Computational Fluid Dynamics Using Finite Volume Method

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Lecture – 10
Finite Volume Method for Diffusion Equation: Discretization of 1D Diffusion Equation

Good morning everyone let us get started. So, we finished 2 chapters that is the review of governing equations and overview of numerical methods or the of the course of the past lectures. These were very short chapters it was kind of an overview and the review of governing equations.

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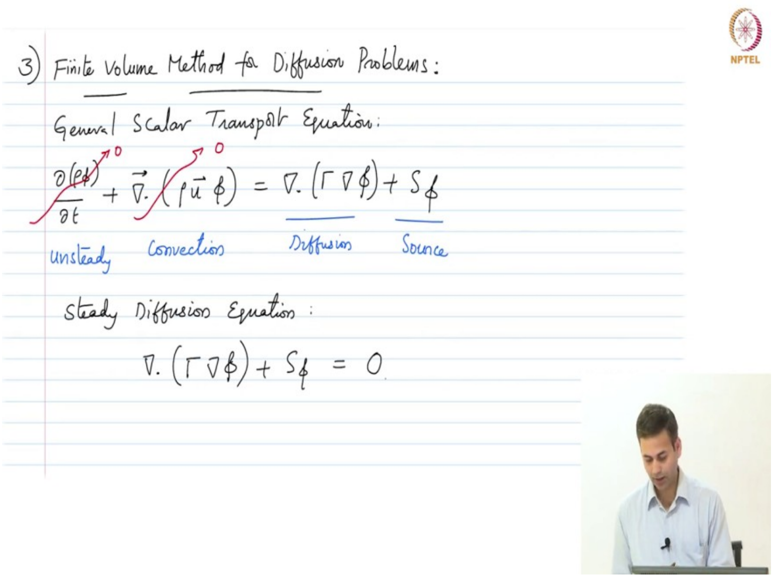

So, we will move on to the 3rd chapter which is relatively takes a while. So, that is the Finite Volume Method for the Diffusion Equation ok. So, this is 3rd chapter the finite volume method for diffusion problems. So, we have kind of visited this part during the overview chapter right. So, we are going to kind of redo some of these stuff, with some emphasis on the source terms and the discretization and things like that.

And then we will depending on the time I take up one problem that is in one dimensions and try to solve at least get the discrete equations ok. So that is the agenda for today's lecture.

So, in order to get the diffusion equation we start off with the general scalar transport equation that we have derived earlier, that i$\frac{\partial}{\partial t}(\rho\phi)+\vec{\nabla}.(\rho\vec{u}\phi)=\nabla.(\Gamma\nabla\phi)+s_{\phi}$do you remember the equation what would the convection term look like? I would del dot rho u bar phi equals on the right hand side you have the diffusion term that is$\nabla.(\Gamma\nabla\phi)+s_{\phi}=0$ some S phi right. So, we have a unsteady term and we have the convection term and we have diffusion and a source term alright. So we have these four terms.

So, essentially in order to come up with a diffusion equation, we just have to set the unsteady term and the convection terms to 0 ok. So, we are going to set the unsteady term to 0 as well as the convection term to 0 to arrive at a steady diffusion equation; which is now has only two terms remaining that is the diffusion term and the source term and the left hand side is now 0.

So, we can write this equation as del dot gamma grad phi plus S phi equals 0 ok. So, this is our steady diffusion equation which is going to be the equation throughout this entire chapter. Of course, we will add the unsteady term little later in the chapter and we look at discretization from 1D, two-dimensions, three-dimensions and on structured and unstructured meshes fine. So let us move on, let us look at a 1D diffusion equation this will be kind of a recap, but with few items embedded in there.

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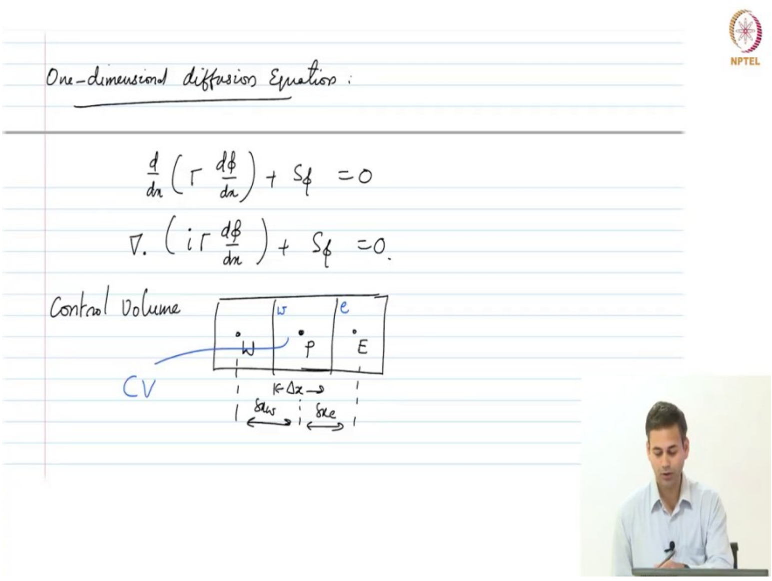

So, this will be like a one-dimensional diffusion equation ok. So, we already saw that the first step is to integrate the governing equation on a control volume ok. So, if I were to write a 1D equation this that would read del dot let me first write it the 1D forms. So, essentially $\frac{d}{dx}\left(\Gamma\frac{d\phi}{dx}\right)+s_{\phi}=0$ and we would write it in a little more convenient form; that is $\nabla.\left(i\Gamma\frac{d\phi}{dx}\right)+s_{\phi}=0$, so that we can apply.

So, essentially we can now integrate this equation on a control volume ok. So, we are going to choose a discretized a domain that is we are going to choose a mesh, that looks like this. These are the let us say the interior cells we call the primary cell P and the cell to the east as E and to the west as W and the faces of the P cell we are going to call it as little e and little w ok.

And we also indicate the distances let us say this width of the cell is delta x and the distance between the centroids of the P cell and the east cell we call it as del x e and the distance between P cell and the west cell we call it as del x w ok.

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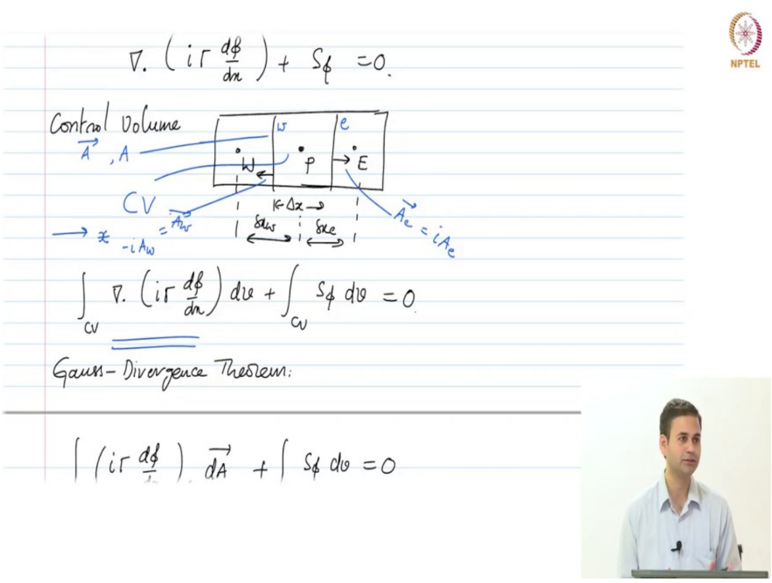

So, we have this kind of notation. Now, so the control volume we are focusing on is this guy this is the control volume, we will integrate our governing equation on this control volume as integral $\int_{cv} \nabla . \left(i\Gamma \frac{d\phi}{dx} \right) dv + \int_{cv} s_{\phi} dv = 0$. Let us also assume that this is a 1D problem, let us also assume that the area of these faces of these east west faces is has a magnitude A ok.

