.

So, how does this term look like, if we were to integrate? Let us say f is 0.2 right, f is 0.2, what will be the value of this integration? How does it look on this graph? Maybe that will make it little more clear.

So, I have I have this graph here, if I take f equals 0.2, what will that be? 0.2 of this guy right or 0.2 times Δt is what I have right. It will be like a step; is not it? You are calculating 0.2 times this quantity right times Δt plus 0.8 times this quantity times Δt right. It is kind of going to cover the area under this curve in two steps right ok. I would leave it for you fine. Then, we are also little lazy to write. So, I we are going to also drop this index 1, which is for $t + \Delta t$ ok.

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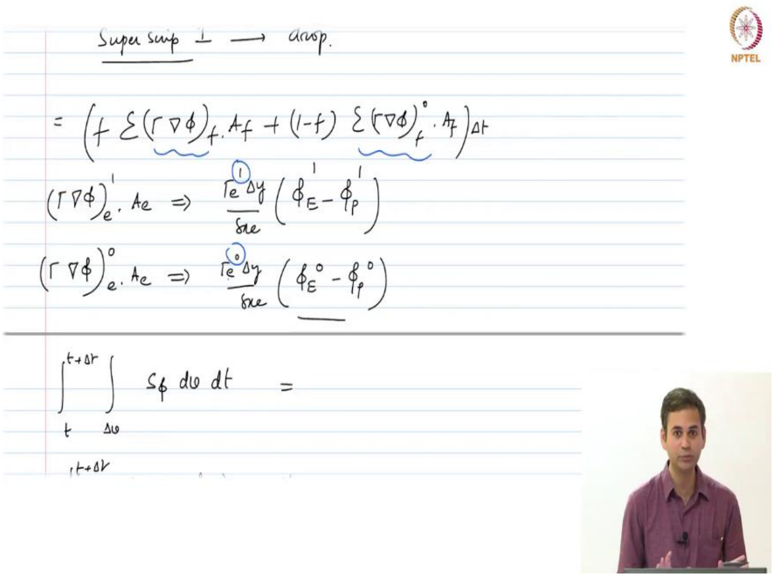

So, I would say I would drop the index superscript 1, I would drop it. So, if there is superscript 0 that is for previous iteration, if there is no superscript that is for current iteration I would. So, we can say have some writing fine. So, that is basically this entire thing rewritten right; $\{f\sum(\Gamma\nabla\phi)_f \cdot A_f + (1-f)\sum(\Gamma\nabla\phi)^0{}_f \cdot A_f\}\,\Delta t$ right ok. So, one is the present value which is not written anymore, all right.

Then, what is the value? Let us say of $(\Gamma\nabla\phi)_e \cdot A_e$. How much would this be? Let us say if we have a two-dimensional problem and we are also back to a cartesian mesh ok, in two dimensions, we have done this right; in diffusion, what will this be? This will be some $\frac{\Gamma_e \Delta y}{\delta x_e}(\phi_E^1 - \phi_P^1)$. These are all evaluated at one, which we are not writing all right ok. Now, what about the?

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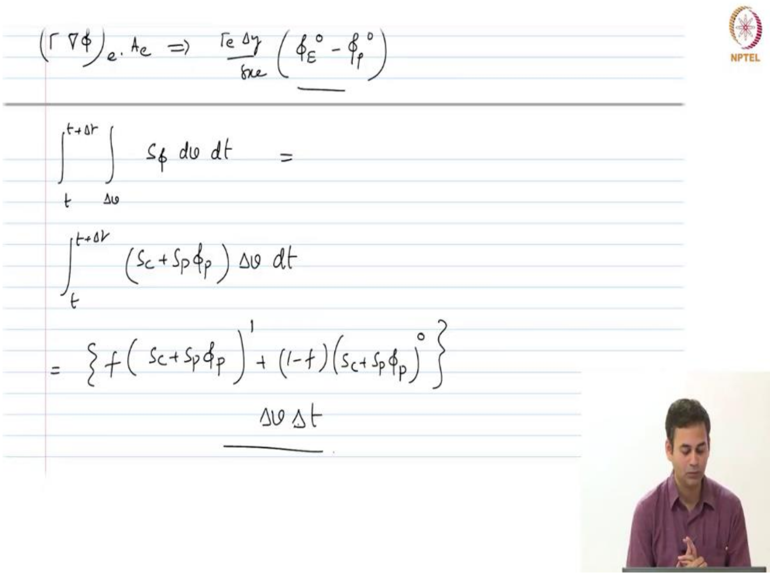


