Basics of Materials Engineering

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Lecture – 53
Problems on Fatigue Failure – 3
(Effect of Notch, Multiaxial Loading)

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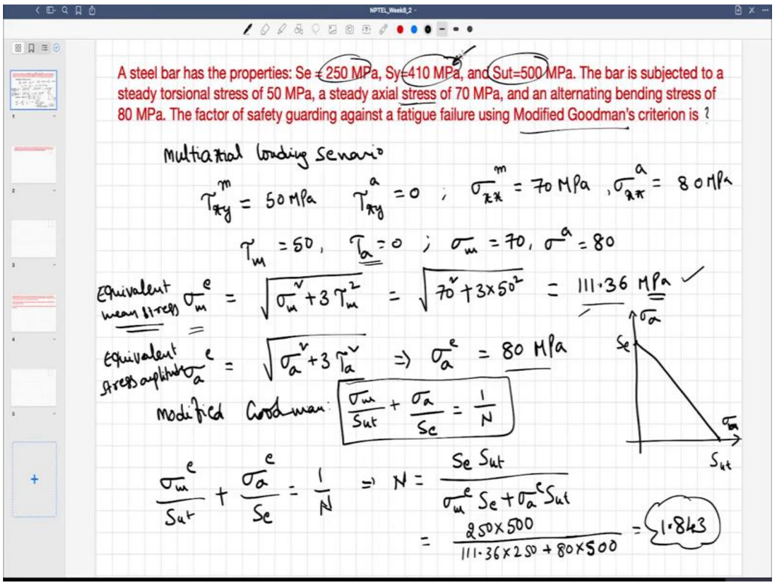

Welcome back to this session on problem solving for Fatigue Design or Fatigue Failure of Materials. So far, we have looked at the situation wherein the loading is one-dimensional - we have looked at the stress amplitude and mean stress for a one given one stress component, primarily the normal stress component; that is sort of a uniaxial loading scenario.

In general, the machine components are subjected to a multiaxial state of loading. In those scenarios how do we actually go about dealing with the problems? So, we have discussed this during the lectures, but now we will see a couple of problems which deal with multiaxial loading.

Here, we have a steel bar that has the properties - endurance strength 250 MPa, yield strength 410 MPa and ultimate strength 500 MPa. It is subjected to a steady torsional stress of 50 MPa, a steady axial stress of 70 MPa and an alternating bending stress of 80 MPa.

Here we can see that the bar is subjected to a multiaxial loading because you have a shear stress as well as normal stress due to axial load and a bending load. The plane in which the normal stress due to bending acts is not given. Hence, we assume that the normal stresses caused by bending as well as the axial stress are in the same direction. In that sense, it becomes a two-dimensional problem.

Basically, we are dealing with a multiaxial loading scenario wherein you have a steady torsional stress, τ_{xy}. Because we are saying steady, it is time invariant. So, we take this as the mean load. The mean load is sort of a static load. So, it does not change.

The torsional stress has a steady component of 50 MPa and the alternating component is 0 which means it is a static load. And then, we have steady axial stress. The axial stress is denoted as σ_{xx}. It can be assumed that the axial stress caused by the axial and bending loads are in different directions; both of them being normal.

Here I am assuming them to be in the same direction. The steady stress is the mean stress that is 70 MPa and the alternating stress is 80 MPa. These are the two things that we have in this kind of a loading scenario, i.e., you have a mean stress and an alternating stress. Now, I will drop the subscript xy and xx because there are only two components.

The mean shear component denoted by $\tau_m = 50$. I am not writing the units because they are all consistent units and $\tau_m = 0$, $\sigma_m = 70$, $\sigma_a = 80$. We know how to deal with the problems when the given state of loading is one-dimensional in nature.

We will now try to convert this multiaxial state of loading into an equivalent one-dimensional stress that we have learned during our static failure case. We can write a multiaxial state of loading into an equivalent one-dimensional quantity called equivalent stress or von-Mises stress.

