Computational Fluid Dynamics Using Finite Volume Method
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Lecture – 16
Finite Volume Method for Diffusion Equation:
Problem Solving using TDMA

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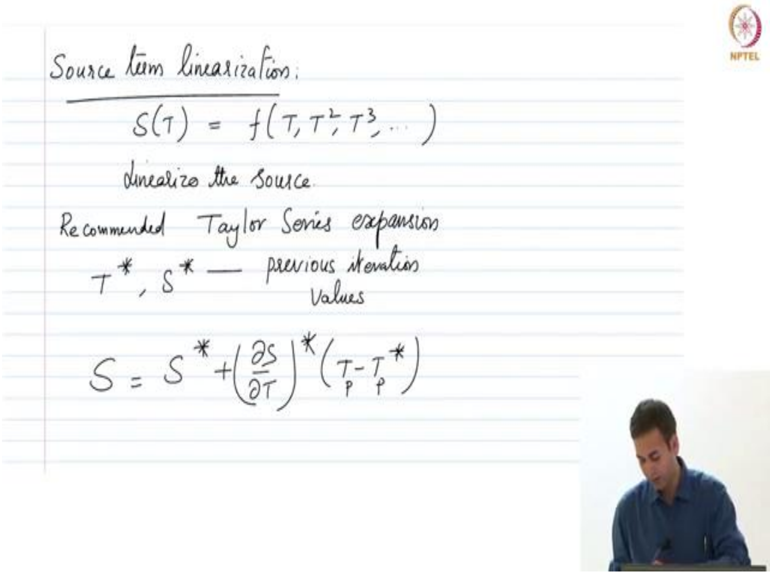

So, we are discussing source term linearization. So, we said if we have a general source term that is S as a function of T which is a non-linear function of, could be a non-linear function of temperature and some constants, then we have to linearize the source, because our nominally linear framework would only allow a linear model to be in there for the source terms and also having a linear dependency on the temperature, on the dependent variable is much better than having a constant in there, in terms of the convergence ok.

So, because of these two constraints and motivation so, we kind of linearize a source term and there are several ways to linearize the term whereas, the recommended one is using.

Student: Taylor. Taylor Series expansion. So, if we say, we know the values of the source term at the previous iteration. So, the star indicates the previous iteration values, then we expand the current value of the source term about the previous values ok, using Taylor series; that means, $S = S^* + \left(\frac{\partial s}{\partial T}\right)^* (T_P - T_P^*)$ S would be expanded as of course, you can go for higher order terms, but that is not required, because we are using a linear model.

So, this is essentially expanding about S star or the star values, the current previous iterate values. Now, of course, if it is a, if you are applying for the cell P, then these would read as Tp and Tp star.

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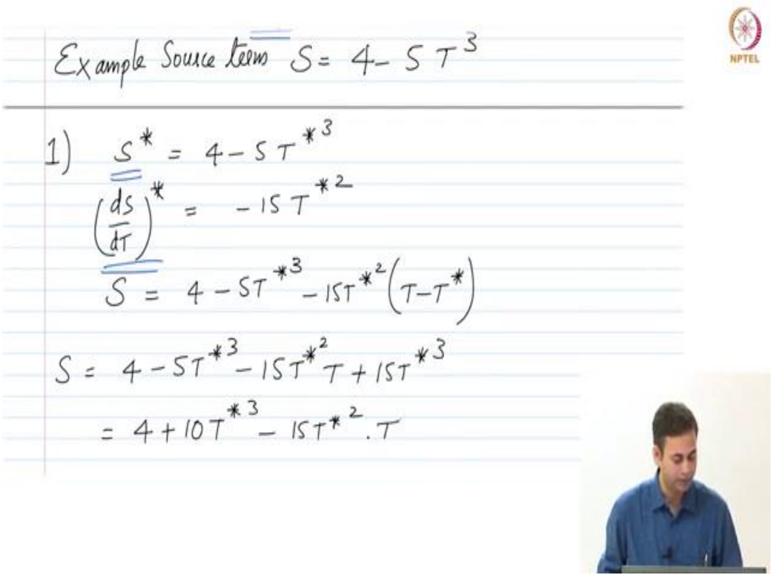

Now, let us consider an example of a source term. So, an example source term, is as a $S = 4 - 5\,T^3$. So, we have a cubic function for S in terms of temperature and some constant. Now, what would be, I would call this as method 1 which is using the Taylor series expansion that we have just discussed.

So, what is S star here? S star would be evaluate S at the T star values right. So, that would be $S^* = 4 - ST^{*3}$ and what would be $\left(\frac{ds}{dT}\right)^* = -I\ 5T^{*2}$dS dT star or partial $S = 4 - 5T^{*3} - 15T^{*2}(T - T^*)$ derivative of this with respect to temperature is how much?

Student: 15.

Minus 15.

Student: T square.

T square. So, that would be T star square, because we are evaluating at star values and the previous values.

So, now S star and dS by dT star, are they known or are they unknown?

Student: Known.

They are known, because they are in terms of the T star values which could be a guess value or which could be the latest value that we have in those locations then, if we plug in what would be the model for S, $S = 4 - 5T^{*3} - 15T^{*2}\ (T - T^{*})$ cube plus or rather , that is what we have. If I plug in for S star from here and dS by dT from here, that is what we have alright.

Then we can rewrite this as S equals, you can rewrite as this $S = 4 - 5T^{*3} - 15T^{*2}\,T + 15T^{*3}$. So, this is nothing, but 4 minus 10 I mean sorry, plus 10T star cube, we have minus 5 T cube and plus 15 T star cube minus 15 T star square times T ok.

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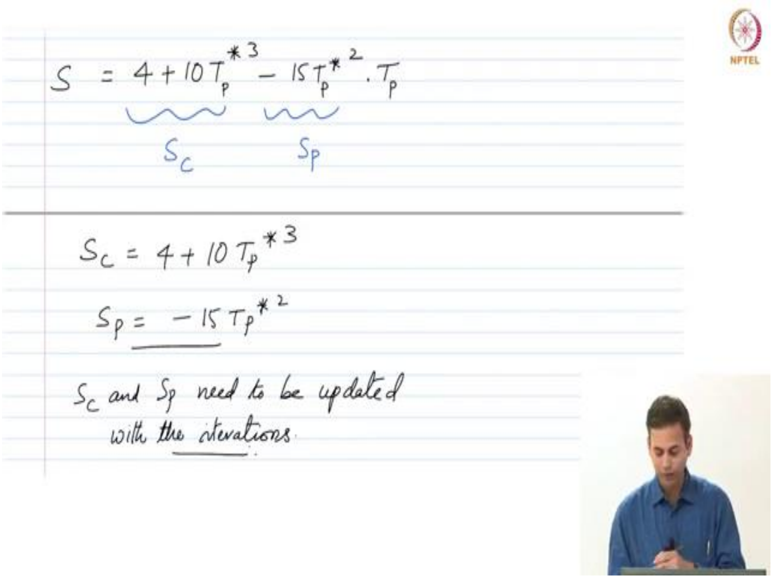

So, can you identify now what is Sc and what is Sp from this relation? $S = 4 + 10T^{*3} - 15T_P^{*2}.T_P$ would be your?

Student: Sc.

Sc the constants source term, because T star cube is known right, T star is known and then what would be Sp? So, Sp would be this guy, because what we have is Sc plus Sp times Tp ok. So, we can even write it as with subscripts p everywhere if you are doing it for cell p ok. So, this could even be Tp Tp star and T right. So, this is your S so; that means, what we have is; we have$S_c = 4 + 10T_P^{*3}$

and $S_p = -15T_P^{*2}$ right.

