Computational Fluid Dynamics Using Finite Volume Method
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Lecture - 11
Finite Volume Method for Diffusion Equation: Discretization of 2D diffusion equation

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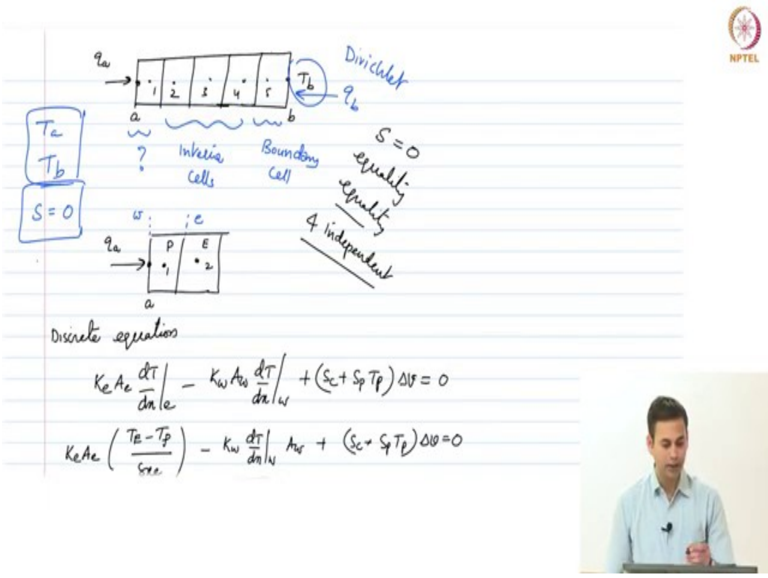

Let us get started. So, we were looking at the solving this 1 D problem which was basically we had a heat conduction problem right, where there are five cells with the given boundary conditions 1, 2, 3, 4, 5. And then we had a isothermal condition on the right hand side and there is a heat flux condition that is given on the left hand side; we called the faces a and b the end faces.

We have looked at how to discretize the interior cells, we have also looked at how to discretize the right boundary cell; the only thing that is remaining is the left boundary cells for which a heat flux condition is given right, that is the only thing remaining in order to solve this problem, ok. So, this one we have not yet looked at. So, let us take a look at this.

So, essentially if I were to redraw this, we have cell 1 and cell 2 ok; where the cell 1 we call it as p cell and the cell 2 is east cell. And this is our face a where there is a heat flux boundary condition that is given, ok. Now if we write the discrete equation, the flux balance equation;

what would that read like? That would read as $K_e A_e \frac{dt}{dx} |_e - K_w A_w \frac{dt}{dx} | w + (S_c + S_p T_p) \Delta v = 0$, that is our discrete equation.

Now, if we look at here; what is our east and west faces here? So, this is our east face and this is our west face happens to be the same as the face label a, right that we have chosen.

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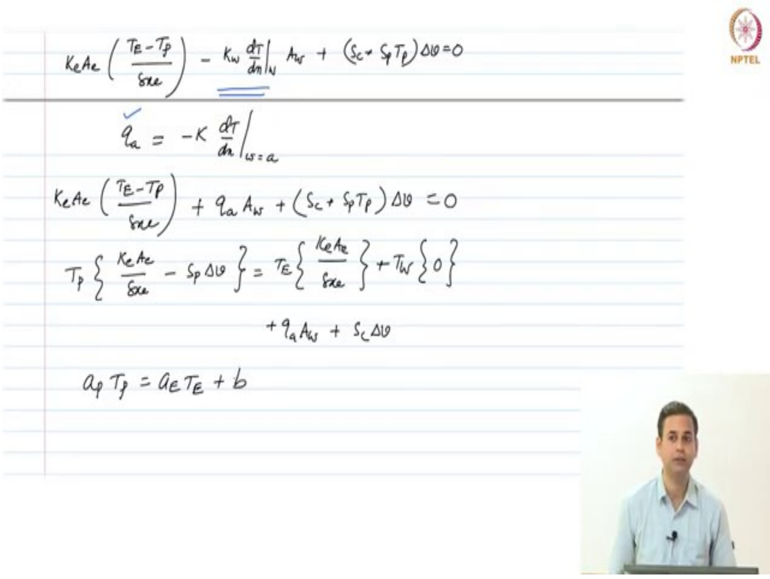

Now, let us go back and again we introduce linear profile assumption for temperature. So, this would be $K_e A_e \left(\frac{T_e - T_p}{\delta xe} \right) - K_w \frac{dt}{dw} | \; A_w + (S_c + S_p T_p) \Delta v = 0$.

Now what is the definition for q a bar or q a? What is $q_a = -K \frac{dT}{dx} |_w = a$ q a?

Student: Minus k.

Minus k.

Student: Student:.

w or basically same as a, right. So, then I can replace the minus K d T d x here with q a right, I can do that. So, if I can do that, then I am going to write the discrete form here which is nothing, $K_e A_e \frac{T_E - T_P}{\delta xe}$¿$+ q_a A_w + (S_c S_p T_p) \Delta v = 0$

So, essentially as you can see, because of the prescription of a heat flux boundary condition; we completely got rid of the flux term here right, we do not have T p minus T west like we had before, ok. So, that is not there. Now if we rearrange in the original form, which is basically T p times the coefficients; that is

$$T_p\left\{\frac{K_e A_e}{\delta xe}-S_p \Delta v\right\}=T_E\left\{\frac{K_e A_e}{\delta xe}\right\}+T_w+\{0\}+q_a A_w+S_c \Delta v$$

We do not have a coefficient to the west right, because that is why the boundary condition is there, plus we have q a times A w; that is the heat flux boundary condition that is already known right, this is already known times the area. Then we have this would be plus S c times delta v right, that is what we have, ok.

So, now if I were to write again in the coefficient form, this would be $a_P T_p = a_E T_E + b$ ok.

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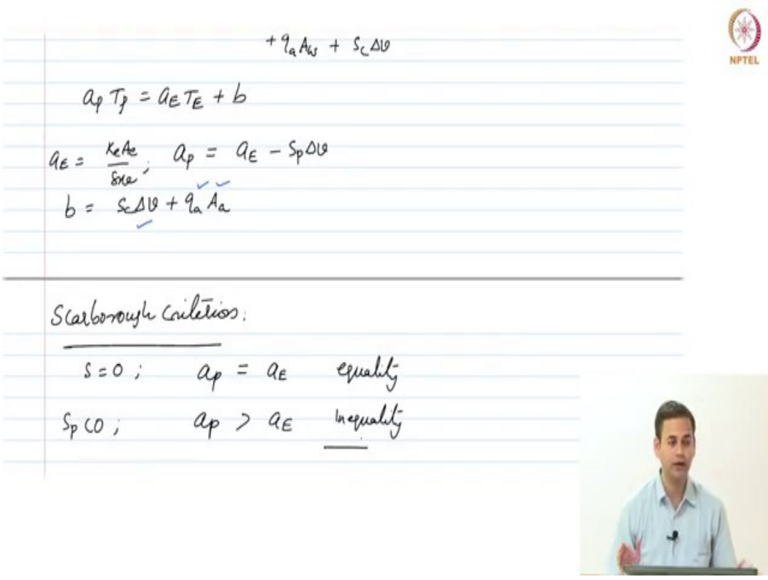

Where a east is nothing, but $\frac{K_e A_e}{\delta xe}$ What would be.