Similarly, now we will try to find out the von-Mises stress equivalent of these two components. So, I need to find out the equivalent mean stress - it is actually computed using the same formula. The regular formula for finding equivalent stress is given by,

$$\sigma_m^e = \sqrt{\sigma_m^2 + 3\tau_m^2} = \sqrt{70^2 + 3 \times 50^2} = 111.36 \text{ MPa}$$

Here, I am trying to find out equivalent mean stress; σ_m^e is called the equivalent mean stress. If you would calculate that, the equivalent mean stress comes out to be 111.36 MPa.

Similarly, I can calculate equivalent alternating stress or equivalent stress amplitude, given by

$$\sigma_a^e = \sqrt{\sigma_a^2 + 3\tau_a^2} = \sqrt{80^2 + 3 \times 0} = 80 \text{ MPa}$$

In this case $\tau_a = 0$, and hence this is equal to 80 MPa.

There are other approaches to solve these multiaxial fatigue problems in different textbooks. However, this is one of the simpler approaches which has an analogy between what we have done in the static failure theories.

Hence, we are adopting this particular approach. There are slight variations in computing mean stress and so on, but we will not discuss that in this class. In this class, we will follow the procedure wherein we find the equivalent stress from the multiaxial state of stress.

Having found this multiaxial state of stress, given these material properties, you need to find the factor of safety according to modified Goodman criterion. So, what is modified Goodman criterion?

The Goodman diagram is shown here. σ_m is on the x-axis and σ_a is on the y-axis. S_{ut} and S_e are marked as shown. The modified Goodman says that,

$$\frac{\sigma_m}{S_{ut}} + \frac{\sigma_a}{S_e} = \frac{1}{N}$$

Since we are dealing with the multiaxial fatigue problems, σ_m with σ_m^e because it is one-dimensional scenario and σ_a with σ_a^e.

$$\frac{\sigma_m^e}{S_{ut}} + \frac{\sigma_a^e}{S_e} = \frac{1}{N}$$

In the above expression, N is the factor of safety and is given by,

$$N = \frac{S_e S_{ut}}{\sigma_m^e S_e + \sigma_a^e S_{ut}} = \frac{250 \times 500}{111.36 \times 250 + 80 \times 500} = 1.843$$

The factor of safety in this particular scenario according to modified Goodman criterion is 1.843. However, you note that you are also given the yield strength of the material. That is additional data that has been provided and need not be used in this particular problem.

Several times when you are dealing with design problems, you may have additional data and you should know that you need to discard that additional data when it is not needed. Sometimes when we are dealing with the design problems, you may have missing data.

That means, some data is not being given to you. Then you need to assume such missing data. In this particular scenario, we only have some additional data and then we need to discard that; we do not have to use that. Whenever there is missing data, you need to make an appropriate assumption for the missing data, ok? I think I hope this is clear - how do we go about solving a multiaxial fatigue problem.

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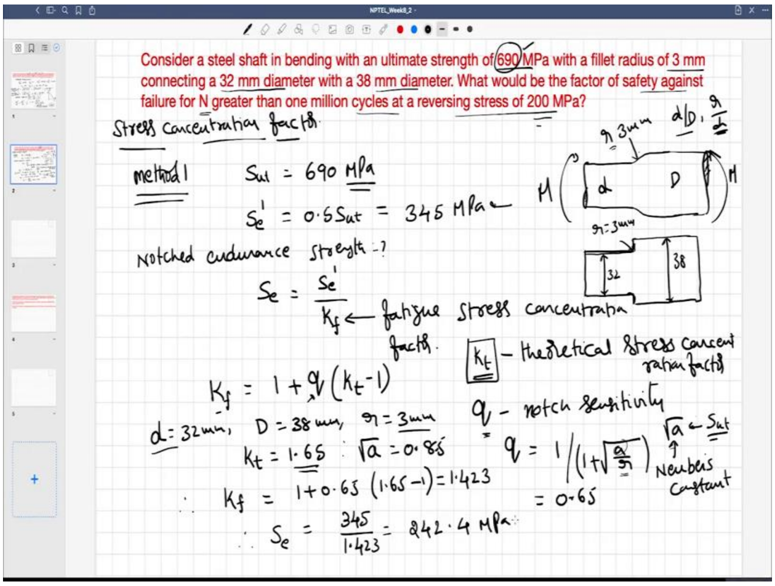

We have seen the case of stress concentration factor being used in the static failure theories. In the fatigue failure theory, you have something called a fatigue stress concentration factor, right? The fact that the loading is dynamic in this scenario, you need to deal with fatigue stress concentration factor.

