Computational Fluid Dynamics Using Finite Volume Method
Prof. Kameswararao Anupindi
Department of Mechanical Engineering
Indian Institute of Technology, Madras

Lecture - 01
Review of Governing Equations: Conservation of Mass

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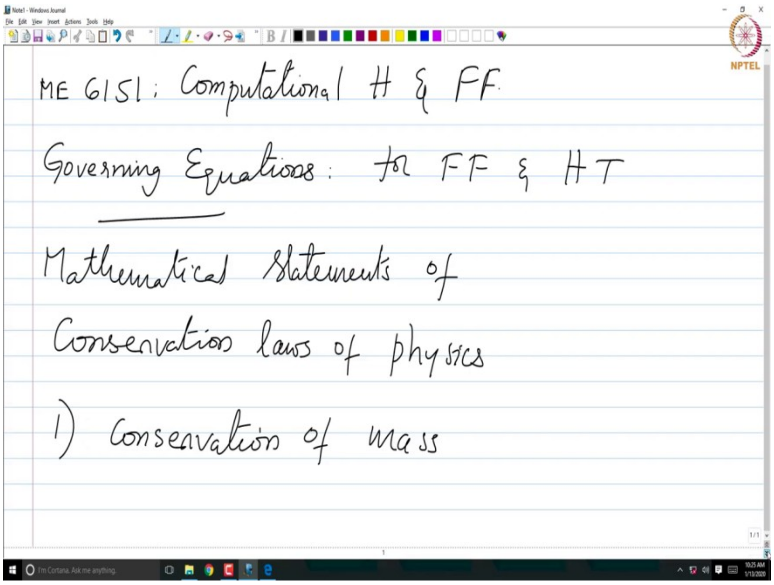

Alright. Is this fine; all of you can read it? Decent size or shall I increase the size? Fine.

Student: Yes.

Very good. So, let us look at the governing equations. So, the governing equations are nothing but mathematical statements right. Governing equations for the fluid flow and heat transfer right; fluid flow and heat transfer.

So, these are nothing but the mathematical statements of conservation laws right of physics ok. So, these are the Mathematical Statements of Conservation laws of physics ok. What are the conservation laws that we know which are relevant here?

Student: Mass (Refer Time: 01:24).

Mass ok; conservation of mass is conserved right and then, the conservation of momentum right. Which is what?

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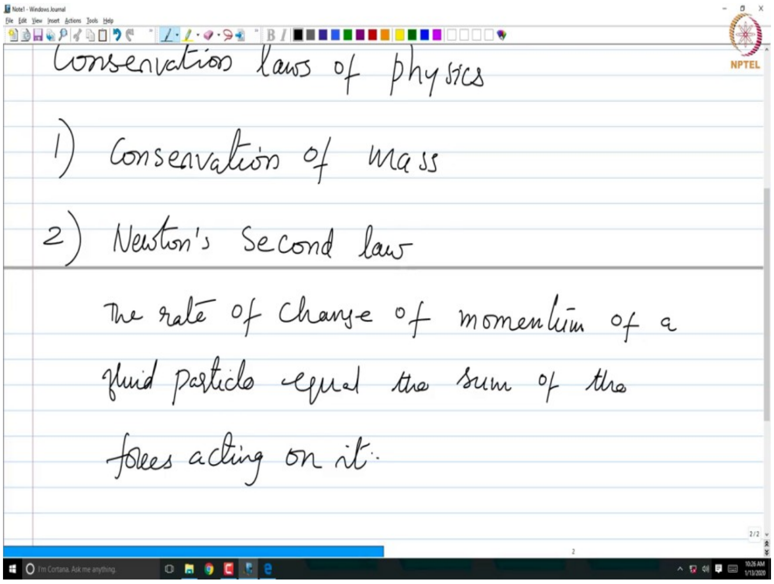

Student: Newton's first law.

Which is what? Which is the Newton's?

Student: Second law of motion.

Second law of motion ok. So, that is Newton's second law which says that the rate of change of.

Student: Momentum.

Momentum of a what? Fluid particle right that we are following of a fluid particle equals what?

Student: Forces (Refer Time: 02:26).