So, essentially now what we realize of course is that Sc and Sp depend on temperature right; that means, they depend on the temperature values at every iteration; that means, they have to be updated as part of your solution loop ok. So, we have linearized it, but Sc and Sp need to be updated with the iterations right. So, essentially these have to be updated as you go along your solution loop as well ok. Now, we had this discussion where.

Student: Sir, (Refer Time: 06:09).

Now, once you converge the T to T star ok, so where do we stop the iterations? That is once you reach a convergence criteria, ok. We will come to that will come to that discussion.

Student: (Refer Time: 06:19).

Yes.

Student: (Refer Time: 06:21).

So, what if you are using a direct method? So, even if you use a direct method, this is this step of linearization comes before you arrive at the system of linear equations right. So, if you are using a direct method a completely direct method right so, essentially you still have to update the coefficients right ok.

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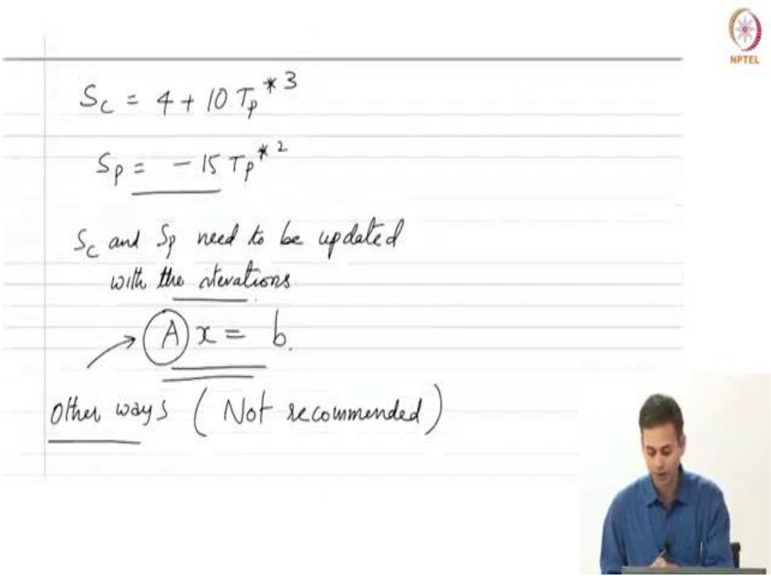

I think now the question is interesting. Essentially, you have Ax equal to b right, where; so, you need to have some kind of a false iteration on this coefficients right, because the coefficients are not correct right, because they are depending essentially your direct method will only solve for a given $A_x = b$ but then A itself is, because of the non-linearity, you have linearized it, that is why A has to be updated as part of a another iteration, we will come to that, we will come to that later.

In a sense you have to iterate the solution for certain reasons right. One of them is of course, if you choose an iterative method right, instead of a direct method the other one is if you have non-linearity, because non-linearity cannot solve directly.

So, either you have to introduce a loop kind of an outer loop right, where you have to update your coefficients of the A, that has to be there, we will come to that discussion little later fine. As far as like this discussion is concerned right now, we are looking at let us say an iterative method fine, other questions, is it clear how to linearize any source term right. We use the Taylor series expansion.

Now, we had this discussion, I think few classes before where would the coefficients be different between cells right in, in that context we said Sp would be, if it is a constant it would not be, but if Sp is a function of Tp, then it would result in different Ap for different cells ok. You see that, because Tp would be different for different cells as a result Sp will come out to be different which will in turn result in Ap being different ok.

So, that will come into play. Now, of course, we have not established that well whether this linearization would give the correct solution right, because somebody gave us 4 minus 5 T cube and then we just said ok, I will use Taylor series, and got, I just change the source term and then said ok, we will just converge this thing. So, we will see that little later.

For now, let us also look at few other linearizations or other ways of doing this which are not recommended of course, ok. Now, you may have therefore, the one the Taylor series one is basically model 1.

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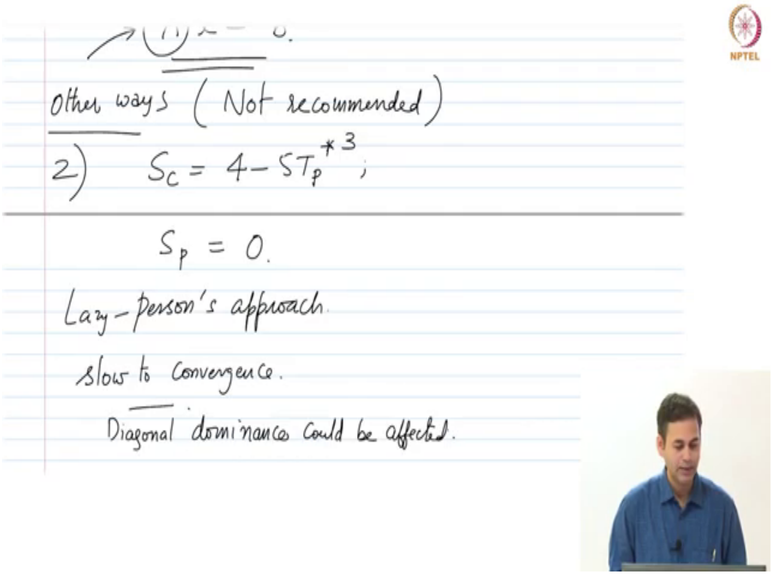

So, let me call it as model 2, model 2 would be you can just say what if I just use $S_c = 4 - 5T_P^{*3}$. And $S_P = 0$ you can of course, do with this right. You just dump everything in the constant and say Sp is 0 ok. So, this is of course, lazy persons approach alright, where you just dump in everything in the source term in the constant ok. Now, does this work or not? Will there be a problem? What do you think would be would might happen in this case?

So, what are the, of course, it will converge hopefully right and then you will get an answer only thing is that it will be slow to converge right. So, essentially the simulation, the convergence will be slow to converge, because we are not using the correct ST relation ok, we are not using the ST relation and also what is the other disadvantage you see with this outright? By making Sp equal to 0 you are losing some very good property.

Student: Scarborough.

This Scarborough right. Essentially, the diagonal dominance is what you are losing ok. So, as a result the convergence might also struggle right depending on if you do not satisfy Scarborough so, essentially the diagonal dominance could be affected, fine. So, alright.

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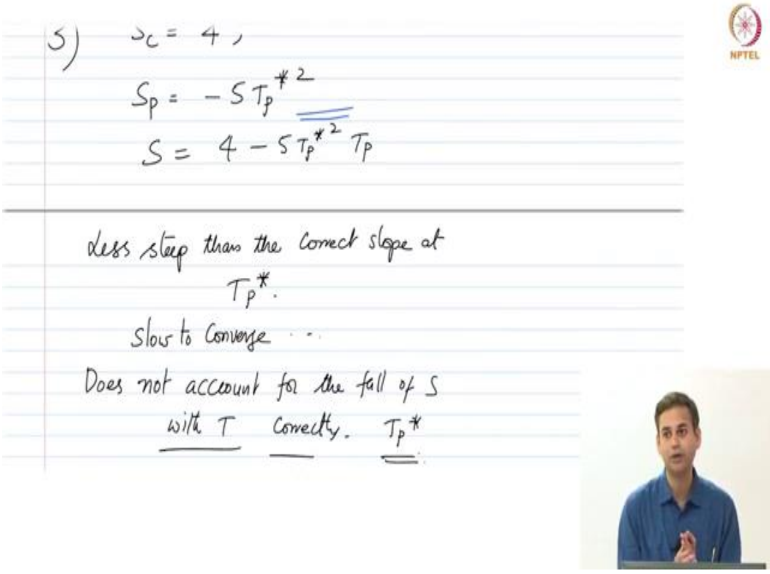

Then of course, there are several different ways, another way to look at is basically, I would say S minus 4 5 Tp star cube. So, I can say Sc equals 4 minus, let us say what we have is you said plus 10 and minus 15 right.