And the if we were to write the vector that will be A bar ok. So, the area vector is A bar and has a magnitude A, fine. So now, what would be the next step? Once you have integrated on a control volume, what do we do in the finite volume method? We essentially invoke Gauss divergence theorem and convert these divergence operator into a the volume integral into a surface integral ok.

So, we are going to invoke the Gauss divergence theorem ok. So, apply Gauss divergence theorem and arrive it essentially convert the volume integral that we have here into a surface integral. That would be $\int_{cv} \left(i\Gamma \frac{d\phi}{dx} \right) \overrightarrow{dA} + \int_{cv} s_{\phi} dv = 0$

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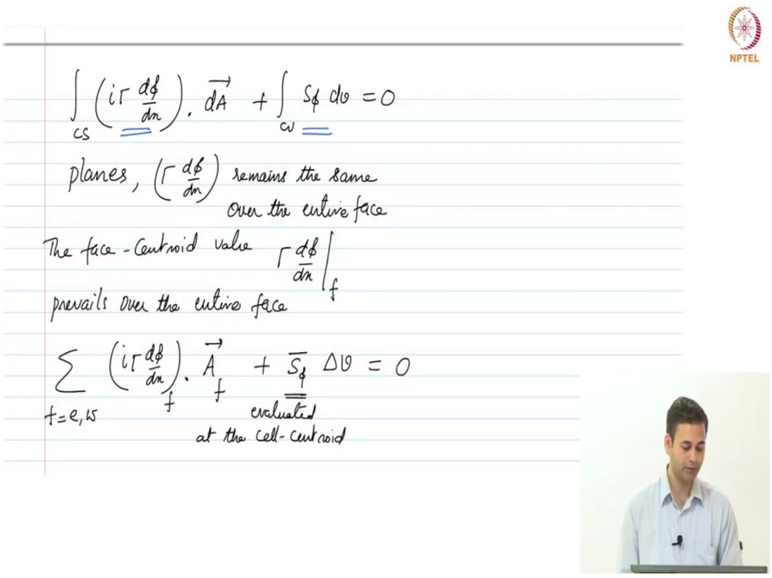

Now, we are going to replace this integration with a summation ok assuming that the faces that make up the control volume are all planar surfaces ok. So, these are all planes and this particular value that we have gamma d phi dx. So, the gamma d phi dx remains the same ok, kind of remains the same or you can have a remains the same over the entire face. And we can essentially the face centroid value that is the gamma d phi dx evaluated as the face centroid we call it sub f ok. The face centroid value prevails over the entire face, so that is the assumption we make.

As a result I can replace this continuous integral with a discrete summation ok. Summation on all the faces as i gamma d phi dx dotted with A bar ok, now these are all for all the faces ok. So, then this quantity has to be evaluated at on the face and this will be on the face ok. So, that is what we have and if we go back how many faces do we have for our cell? We have only two faces this is kind of a 1D problem.

So, we have east face and west face. So, this summation has to be done on f equals e and w ok, little east and little w plus I am going to again say that we can replace this S phi d v with a cell centroid value ok. So, we have some kind of an average value which is denoted with the S phi bar ok. Again this is evaluated at the cell centroid and then we multiply with the delta v that is the volume of the cell S phi bar delta v.

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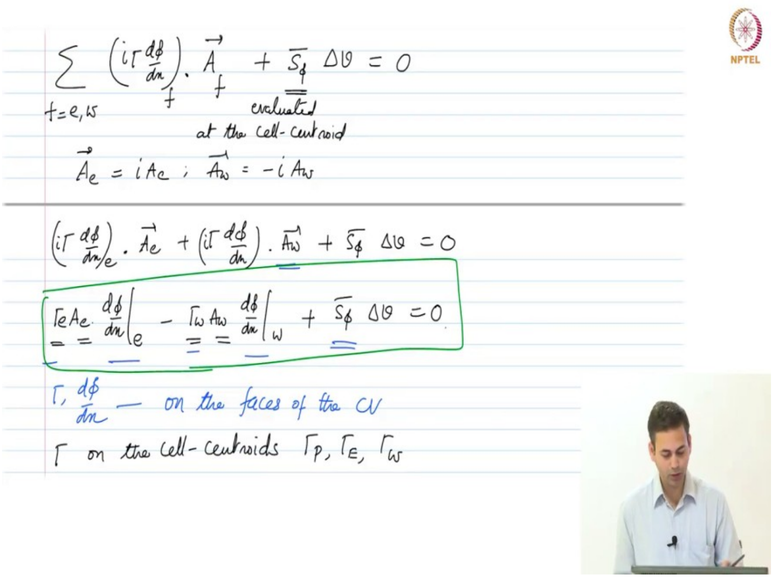

Now, what are the area vectors that we have? A east is what? A east is facing in the positive x direction if you were to denote our x axis in this direction what would be A east? A east is this way right and A west would be this way right. So, this is our A east and this is our A west ok. If we were to write A east bar ok, so this would be how much? This would be i A little e right. So, where A east bar is the vector A is the magnitude and A west would be minus i A west right, A west bar would be minus i A west.

Of course I am using this east and west subscripts, but we know that all are the same right A east equals A west equals A alright. Then I am going to substitute for these. So, A east would be A east bar a is nothing but i A east and A west bar would be minus i A west ok, then if I evaluate this what we are going to get is gamma d phi dx on the east dotted with Ae bar plus gamma d phi dx I am sorry it should be in i here i dotted with A west bar plus S phi bar delta v equals 0 ok.

Now, if you plug in what is A e bar as i A e we are going to get gamma east A east d phi dx evaluated on the east, then from the second product that is A west bar would be minus i A w. So, this would give you a minus, which would be minus gamma west A west d phi dx evaluated on the west face plus S phi bar times delta v equals 0 so far so good. Now, we obtained gamma and the gradients on the faces right, all these things the gamma and the gradients d phi dx needs to be evaluated on the faces of the control volume.

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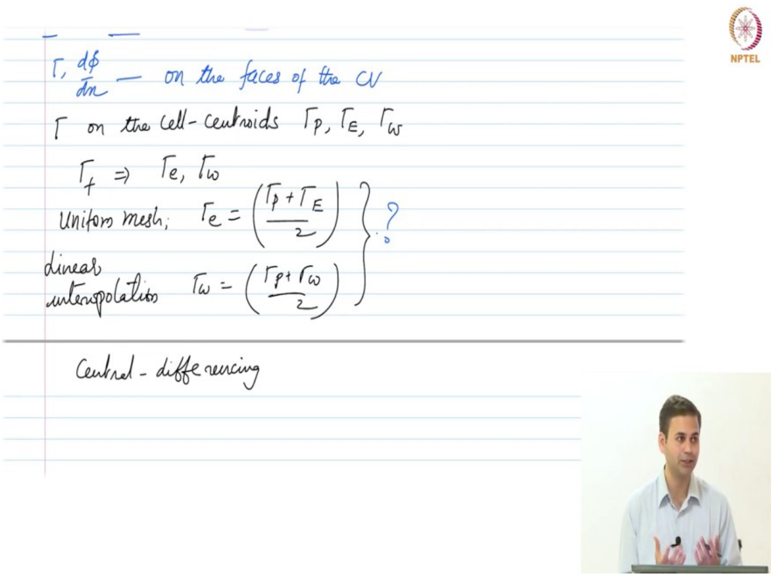

Then assuming that we only know let us say we only know the diffusion coefficient on the cell centroids. If we know gamma only on the cell centroids; that means, we know gamma values on the gamma P gamma capital E gamma capital w, then we need some kind of an operator to calculate what would be gamma on the faces right that is what would be gamma on the little e and gamma on the little w ok.

Assuming that we let us say we have a uniform mesh, the easiest thing would be to do a linear interpolation right and obtain the values of the gamma on the faces from the values of the gamma on the cell centroids ok. So, I could just write what would be $\Gamma_e = \frac{\Gamma_p + \Gamma_E}{2}$ and

$\Gamma_w = \frac{\Gamma_p + \Gamma_w}{2}$ Now, we just used linear interpolation and if you use a linear interpolation like this then this is called central differencing in the context of finite difference methods. Now, we are not saying whether linear interpolation is the right thing to do or not ok. As of now we are saying if you have a uniform mesh we could obtain the values of the properties on the faces from the values on the cell centroids using linear interpolation ok.