The sum of the forces right. This resultant of the forces, sum of the forces acting on it right, acting on the fluid particle ok. So, that is Newton's second law of motion. What else?

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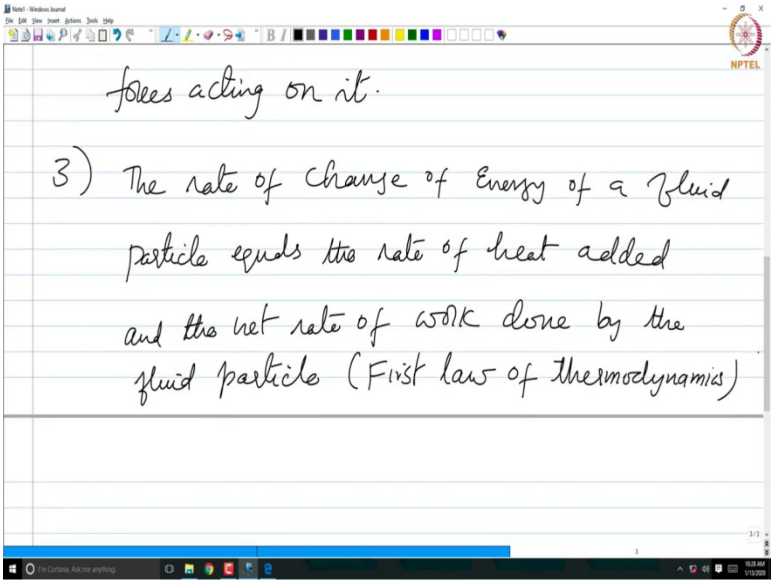

Student: Conservation of energy.

Conservation of energy ok. So, that is conservation of energy that reads as the rate of change of energy right of a fluid particle equals what? Equals the rate of heat added right ok; equals the rate of heat added and the net rate of work done right by the fluid particle ok, work done by the fluid particle. So, this is conservation of energy which is also is a law of thermodynamics right. Which law of thermodynamics is this?

Student: First law.

First law ok. This is the first law of thermodynamics; fine. Alright ok. So, we have these three conservation laws which are kind of relevant for what we kind of do ok. So, you will be writing them in mathematical statements and we will discretize them and then, solve them and obtain what we need alright. So, you will be regarding the fluid as a continuum.

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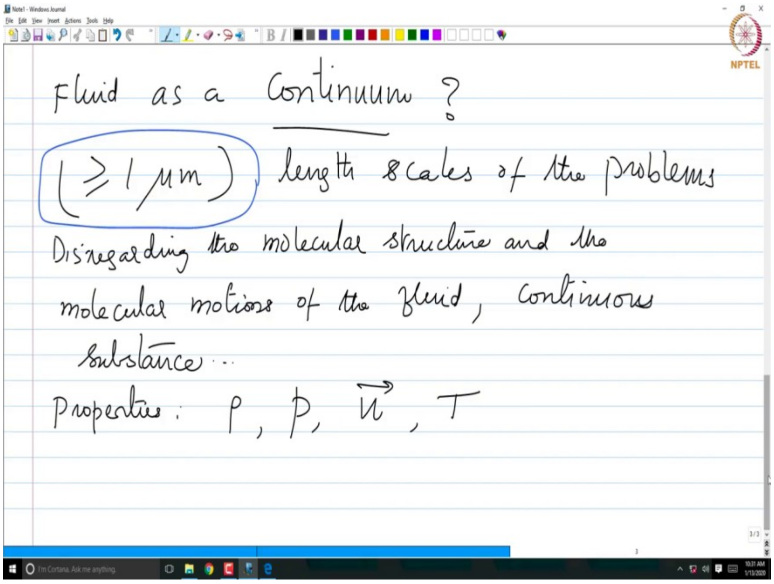

Now, what is a continuum? For the kind of length scales we are looking at right, for the kind of problems we are looking at which are all greater than let us say 1 micrometer ok, for all these length scales of the problem right, we say that the fluid that we have can be treated as a continuous field right by disregarding the molecular structure and the molecular motions of the fluid ok.