So, I can also have I can say, Sc equals 4 ok, $S_C = 4$ and $S_P = -ST_P^{*2}$ can I say this right. Essentially; that means, my Sc $S_C = 4 - 5T_P^{*2}T_P$ that is my old original model, I just say 4 is my constant minus 5 Tp cube I would just get minus 5 Tp star square and then this is my Sp. Can you use this model, you can use it, but only thing is that this particular curve that you are talking about this particular line has is kind of less steep than what it actually is ok.

So, essentially the first model that we had, what was the slope of the curve that we had? Minus 15 Tp star square ok, but so, essentially this is like a tangent to the curve at Tp star right. What will be the slope of this guy? It is only minus 5 Tp star square. So, this is less steep than the original model right. So, essentially less steep than the?

Student: (Refer Time: 12:18).

Correct slope right, slope at Tp star. As a result this would this would also converge the only thing is that it will again may not. So, it will be again slow to converge. Fine and it also does not account for the fall of S with T right, correctly right. It does not correctly account for the amount of fall of S with respect to T at Tp star ok. So, as a result this would also; would be slow to converge, yes.

Student: (Refer Time: 13:12) can we change source or change the (Refer Time: 13:17).

So, the question is I am just changing the source, are we not solving different problems? I will come to that question little later.

Student: Why are we?

So, essentially we will come to that in a little while ok. So, each time we change the source, are we not changing the solution also? Ok, that could also happen, right I will come to that which method kind of produces the correct answer when you converge ok. Essentially, we should be solving the same problem that is given to us, but not something else right fine.

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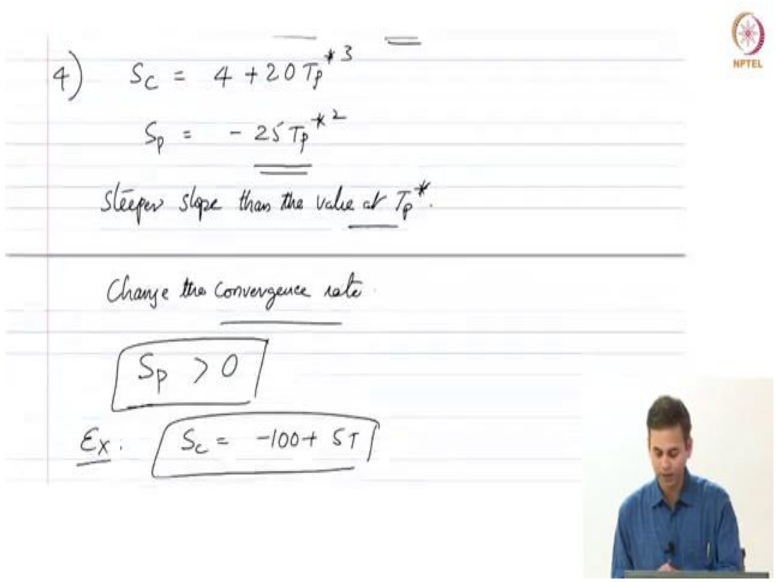

So, we can of course, have another model where Sc could be $S_C = 4 + 20T_p^{*3}$ and $S_P = -25T_P^{*2}$ or should be the other way around minus 20 Tp star square would this, would this work right or 20 and 25 should be exchanged is it? What will Sp? Let us see. So, I have how much is the value I had minus plus 10 and minus 15. So, I would make it, plus 20 and minus 25 right, that would be fine, right? I can come up with infinite number of ways right?

Now, what would be the slope here? Slope is larger than the slope it is supposed to be right. So, this is as a steeper slope than the value at Tp star ok. So, this is kind of acceptable, but again it will it might kind of slow down the convergence ok. So, essentially the best way to look at is you should take care of from the tangent to the curve ok.

So, this also would work, but it might also kind of change the convergence rate. Now, let me look at how does these graphs yes, essentially you are not having the correct slope of the source with that with the temperature. As a result you are not capturing the correct S, S to T relation. So, as a result it might slow down the convergence ok. So, this is again an observational fact there is its not coming from the equation as such.

Student: (Refer Time: 15:32) guaranty that it will?

Yes, it need not, it need not actually give you the solution quickly right, but of course, it is good, because you have now more diagonal dominance right. You have the Ap is much larger now right, but that does not guarantee that you will get the solution quickly ok. Other questions? Yes.

Student: (Refer Time: 15:52).

So, question is will Sc and Sp change from cell to cell?

Student: Yes.

Yes, they change right. Do they also change from iteration to iteration?

Student: Yes.

Yes, they change ok, fine. Other questions, yes?

Student: (Refer Time: 16:08) as minus (Refer Time: 16:09).

Just out I mean, like curiosity that is all. It can be anything you want to have 40 and 45, it can be anything infinite number of ways. I just put something there which is greater than minus 15 Tp star square.

Student: (Refer Time: 16:23) ok.

Yes that is true, but then you cannot have any random numbers here, because the difference has to be minus 5 step correct. So, the question is ok, only if am lucky the slope will be correct. So, we do not have to be lucky we can use Taylor series expansion that is all ok. Other questions, yes.

Student: Sir, so what if (Refer Time: 16:43) source term are both minus (Refer Time: 16:46) at least 4 plus 5T.

Right. So, essentially the question goes back to the assumption, we made which is Sp is less than or equal to 0 what if, if Sp is positive right or S is a positive function. Essentially, we the whatever we are talking about I am just calling it as source, but actually it is a sink right, because I have minus in there, would you get such a problem in like that is physically possible, what will be the solution for if you have a positive feedback loop? What will be the solution for a positive feedback loop?

So, essentially you have some temperature 100, your source becomes 100 cube right plus and then you have some solution. So, what will be, is there a physical problem where you can get a positive source? So, essentially a positive feedback loop right. The more, the higher the answer, the more will be the source. What will be the solution for that?

Student: (Refer Time: 17:41).

Infinite everywhere right, that is all.

Student: Net value of the source need not be more (Refer Time: 17:45).

Net value of the source ok. So, locally you may have some source which can be larger plus so say it again.

Student: Minus 100 plus 5T.

Minus 100 plus 5 T.

Student: Yes.

So, in that case.

Student: (Refer Time: 18:00).

Sc equals so, this is an example that is suggested which is minus 100 plus 5 T. So, in this cases ok, in this cases the argument is that ok, the source will has a sink term right, but we have a 5T that is kind of a source right.

So, with this we still get a bounded solution, right not an infinite solution, but then the problem is that, diagonal dominance is not satisfied like how do I deal with this thing right? So, to deal with these kind of problems, we have something known as relaxation, which is kind of artificially changing the diagonal dominance ok. We will come to that little later ok, but you would not get a problem where it will be just 5 T. You do not have a kind of a positive feedback loop physically possible.

But as he said you can have a different model or you can have a transient model where you may have, let us say the source is switched on for some time and then it kind of switches off, before it reaches infinite right, that is possible if that is there for these kind of problems we would have something known as relaxation under relaxation where we kind of change the coefficient of Ap a coefficient of phi p which is Ap by certain amounts ok, to deal with the diagonal dominance issues yes.

Student: Sir, is it T star and T neighbouring cell temperature or like (Refer Time: 19:18).

Say it again.

Student: T star.

T star and T for the same. So, is the T star and T for the neighbouring cell temperatures or for the same they are for the same cell right.