So, this would be a discussion for another day, where is this correct or not we will follow up later fine. Now, somehow somebody has given you the gamma diffusion coefficient on the cell centroids, you know how to calculate on the faces and you would use those values right here in these equations ok. Now, A east A west these are also known now again we know that we need to evaluate d phi dx and d phi dx on the east and west faces.

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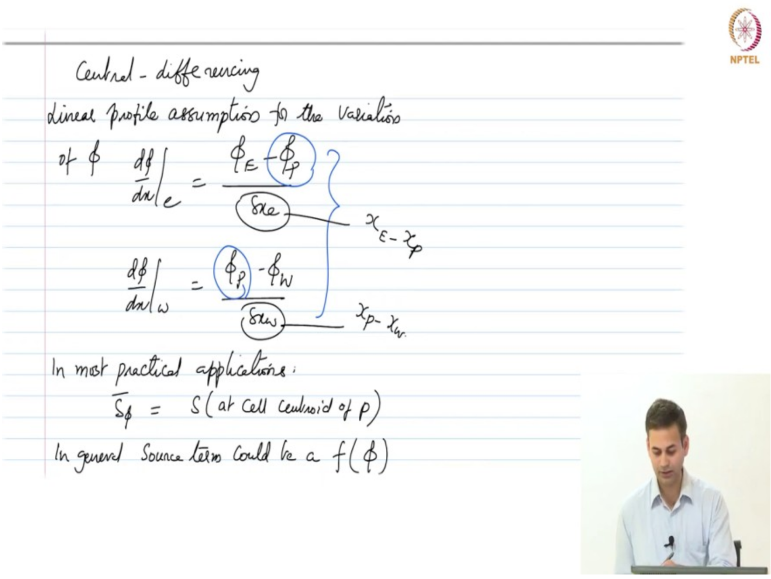

We said again we would use a linear profile assumption for the variation of phi ok, with which we can say that we can approximate $\frac{d\phi}{dx}\Big|_e = \frac{\phi_E - \phi_P}{\delta x_w}$. And $\frac{d\phi}{dx}\Big|_e = \frac{\phi_P - \phi_W}{\delta x_w}$.

So, we have these two linear profile assumptions with which we can evaluate the fluxes on the faces of the control volume all right so far so good. Now if we go back to the equation we have the S phi bar term ok. So, we have S phi bar so in most practical applications S phi bar is basically value of the source term evaluated at the cell centroid ok. So, this is basically value of source evaluated at the cell centroid at cell centroid of P ok.

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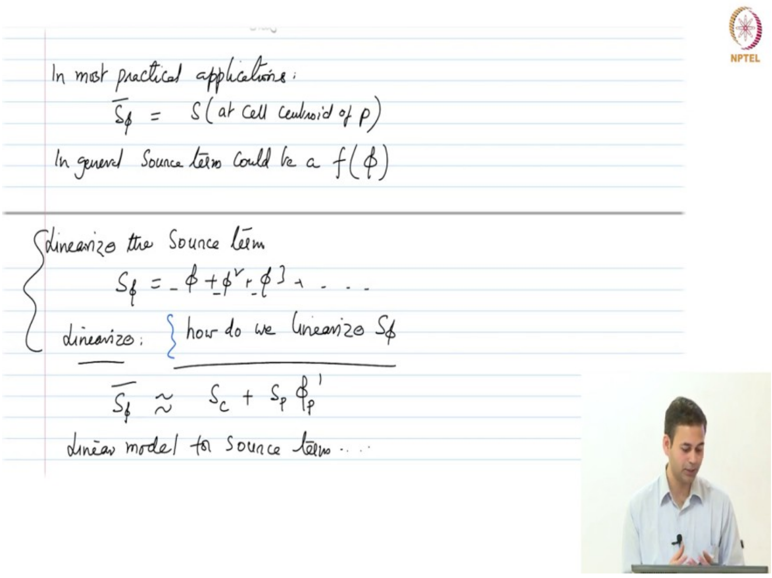

Now, this would be a non-linear function ok, in general this would be in general this source term could be a function of the dependent variable ok. So that means, this could be a function of phi itself ok, the S phi that we have could be a function of phi or it could be constant or it could be a non-linear function of phi as well. Now, one assumption we make here is that we have to linearize the source term.

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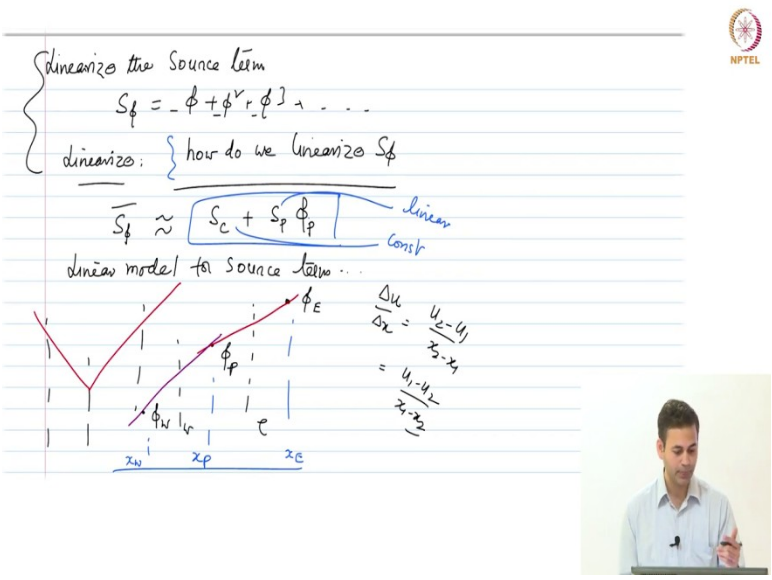

So that means, what we are saying is let us say we give you an expression I give you an expression to you as S phi as some non-linear function of f of phi. That means, let us say this is some phi plus phi square plus phi cube or something like that with some constants in the front. Now, we cannot use this form directly in the discretised equations. Why it is? So, why is that because the small changes in phi would result in a very large changes in the source term, as a result this all were you are going to use may not work it might diverge ok.

As a result we have to linearize the source term and if we say linearize that means S phi can be at most a function of phi to the power 1 or it could be a constant ok. Now, again this will be how do we linearize how do we linearize S phi this is again a topic for another day ok, we are not going to discuss it now. But as of now I am going to give you a linearized model ok. Will the linearized model work or it will not work all these things will discuss on another day ok.

As of now you would take it for granted, that given any source term we can linearize it to be evaluate at the cell centroid as some constant plus that is S sub c plus S p times phi P ok. So, essentially this is to the power 1 right. So, this is a linear model so this is a linear model ok, which I have approximated this is a linear model for source term.

Now, one question that pops up in your mind is that ok, you have just changed a big non-linear function into a linear model do I get the same result right. Because we have changed the physics you will get the same result in the end when everything starts converging ok, it only changes the path to the solution ok. So, that we will see later on, we will justify that it will only change path of the solution as long as we are going to recover the same source term once you reach convergence ok.

So, that we will discuss in some other lecture yes ok; so, the question is about the way in which we have written the fluxes right, the face fluxes d phi dx on east and west. Why is essentially phi P coming in the as a minus here and why is phi P coming in the positive here, that is to do with the linearization right. Essentially you have a linear profile. So, if you if you essentially write delta x e as what x east minus x P right and this as x P minus x west, then you would get this as phi P minus phi W right.

It has to be always between two points right and the points happen to be 1 and 2 and 3 right essentially you have west P and east right. The question is let us say I have some value this is phi east some value phi P another value phi west and these are at different x locations right. So, this is x west x P x capital E how do you write a linear profile assumption, how do you calculate the slope? Phi east minus phi P by x east minus x P that will give you positive here and similarly you would calculate the other slope as phi P minus phi W upon x P minus x W right.