Student: a.

a p? $a_p = a_e - S_p \Delta v$; there is no a west, a west is 0 right, there is no a west is a 0. So, this will be a east minus S p times delta v. What would be b? $b = S_c \Delta v + q_a A_a$.

Student: Plus.

Plus q a times A w or A a right, both are the same that is what we have; because q is known, cross section area is known these all go to the right hand side, right these are all known, so far so good. Now, what about the Scarborough criteria? If we have Neumann boundary condition; what about Scarborough criteria? For a particular cell that has a heat flux boundary condition. What is a p now? Is it greater than, let us say if we have a source is 0, s is 0; in the absence of sources, what is a p? $a_p = a_E$

Student: It is equal to.

Equal to.

Student: (Refer Time: 06:21) a E.

a E, right. So, this only satisfies in equality; does not satisfy in inequality, right. If there was a source, then this would be a p would be greater than a east right; because S p, if I assume that S p is less than 0, in which case a p would come out to be greater than a east right which it satisfies in inequality as well, ok.

Now, what does it mean? Let us say if we have a problem ok; if I kind of go back to the problem statement and I say that instead of the constant temperature condition here, that is instead of this Dirichlet boundary condition; if we also specify a heat flux here, if we specify q b, ok. If I specify q b here being applied on this right face b; then what will happen to the Scarborough criteria for all these cells?

For the interior cells we said, if let us say source is 0 ok; for all the interior cells if the source is 0, Scarborough is satisfied in what?

Student: Equality.

Equality right; it satisfies in equality. What about the boundary cells now?

Student: (Refer Time: 07:37).

It will also only satisfy in equality, right. So, then are we actually satisfying Scarborough criteria in inequality anywhere, for any of the cells; we are not, ok. Then of course, the question that pops up is well will this converge right and it is completely physical right; it is physically possible to have two heat flux conditions, right. Let us say you have insulated on one side and providing heat flux on the other side; then if I want to solve it numerically, can I solve it? Yes, you can.

So, what you realize is that, out of the two Neumann boundary conditions that you have given; there will be one less equation, essentially one of these equations will become dependent on the other one, ok.

So, out of the five cells you have, up till now you have five independent equations for all the five cells; then what you realize is the moment you specify all the heat flux boundary conditions for the problem, you will get a one you will get a rank deficiency, ok. Essentially that one of the equations that you get out of the five will be dependent on the other equations, ok.

So, as a result you will only have four independent equations and you will be able to solve; then as a result you will satisfy Scarborough, because one of the cells would come out to be a known value ok, in that context you will be able to still satisfy Scarborough and get an answer using an iterative method or a direct method,. This we will probably see in a particular assignment problem or something like that ok; I will try to set up an problem like that, fine is it clear.

Questions till now, if I have two Dirichlet boundary conditions; let us say if we specify T a and T b what will happen? And in the absence of sources, do we satisfy Scarborough in inequality and equality?

Student: (Refer Time: 09:21).

Yes we do right, all the interior cell satisfy inequality and the boundary cells will start satisfying in inequality and the problem converges without any issues right; because you get a diagonally dominant system, alright let us then move on.

So, what I plan to do is, we look at the steady diffusion equation in two dimensions ok; then also look at the boundary conditions there off, then we will come back to a discussion on how to calculate diffusion coefficient on the faces, that we will come back once we finish the diffusion equation in two dimensions.

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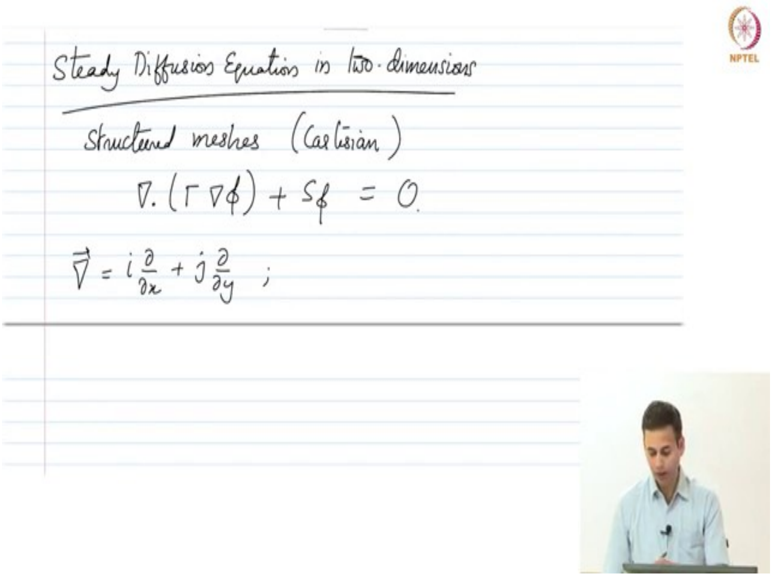

So, this is a steady diffusion equation in two dimensions and we are still looking at structured meshes. We have not discussed, but we are actually looking at structured Cartesian meshes; so essentially structured Cartesian meshes are is what we are looking at implicitly, right.

So, how does the steady diffusion equation in two dimensions read? It will read the same as the previous one; essentially what you have is, you have $\nabla.(\Gamma \nabla \phi)+s_\phi=0$, where your del bar is now, because your in two dimension, this will be $\vec{\nabla}=i\frac{\partial}{\partial x}+j\frac{\partial}{\partial y}$ that is your nabla operator.

Then we have to have a domain to discretize this equation on the particular cells. So, I am going to draw a particular domain here.

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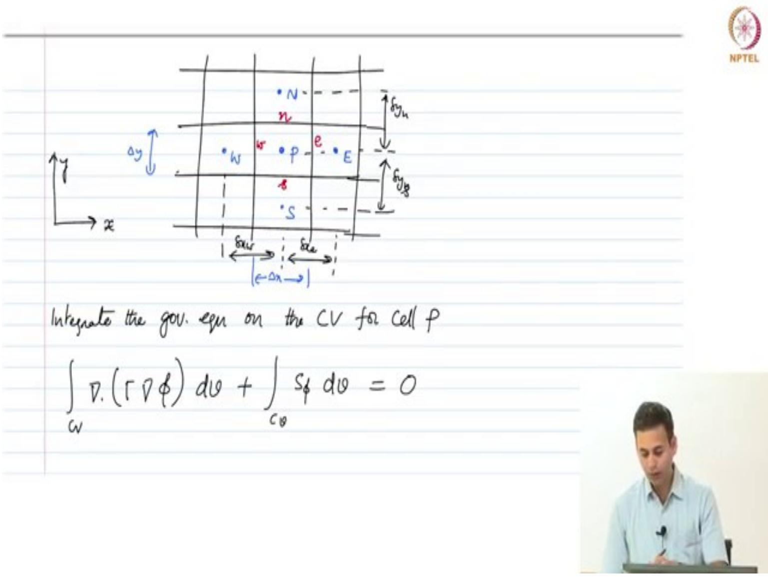

The domain we choose looks like this, ok. So, this is my. So, this is a domain that is discretized, ok. So, this is the mesh, just like before we would like to call our cells. So, we call this as P cell and we call this cell as East, this as West, this as our North, this as South.