Here is a problem - it is of course, given as a one-dimensional problem. But the idea of this problem is to see how we can apply stress concentration factor in the case of fatigue loading -

that is the objective of this problem. The problem can be considered as fatigue stress concentration factor; that is the idea to show this example.

Consider a steel shaft in bending with an ultimate strength of 690 MPa, with a fillet radius of 3 mm connecting 32 diameter with a 38 diameter. You have a steel shaft which is having 32 mm diameter and it is connected to a 38 mm diameter using a fillet.

A 32 mm shaft connected to a 38 mm part. Let us say this is 32 mm and that is 38 mm and it is connected with a fillet; that fillet radius is 3 mm. You do not actually connect like this; you will have a fillet. The fillet radius, $r = 3$ mm is shown here.

Now, the question is - what would be the factor of safety against failure for life N greater than one million cycles at a reversing stress of 200 MPa? The shaft is subjected to a reversing stress of 200 MPa. How do we go about finding the factor of safety using the known information?

Let us look at two methods. In method 1 we know $S_{ut} = 690$ MPa, and $S_e' = 0.5S_{ut} = 345$ MPa.

One way to do that is to calculate the notched endurance strength/notched endurance limit S_e from the unnotched endurance strength $S_e{}'$.

$$S_e = \frac{S_e'}{K_f}$$

In the above equation, K_f is the fatigue stress concentration factor. How do we go about calculating K_f?

$$K_f = 1 + q(K_t - 1)$$

Here, K_t is the theoretical stress concentration factor and q is the notch sensitivity. The shaft is subjected to pure bending as the loading is given as reversing bending stress.

It is subjected to M and you have inner diameter d, outer diameter D and this is r. By knowing d/D and r/D, one can get K_t from the charts.

In order to calculate fatigue stress concentration factor, you need to know q.

$$q = \frac{1}{\left(1 + \sqrt{\frac{a}{r}}\right)}$$

In the above expression, $\sqrt{a}$ is known as Neuber's constant which depends on the ultimate strength tensile strength of the material for steel.

For the given dimensions, we know that $d = 32$ mm, $D = 38$ mm and $r = 3$ mm. From the stress concentration factor charts available in various design data handbooks, you can see that for the choice of r/D and d/D , you will find that $K_t = 1.65$.

You can look at the charts and find out this value as we have done in the class. By knowing the ultimate strength of the material to be 690 MPa you can find the Neuber's constant - that is one way.

Otherwise, by knowing the r/d value, you can look at the chart and directly get the q value. In the former case, plug in the value of Neuber's constant and radius of the fillet and then from that you can calculate q.

For this given data $\sqrt{a} = 0.85$. q can then be calculated from the previous equation and is found to be 0.65.

$$\therefore K_f = 1 + q(K_t - 1) = 1 + 0.65(1.65 - 1) = 1.423$$

$$\therefore S_e = \frac{S_e'}{K_f} = \frac{345}{1.423} = 242.4 \text{ MPa}$$

The notched endurance strength of the material is 242.4 MPa. Now, we need to find out the factor of safety.

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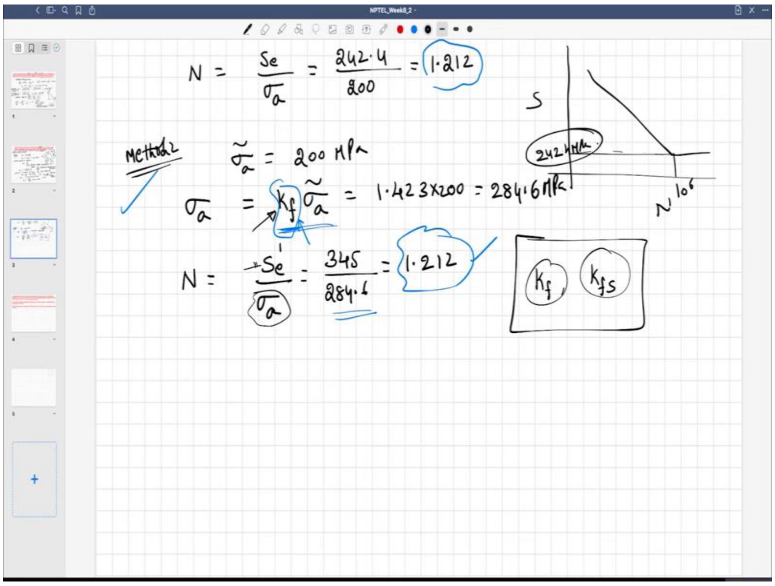