Or if you want to write it phi W minus phi P then you get a minus in the denominator, so because we have taken away that minus with the delta x right. So, delta x is always positive as a result the numerator changes right, that is all we calculating. What is the value of this slope at the midpoint that is on the east face and on the west face.

Now, it need not so the question is we are not assuming that P is increasing P could decrease as well right. It is how do you calculate some change in the numerator, let us say some change in delta u by some change in x. This is always between two points right, you would write this as u 2 minus u 1 by x 2 minus x 1 or you will always write it as u 1 minus u 2 by x 1 minus x 2 right. You cannot change the point's one for the numerator one for the denominator right that is what we are doing.

So it need not increase this way, so you could even have your variation of phi as it could even go this way right it could go in anyway right the definition remains the same. So, essentially what I have drawn here is for different location. So, we have let us say phi west phi P and phi east could be even this way that you can check that you can check later after the class. Other questions we are only assuming linear variation between east and P and so between the cell centroids phi varies linearly ok.

That is not compulsory, but most of the general finite volume methods that are used in you know open source and commercial software you assume it linear. Because you get a smaller system and as you make yourself size smaller and smaller, these linear variations would not affect your solution much unless you want a higher order accuracy. The question is the changes in the slope can only happen at the centroids, well the values of phi are defined only at the cell centroids right. So, you can only define slope between the cell centroids right.

But if you want to define let us say slope at this cell centroid, then there is nothing fixed we will see how to calculate that ok. Then you can use the face values interpolated face values to calculate the value at the cell centroid and things like that right.

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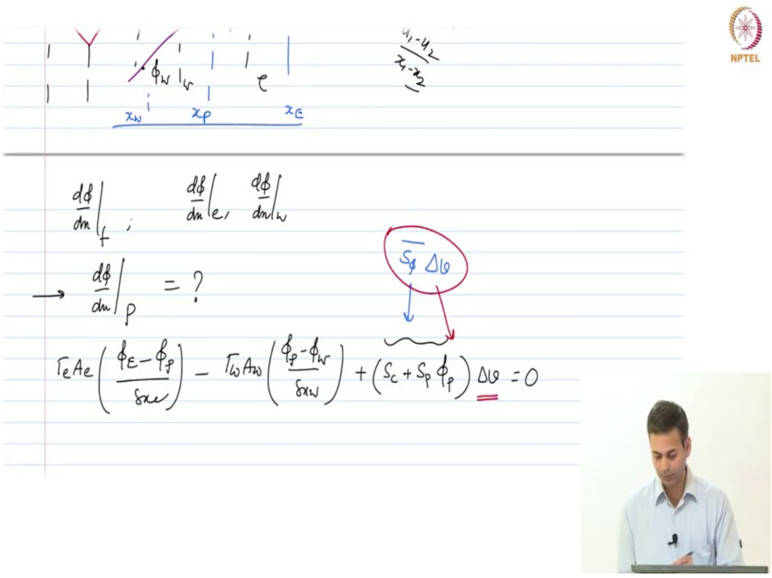

For example, so the thing is you have up till now we have been only writing d phi dx on the faces right, that is on the east little e and little w little e and little w. Now, what if you were to get d phi dx at the P itself right; we will see how this to be done one way is to obviously use interpolate phi to the faces and then use those values and calculate it ok. So, these things we would not get it here, but whenever we get we will come to the discussion. Other questions fine ok, we have kind of side line a little bit I will go back to this.

So where were we so we have now linearized, we have introduced a linear model for our source term. So, it does not matter whatever would be the variation of the source term in terms of phi, we are always going to use this standard recipe that is some constant. So, this is a constant part and this S p would be a coefficient of the linear part ok, which will multiply with phi P prime that is not phi P prime it is just phi P. I just have this phi to the power 1 ok.

So where phi P is the value of phi at the cell centroid ok, we have introduced linear model. Now will the linear model work will it give the same results we are not discussing in today's lecture we will do it little later ok; until then you would take this for granted all right. Now let us go back and substitute all of these in the discrete equation that is that is this equation right ok.

So, we are going to use this equation over and over. So, I am going to plug all of these in which will give me $\Gamma_e A_e\left(\frac{\phi_E-\phi_P}{\delta xe}\right)-\Gamma_w A_w\left(\frac{\phi_P-\phi_w}{\delta xw}\right)+\left(s_C+s_p\phi_p\right)\Delta v=0$. So, that is what we have the two terms and then what else we have, we have the source term that is a plus S phi would be Sc that is some constant 10 20 whatever; plus some S phi this is also a constant it will be some 200 300 whatever times phi P ok.

So, this is a model we have introduced for our S phi bar ok, so that is a linearized model. What else we have? We have this multiplication with delta v equals 0 ok. So now, if you take some books maybe west you can or else it would probably include the delta v also as part of the S phi bar ok. So, they probably would not introduce a model for S phi bar delta v ok, for the entire thing they would introduce S c plus S p phi P. We are not doing that we are only doing a linear model for S phi bar ok.

So that means, we still have this delta v term hanging or multiplying our S c plus S p phi P ok. S c and S p are constants within the cell they could within the cell they are definitely constant, within the entire domain they could be constant or they could vary depending on the kind of source terms you have. For example, you have put in heaters which will essentially produce let us say the same kind of heating capacity everywhere then S c and S p would remain the same ok.

But the overall source term S bar would change; this would change depending on the location because now you have a dependency on phi P right. As a result that would change with phi P values ok. But S c and S p most likely there will be kind of constant or they could be functions of space as well depending on the problem alright. Let us now rearrange all these terms in a form that looks more appealing.

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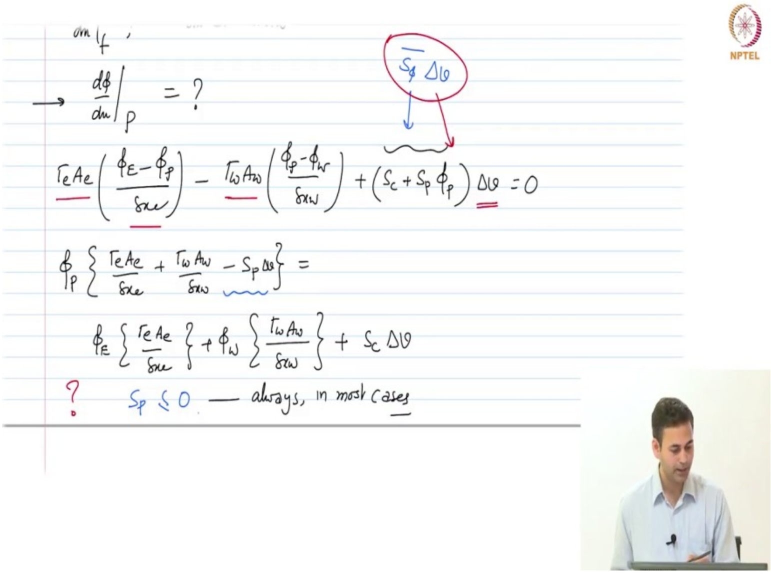

So, that would be you would have collect all the coefficients for phi P ok, phi P coefficients are what our gamma east A east by delta xe with a minus. So, we kind of send this to the right hand side. So, that would be $\phi_p \left\{ \frac{\Gamma_e A_e}{\delta xe} + \frac{\Gamma_w A_w}{\delta xw} - s_p w \right\}$ We have one more term which is multiplying phi P that would become minus S p delta v equals, what do we have on the right hand side on the left hand side. Now essentially that would be $\phi_E \left\{ \frac{\Gamma_e A_e}{\delta xe} + \phi_w \left\{ \frac{\Gamma_w A_w}{\delta xw} \right\} + s_c \Delta v \right.$

.