And then essentially we have x and y coordinates aligned in the Cartesian axis, alright. Then we also of course, need to have a dimensions for the widths of these things. So, this is essentially we have a width of delta x for the P cell and delta y in the y direction for the P cell, ok.

We also need other dimensions that is this distance that is between cell centroid of P and capital E would be del x e just like before and this distance between P and w would be del x w. Similarly the distance between cell centroid of P and North would be del y North, and between P and South cells centroid would be del y South, ok. So, we have all these widths for the cell P and for the distances between the respective cell centroids.

Of course we have to also identify the faces which lie between these cells and they happen to be I will use. So, this will be face e east which lies between P and capital E, and this would be the west face little w; this will be the north face and this will be the south face, ok. So, we have east west, north, south faces which form this cell, primary cell P, that is all, ok.

Now we are ready to start discretizing the equation, that is first step is to integrate the governing equation on the control volume for cell P, ok. So, essentially that leads us to

$$\int_{cv} \nabla . (\Gamma \nabla \phi) dv + \int_{cv} s_{\phi} dv = 0$$

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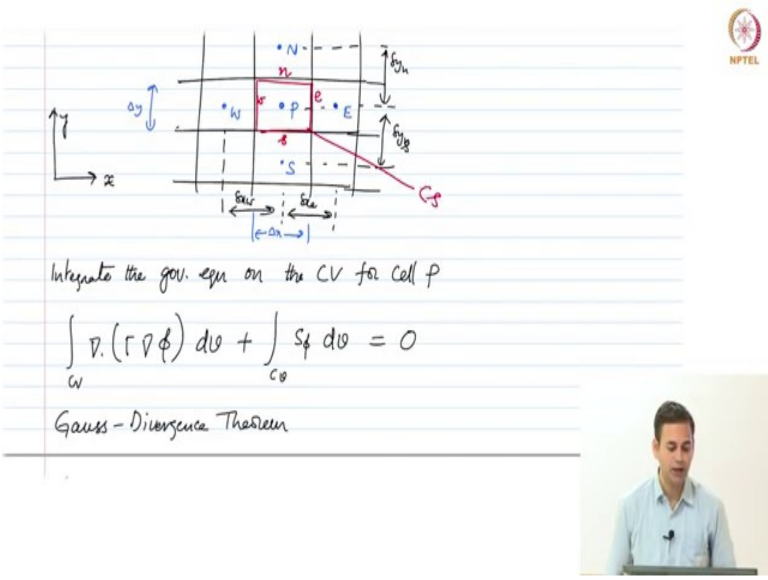

Now, what do we do, we invoke Gauss Divergence Theorem and going to convert this volume integral into a surface integral. So, this will be control surface gamma grad phi dot d A bar plus integral control volume S phi d v equals 0, that is what we have. Now what is this control surface in this context? Control surface is now made up of.

Student: Four faces.

Four faces right, the four faces; that is the east face, the west face, the north face and the south face, right. So, this is our control surface, ok.

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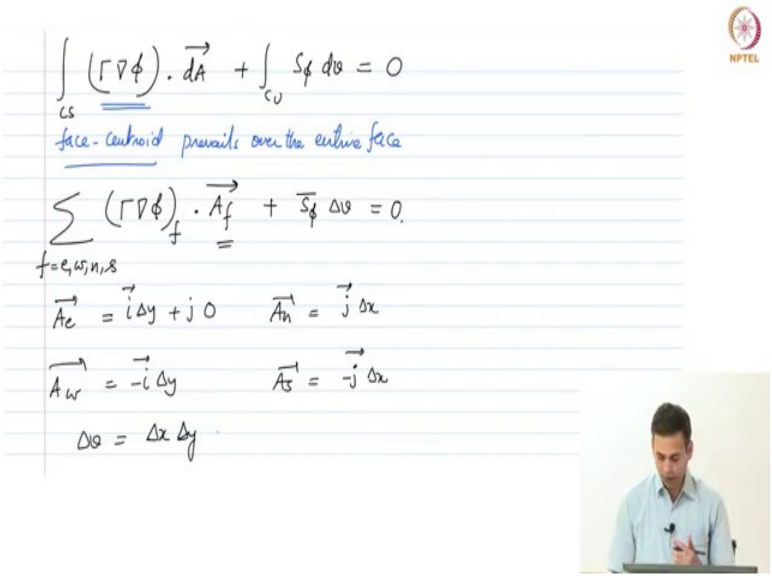

And we assume that the control surfaces are planar and this value the gamma grad phi that we have here ok; if we evaluate at the face centroid and that value prevails over the entire face, ok. If I assume that, then this is a constant for a particular face, ok.

Then that constant I can integrate and then replace this integration with a summation ok; and that summation would lead to now a discrete form of this integration that is nothing, but sigma on all the faces, $\Sigma\left(\Gamma\,\nabla\phi\right)_f.\overrightarrow{Af}+\vec{s}_{\phi}\Delta v=0$Now what are the faces that we have; f goes from little east, west, north and south, ok. So, we have these four faces plus we have already also said that S phi has to be evaluated for the entire cell right; we need to calculate what is S phi d v.

And if we assume that S phi, the value of S phi at the cell centroid prevails all the entire cell; then this can be written as some average value of S phi bar calculated at the cell centroid times delta v equals 0. Now what is. So, each of these will be, here the sub f here would change from east, west, north, south, ok. So, what will be vectors A east, A west, A north and A south? So, if I go back, what will be the vector A east bar?

Student: (Refer Time: 16:54).

So we assume that all the area vectors point outside ok, out of the cell. So, what will be A e bar?

Student: i (Refer Time: 17:02).

A e bar would be?

Student: i e bar (Refer Time: 17:05)

I would be the point direction which is pointing; what will be the magnitude of th?

Student: (Refer Time: 17:11).

Delta y right, delta y is the area right; delta y times i, right. So, $\vec{A_e} = \dot{i}\Delta y + j0$ it is pointing in the positive x direction; is there a component in the j direction?

Student: No.

No. So, essentially 0 right, we do not have a component. So, that is 0 plus j times 0. What will be A west bar?

Student: Minus.

Minus, i times the; what is the magnitude?

Student: Delta y.

Delta y right, it is the same square cell a rectangular cell, ok. So, this would be $\vec{A_w} = \vec{-i}\Delta y$ minus i times delta y. What about a north?.

Student: (Refer Time: 17:54).

Delta x is the magnitude, ok. So, we have heat here, delta x is the magnitude. And it is pointing in which direction?

Student: j direction.

j direction, so this is A north bar. So, this would be, if I were to write this will be j times delta x plus i times 0, right. We do not have any component in the i direction. What about A south? It will be minus j times delta x, right. If you like a bar, I will put a bar on this, ok. So, these are the area vectors for the primary cell p, ok. Now what about delta v? We have this guy delta v which is the volume of the cell in two dimensions, this would be delta x times.

Student: Delta y.