In the S-N diagram, this is 10^6 cycles. The endurance strength of the material is 242.4 MPa. This is the failure stress at 10^6 cycles. The factor of safety is,

$$N = \frac{S_e}{\sigma_a} = \frac{242.4}{200} = 1.212$$

This is one way to solve the problem, wherein, by using the notch, you are correcting for the endurance strength of the notch. There is another way to solve this problem, i.e., method 2, wherein, instead of taking the effect of notch on the material property, we will bring in the effect of notch on the stress concentration.

The far-field stress that is applied, $\widetilde{\sigma_a}$ is 200 MPa. The actual stress experienced by the material point, σ_a will be fatigue stress concentration factor times far-field stress. Fatigue stress concentration factor was 1.423.

$$\sigma_a = K_f \widetilde{\sigma_a} = 1.423 \times 200 = 284.6 \text{ MPa}$$

$$N = \frac{S_e{}'}{\sigma_a} = \frac{345}{284.6} = 1.212$$

The uncorrected endurance strength is used and not the notched endurance strength while calculating the factor of safety, as the effect of notch has been taken into the applied stress. So, you do not do this double counting.

Because the effect of notch has been taken into the applied stress, you do not have to take that into the account of endurance strength of the material. The factor of safety is 1.212.

Whether you take approach 1 or approach 2, you will get the same factor of safety. In one case you are accounting for the effect of notch in the material property itself; that means, you are modifying the material property while keeping the far-field stress as the stress which being applied. Another way to do it is to account for the stress concentration through the applied stress.

So, you have some far-field applied stress, but the actual stress experienced by the material is much higher. This is a better way to approach the problem. You have a material property and for that material property, you have an increased stress and then this is how you calculate the factor of safety.

This is how we are going to account for factors of safety. Please note that here, the stress concentration factor is given only in bending. In general, for a multiaxial loading scenario, you may have fatigue stress concentration factor for bending and fatigue stress concentration factor for torsion or shear.

In such scenarios, you need to account for them separately. You need to be very careful when we dealing with multiaxial loading with stress concentration factor as you have to multiply the appropriate stress concentration factor with the corresponding type of stress - that is very important to realize. Let us now look at the next problem.

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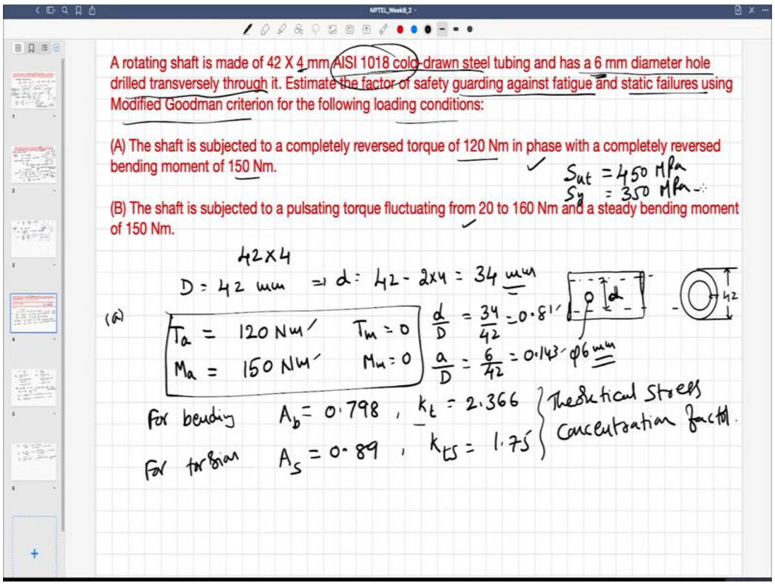

A rotating shaft is made of 42 × 4 mm AISI 1018 cold-drawn steel tubing and has a 6 mm diameter hole drilled transversely through it. Let us draw a schematic of the shaft. This shaft is hollow - it has dimensions 42 × 4 mm; that means, the outer diameter is 42 mm.