Now, I kind of throw in another thing in here, I will say that S p would always be less than or equal to 0 ok. I am kind of again stating this thing ok, we already have a minus in front of S p and I am saying the value of S p would be less than or equal to 0 ok. I am saying this would be in always or in most cases ok. That means, what will happen to the value of this entire term here minus S p delta v, delta v is always positive what would be minus S p delta v.

Student: Positive.

This is always to come out to be positive that is what I am trying to say. Now you may have a question why not S p be positive ok, that is for you to think and come back to me otherwise we will discuss it later ok. So, this is for you to think why S p is not positive and I say that it will always come out to be 0 or less than 0 ok, always negative also fine. So, then let us write this in a little different way. So, I would like to call these values in the parentheses here at the curly braces as some coefficient A ok.

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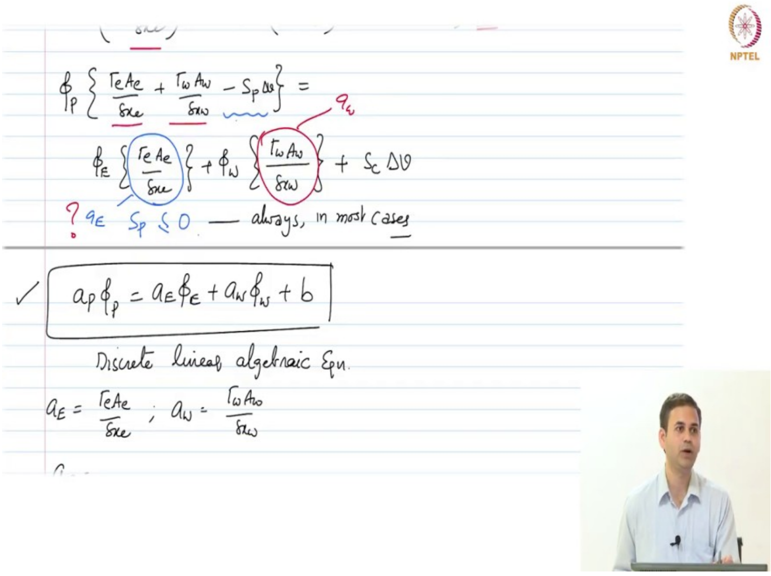

So, then this will be a P phi P equals a capital E phi capital E plus a west phi west plus b ok. Now this is our discrete linear algebraic equation ok. We will use only this form throughout the entire course, does not matter whether we are looking at a diffusion equation or a convection diffusion or Navier-Stokes anything we will always or an unsteady diffusion will always come back to this particular form ok. So, this is important all right.

Now what about the coefficients that we have what about A east, A east is of course, gamma east A east by del x e right. If we go back here this is our A east term ok. Similarly what would be our A west?

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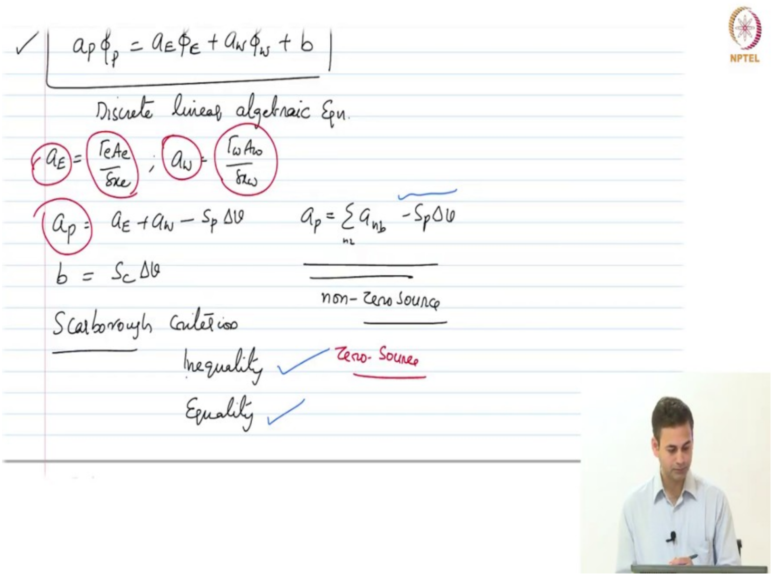

Gamma west A west by del x w that is this is our a west. Now, what about a P? If you look at the coefficient what does a p contain, a east a west and minus S p delta v ok.

So that means, we can write simply it a simply this as a east oh sorry, here we can simply write this as A east plus a west minus S p times delta v that is our a p ok. Now what about b, b is Sc times delta v right that is the only term remaining here. This is only S c times delta v very good. Now, so now you see now if we go back to the discussion on Scarborough criteria, we had two things right we had an inequality and an equality right.

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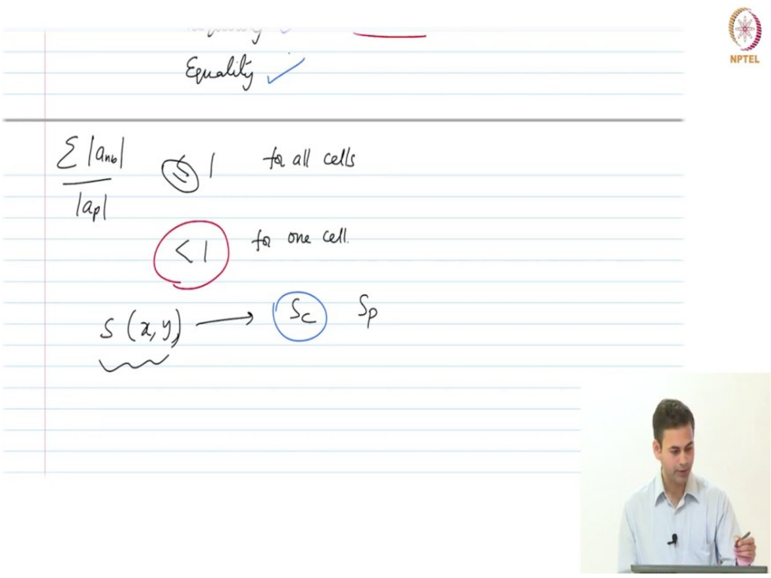

We said we said the summation modulus of a and b by modulus of a p should be less than or equal to 1 for all cells and should be at least less than 1 for 1 cell in the domain that is what we said, only then the convergence will be guaranteed. Now it is somewhat easier to satisfy Scarborough criteria in statistical Scarborough criteria inequality ok.

So, this inequality satisfaction is sometimes not possible that is what we have to look for. So, I will be using this equality and inequality ok. So, equality is I mean by this equal to inequality is this one or the other one appearing here. Now, with our discussion we have to look for the coefficients a p a east a west right, essentially we have to look for this guy this guy and this guy for every cell in order to look for their magnitude to evaluate this Scarborough criteria ok.

Now, I also said that S p is negative right; I said S p is negative the source term. So, what would be now a p here? Sum of a the neighbours right minus S p delta v. So, this is does it satisfy? Let us say if we have a non-zero source right. We have a non zero source, then does it satisfy is Scarborough criteria it does satisfy, because it is greater than it is neighbours why the amount of this minus S p delta v.

If we have a non-zero source yes gamma e or gamma w can the diffusion coefficients be negative, the physical properties will not be negative like viscosity thermal conductivity these are never negative right they are always positive right, delta x and the area magnitude these are always positive ok. So, so these can never be these are always positive fine. So, it satisfies if we have a non-zero source Scarborough criteria is satisfied in what in equality or inequality in inequality ok. So, it satisfies in inequality.

Now if we have a zero source and so if we have a zero source term, then what will happen? It only satisfies in equality ok, because then a p would be sum of sigma a and b or modulus of sigma a and b then it only satisfies an equality, then we have to look for some other sources where it could satisfy in inequality. For all the interior cells it only satisfies in equality, if we do not if we have a zero source term only the equals is satisfied not less than condition fine all right.

So, by the way where do we look for if it does not satisfy Scarborough in the interior cells boundary conditions right. The source terms would so essentially the would source terms would help you make the system diagonally dominant right and the boundary conditions will also make the system diagonally dominant right. Because they will produce in this less than less than a satisfaction right the inequality will be satisfied as well ok.