Delta y times 1 right, it is anyway two dimensional problem. So, essentially we are doing this, because we have to now evaluate this dot product of gamma grad phi on, let us say on the east face and for the west face and so on, right. And then we have to make a summation here, so that is why we are doing all this.

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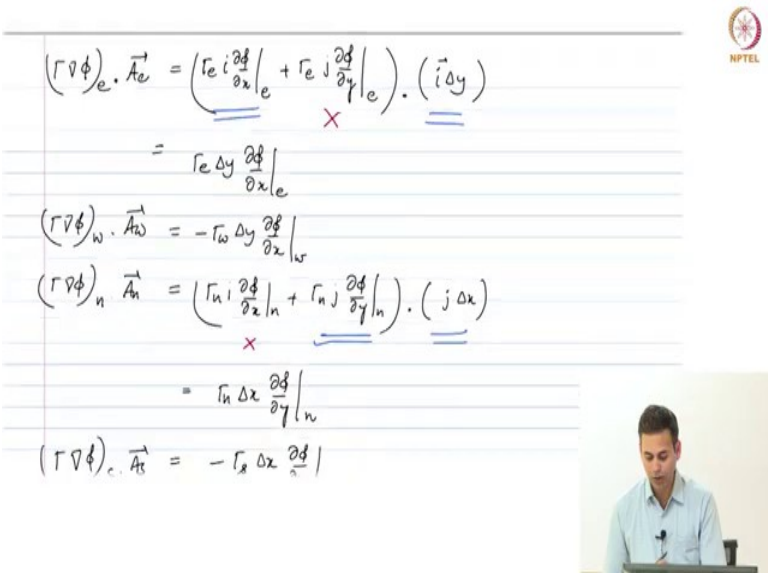

So, let us look at the first component. So, that is in the summation that is $(\Gamma \nabla \phi) . A_e =$¿)_

This is the first term that we get in the summation. What about gamma grad phi east? This is nothing, but gamma east; grad phi is i partial partial x plus j partial partial y. So, this will be i partial phi partial x evaluated at east and then we have plus gamma east j times partial phi partial y; evaluated at what?

Student: East.

East right times, what is A e bar?

Student: i times.

i times delta y, ok. So, which term survives out of this inner product?

Student: (Refer Time: 19:46).

Only the first one right, we do not have a j component here, so essentially it is a dot product; only this guy survives, we do not this guy will not survive, because there is no j component there e c. Then this will be what? Gamma east delta y partial phi partial x e times i dot i that is 1, ok. So, we have this term that survives simple.

Now without actually calculating this way, you can now guess what would be gamma grad phi west dot A west bar, gamma west.

Student: Delta y.

Delta y.

Student: Delta y.

Partial phi partial x evaluate at w.

Student: Minus.

With a minus, because this comes with a minus i bar delta y, ok. So, we can write that, ok. Now very good, let us move on; what would be gamma grad phi north dot A north bar? So, if it comes to north, I can write it here just for convenience. So, this would be gamma north i partial phi partial x north plus gamma north j partial phi partial y north dot. What would be A north?

Student: j times.

j times delta x, right. So, the only term that survives is the j component term, ok. So, this term does not come into picture. So, as a result what would be gamma grad phi north dot A n bar? Gamma north.

Student: a bar.

Delta x.

Student: A bar.

Partial phi partial y evaluated on.

Student: North.

north, ok. So, similarly gamma grad phi s dot A s bar is would be how much? If you can, it will be a minus gamma little south times delta x times partial phi partial phi, partial phi partial y evaluate on the south face right; is this correct? Yes, ok. So, essentially what we did is, we have evaluated all the four terms for all the four faces that come out of this gamma grad phi f dot A f bar, that is what we have done.

We have to basically sum all these guys ok, and then I am going to use the same linearization for the S phi bar, ok.

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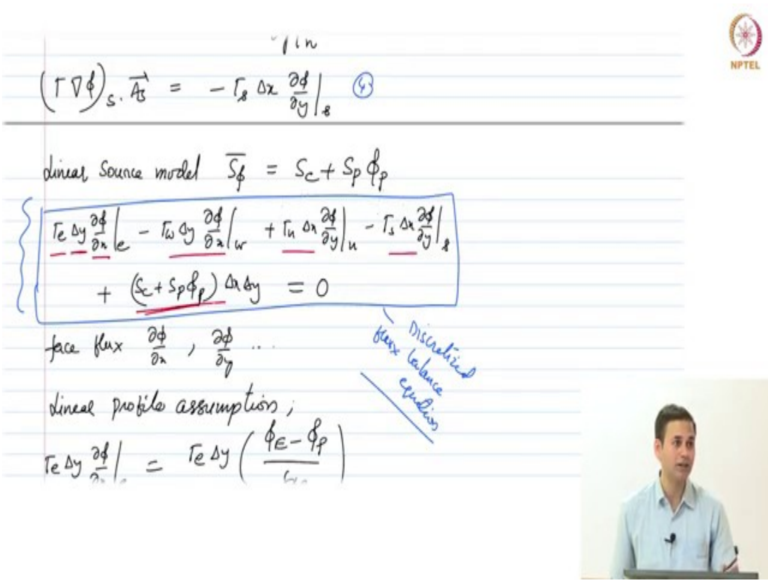

So, I am going to use a linear source model ok, which would tell me what is $\overrightarrow{S_\phi}=S_c+S_p\phi_p$ So, we are going to use this source model, fine. So, if I were to put all of these back; what would that be?

The equation would be $\Gamma_e \Delta y \frac{\partial \phi}{\partial x}|_e - \Gamma_w \Delta y \frac{\partial \phi}{\partial x}|_w + \Gamma_n \Delta x \frac{\partial \phi}{\partial y}|_n - \Gamma_s \Delta x \frac{\partial \phi}{\partial y}|_s + (S_c + S_p \phi_p)\Delta x \Delta y ¿ = 0$ that is what we have here right, the second term is here. This is the first term, this is the second term, third term and the fourth term right in the summation that we have. That is the first term, plus we have right. Is it correct or any mistakes in here? Fine, alright.

Now what do we do? We have to introduce, so we got the face fluxes; the face fluxes which are the partial phi partial x partial phi partial y terms, these we have to express them in terms of the cell centroid values, ok. For that we are going to make a linear profile assumption ok, and express these guys; that is $\Gamma_e \Delta y \frac{\partial \phi}{\partial x}\Big|_e = \Gamma_e \Delta y \frac{\phi_E - \phi_p}{\Delta xe}$ gamma p upon, what? What would be the distance by which this is changing?

Student: Del x e.

because that is the distance between the cell centroids; we are looking at phi capital E, we are looking at phi capital E here and phi p minus the distance is del x e, alright.