So, essentially the fluid is composed of some molecules right. So, we are kind of disregarding the molecular structure and the molecular motions right, of the fluid such that we can consider this as a continuous substance right, without any voids that are present between the within the molecules right between the atoms and things like that.

So, that we can kind of treat it as a continuous substance ok. So, that is a continuum. Now again, this length scale approximation has to be kind of remembered all the time right. So, we are not looking at something that is very small. We are not looking at something that is very small, where the properties could be affected by the number of molecules that are moving in and out of the fluid element that we have considered ok.

So, but we are still looking at something that is very small such that right or something that is very large, such that where the number of molecules that are there kind of represent the particular property for that fluid element ok. So, when you invoke the continuum assumption, you can kind of characterize a fluid or describe a fluid in terms of its properties right ok. So, what are the properties for a fluid? You can describe your fluid in terms of its.

Student: Density (Refer Time: 06:53).

Density ok; density or?

Student: Pressure.

Pressure right. You have a pressure is a and then, you have a velocity vector right. The fluid flow velocity vector describes the fluid or temperature and so on right. All these properties are continuously varying across the fluid domain that we have considered ok, without any voids in the domain ok. Now, alright. So, but these are also could be functions of both space and time right.

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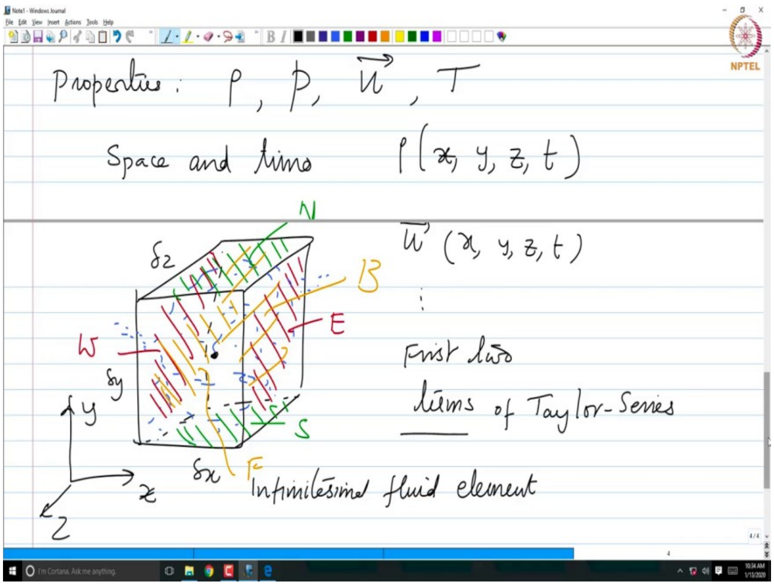

So, each of these properties that we have are functions of space such that is rho is a function of x, y, z, t. Similarly, the velocity vector u is a function of x, y, z, t and so on right. So, these are all functions of both space and time. Now, the fluid element that we consider here let us say I consider a fluid element like this. So, this is an infinitesimal fluid element. So, we have x axis, y axis and z axis. So, when we say x, y, z, we are looking at the centroid of this fluid element ok.

Now, this is an infinitesimal fluid element with dimensions delta x, delta y and delta z in all the three directions ok. Now, this is very small or an infinitesimal fluid element. However, the number of molecules that are there in this are still ok, these are let us say these are all the molecules that we have in the fluid element.

So, the number of molecules that are there inside are quite large in number right such that if we have some molecules going out or some molecules coming in, they are not affecting the fluid properties of this fluid element or particle that we have considered ok.

So, that is kind of good to know. Now, but this fluid element is small enough such that the properties of this fluid at on the faces for example, on this face let us say this is we call it as a east face or we call this as a west face right. On the faces, we can obtain the properties using just the first two terms of the Taylor series expansion that is because the fluid element is quite small ok. So, just by using the first two terms of Taylor series, we can obtain the properties on the faces.

So, for that matter, we can denote let us say the faces as east and west in the x direction. We could call this as a north face and this as a south face and similarly, we can call the other faces which are let us say the front face. So, this is a front face and the one at the back as a the back face ok. So, we could call east, west, north, south, front and back; about six faces alright.