Now, what boundary conditions satisfy, what boundary conditions do not satisfy these things we will look into that fine. So, essentially we have arrived at this equation and kind of finish the discussion. Now, essentially we write the this kind of a P phi P equals a east phi east plus a west phi west plus b. So, this linear equation is what we have to write for every interior cell that we get and we have to obtain a similar equation for all the boundary cells that we have ok. Questions still now? Ok so we will come back to that essentially in a minute when we do the problem question yes.

Student: (Refer Time: 34:01).

Yeah it is probably not a function of phi it is just a essentially, that that means it is it only satisfy this inequality.

Student: (Refer Time: 34:11).

Yeah it could be a function of x y as well, in which case it will not show up as a in which case where will the source term be. The question is what if the source term is only a function of x y , but not a function of phi right, then where you look where will you put this guy which term do you put this guy in S c or S p.

Student: S c.

Sc right because this is does not depend on the dependent variable, it has to go into Sc right it will become it is only a function of the independent variables then it will go into the constant term ok. Other questions it will be actually I think in the assignment it is a function of xy only write your assignment problem.

Other questions fine so let us move on let us do a simple 1D problem and try to formulate the system and later on we will see how to solve it. Regarding the assignment I am going to probably discuss it on Wednesday or so ok. In the next to the next lecture just to kind of go through it in a overview, so that you can probably solve it and answer your questions. Until then I want you to take a look at it and understand how to solve it fine. We will have an overview on how to solve it on Wednesday or so and we will move from there ok.

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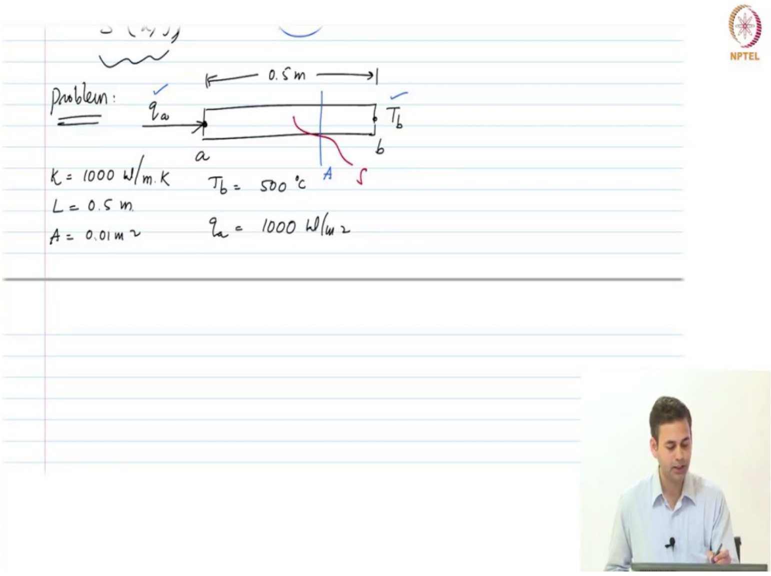

Let us pick up a problem. So, let us say we have a 1D domain ok, we have a 1D rod or a beam on which the length is let us say 0.5 meters and the boundary conditions are on the left end which we call it as a and the right end which we call it as b; left end there is a heat flux given by q a ok.

So, this is known a known heat flux q a and the right hand side is maintained at a constant temperature T b ok. This is also known fine. Then the length is given let us say the thermal conductivity is 1000 watt per meter Kelvin, length is 0.5 meters and the cross section area A; A is 0.01 meter square and let us say the temperature T b is around 500 degrees Celsius and the heat flux q a is let us say 1000 watt per meter square and let us also introduce a source term ok.

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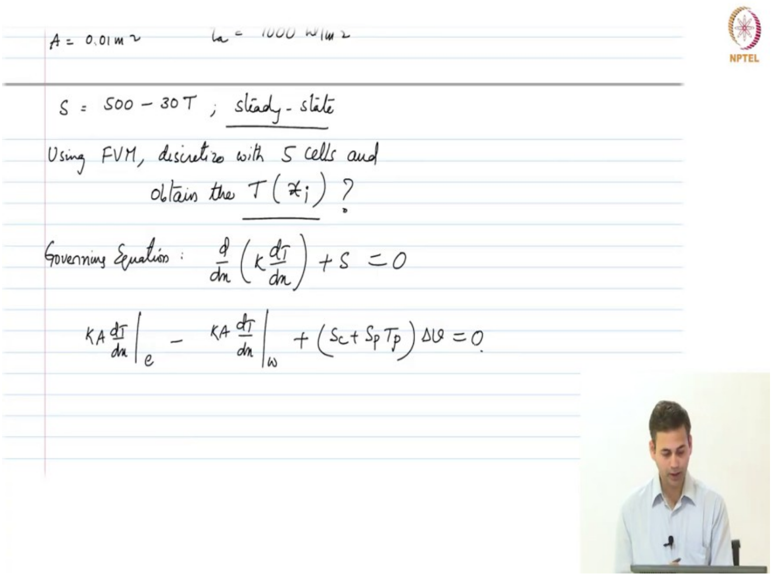

So, there is a source term there is a source term S and this S is given by let us say 500 minus 30 T ok; 30 it is a linear source term essentially your Sc is 500 S p is minus 30 ok. Now, we have a source term as well that produces heat generates heat in this beam ok. Now, the question is using Finite Volume Method, discretize with 5 cells and obtain the temperature distribution at these 5 cells at these 5 discrete cell centroids T of x i ok.

How do we do this? So, essentially you have to use the 1D diffusion equation and then solve for this problem ok. So, what is the Governing Equation, the governing equation is what d by dx of k dT dx plus some S equals 0 ok. This is a steady state problem, let us say this is given in the problem statement there is some steady state solution available ok. So, we have d by dx of k dT dx plus S equals 0.

Now, we will not do all the steps that we have done before ok. So, I am going to kind of walk you through all these steps and write only the necessary one ok. So, essentially what do we do we integrate this on a control volume, change it to a surface integral the volume integral to surface integral then the surface integral has to be valid on the faces east and west and we do all this and arrive at one equation right. That is a equation of balance of fluxes with the source term and that if you recall would look like k A dT dx on the east minus kA dT dx on the west plus S c plus S p T p times delta v equals 0 ok.

So, this is basically all the things we have done at the beginning of the lecture and we have arrived at this particular equation. Do you all agree ok; are there any mistakes in this equation looks fine. So, we have now reached this one all right. Then we have to also discretize our domain right which we have not done till now.

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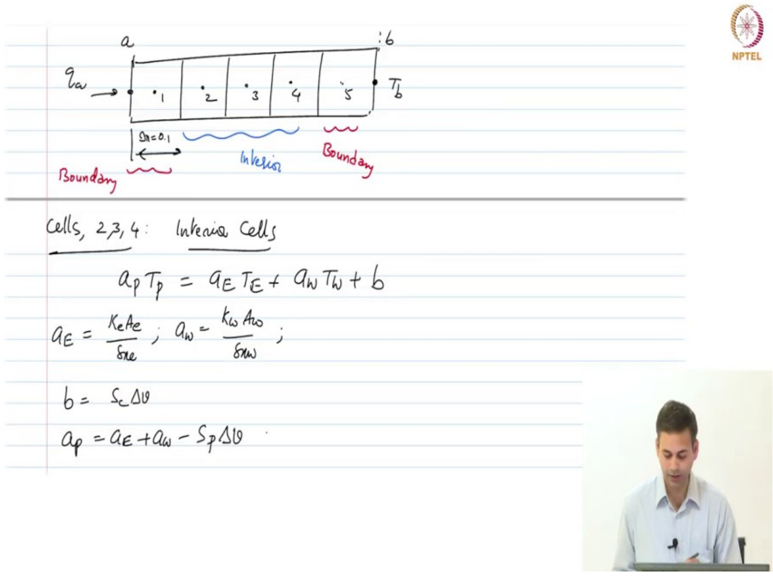

So, that is looks pretty uniform. So, we have this is 1st cell, 2nd cell, 3rd cell, 4th cell and 5th cell. What would be the width of each cell here delta x would be 0.1 and the boundary conditions are also known this is Tb, this is q a ok. Let us call this face as a; this face has a and this face as b a face and b face.