The inner diameter is denoted by d, 4 mm is the wall thickness. 42×4 means that $D = 42$ mm and the wall thickness is 4 mm. Hence, $d = D - 2 * 4 = 34$ mm.

There is a hole drilled in the transverse direction and the diameter of the hole is 6 mm. This is a little bit more involved problem; we need to be very careful in understanding this problem. We need to estimate the factor of safety guarding against fatigue and static failures, i.e., estimate the factor of safety guarding against fatigue and static failures using modified Goodman criteria for the following loading conditions.

In loading condition A, the shaft is subjected to a completely reversed torque of 120 N-m, in phase with a completely reversed bending moment of 150 N-m.

In the load case A, torque $T = 120$ N-m; completely reversed means alternative torque, and mean torque is 0. Similarly, the bending moment is completely reversed; that means, only alternating stress is present, and there is no mean component.

$M_m = 0$ as only the alternating component is present. The fact that there is a hole in the shaft, there is going to be a stress concentration factor and hence you need to worry about the fatigue stress concentration factor in this particular scenario.

We need to compute the fatigue stress concentration factor and the stresses experienced by this system. I will just show you how these tables look like.

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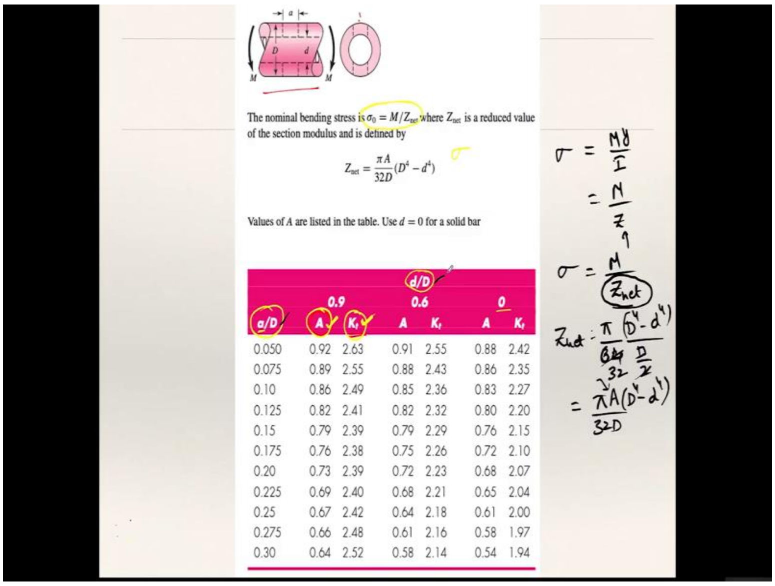

	d/D					
	0.9		0.6		0	
a/D	A	K_t	A	K_t	A	K_t
0.050	0.92	2.63	0.91	2.55	0.88	2.42
0.075	0.89	2.55	0.88	2.43	0.86	2.35
0.10	0.86	2.49	0.85	2.36	0.83	2.27
0.125	0.82	2.41	0.82	2.32	0.80	2.20
0.15	0.79	2.39	0.79	2.29	0.76	2.15
0.175	0.76	2.38	0.75	2.26	0.72	2.10
0.20	0.73	2.39	0.72	2.23	0.68	2.07
0.225	0.69	2.40	0.68	2.21	0.65	2.04
0.25	0.67	2.42	0.64	2.18	0.61	2.00
0.275	0.66	2.48	0.61	2.16	0.58	1.97
0.30	0.64	2.52	0.58	2.14	0.54	1.94

You have a shaft as shown here with the applied bending moment. The outer diameter is D and the inner diameter is d. The transversely drawn hole is shown here, whose outer diameter is a. Now, there are a couple of things. Here you are actually looking at the stress concentration factor and there is another parameter, A.