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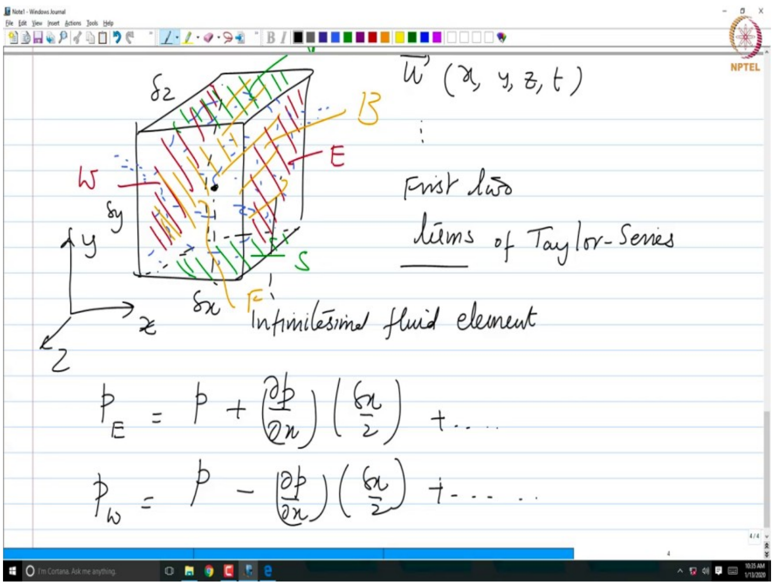

Now, so, if we want to construct the properties, let us say we want to construct pressure on the east face ok, we could do write it as p right. This is p at x, y, z that we have. On the east face E is at a distance delta x by 2 right. So, I could obtain using Taylor series as

$$p_E = p + \frac{\partial p}{\partial x}\left(\frac{\delta x}{2}\right)$$

and I do not need the higher order terms because the element is so small that this is accurate enough to represent the face value of the pressure.

Similarly, I can construct what is the value of the pressure on the west face by using,

$$p_W = p - \frac{\partial p}{\partial x}\left(\frac{\delta x}{2}\right)$$

alright plus minus and so on right. So, these kind of give us the values on the different faces. Similarly, we can construct it on the north face south face front and back and so on alright.

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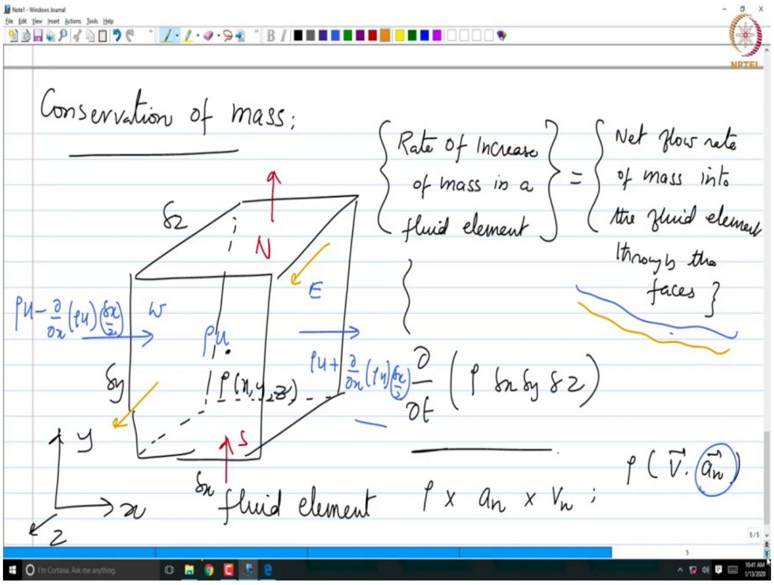

Look at conservation of mass. So, conservation of mass, we are looking at again a fluid element ok, something that we just drew above. What will be a statement of conservation of mass? We say that if we have a fluid element something like this ok. We have x, y, z. So, this is a fluid element alright. So, the conservation of mass, a mathematical statement is the rate of increase of mass within this fluid element should be balanced by the net rate of mass, that is entering through the faces ok.

So, if I were to write it, so this is basically rate of increase of mass in a fluid element right is balanced by the net flow rate of mass into the fluid element through the phases alright. So, what will be a rate of increase of mass in a fluid element?