Similarly, what about $(\Gamma\nabla\phi)^0{}_e \cdot A_e$, that is basically would come from this term here right. This will have four terms and this will have four terms right. How much would this be? This will be, it will be the same, but there is one difference, phi 0 right. These are all evaluated ϕ^0, instead of ϕ^1 ok. So that means, $\frac{\Gamma_e \Delta y}{\delta x_e}(\phi_E^0 - \phi_P^0)$ right. So, what is our objective? We are doing all this superscript 1, superscript 0, all these things; but what is our objective? Given a time level t right or an initial condition, we want to calculate ϕ at the next time level. Essentially, what is the unknown in this case; in these cases?

Everything that sub superscript 1 or no subscript right that is the unknown and everything with superscript 0 are all.

Student: Known.

Known and we have the initial condition to begin with right fine ok. What is the other term that is remaining in the equation?

Student: Source term.

The source term right. We have the source term; it needs to be also considered. So, that is the source term. So, we will use a similar to the diffusion, we will also assume that the source term also has a linear variation in time ok.

So, essentially that means integral t to $t + \Delta t$ over volume, what is that that we have? So, I would write it as just the $S_{\phi} dVdt$ right, that is what we have in the original equation and this is nothing but integral t to $t + \Delta t$, $(S_C + S_P\phi_P)$dVdt. This is the average value times dVdt right that is the value of the source term.

Now, we again use a interpolation factor f and we would write this as $f(S_C + S_P\phi_P)^1\}$ all right ok. I will put the 1 outside because S_C could also be dependent on the ϕ_P all right; as a result, that could also change. I will put it on top, then we have plus $(1-f)(S_C + S_P\phi_P)^0$ right and both of them have, what is the multiplication both of them have?

Student: ΔV.

ΔV times.

Student: Δt.

Δt ok, where we have not defined what is f still ok. f is some linear interpolation between 0 and 1.

If I choose a scheme.

Student: (Refer Time: 34:00).

So, the question is will the f be the same between the source term and the diffusion term, if we choose a scheme. Now, that is something usually when you say when you choose a particular scheme, the same interpolation we applied for source terms and diffusion terms ok; but it need not be the case.

For example, right now we do not have convection terms right say and let us say you realize that the source terms are highly non-linear. As a result, you may want to choose a different f for source terms and a different f for the diffusion term r for the convection terms that is very much possible ok.

But if you do it that way, then or in either case you have to specify how did you choose the time discretization ok. What is the f value you have used for diffusion, for source terms and for convection term has to be specified, when you solve any problem. Many a times, it is probably required that you choose different f for different terms in the equation ok. That means, you are kind of controlling what factors would come into the equations, we will come to that once we define what is f, we will see how these terms will kind of come into play ok.

Now, we will also see that depending on how you choose f, these terms would either go into the b term or they would either go into the ϕ_P term that is also happens ok, that is there, we will come to that. As of now, any other questions?

Student: (Refer Time: 35:25).

Yes.

Student: (Refer Time: 35:27).

No, nothing. I just instead of writing 1, I just I am lazy and not writing 1, that is all ok. If I do not write 1, just make sure this just think that is actually the current time level, that is all ok. So, here I have put it because for the sake of completeness that is all; Yes. 1 is something that is unknown; 0 is something that is known from the previous time step. Other questions yeah?

Student: (Refer Time: 35:51) change in diffusion (Refer Time: 35:53).

A change in the diffusion coefficient between time steps ok. So, I am not considering the change in diffusion coefficient in time ok, that means, essentially Γ is not dependent on time ok, its only dependent on space and that could be dependent on ϕ as well ok. So, if that is the case, then Γ also has to have a superscript of 1 ok.

If that is the case just like a ρ ok, I am not considering at the moment fine. Other questions? That can be done; is not it? It does not, it will be the same. You just have to put 1 and 0 on the superscript for Γ like we have done it for the source terms that is possible.