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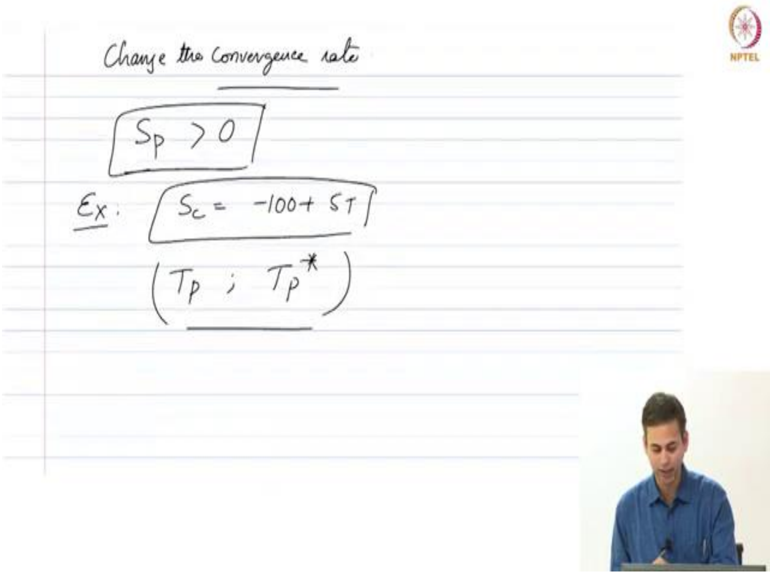

Essentially, we are talking about Tp and talking about Tp star both are evaluated for the same cell p ok. The previous iteration value and the current value that you want to calculate.

Student: Ok.

Other questions ok. So, we saw all these things now, let us kind of plot all these 1, 2, 3, 4 the 4 examples we have; for this source sum linearization on an ST relation ok.

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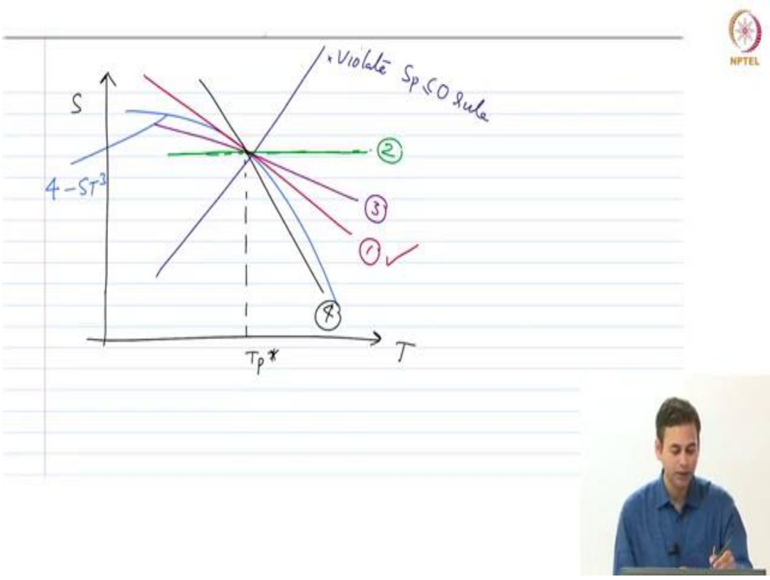

So, that is basically we have the source term on the y-axis and temperature on the x-axis let us say, the blue curve is my model that is S this is essentially, what was the model that is given? 4 minus 5T cube right that is the model I have and let us say this is Tp star, that is Tp star and what would the first model the Taylor series expansion produce a line that looks like.

Student: (Refer Time: 20:36).

It should be tangent to the curve right, because it is a dS dT times T minus Tp star square. So, that would be a tangent. So, if I use this curve ok. So, this is my first model right, which has minus 15 as the slope. Now, anything on this curve if you have lines that have a positive slope right; if you have a line that is like this what does these indicate.

These indicate is usually a positive feedback loop as such right. So, these are all has a they violate your Sp less than or equal to 0 rule right we, said it Sp has to be less than or equal to 0. If you have a positive slope, they all have plus right. Sc is a positive value. So, we will not talk about them at the moment ok. Now, what is the other one we had? The other one we had was the lazy persons approach which is Sc is just 4 minus y Tp star cube. So, that is essentially constant right, slope is 0.

So, this is your second model right, where it is just a constant value. What are the other two models we had? The third one is an equation 4 minus 5 p T star square is what is the slope.

So, that is less steeper than this alright or; that means, how does that look? That we look between 1 and 2 is not it, that will be somewhere between 1 and 2. So, that would be this model. This is your equation 3 right. It is all and then how does the fourth one look like? Fourth one has minus 20 minus 25 Tp star square right, this is the slope. So, how does the last one look like (Refer Time: 22:20) I will use the black. It would go like this right, other than this, so this is your fourth one.

So of course, you can draw as many as you want on this graph, but the correct one, the recommended one is this one that is based on the Taylor series expansion ok. Now, we will come back to this question of well we have arbitrarily changed the source by using Taylor series expansion. Now, is that correct to answer that we will look at the expansion? We have S equals S star plus dS dT or partial S partial T star times T minus T star ok. So, if we consider any cell this would be Tp minus Tp star times dS dT star. Now, if we have if you attain convergence, what will be the difference between what will be the values of Tp and Tp star?

Student: (Refer Time: 23:09).

At convergence.

Student: 0

0, right?

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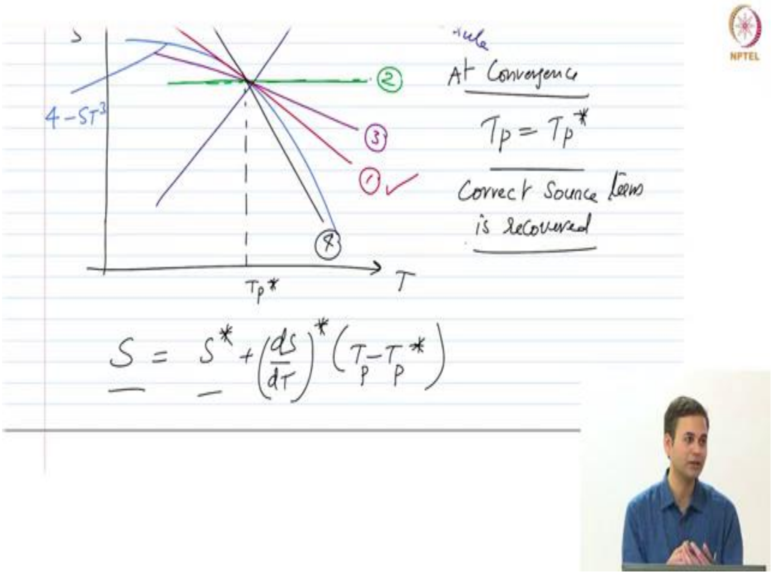

Essentially, the difference would be of the order of epsilon right, very small. Essentially, $T_P = T_P^*$ at convergence right, by the time we reach convergence then what will be S and S star would be?

Student: Same (Refer Time: 23:24).

Will be the same right essentially, it will the same; that means, we are recovering the same source term that is given in the problem ok. So, the correct source term is recovered, at convergence.

Now, what about the other models we have seen? Would they also let us say we have used this 2 3 4 or something and then found a solution in the end, would they also satisfy would be that will also model the correct source term not?

It will right, because the slope does not matter anymore because you are multiplying with Tp Tp star, if they reach a convergence value then slope would not matter right, is not it. So, that those are all also if they converge the value you would get would be correct value right, only thing is that we might the solver might struggle to kind of converge that is the only issue fine, question till now?

Student: Sir.

Yeah.

Student: (Refer Time: 24:25).

Yes.

Student: (Refer Time: 24:31) separate (Refer Time: 24:32).

So, essentially the question is the we are neglecting the higher order terms right. There will be some error right, how do we quantify? So, there will be certain order of accuracy for modelling this source ok. We will show that as second order accuracy which would be the same as the other special discretization accuracy ok.