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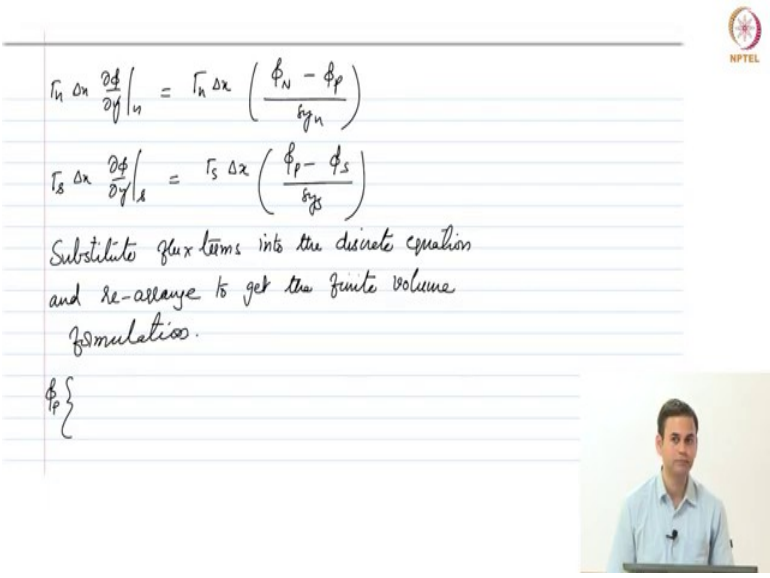

So, similarly what about the gamma w delta y partial phi partial x west? This will be gamma west delta y phi p minus w upon del x w, ok. What next? $\Gamma_n \Delta x \frac{\partial \phi}{\partial y}\Big|_n = \Gamma_n \Delta x \frac{\phi_N - \phi_p}{\delta y_n}$

Gamma north del x partial phi partial y north right; this would be how much?

Student: Gamma north.

Gamma north delta x times phi.

Student: north minus.

north or p?

Student: p.

Student: North.

North minus.

Student: Phi p.

Phi p upon.

Student: Del del y.

Del y north, ok. What about south $\Gamma_s \Delta x \frac{\partial \phi}{\partial y}\Big|_s = \Gamma_s \Delta x \frac{\phi_p - \phi_s}{\delta y_s}$

Student: Gamma s.

Gamma south.

Student: Delta x.

Delta x times.

Student: Phi p.

Phi p minus.

Student: Phi s.

Phi south upon.

Student: Del y.

, is this clear or any questions in this part, essentially did same linear profile assumption; instead of in the x direction, we have now applied in the y direction as well,. Alright this is simple, now we all have to do is, we have to plug these things back into this equation, ok. So, this is our discretized equation. So, this is our kind of discretized or the flux balance equation which we will come back little later, ok.

So, as you can see there is a minus for the west and the south terms, so that we have to consider while we plug in these quantities, ok. Now can you plug in these quantities into the equation and club or essentially separate the terms as the coefficients for phi p and the coefficients for phi east phi west phi north phi south and write it in terms of a phi p equals a east phi east and so on, right.

Can you work on that and tell me? So, plug in all these quantities back into the discrete flux balance equation and rearrange the terms, such that we get our original equation, ok.

So, essentially substitute the flux terms into the discrete equation and rearrange to get the finite volume formulation. What should be the coefficients for phi p? So, essentially collect all the coefficients for phi p and then send them to the right hand side.

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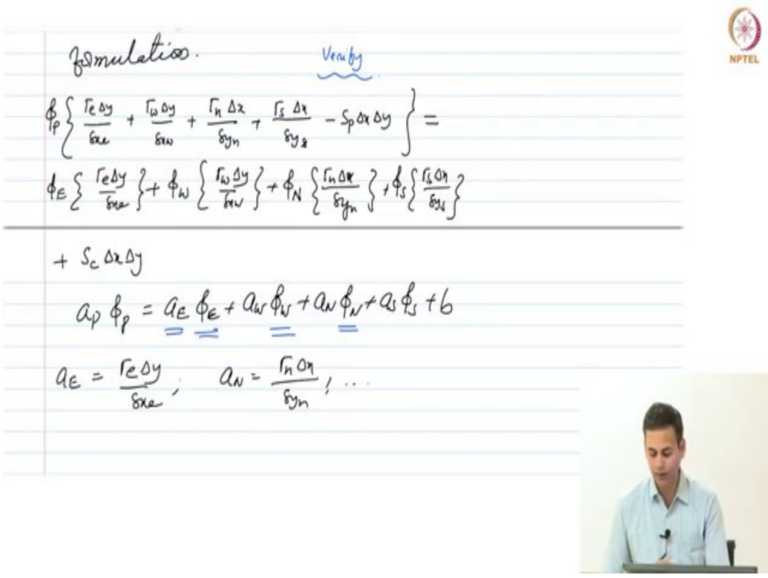

What are the coefficients anybody for phi p?

Student: Gamma east.

$$\phi_p\left\{\frac{\Gamma_e \Delta y}{\delta xe}+\frac{\Gamma_w \Delta y}{\delta xw}+\frac{\Gamma_n \Delta x}{\delta yn}+\frac{\Gamma_s \Delta x}{\delta ys}-s_p \Delta x \Delta y\right\}=\phi_E\left\{\left\{\frac{\Gamma_e \Delta y}{\delta xe}\right\}\right\}+\phi_w\left\{\frac{\Gamma_w \Delta y}{\delta xw}\right\}+\phi_N\left\{\frac{\Gamma_n \Delta x}{\delta yn}\right\}+\phi_s\frac{\Gamma_s \Delta x}{\delta ys}+s_c \Delta x \Delta$$

Gamma east delta y upon del x e right.

Student: Plus.

Plus gamma west.

Student: Delta y.

Delta y by.

Student: Del x w.

Del x w plus.

Student: Gamma n.

Gamma north.

Student: Delta x.

Delta x upon del y north plus gamma south.

Student: Delta x.

Delta x upon.

Student: Del y.

Del y south ok; now if you write the remaining east, west, north, south on the other hand.

Student: (Refer Time: 28:36) plus source.

Plus source term.

Student: Minus sign minus Sp.

Minus very good. So, this will be minus S p times delta x delta y; is that correct?

Student: Yes.

Fine and equals on the right hand side what we have, phi east times.

Student: Gamma east.

Gamma east delta y by del x e plus phi west times.

Student: Gamma.

Gamma west delta y by del x w plus phi North times.

Student: Gamma.

Gamma north delta x by.

Student: Del y.

Del y north plus phi south times gamma south delta x by del y south, right plus.

Student: S c into.

Sc times.

Student: Delta x.

Delta x delta y, ok. So, these are that is alright, that is all we have; we do not have any other minus other than the minus S p delta x delta y term, ok. So, this if you have not done, you can do it, and then check this part ok; essentially verify that you get this equation, fine. So, we want to put this equation back in the original form, that is a p phi p equals a East phi East plus a West phi West plus a North phi North plus a South phi South plus b, ok.

So, we have written all these equations. Now one thing we have to keep in mind is that, we are multiplying the neighboring values; that is the phi east phi west all these things right, these are all the cell centroid values.

The coefficients when I am writing, I am using a capital E to indicate the coefficient for phi capital E; but we realize that this coefficient actually depends on where, on the face right, because it contains $a_E = \frac{\Gamma_e \Delta y}{\delta xe}$. So, this actually is evaluated the faces ok, that have to be kept in mind, ok.

Now, what is; of course, we can explicitly write it down, I am not going to write all of them, but going to write a East a North is$a_w = \frac{\Gamma_n \Delta x}{\delta yn}$ and so on for the other coefficients.