Other questions? Clear; what we are doing with Δt. Yes?

Student: (Refer Time: 36:40) known or unknown?

It should be still be known because Γ usually for a problem diffusion coefficient would be known. So, you have a substance which is which has a time varying diffusion coefficient, then it would be known all right. Just like you know the value of Γ as a function of determinable ϕ, you would also know how it behaves with time.

Student: (Refer Time: 37:06) density.

Density; the same thing you should know what is the material density right depending on other properties.

Student: (Refer Time: 37:12) superscript of Γ and (Refer Time: 37:18).

Yes, I have here I have kind of neglected it, but otherwise this would this was already 1 and 0.

Student: So, Γ (Refer Time: 37:27).

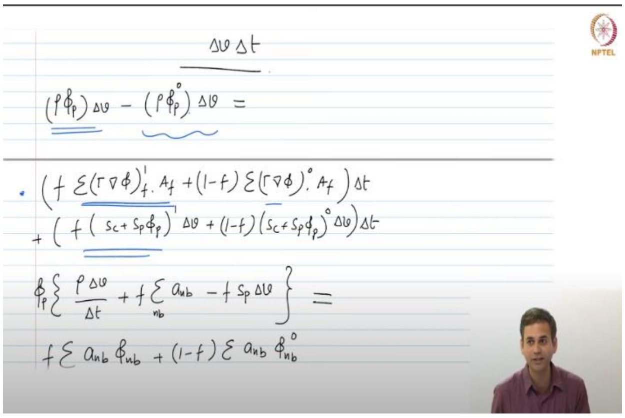
Yes, I mean what his question is essentially I did not put 1 here and 0 here. His question was why did I not put 1 here and 0 here; but otherwise, they are considered ok. So, I have assumed that Γ_e is there ok, but these will eventually come into the coefficients. So, at that point we may need a different set of coefficients for these right because now they are the same, I would call the coefficient as a_E for both ϕ_E^1 and ϕ_E^1 because they are the same right; but otherwise, they would not be right.

Other questions.

Student: No.

No ok. Now, then, we have this big task ahead of you is to know combine all these things and tell me, what will be the equation? Ok. $a_P\phi_P$. So, or else, you can help me write it, probably you can take a look and help me write it.

(Refer Slide Time: 38:30)



So, essentially let us first consolidate the equation ok. What is the consolidated equation on the left hand side what do we have? $(\rho\phi_P)\Delta V - (\rho\phi_P^0)\Delta V$ right that is all on the left hand side.

Student: Yes.

Yes.

Student: Yes.

Then, on the right hand side, we have; what do we have? We have.

Student: (Refer Time: 38:55).

f times ok, I will just write down whatever we have written that is yes, $\{f\sum(\Gamma\nabla\phi)^1{}_f \cdot A_f + (1-f)\sum(\Gamma\nabla\phi)^0{}_f \cdot A_f\}\ \Delta t\ +\ \{f(S_C + S_P\phi_P)^1\Delta V + (1-f)(S_C + S_P\phi_P)^0\Delta V\}\,\Delta t.$ Is that correct?

Student: (Refer Time: 39:44).

Oh sorry, this is the this should be 1 or none and this should be 0 ok, 0 fine 1 and 0. Is that correct otherwise?

Student: (Refer Time: 39:54).

Ok. Yeah.

Student: (Refer Time: 39:58).

Now, this $(\Gamma\nabla\phi)_f \cdot A_f$ would give rise to what terms? $\phi_E - \phi_P, \phi_N - \phi_P$ and so on right. Essentially, you would get all your a_E, a_W, a_N, a_S from these quantities as well as the second one that is 1 minus f right. So, you would get from here as well as from here ok.

Then, let me divide everything with Δt ok; let me divide everything with Δt and collect the terms for the coefficient ϕ_P on the left hand side ok. Because we always want coefficients of ϕ_P on the left hand side right, even when we had the diffusion equation alone.