How do we write it mathematically? We have a rate of increase, so we invoke the partial derivative with respect to time right. What will be the mass of this fluid element? So, let us say this is delta x, delta y and delta z and the density at the centroid which could be taken as uniform representation for the entire thing or the density in the fluid element right.

Now, what will be the mass of this fluid element? That will be the density times the volume right. So, essentially, we have the density rho and the volume is delta x, delta y delta z right. So, this is the term on the left hand side that we have. Partial partial t times rho times delta x, delta y, delta z ok. Now, we have to look at what is the net mass flow rate right net flow of mass into the fluid element through the faces.

What will be a mass flow rate across a particular face? What will be an expression for mass flow rate in terms of let us say the density right and the velocity of the fluid and the area of the surface that we are looking at? The mass flow rate would be?

Student: (Refer Time: 14:31).

Essentially, density right; times the cross-sectional area times the velocity component that is normal to the surface that we are considering right because whatever is not normal, would not enter the fluid element right. So, essentially so that means, rho times right rho times we have the cross sectional area, I would try to write it as a n that is the normal cross sectional area right times what we have? Times the velocity that is normal to the surface right.

So, essentially that is V n right, some kind of velocity. Now, if you were to write this thing, this is essentially nothing but rho times V bar dot a n bar right that is in terms of a vector notation ah. So, density times the velocity times the cross sectional area.

Now, what will be the density on this. So that means, I can write this as the cross sectional area that is a n right which is any of these cross sectional area that we have times rho times the V bar that is rho u would give me the momentum right into coming into the face right.

So, what would be the rho V bar dot a n? So, what will be the if the momentum at the center is rho u right that is in terms of x, y, z, what will be the momentum on the west face? I can use the Taylor series expansion and constraint again.

So, this would be¿rho u mi right that is what we have. Similarly, a momentum on the east face would be rho u plus partial partial x rho u times delta x by 2 right that is what we have on the east face. So, this is on the east face, this is on the west face ok.

Now, ah. So, we are looking at this second term essentially, we are calculating what is this net flow rate of mass into the fluid element. So, the fluid into the fluid element through all the six faces, we can have mass that is coming in and out right. For example, through the west phase, we have mass that is entering through east face, we have mass that is leaving.

Similarly, through the let us say north face the mass is kind of leaving and through the south face, we have mass entering Similarly, through the back face and the front faces, we have mass that is entering and leaving. So, we have to consider all of these sum to get to evaluate this second term here on the right hand side ok. So, that is what we do now ok.

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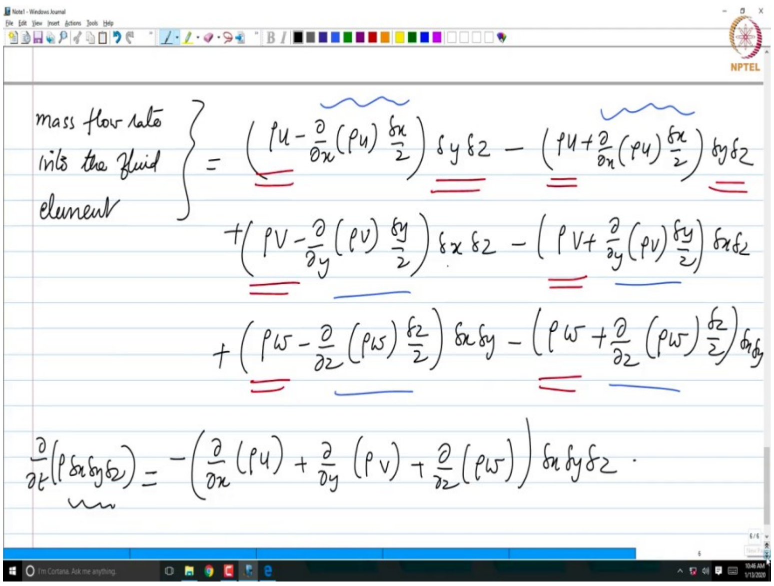

So, then what will be the mass flow rate into the fluid element? What will be the mass flow rate? So, whatever is flowing in we call it we represent it with positive right. So, from the west face we have $\left(\rho u-\frac{\partial}{\partial x}(\rho u)\frac{\delta x}{2}\right)$ right. So, this is the density times the velocity. What will be the cross sectional area of the west face?