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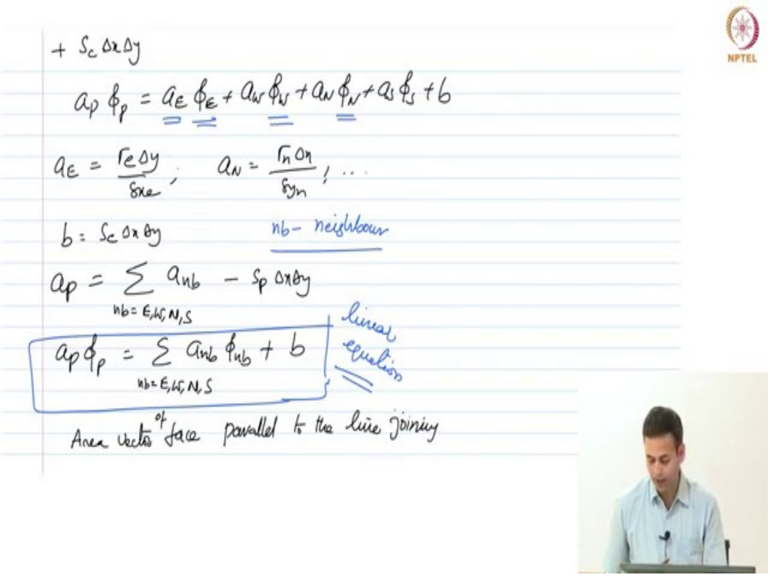

And what is b? B is $s_c \Delta x \Delta y$Now what is a p? A p is if you look at here it is a east plus a west a north plus a south minus S p delta x delta y, ok. So, this is nothing, but sigma a n b, where n b I would like to write it as a neighbor; East, West, North, South ok, sigma a n b minus S p times delta x delta y ok, where I am writing n b as a short form for neighbor, is that correct.

$a_p = \Sigma a_{nb} - s_p \Delta x \Delta y$; of course we can also simplify our equation in terms of n b, I can write this as$a_p \phi_p = \Sigma a_{nb} \phi_{nb} + b$, where n b is basically capital E, W, North and South, ok.

So, this is our final linear equation, the discretized linear equation; which we have to write one such equation for every cell, and then we have to use some linear solver to obtain the solution for phi all over the domain. Questions still now; is this part clear now. Now what you see is; because all the faces are aligned with the x and y axis, we did not get any cross terms, right.

For example, east is always evaluated; you know the east face always had this x derivative, it never had the y derivative, right. These things never came right; otherwise earlier when we started off, we had these terms, right. We had, on the east face we had a y derivative, but that never showed up, right.

Similarly, on the north face, we had a x derivative and a y derivative; but the x derivative never showed up right, because they all got, they all kind of disappeared. Why did all these terms disappear? Because the mesh is.

Student: Structured mesh.

Not only structured, because the mesh is orthogonal, ok.

Student: Cartesian.

The essentially it is orthogonal, it is kind of Cartesian. Cartesian meaning it is orthogonal; that means your area vector right that you have is aligned with one of the coordinate directions, right. That mean the line joining the cell centroids right, that is your area vector is aligned with one of the coordinate directions, ok. Or in other terms essentially the area vector of the face ok, is parallel to the line joining cell centroids, ok.

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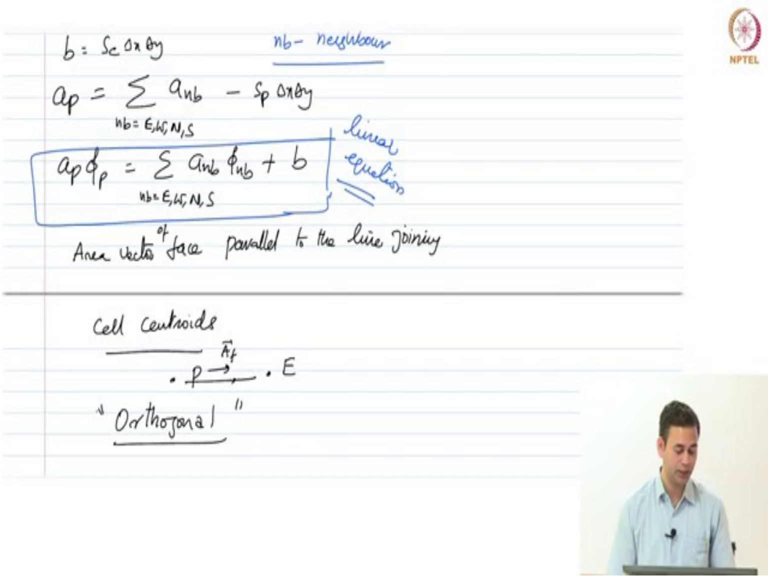

That means if you are going to draw a line between P cell and East cell between the cell centroids, this line is aligned with the area vector that we have, ok. It is parallel; so these kind of meshes are known as orthogonal meshes.

We will come to this little later as we look at the unstructured meshes, ok. If these two are parallel, then we have an orthogonal mesh; as a result you will never get these cross terms, right. They will always drop out; otherwise you end up with these cross terms as well, ok. So, that is one thing we have to keep in mind, alright.

Let us look at some comments on what we developed, ok. So, let us look at some comments for the 2 D diffusion equation, ok. What is the first comment? The first comment is of course similar to the 1 D case if we go back; the discretized equation that we have here. What is this? This is an equation, this has some physical significance, right.

What is it indicating? It is indicating the flux that is gamma east times partial phi partial x, times the area, flux times the area terms, right. So, this is all flux times the area, is balancing the source that is getting generated, right; that means the flux that is leaving times the area out of the east face minus what is entering through the west face and whatever is leaving through the north face minus whatever is entering to the south face is balanced by whatever is generated inside, ok.

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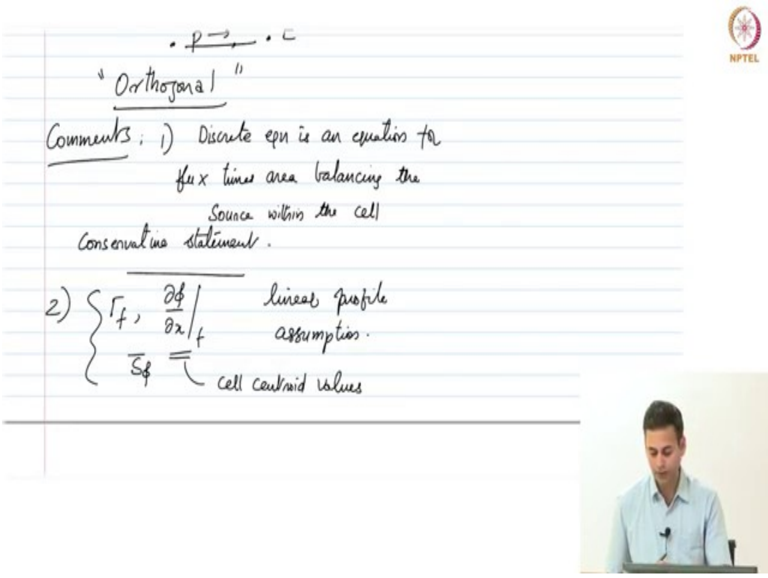

So, that is basically we have a the, there is a the discrete equation is an equation for flux times area balancing the source within the cell, right. So, this equation is a conservative statement or a statement of conservation ok, which tells whatever is leaving or entering through the cells is balanced by whatever is generated in within the cell, ok. So, that is something that we need to look at.