We are not considering that, because why? A linear model, if you consider then it will be same as what is given right. You will have the T square term and then you will get the T cube term and then it will be the same as what the model that is given as.

Other questions; no, clear? Ok. So, then there is one more thing that we have to discuss that is basically under relaxation, because now that we have touched upon it, but before we do that, I want to go back and kind of answer this question that somebody had yesterday, can we do a solve an example problem of the TDMA and the line by line TDMA ok.

So, we will just quickly go through that. They probably do 1 D problem for calculating the TDMA and then we will probably look at a 2 dimension problem as well ok. I think this is somebody asked yesterday. I think you asked this question yesterday, right. So, shall we do that and then we will come back to discussion on under relaxation in the after we finish this fine.

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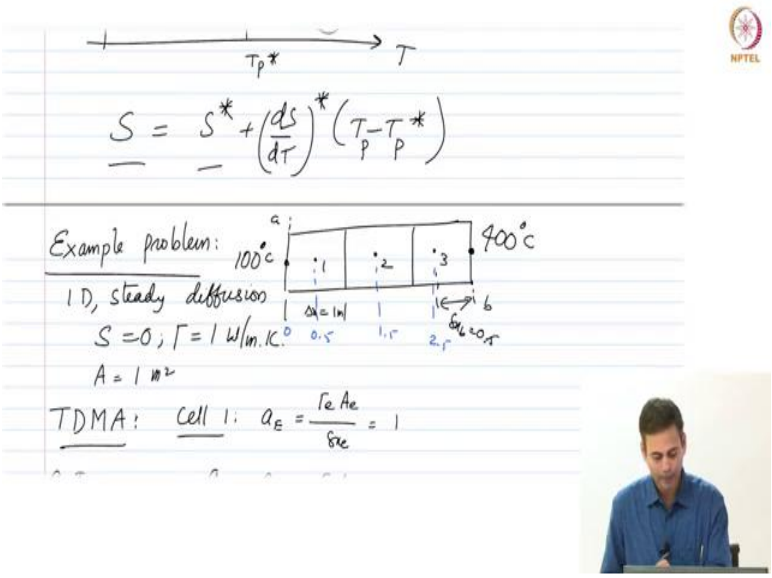

So, let us look at a an example problem, this is very straightforward. So, example problem is we have let us say the boundaries, let us say we have 1D steady diffusion. Let us take source equal to 0 for the sake of simplicity and we have three cells ok.

So, delta x let us take as 1 meter and then the temperatures would be 100 degree Celsius and 400 degree Celsius and then I have three cells which is 1, 2 and 3 source is 0 and then let us take gamma or k as 1 Watt per meter Kelvin ok. Essentially, it will just easy for us, it will be easy if we take numbers like this and then let us also take area as 1 meter square that is the area of the faces. So, 1 D problem anyway fine.

So, what do we do, what is the first step? You generate the discrete equations right ah, discrete equations and if you are using a Gauss Seidel you would again guess values for 2 1 3 T 2 and T 3 and calculate the value of T1 equal to T2 and so on that you are familiar with right, Gauss Seidel.

Now, let us look at how does this look work in TDMA ok. So, we are applying a direct method, but first we need the equations. So, what would be the equations? Can you tell me what would be if I take cell 1, what would be a east? $a_E = \frac{\Gamma_e A_e}{\delta xe} = 1$

Student: (Refer Time: 27:46).

by?

Student: (Refer Time: 27:50) Del x e.

. So, gamma is 1 A is 1 del x e is?

Student: 1,1.

One all of them are 1 a east is 1. What would be a west? We do not have a west, we have something known as a, if we call this as a and if we call this as b, we have something known as a a right. This is our a west. What would this be?

Student: Gamma (Refer Time: 28:13).

Gamma west, a west upon del x b or del x a right, what will this be?

Student: [noise] 2.

2, because this is gamma is 1 a is 1 and delta x a is half alright. So, this is 2 ok, then source is 0, then what would be the final equation?

What will go into ap?.

Student: a east.

a east right.

Student: (Refer Time: 28:38).

a west would, a west is not there, but would a go into ap?

Student: Yes.

Yes, it will go because the boundary condition you given is a Dirichlet boundary condition, it will go into ap. So, what will be a p?

Student: [noise] 3.

3 ok, 3 because Sp is 0, 3 times T 1 equals what would be the equation?

Student: a E (Refer Time: 28:59).

a E times what?

Student: (Refer Time: 29:02).

T 2

Student: T2.

That means, 1 times T 2 right that is a east times T 2 plus?

Student: (Refer Time: 29:09).

2 times.

Student: 2 (Refer Time: 29:12).

100 right. Essentially, this is a w times Tw, T w is nothing, but T x. So, that is what we have written here is a times T a right that is all. Do you all agree with this or no? Agree, easy.

Do you already see the diagonal dominance? 3 fine. Then let us write the other equation now, it is very easy just by looking at it you can tell me what would be the equation for cell 2.

Student: (Refer Time: 29:41).

So, for cell 2 would you get this kind of term like 2 no, because you do not have boundaries. You have 1 and 3. So, east and west would come out to be one each.

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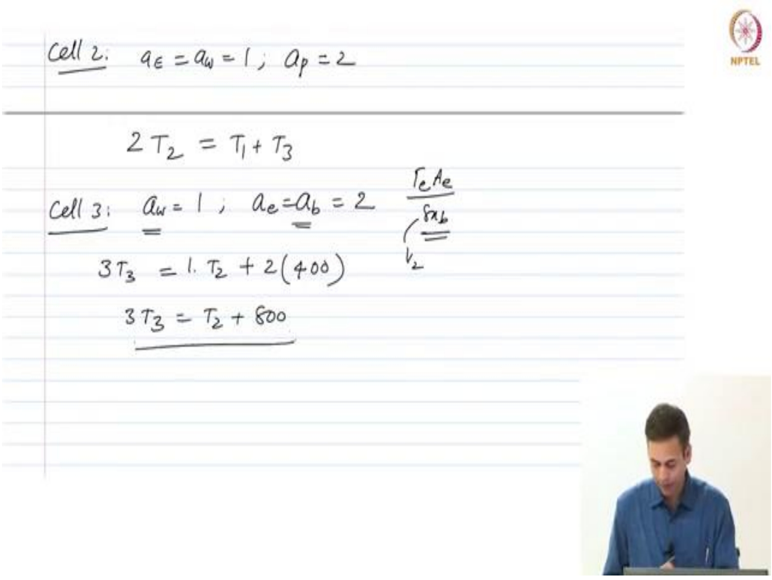

So, essentially for cell 2$a_e = a_w = 1, ap = 2$ and what would be a p?

Student: (Refer Time: 30:00).

; that means, what would be the equation for cell 2? $2T_2 \; = T_1 + T_3$.

Student: (Refer Time: 30:06).

, source is not there, boundary condition is not there, nothing is there ok. What about cell 3?

Student: (Refer Time: 30:17).

Cell 3 would have a west as how much?

Student: (Refer Time: 30:21).

$a_w = 1$.

Student: a east (Refer Time: 30:25).

$$a_e = ab = 2$$

Student: 2

because we have$\frac{\Gamma_e A_e}{\delta xb}$ This is one half right, because we only have half the cell here right. So, this is del x b equals 0.5. So, as a result a b will come out to be 2, then what will be the equation for a cell 3?

Student: 3 T 3.

$3T_3 = 1 \; T_2 + 2\,(400)$ 3 times T 3 equals

Student: (Refer Time: 30:59).

1 times T 2?

Student: (Refer Time: 31:00) .

Plus.

Student: 2 times (Refer Time: 31:04).