Student: d by (Refer Time: 17:51).

Delta?

Student: Delta y by (Refer Time: 17:52).

y delta z right. So, that would be the cross sectional area right. So, that is your delta y delta z that is this area right, this times this alright and what about the mass that is leaving through the east face that will be with the minus right.

So, then you have $\left(\rho u+\frac{\partial}{\partial x}(\rho u)\frac{\delta x}{2}\right)$

times delta y delta z right that will be the same thing we have. Is that all? No, we have all the six faces. So, we have to add plus, what will be the mass flow rate coming through the entering through the north face? Sorry, entering through the south face.

Student: (Refer Time: 18:41).

So, we were to consider the velocity that is normal to the surface normal to the surface the velocity component is v right and then, v is represented here. So, it will be I am coming in the y direction. So, that would be $\left(\rho V-\frac{\partial}{\partial y}(\rho v)\frac{\delta y}{2}\right)$rho right times what would be the cross sectional area?

Student: (Refer Time: 19:04) delta x delta z.

Delta x delta z right. So, that is essentially this cross sectional area that we have that is this guy right. So, delta x delta z right minus we have the mass flow rate that is leaving through the top surface is rho v plus partial partial y rho v delta y by 2 times delta x delta z.

Similarly, if I extended to the back face and the front face on the back face, we have rho w minus partial partial z rho w delta z by 2 times what would be the cross sectional area? Delta x delta y right minus we have rho w plus partial partial z rho w times delta z by 2 times delta x delta y right. So, that is all. So, this is the mass flow rate that is there on the right hand side.

Now, what do you see? Of course, you see that we have a rho u times delta x delta y delta z right times delta y delta z here and rho minus rho u times delta y delta z here right. So, these terms get cancelled right and then, we have these terms, this guy and this guy are one and the same and they represent two halves right. So, this is delta x by 2, this is delta x by 2 times we have the same things. So, these two have sum to 1 right and then, have we have 3 such terms; one here and one here.

So, we just have to kind of write it as minus partial we can write it $-¿$ .So, we have these three terms on the right hand side and on the left hand side, we had $\frac{\partial}{\partial t}(\rho\delta x\delta y\delta z)$partial partial t that is the net rate of increase of mass within the control volume. Now, this is an equals here right. I would send this guy to the left hand side right and also get rid of the assuming that the fluid element is not changing its size right.

So, I can get this with time, I can get this out and then, we can get these cancel these two terms. So, what we end up here is? So, I whatever end up here is $\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0$ partial rho partial right?

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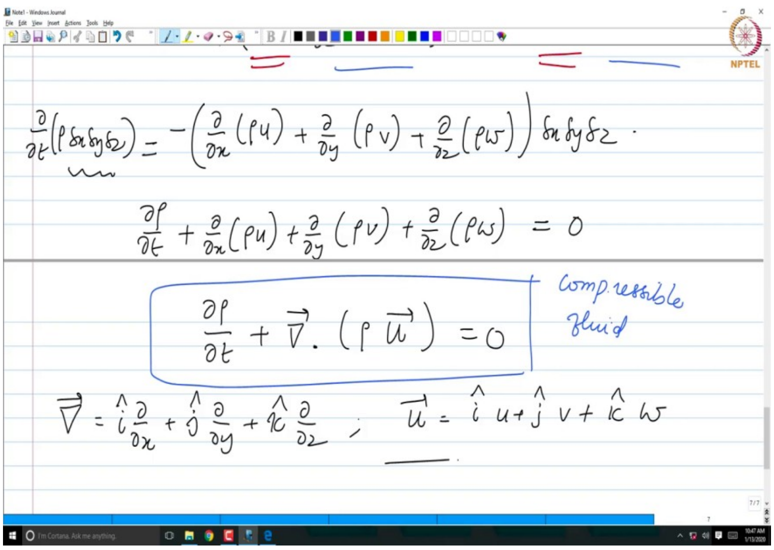
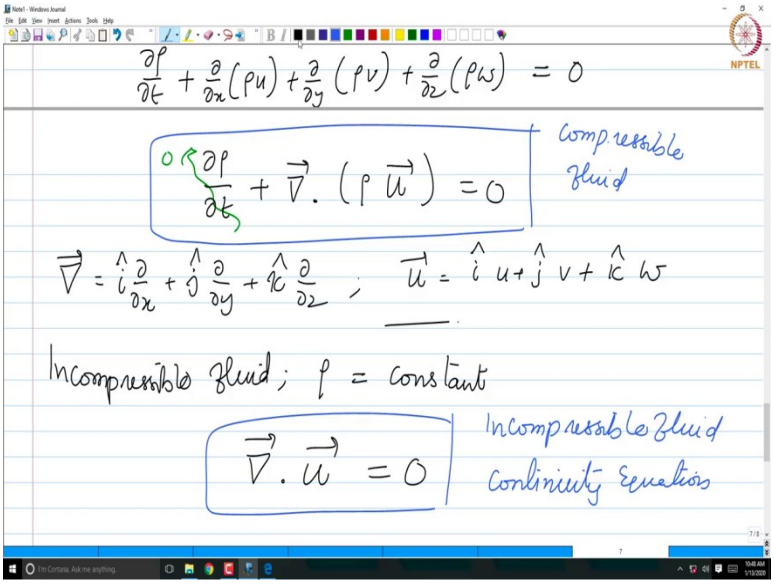