Now, we are still going to use the east west face notations for every cell ok. So, e and w little e and little w will be for that particular cell ok. I am not introducing all the faces here ok. Now, let us look at first the interior cells we call 2, 3, 4 as interior cells and we would like to call the 5th cell as a boundary cell as well as the first cell as a boundary cell, meaning that these cells have at least one face common on the boundary ok.

So, first we can look at a 2, 3 cells 2, 3, 4 ok. So, that is for cells 2, 3, 4 for the interior cells, we can again use linear profile assumption for temperature ok. And I can write one equation by collecting all the terms I can come up with this linear algebraic equation. The discrete linear algebraic equation that look like $a_p T_p = a_E T_E + a_w T_w + b$. Where you are going to tell me what would be a east; $a_E = \frac{k_e A_e}{\delta xe}$ in which case del x e equals 0.1 right del x e equals delta x equals 0.1.

What would be $a_w = \frac{K_w A_w}{\delta xw}$ What would be b? b would be Sc times delta v and ap would be a east plus a west minus S p times delta v right. If we were to apply this to let us say cell 2. What would be the p we would substitute p for 2.

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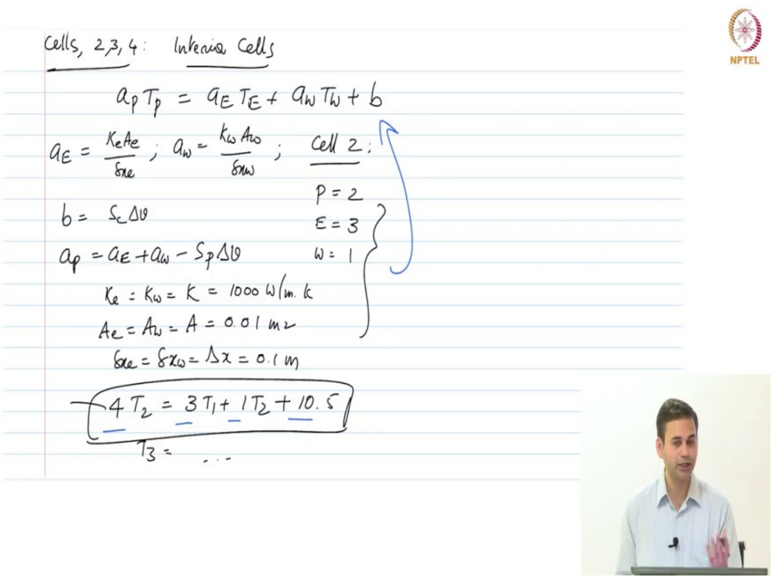

E for 3 right, east for cell 2 east is three and west would be.

Student: 1.

1 ok. So, you would substitute these things right and essentially do you know what is k right, k east equals k west equals constant that is already given 1000 Watts per meter Kelvin and we also know what is cross section area A delta x is known.

So, A east equals A west equals A equals 0.01 meter square. What will be del x e? Del x w equals del x equals 0.1 meters. So, everything is known you can evaluate what is a east a west a p as well, because S c and S p are also known. You use all these values and plug in all these things back into this equation right. You would get an equation in terms of what? T 2 T 1 and T 3 right, because your you will replace what is T p T east and T W.

You will get an equation in terms of T1 T2 T3 right for cell 2, similarly for cell 3 you can do the same cell 4 you can do the same. Now, what would change for cells 2 3 4 in these things or what which of these quantities will not change? k is a constant a is a constant delta x is constant. So, the coefficients are not going to change right, a east and a west are not going to change will ap change from cell to cell? Yes, ap changes because ap is a summation of a neighbours minus Sp.

Sp changes because it depends on temperature; if source were also 0 then ap would also have been the same for all the 3 cells ok. What about b would b change or cell to cell for 2, 3, 4 it would not change because S c is constant S c times delta v is the same right. So, you can think through and tell me that there are essentially you will get 3 equations ok, which would look something like maybe the coefficients are completely wrong. But I would like to write this as a 4, 3, 2 equals some 3 T 1 plus 1 T 2 plus some 10.5 the coefficients are all in correct.

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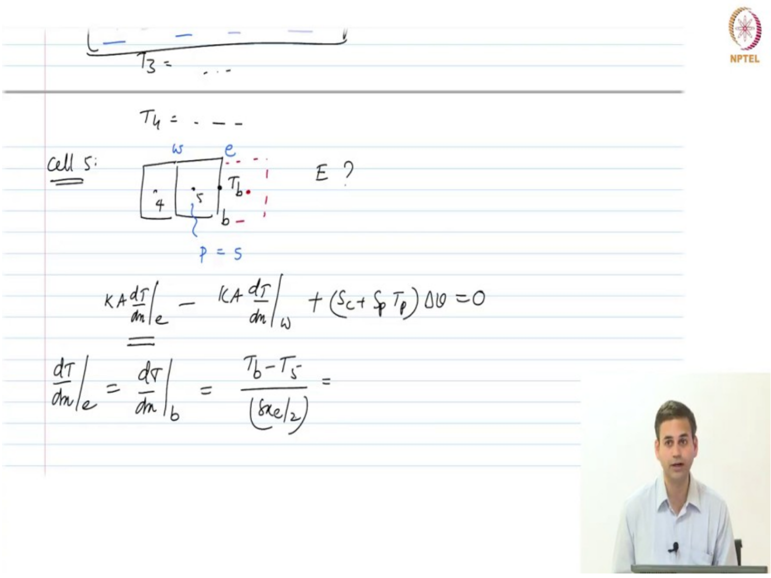

But you get some equation like this right and you get two more equations for T 3 and T 4 right ok. These coefficients are of course not correct I just wrote to indicate something alright. You have now 3 equations for the interior cells that is 2, 3, 4 fine ok. Now, we had look at the boundary values right, we have to look at the boundary values questions till now.

So, we have 3 equations for the three cells right we just need two more equations S p. So, the question is will why not a p a constant for all the cells, well the thing is yeah I think a p will also be constant because S p is also constant here right. If S p were a function of x y it would be different you are right. So a p will also be the constant for all these things right, because the T p is going into the unknown right, it will also remain the same. So, ap also would be the same for all these 3 correct. If it was a function of xy like what minus had before, then it would have been varying from cell to cell ok.

So, in your assign problem it is different now ok. So, let us look at the boundary cells, then let us look at the fifth cell first. So, this is cell 5 oh sorry cell 5. So, cell 5 how does it look like? It looks like this, this is 5 this is constant temperature and the neighbour is what neighbouring cell is 4 and we have we call this as the faces are we this east and west. But we know that the east face is nothing but face b that is what we know. Now what is our P our P is 5 right P is 5 we are writing for this ok.

We will go back and start off with our discrete equation ok, that was what $KA\frac{dt}{dx}\vee e-KA\frac{dt}{dx}\vee w+\left(s_c s_p T_P\right)\Delta V=0$ that is what we have. Now, for the east face, if we were to calculate $\frac{Dt}{Dx}\bigg|e=\frac{dT}{dx}\bigg|b=(T¿¿b-T_5)/\delta xb/2¿$ would be dT dx at b would be what not 0. Why is it 0? It is an isothermal condition right it is not 0, only thing is that we do not know the cell values of for the e cell right. We do not know what is the cell value for e because we do not have such a cell here right.

If it were there we would have calculating this as some T 6 minus T 5 by delta x right. But do we have any other value?

Student: T b.

We have Tb right; I can do Tb minus T5 to calculate the gradient. But what would be the distance between these two guys then?

Student: (Refer Time: 47:29).

So, then this will be written as T b minus T 5 by the distance, distance would be del x e by 2 right. So, this would be the value for the gradient ok. What would be the for the west cell?