2 times that is a b times, Tb that is?

Student: 400.

400. So, this is nothing, but $3T_3 = T_2 + 800$

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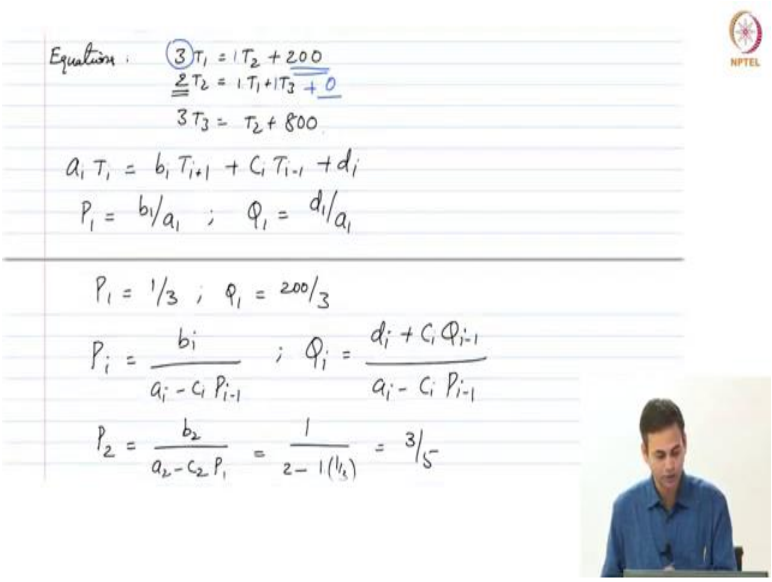


So, essentially we obtain all the three equations. I would just write down the equations. Equations are 3 T1 equals T 2 plus how much?

Student: 200.

200

Student: 200.

Then 2 T 2 equals T1 plus T 3 plus 0, anything else here, ok. What about the third one; 3 T 3 equals T 2 plus 800 fine. So, we have these three equations. Now, we will apply TDMA ok.

So, we apply TDMA. So, in our TDMA we had some abcd right. So, let me write down those things otherwise, it will be difficult. So, we have $a_iT_i = b_iT_{i+1} + C_iT_{i-1} + d_i$ ai Ti equals bi Ti plus 1 right, was it bi divided with the each coefficient Ti plus 1 plus Ci Ti minus 1 plus di right, this is what we had and we also had $P_1 = \frac{b_1}{a_1}$ and $Q_1 = \frac{d_1}{a_1}$ right that is what we had. So for then what would be?.

So, we are essentially applying TDMA in the positive x direction what would be P 1? P 1 would be for cell 1 right, that is b 1 by a 1.

Student: (Refer Time: 32:35).

What is b? b is 1 right, because Ti plus 1 is T east b is 1. So, this is$P_1 = \frac{1}{3}$ 1 upon, what is a 1?

Student: 3.

because a is this guy right, that is what about $Q_1 = \frac{200}{3}$ Q 1?

Student: 200.

200 a 1 is?

Student: 3.

. So, this would be 200 upon 3 right that is what we have P 1 Q 1 are know what is the next step in TDMA? Calculate P i Q i right essentially. So, what is the relation $P_i = \frac{b_i}{a_i - C_i P_i - 1}$

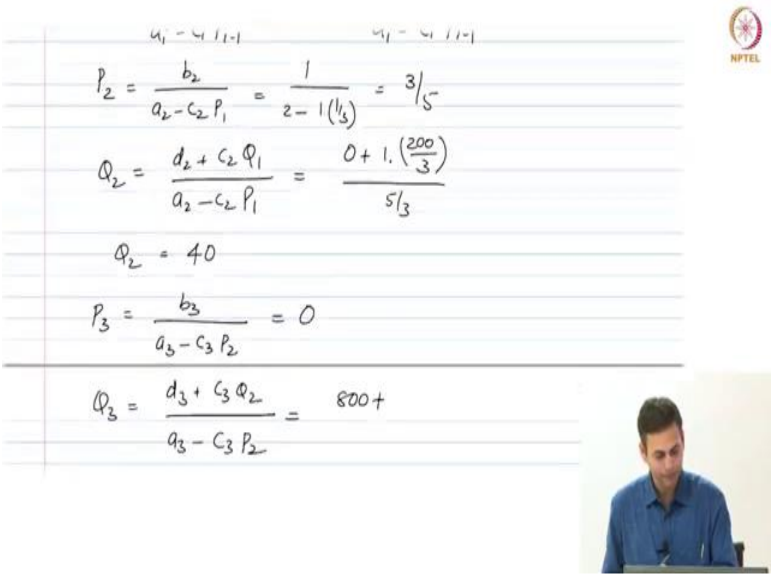
Student: bi.

Student: Yes.

And then what is Qi? $Q_i = d_i + C_i \ Q_{i-1} / a_i - C_i \ P_{i-1}$Ti plus, can you look it up from your notes and confirm?

That is correct ok.

(Refer Slide Time: 33:43)



Then what is so, we are now at cell 2. So, what is P 2$P_2 = \frac{b_2}{a_2-c_2P_1} = \frac{1}{2-1(\frac{1}{3})} = \frac{3}{5}$ P 2 is b 2 by a 2 minus c 2 P 1. So, what is b 2? b is the coefficient of what? For cell 2 of T 3 right. So, that is 1 is not it, because b i T a plus 1. So, this is 1 upon, how much is a 2? a 2 is the coefficient of T 2.

how much is c 2? c 2 1 right, because it is coefficient of Ti minus 1. So, this is 1 times how much is P 1.

Student: One.

o, this is how much ?

Student: 3.

Three fifth is it 2 minus one third 6 5 3 fifth is that correct ok, then what about Q 2?$Q_2 = d_2 + \frac{C_2Q_1}{a_2-c_2P_1} = 0 + 1.\left(\frac{200}{3}\right) 5/3$. How much is d 2? d 2 for cell 2, how much is it?

Student: 0.

0, because we do not have a. So, this is 1 catch ok, we will come to this and when we do 2 D. So, in 1 D, you have 0. So, this is 0 plus, what is c 2?

Student: 1.

c 2 is.

Student: 1.

1 times, what is Q 1?

Student : 200.

200 upon 3 upon a 2 minus c 2 P 1 is again how much? Five-thirds right, that is five- thirds. So, this is a 40, is it 40? Sorry, 40 200 by 3 by 5 by 3 ok. So, this is Q 2 fine, all right. Now, what about next values? $P_3 = \frac{b_3}{a_{3-c_3P_2}} = 0$

Student: (Refer Time: 35:34).

b 3. So, b is the coefficient of Ti plus 1. So, we are looking at 3 T 3, what would be coefficient of T 4?

Student: (Refer Time: 35:46) 0.

0 right, you remember we said P n equals b n equals 0 right. We cut off the links to the outer cells. So, this is how much? 0 right, because b 3 is 0. What about Q 3? $Q_3 = d_3 + \frac{C_3Q_2}{a_{3-c_3P_2}} = 800$

So, this is 800 plus c 3, c is the coefficient of Ti minus 1 right; that means, how much? 1 ok. So, this is 1 times, how much is Q 2? 40 upon a 3 minus c 3 P 2. How much was a 3? 3 c 3 1. How much was P 2? Three fifth ok.