Then we have of course introduced for the gammas and for the fluxes that we got on the faces. So, we have introduced a linear profile assumption, right. Of course, we have also introduced a profile assumption for averaging for S c bar also; we have also linearized this thing.

So, the diffusion coefficient, the fluxes and the source terms; we have introduced a model, that is a linear profile assumption or a model for all of these things. And we connected the face fluxes that we got; similar to the 1 D case, we connected it to the cell centroid values, right. By introducing this thing, we connected this to cell centroid values, fine.

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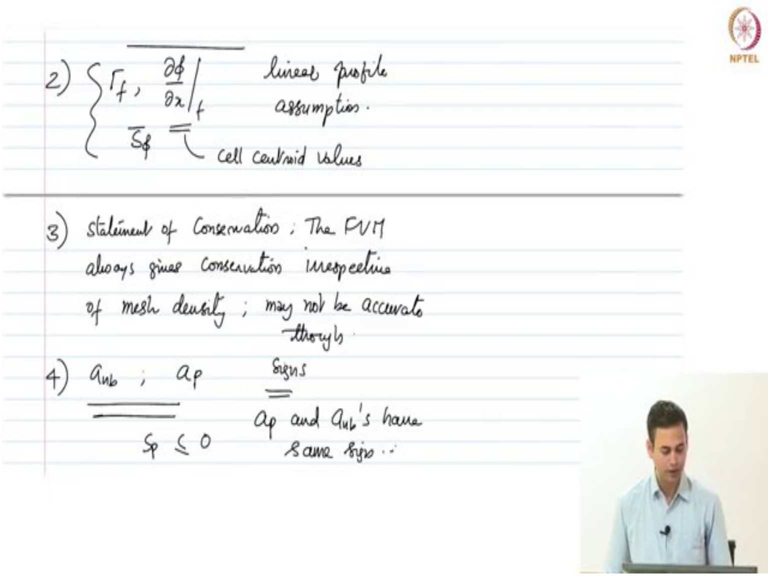

Now, because we have a statement of conservation, the finite volume method always gives conservation irrespective of mesh density, right. If we take core cells or if we take fine cells, it is always going to give conserved solution ok; but of course it may not be accurate.

So, that we have to see, may not be accurate though, that we have to see depending on the accuracy that we want; but otherwise, because phi is found from this by satisfying this discrete equation, it always gives a statement of conservation ok, alright.

So, we have a statement of conservation here. Now, what else do we have? We have in terms of properties. Then what about the, these coefficients; the coefficients that we have that is a east, a west, a north, a south and a p. What is the, what are these signs of these, these coefficients? a and b are always.

Student: Positive.

Positive. What about a p? a p is summation of a and i bus. And there is minus S p and we said S p is always, I am assuming S p is negative. So, this is. So, they have the same sign, ok. So, essentially a p and a n b's have same sign, ok. Now is it what does it mean if they have the same sign?

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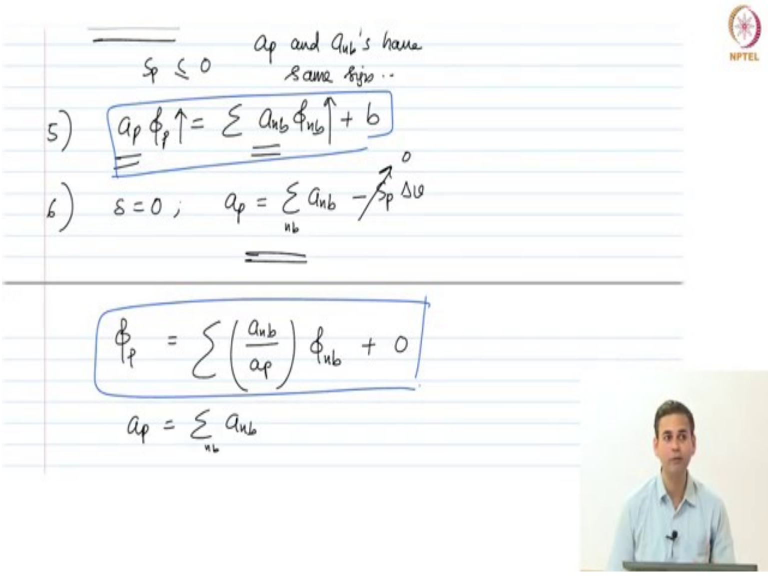

If they have the same sign, because we have$a_p \phi_p \uparrow = \Sigma a_{nb} \phi_{nb} \uparrow + b$; if I have the same sign for a and b and a p. If my phi n b, that is the phi for east, west, north, south or something it goes up; what will happen to phi p? Would it increase or decrease?

If these increases, they have the same sign; what will happen to phi p also? Increases; so essentially if this increases, this increases, right. This is a property that has to be satisfied for elliptic equations. For example, in an elliptic equation we have discusses you know in the initial lectures; that if there is a disturbance somewhere, it has to propagate in all directions and the properties should be the final phi should be smooth everywhere, ok.

So, as a result because these have the same sign; as phi n b goes up, phi p goes up. So, this is the property of elliptic equations which is now satisfied by the discrete equation that we have developed. That is one property. The other property is, in the absence of source terms, let us say$S=0$ what is a p? $a_p = \Sigma_{anb} - S_p \Delta v$; but we have said that the source is 0, S is 0.

So, what is a p? a p is simply summation of sigma a n b, ok. Now what does that mean? What that means is, if we rearrange this equation. If I were to write $\phi_p = \Sigma\left(\frac{a_{nb}}{a_p}\right)\phi_{nb} + 0$; I do not have source term, so this is 0, ok. Now I got some equation here; but what is my a p? Sigma a n b. Now what does this equation tell us? This tells us that phi p is a.

Student: (Refer Time: 41:22).

Linear combination of all the phi n b's right; and because a n b by a p, this will always sum to what?

Student: 1.

This will always sum to 1. So, phi p is always bounded by p n b's, right. Now I use this word called boundedness.

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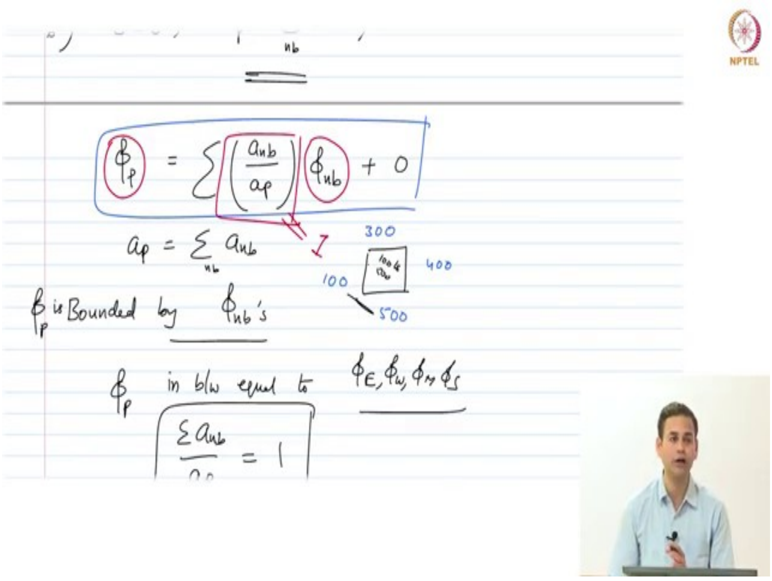

So, this is bounded. So, phi p is bounded by phi n b's. What does it mean? That means, the phi p value that I am going to get would be, will be somewhere in between or equal to phi East, phi West, phi North, phi South, right.