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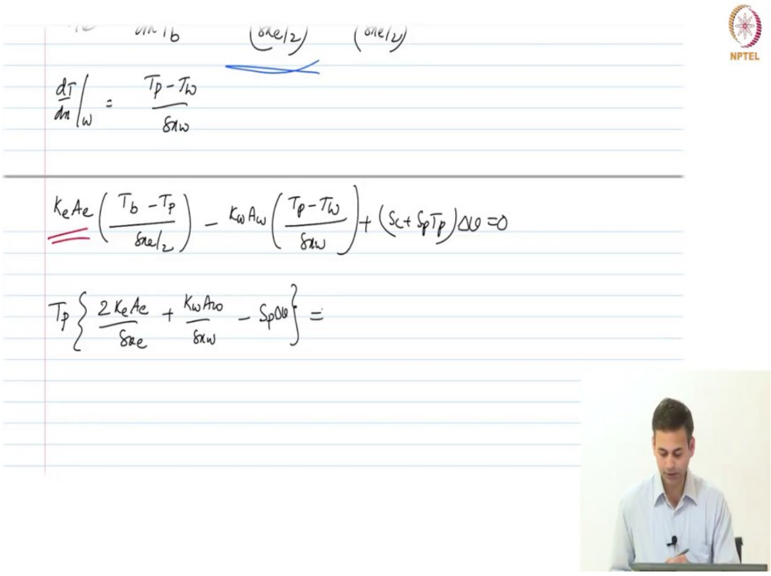

$\frac{dT}{dx} \vee w = \frac{T_p - T_w}{\delta xw}$f for the west cell is what, it will be same as before this will be that would remain the same. That means, T 5 minus T4 by delta x ok.

That is the same thing fine we will plug these in back here, this will give us

$K_e A_e \left(\frac{T_b - T_p}{\frac{\delta xe}{2}} \right) - K_w A_w \left(\frac{T_p - T_w}{\delta xw} \right) + \left(s_c + s_p T_P \right) \Delta V = 0$. Let me write T 5 as T p this is all right.

Now, if you were to collect these terms, what do we get? You have a del x e by 2 this goes up, then this coefficient will become 2 K a by delta x right on the denominator ok. So, then what will be the coefficients for T p, you send all the coefficients of Tp to the right hand side you will get$T_p \{ 2 \frac{K_e A_e}{\delta xe} + \frac{K_w A_w}{\delta xw} - s_p \Delta v \}$. Because the denominator the two half in the denominator goes to the top plus we have K what do we have on the right hand side on the left hand side.

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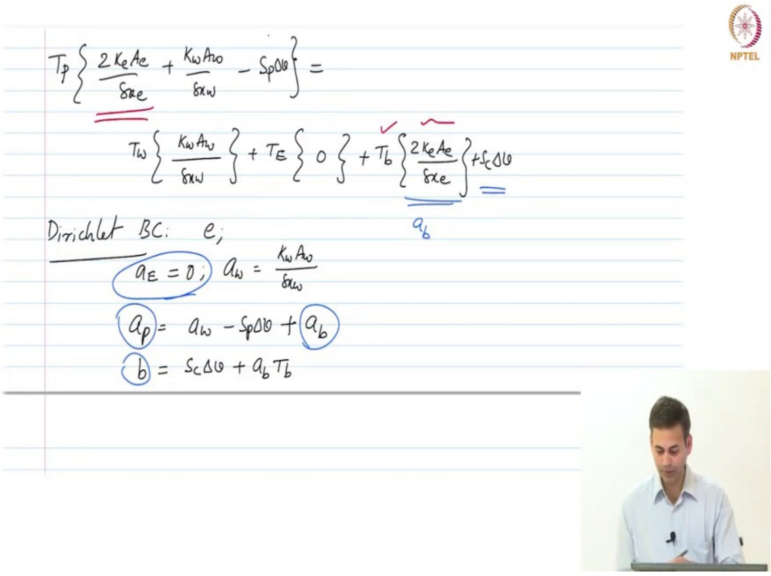

Now from here this would be $T_w\{\frac{K_w A_w}{\delta xw}\}+T_E\{0\}+T_b\{2\frac{K_e A_e}{\delta xe}\}+s_c\Delta v\}$. What would be the coefficient for T east T capital E? we do not have anything plus. What about T b? T b has the same coefficient as that went into do you all see this is correct alright ok. If you were to call this as some coefficient a b ok, then if we have a constant temperature right; we have specified the temperature value that is we have specified a what kind of boundary condition? Dirichlet boundary condition.

Then what do we have we have if we have Dirichlet boundary condition on the east face little e, what would be the coefficient a east? 0. What will be the coefficient a west? Remain the same K w remains the same del x w, what would be the coefficient $a_p=a_w-s_p\Delta v+a_b$a p? plus this guy which is nothing but a b. What about the coefficient? What about the value for b.

Now$b=s_c\Delta v+a_b T_b$ is was b till now, what about T b is T b known T b is known right. What about this coefficient is it known? So, where will this guy go here is an unknown or known? This will be an unknown. So, where will they should be this put up in as part of.

Student: b.

b right this will be part of b. So, b would be S c; S c delta v that is what we had till now plus what else a b times T b alright, a b times T b. So, what have we done we have what we have done is essentially we have cut off the link to the east cell right, by setting that coefficient equal to 0 right. By setting this to be equal to 0, we have cut the link to east cell because we do not have an east cell for the boundary face 5 right.

And included the other contribution that is coming from the face into the a b value, that is in to be as well as into a p ok. Now, I will kind of finish up here.

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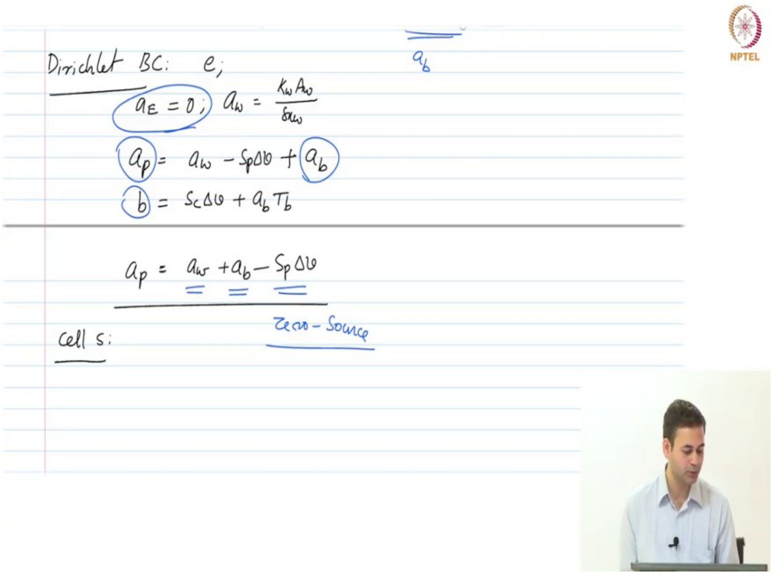

Now what is a p here? $a_p = a_w + a_b - s_p \Delta v$

Now, does it satisfy Scarborough, for the 5th cell how many neighbours are there?

Student: (Refer Time: 52:34).

Only one neighbour right, 4 is the only neighbour right, a west is the only neighbour. So, a p should be equal to a west right. But in addition we have what else?

Student: a b.

We have of course a b and we have a source term of course, let us say we have a zero source term. We have a zero source, then does it still satisfy Scarborough or not? It will still satisfy because it is now higher than it is neighbours by a value of a b right. So, it still satisfies Scarborough in inequality even if we have a zero source term. Now, this is happening because of the type of the boundary condition that we have which is at Dirichlet boundary condition ok.

So, if we have a Dirichlet boundary condition it will help you satisfied Scarborough criteria in inequality ok. We will look at this again a later on in the two-dimensional problems as well. So, I am going to stop here; we will pick up the heat flux boundary condition in tomorrow's lecture right ok.

Thank you.