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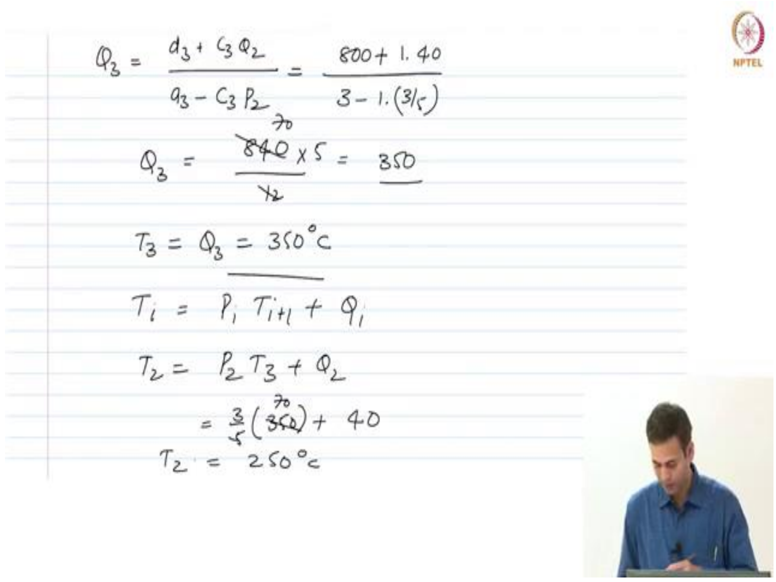

So, what should be this value? 840 upon 12 by 5. This is 12 by 5. How much would this be?

Student: 350.

350 12, 70s ok. So, we have Q 3 is 350. So, Q 3 is 350. Now, what is the next step?

Student: (Refer Time: 37:03).

T 3 is how much?

Student: Q 3.

Q 3. So, T 3 is 350 degree Celsius. So, we already started back substitution right, then how do you what do you use? You use the relation Ti equals Pi Ti minus 1 plus Qi and calculate all the other values right you go back substitution.

So, what is T 2? T 2 T 3 plus Q 2. How much is P 2? Three fifth, T 3 just we got it as 350 plus Q 2 40 ok. So, this is 40, so, we have 70. So, this is 250 alright, 3 70s 210 plus 40. This is 250 degree Celsius ok.

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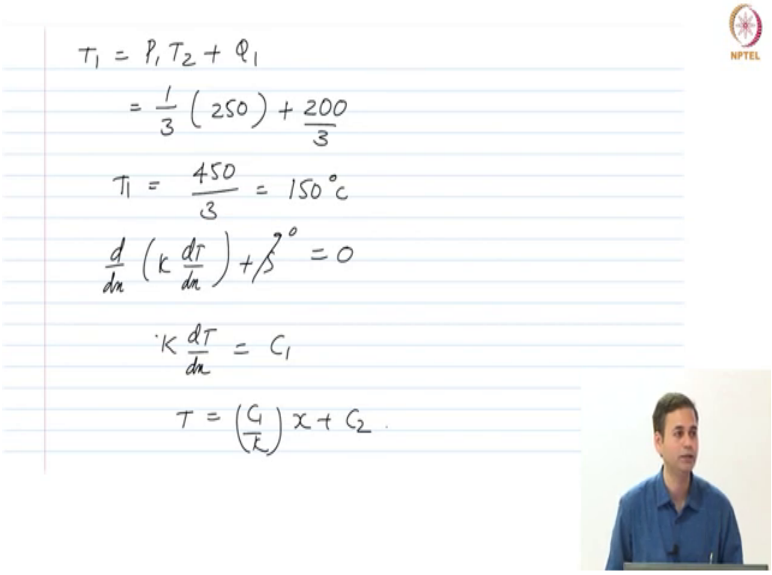

What about the last one? $T_1 = P_1 T_2 + Q_1$. How much was P 1? How much?

Student: Five thirds.

Five thirds ok, five thirds of 2.

Student: 1 One third.

$\frac{1}{3}(250) + \frac{200}{3}$ how much is Q?

Student: (Refer Time: 38:15).

200 is P 1, correct.

Student: 200 by 3.

200 by 3, 200 by 3 that makes sense ok. So, we have $\frac{450}{3} = 150°C$. This is 150 degree Celsius that is T 1. So, essentially we have 100 and 400 as boundary conditions, we got 150, 250 and 350 right from our TDMA that is just like one go ok.

Now, is this correct? We can of course, solve it analytically, but if you do it in the exam unless, it is asked you do not get points ok. So, we use $\frac{d}{dx}\left(K\frac{dT}{dx}\right) + S = 0$ then what would be the solution for this integrate it 2 times? What will be T? $K\frac{dT}{dx} = C_1$, $T_K = \left(\frac{C_1}{K}\right)x + C_2$

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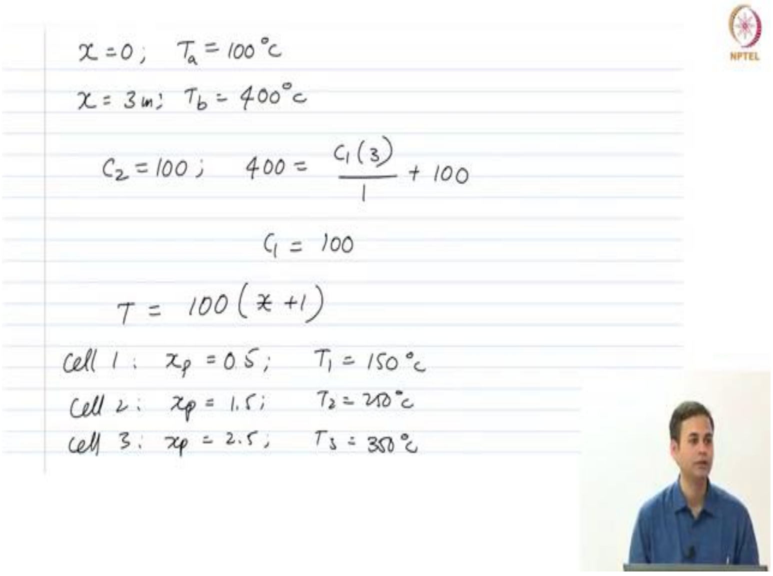

That is what we get and what would be your C 1 and C 2? x equals 0, what is the temperature given? 100 degree Celsius, x equals 3, 3 meters. What was Tb?

Student: 400.

400 degree Celsius, if you plug it in x equals 0 gives you.

Student: C 2.

C 2 equals 100 x equals 3 gives you 400 equals C 1 times 3 upon 1 plus 100. So, C 1 is also equal to.

Student: 4 (Refer Time: 39:43).

To 400 minus 100 300 by 3, this is also 100 ok. This is also 100, then what is the final answer? T equals 100 x plus 100. So, 100 times x plus 1 right, that is 100 x plus 100 ok. So, what is per cell 2? what is x? 1.5 right or for cell 1 what is x? 0.5 right. So, then what we get is a T 1 equals 150 cell 2, x p is 1.5 right. It is half way, is not it.

So, go back, done a lot. So, this is how much? 0.5, 1.5 and 2.5 multiply 2.5 would give you how much?

Student: (Refer Time: 40:51).

250 degree Celsius, cell 3 would give you xp is to 2.5, T 3 is how much?

Student: 350 degree.

350 degree Celsius, fine. Now, we just have to write a code for this same, it is very straightforward fine, happy. Now, we will do a 2 D problem as well fine ok. Questions on this, no are you able to follow right, able to think through ok.

Now, let us move on with a 2 D problem again, a very simple problem. So, essentially, I want to demonstrate how do you do the line by line method ok, because we said we will do TDMA in the y direction and x direction, we would do some kind of an a Gauss Seidel kind of thing.