That is because the fraction, you know the coefficient that it is multiplying is a satisfies this equation right; sigma a n b by a p this equals 1. That is why this is bounded right; that means if you have let us say several values. Let us say you have 300, 400, 500 and let us say 100; what will be a possible value for phi p? Can phi p be 700?

Student: (Refer Time: 42:36).

It cannot be, because it can never be greater; because we do not have a source term. So, this will be somewhere, this cannot be greater than what?

Student: 500.

It cannot be greater than 500, right, and can it be less than 100.

Student: No.

It is not allowed to be less than 1. So, it has to lie somewhere between 100 to 500, right.

Student: Yes.

Between the maximum mean, it has to be bounded somewhere between 100 to 500. That is what this equation ensures. Now this is another property of the elliptic equations right; because you have the smoothness, as a result of which in the absence of source terms your solution should be bounded by the neighboring values. You cannot all of a sudden get some other value which is unphysical.

So, these are the two properties of elliptic equation which are now satisfied. These are the properties of elliptic equations which are now satisfied by the discrete equations that we have developed. Elliptic equation now these satisfied by the discrete equations developed. Now if we have a source term; would it be, can it be larger?

Student: Yes.

Yes it can be, because a p is now not just sigma a n b; but you have sigma a n b plus something right, that is minus S p delta v. Is that physical? That is physical, let us say you have a heater somewhere in the room and those places is where it will be larger than the neighboring values, right. That is completely physical, ok. So, that is in the presence of a source term, it is physical, alright.

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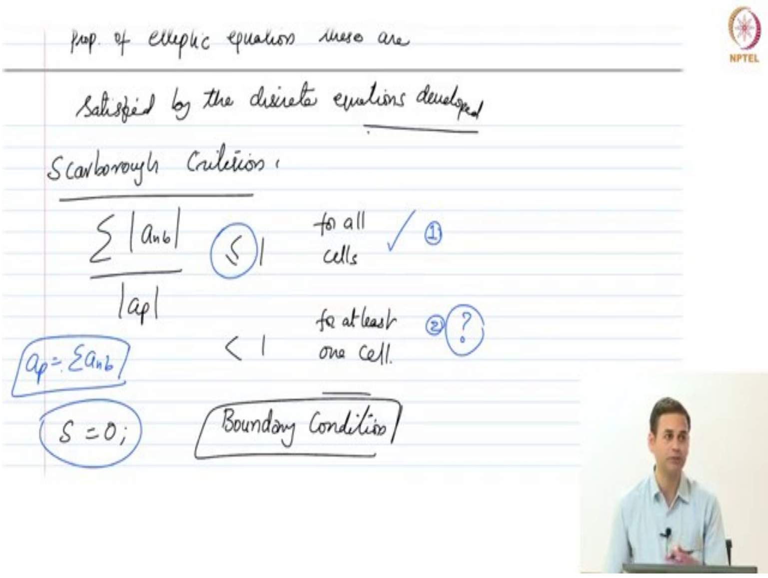

Now, let us look at the Scarborough criteria. What does Scarborough criteria say? It says that sigma modulus a n b upon mod a p has to be what; less than or equal to 1 for all cells and less than or equal to 1 for at least one cell, right.

Let us say we do not have source ok, let us say S is 0. Source is 0; then which of these is satisfied, the first one or the second one, by the equations that we have developed? So, if we do not have a source term; what is a p? $a_p = \Sigma a_{nb}$that is all. So, only equality is satisfied, ok.

So, essentially this is satisfied. What about inequality? If you have a source term, it will be satisfied; if you do not have a source term, it will not be satisfied, ok. So, in the absence of sources, the inequality is not satisfied. Now, how do we make this satisfy?

Student: (Refer Time: 45:41).

Through the boundary conditions that we have already seen like yesterday, ok. So, if you have boundary conditions, then it may be possible to satisfy the second condition ok, so that this Scarborough criteria is satisfied as well. Questions till now.

Student: (Refer Time: 46:02).

These Scarborough criteria, ok. These Scarborough criteria explanation essentially you have, you know the condition right; sigma modulus a n b upon mod a p should be less than or equal to 1 for all the cells in the domain and should be less than 1 at least for one cell, ok. Now in the absence of sources that is $S=0$ we know that what we have developed is a p equals sigma a n b, right.

This is what we have; that means, we are only satisfying this less than or equal to condition, right. So, for all the cells, it will be equal to, a p would be equal to sigma a n b. But, for none of the cells a p would be greater than its neighbors values right, in the absence of source term. If there is a source term, then a p would be greater than sigma a n b by minus S p times delta x delta y, or if there is a boundary condition a p would be greater, ok.

So, in that context this Scarborough will be satisfied for both 1 and 2 conditions; Otherwise it will be only satisfied for the condition 1 right, it will be only less than or equal to mod a p or less than or equal to 1. Other questions.

Student: If boundedness less than (Refer Time: 47:12).

Boundedness. Question is about explain boundedness? So, take for example, few numbers, ok. Essentially, you have a n b's; choose some numbers and choose a p as sum of all of them, ok. These are all positive and the a p is summation of all the numbers. Now what I am saying is, the value of phi p that you are going to calculate from phi East, phi West, phi North, phi South, would always be bounded by these neighbors; because the coefficients with which you are multiplying sum to 1, right.

Because this sigma a n b by a p is how much? This sums to 1. As a result, if you have taken some numbers, it will then this phi p would always come out to be limited by the neighboring values that you have, right.

So, what I suggest is you can choose some a n b's and then see for yourself that these fractions would always make it to come between these values only; it will never be higher than this or it will never be lower than, it will never be lower than 100 or it will never be higher than 500, ok. That is because sigma a n b by a p sums to 1.

As a result, we say that the value for the cell centroid is always bounded by the neighbors ok. It is not more than it is cannot be more or it cannot be lesser than the lowest value, ok. That is what I call boundedness, which I may refer to it from time to time, ok.

Other questions clear, ok. One thing we have not discussed is the problem, the 1 D problems; essentially, we have now set up equations for 1, 2, 3, 4, 5 cells, right. So, you have 5 equations and then you have to solve those equations using some method; that could be a Gauss Seidel or direct method and things like that, ok. Then tomorrow, I am going to look at the assignment; essentially quickly go through how to solve the assignment.

And then, I hope by tomorrow you will take a look at the assignment and come back with questions; then we can answer all the questions, then we will start off with the boundary conditions in two dimensions, ok. So, we are going to look at the Dirichlet, Neumann and mixed boundary conditions and then move on from there, alright.

Thank you.