So, and also divide everything with Δt. So, what will be the coefficients on the left hand side for ϕ_P? First term is of course, this guy $\frac{\rho\Delta V}{\Delta t}$ ok that is first term. Now, where do you get other terms from for ϕ_P? So, ϕ_P is what ϕ_P^1 all right, which terms can contribute to that. Can the first term contribute this one?

Student: Yes.

Yes, it will right. What about the second term?

Student: No.

It will not. What about the third term?

Student: Yes.

Yes. Fourth term?

Student: No.

No ok. Then, let us consider these two and then, can you tell me what will be that? From the first term, you would of course get $\sum a_{nb}\phi_P$ right because you will get $a_E\phi_E$ and then, you have another term which is $a_E\phi_E$ right, we can we turn to the other side. So, similarly you have all other terms right.

So, that will be and they all come with a minus right, when they go to the left hand side, they will all become plus. So, this will be plus $f\sum a_{nb}$, on all the faces times ϕ_P that is outside. Anything else that will come from the first term on the right hand side? Nothing that is all right. What about the third term? Which one will go into?

Student: (Refer Time: 42:02).

$fS_P\Delta V$ right. We have already divided with Δt. So, this will be plus or minus?

Student: Minus.

Minus right; this will be $-fS_P\Delta V$ that is all, on the left hand side right. So, we have successfully written ϕ_P coefficient right. Correct or any other term would contribute? That is all, nothing else ok. On the right hand side, what do we have? Right hand side of course, from the first term if I were to calculate what is from the first term on the right hand side, what will what will be there?

$f\sum a_{nb}\phi_{nb}$. These are all 1 right; these are all calculated 1 plus what else? From the second term, second term you will have; how many terms will you have from the second term? Second term will have a_E, ϕ_E, ϕ_W all these things and then, will it also have ϕ_P^0?

Student: Yes.

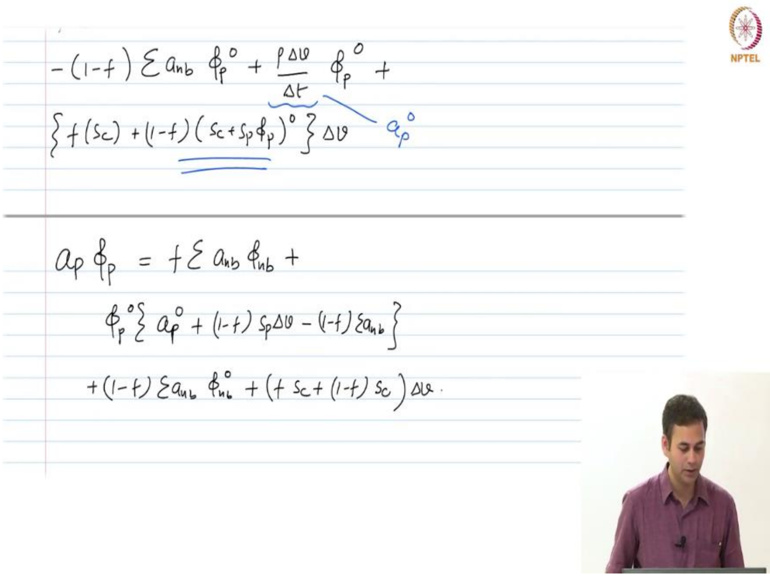
It will have all those things; we cannot send it anywhere right because those are all unknowns or knowns?.

Student: Knowns.

Knowns. So, they should stay on the right hand side.

Student: Yes.

(Refer Slide Time: 43:34)



Right. So, essentially, we have now two set of terms coming from the second term on the right hand side, those are $(1-f)\sum \mathrm{a_{nb}}\phi^0{}_{nb}$ right. These are all the neighbouring coefficients and then, we what else we have?

Student: ϕ_P.

We have ϕ_P, but that will that come with a plus or a minus?

Student: (Refer Time: 43:53).

It will come with a minus. So, that will be $-(1-f)\sum \mathrm{a_{nb}}\phi_P^0$ times how much?

Student: ϕ_P^0.

ϕ_P^0right because ϕ_P^0 will have east, west, north, south all of them clubbed right. Do you see that ok; then of course, we have this trailing term right, we have this guy which is which we did not talk about on the left hand side, that also will come to the right hand side right because this is a known or unknown?