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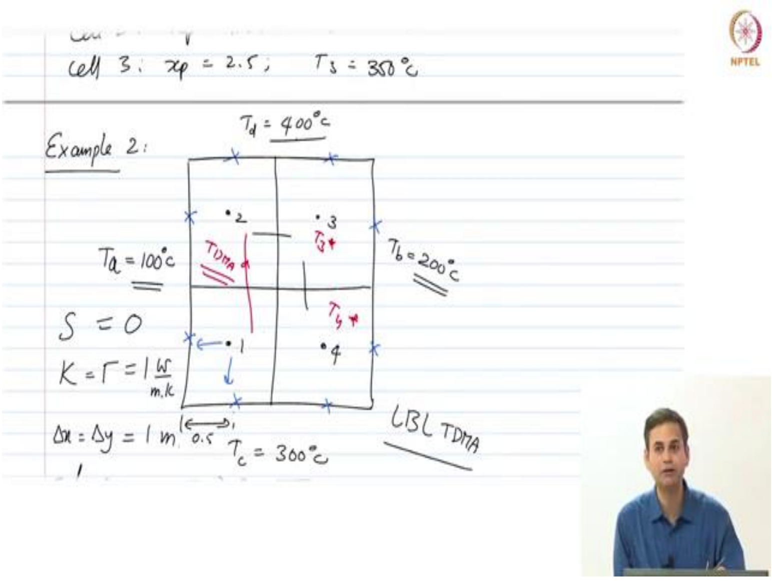

Let us look a 2 D problem. So, example 2 again, let us consider at domain 2 D domain, we have only 4 cells. Let us call cells as, I will call it as 1 2 3 4 ok, that probably would be easy and then of course, we have boundaries. These are the boundaries of these faces ok.

Now, the boundary condition on this side let us call it as T 1 as 100 degree Celsius. Let us call T a. So, for the entire face. It is 100 degree Celsius, on this side we have T b equals 200 degree Celsius and bottom is T 3 equals 300 degree Celsius and T 4 is 400 degree Celsius fine. So, essentially call it as abcd, you know. Let me change it I call it Ta Tb Tc Td ok.

So, now again if we assume source equals 0 K or gamma equals 1 Watt per meter Kelvin and what other values we need? We need delta x delta x equals delta y equals 1 meter ok.

Everything is we are setting it as 1. Then what is the problem here? Calculate T 1 T 2 T 3 T 4 ok.

So, essentially calculate T 1 to T 4 ok, using finite volume method and using line by line TDMA we have to calculate, we will just do like one iteration of this sweep or maybe not even one ok. Just one solid solution, fine looks this problem.

Now, you have to help me set up the discrete equations ok. Very easy, because now that you have done the 1 D, you will easily be able to tell what are the coefficients.

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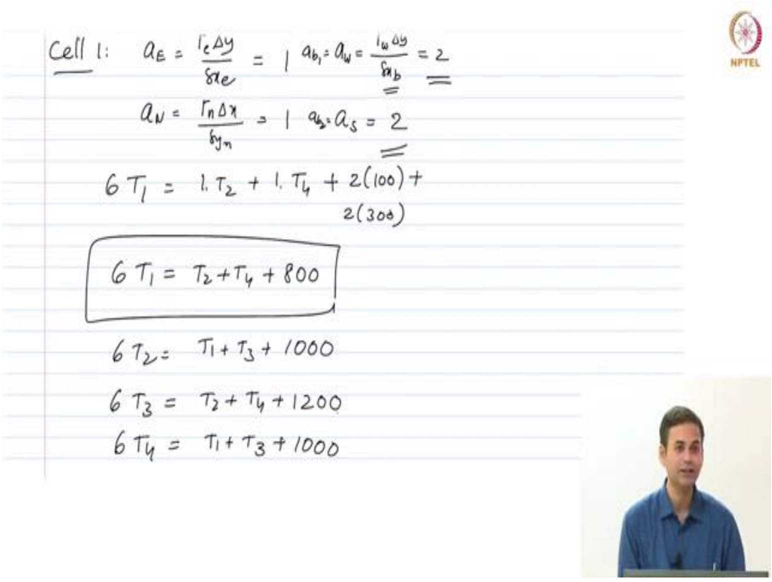

So, let us say a cell 1. What will be a east for cell 1 ok? Area vectors would again be same as delta y. So, we do not have to worry what will be a east? $a_e = \frac{\Gamma_e \Delta y}{\delta xe} = 1$ for cell 1 right. What would this come out to be 1 ok, very good what about a north gamma north delta x by l y north what would this come out to be? 1 very nice. What about a west? $a_w = \frac{\Gamma_w \Delta y}{\delta xb} = 2$

Student: 2.

would become half right, because it is only this much. So, this is only 0.5 as a result this will be 2 and you have if you apply the same logic. What would be a south?

Student: 2.

2, because a west and a south are for these two face centroids. So, this will also come out to be value of 2 ok, very good, very nice. Now, what will be the final equation? a p will have contributions from a east, a west, a east, a north and these two are actually a west, and a south are what? Are kind of boundaries right.

So, this is actually a b 1 and this is some kind of a b 2 right. These are two boundaries one west and one south. So, these coefficients will go into two places; one is the contribution of a p, the other one is when they are multiplied with the corresponding temperature that will go as a?

Student: (Refer Time: 45:16).

b term, source term right, b term on the right hand side. So, what will be the final equation for cell 1? 4 or 6?

Student: 6.

6 right, because you have 1 plus 1 plus 2 plus 2 right. So, this is 6 times$6T_1 = T_2 + T_4 + 2(100) + 2(300)$ T 1 equals what neighbours will be there?

Student: T 2.

T 2 and?

Student: T 4.

T 4 ok. So, that is how much will be coefficients for 2?

Student: 1

1 times T 2 for 4?

Student: 1.

1 times T 4 plus, what else will you have?

Student: 100.

Student: 300.

300 right, that is basically you are these two. So, that is $6T_1 = T_2 + T_4 + 800$ That is your equation for cell 1.

Now, you would directly tell me what would be the equations for cell 2, cell 3, cell 4 without actually calculating them. So, that is the beauty. Now, what is the equation for cell 2?$6T_2 = T_1 + T_3 + 1000$ 6 T 2 equals. What will be the neighbours?

Student: (Refer Time: 46:26).

, how much will be the boundary terms?

Student: 2 into 400.

2 into 400 that would be 800 and 2 into 100 that will be?

Student: 1000.

So, this will be 1000 fine and then what is the equation for cell 3? They are all symmetric right, all the cells share two faces. So, because of that you would get two integer coefficient which is 1 1 and 2 boundary coefficient which is 2 2 right, as a result they all add up to 6. So, this will be $6T_3 = T_2 + T_4 + 1200$ 6 T 3 equals, what are the neighbours for cell 3.

Student: (Refer Time: 47:00).

Cell 3 has 2 and 4 right. So, this will be T 2 plus T 4 plus how much will be the boundary values?

Student: (Refer Time: 47:08).

that is how much?

Student: 1200.

, then for the final cell $6T_4 = T_1 + T_3 + 1000$

Student: (Refer Time: 47:22).

What are the neighbours for T 4?

Student: 1 and 3.

1 and 3. So, that is what will be the boundary value?

Student: 1000.

1000 that is because 200 into 2 plus?

Student: 300.

300 into 2 right that is 500 into 2 that is 1000 fine. So, we got all the four equations. Questions on this part clear, easy right.

Now, of course, if there is a source term you have to be careful you have to add the source term and if there is a dependency on x or T, then you have to be careful, fine ok. Now, of course, we have these equations, what we have to do is we have to apply TDMA in the y direction that is for cell 1 and 2 we have to apply TDMA and then you would use the guess values for T 4 and T 3, while you collect coefficients for these things right that is what we would do and where would the coefficients.

Where would these T 3 T star, T 4 star go while writing these equations for TDMA? They will go into the constant on the right hand side on the boundaries. So, we are kind of out of time. So, I will stop here. Tomorrow, if you can let me write these equations again, we will do a TDMA loop and then move on with under relaxation and other things, fine.