So, that is the conservation of mass. In a shorthand notation, we can write it as $\frac{\partial\rho}{\partial t}+\vec{\nabla}.(\rho\vec{u})=0$, where$\vec{\nabla}=\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}$ and your$\vec{u}=\hat{i}u+\hat{j}v+\hat{k}w$. So, we can write this. So, this is nothing but your conservation of mass for conservation of mass for a compressible right; the unsteady conservation of mass for a compressible fluid ok. (Refer Slide Time: 23:01)



Now, let us say we have a an incompressible fluid, which is where the density is what?

Student: Constant.

Is a constant ok. If we have density is a constant, then the first term drops out right, then this drops out, then rho can be taken out of the divergence because density is constant. So, then all you end up is $\vec{\nabla}.\vec{u}=0$. So, this is your for an incompressible fluid. This is conservation of mass that is also known as the what continuity equation? Why is it called continuity equation?

Because it defines the continuity of the velocity field right, you are taking the divergence of u bar and the continuity is being defined here right. It has to be a continuous field such that it would be delta t bar equal to 0 right, that is why it is a continuity equation. So, this is in a short form.

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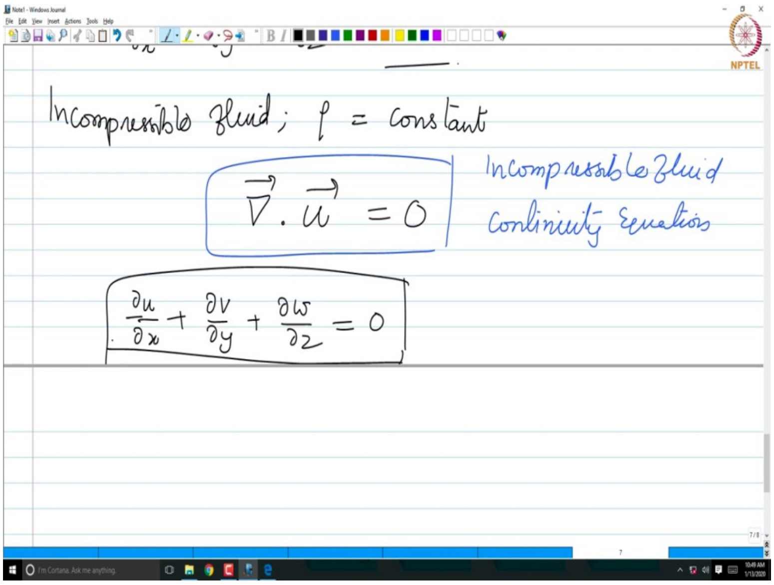

And if you were to take it in a long form this would be $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$. So, this is your continuity equation in long form right alright. Then, I am going to kind of stop here ok.

Thank you.