Student: Known.

Known. So, we will send that guy to the right hand side that will be how much?

Student: Plus (Refer Time: 44:30).

That will be a plus $\frac{\rho\Delta V}{\Delta t}\phi_P^0$ ok, fine. What else?

Student: (Refer Time: 44:42).

Then, we have a bunch of source terms that are coming from 0 right, as well as the Sc of the one level right ok. Then, let us put all of those together that will be

$$\{f(S_C^1) + (1-f)(S_C + S_P\phi_P)^0\}\Delta V,$$

is that correct? Anything else? That is all right; that is all fine. Anything we did we miss out or no? Ok fine yeah.

Student: S_C^1 (Refer Time: 45:41).

S_C^1 is well ok; S_C^1 is a good question. So, if we have; so, what will be S_C^1? It will be evaluated at.

Student: (Refer Time: 45:56).

At ϕ at the previous at the current value right; but you have to do you have to introduce a model for it, you have to this will be valid at zeroth level right. So, whatever I have written S_C^1 actually corresponds to 0 only right because you would introduce the star values for that S_C right. So, I would then drop this guy I wrote S_C and this is also.

So, this is basically known right that is why it is on the right hand side. So, I will write this as I mean I will leave it as S_C^1, but you understand that this is actually a S_C for the next level which is evaluated using the current level values of ϕ ok. If you have a non-linear variation of S, that is dependent on ϕ ok.

Student: S_C will be same (Refer Time: 46:42).

Sc will be the same actually right.

Student: (Refer Time: 46:46) constant (Refer Time: 46:48).

It could be it will be just a constant. So, it will not be. So, it will be anyway constant. So, you do not need this 1 here that is basically not required fine. S_P could be a function of ϕ right. As a result, I would need something there fine, but S_C and S_P could be functions of space which is fine which is nothing to do with the time level ok. Other questions?

Student: (Refer Time: 47:16).

It is good. So, I would like to kind of rewrite it little bit such that we will kind of recognize it. So, I would like to write it as our old thing that is $a_P\phi_P$ ok, where a_P is what everything that is in the curly braces equals equals f times ok. So, I would like to write it as we have ϕ_{nb} coming here what other ϕ_{nb} are here. So, we do not have any the other thing. So, this will be $f\sum a_{nb}\phi_{nb}$right that is what we have plus we have a lot of terms here. Can I combine any of these things ϕ_{nb}^0 is there, ϕ_P^0 is there all right.

So, then, I would write it as I will call this term as some a_P^0 because its multiplying ϕ_P^0 ok. So, we have that term. So that means, I have plus ϕ_P^0; can we collect the terms for ϕ_P^0on the right hand side? What will be there for ϕ_P^0? This a_P^0 that I just wrote plus?

Student: S_P is there.

Yeah, S_P is there and then, you have this guy right that would be plus $(1-f)S_P\Delta V$ right. Coming from here, I am multiplying with collecting terms of ϕ_P^0 ok. Minus or plus?

Student: Sir, above.

Above would be $-(1-f)\sum a_{nb}$ that is what we have yes.

Student: (Refer Time: 49:22).

Ok. No, it should be fine because we do not have yeah I mean a_P^0 is something that is coefficient of ϕ_P^0 that is all we do not have zeros and ones for the coefficients anywhere right. These are unless you have a different notation right. So, a_P and a_P^0 essentially, it denotes another constant is multiplying ϕ_P^0 that is all fine. What else is remaining? What else is remaining is basically the ${\phi^0}_{nb}$. So, that is plus $(1-f)\sum a_{nb}{\phi^0}_{nb}$ right plus this entire thing which we would call it as some b term right, that is basically fS_C plus $(1-f)S_C$ times ΔV right is that what we have fine.

Student: (Refer Time: 50:28).

This should be just as c ΔV is not it? We choose some f here. So, did we get the $a_P\phi_P = \sum a_{nb}\phi_{nb} + b$?. We have got it right. I think this is the working equation now. Is this fine? Ok

Now, in the next lecture, we will see how to obtain different time stepping schemes by setting different values for f ok that is what we will see in the next lecture. As of now, this is the general equation for any time stepping scheme ok.

Thank you.