The parameter A as a function of d/D and a/D is shown here in this table. a/D is the ratio of the transverse hole diameter to the major diameter of the shaft. d/D is the ratio of the minor diameter of the shaft to the major diameter of the shaft. For different values of d/D - 0.9, 0.6 and 0 you have these values and you need to you need to compute these two parameters A and K_t.

Actually, we only need the stress concentration factor K_t, but we are also getting another parameter A. Because of the fact that there is a hole, the amount of cross-sectional area available near the hole is not the same. Hence, the nominal bending stress should not be calculated based on the total area.

You should take into account the presence of this transverse hole. Normally, the bending stress is My/I, where M/I is called as section modulus, Z.

In this particular scenario, when you have a transversely drilled hole, σ will be a function of Z_{net} as the area available and section modulus change.

$$Z_{\text{net}} = \frac{\pi A}{32D}(D^4 - d^4)$$

Usually, the section modulus is given by,

$$Z = \frac{\pi}{32D}(D^4 - d^4)$$

However, due to the presence of the transversely drilled hole, the section modulus must be multiplied by the parameter A to obtain Z_{net}. One needs to take into account of the factor A based on the values of small a/D and d/D - that is for bending.

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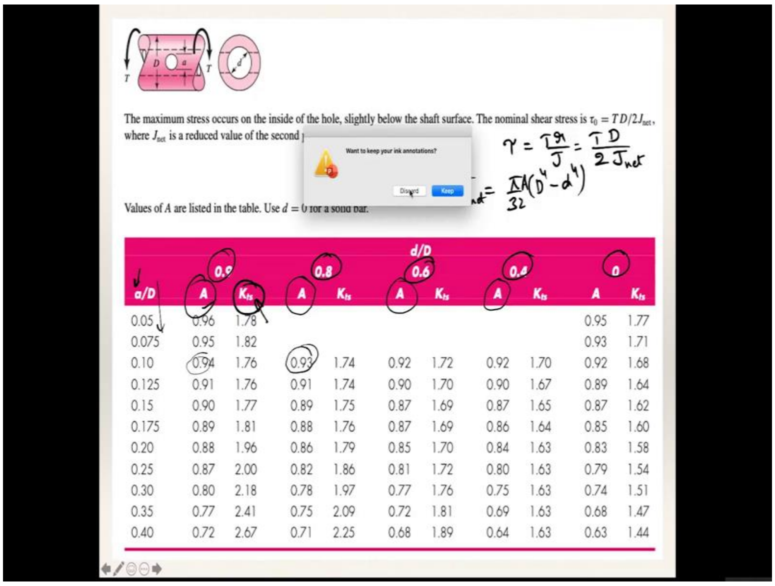

a/D	d/D = 0.9 A	0.9 K_{ts}	0.8 A	0.8 K_{ts}	0.6 A	0.6 K_{ts}	0.4 A	0.4 K_{ts}	0 A	0 K_{ts}
0.05	0.96	1.78							0.95	1.77
0.075	0.95	1.82							0.93	1.71
0.10	0.94	1.76	0.93	1.74	0.92	1.72	0.92	1.70	0.92	1.68
0.125	0.91	1.76	0.91	1.74	0.90	1.70	0.90	1.67	0.89	1.64
0.15	0.90	1.77	0.89	1.75	0.87	1.69	0.87	1.65	0.87	1.62
0.175	0.89	1.81	0.88	1.76	0.87	1.69	0.86	1.64	0.85	1.60
0.20	0.88	1.96	0.86	1.79	0.85	1.70	0.84	1.63	0.83	1.58
0.25	0.87	2.00	0.82	1.86	0.81	1.72	0.80	1.63	0.79	1.54
0.30	0.80	2.18	0.78	1.97	0.77	1.76	0.75	1.63	0.74	1.51
0.35	0.77	2.41	0.75	2.09	0.72	1.81	0.69	1.63	0.68	1.47
0.40	0.72	2.67	0.71	2.25	0.68	1.89	0.64	1.63	0.63	1.44

The shear stress under torsional loading is given by,

$$\tau = \frac{Tr}{J} = \frac{TD}{2J}$$

$$J = \frac{\pi(D^4 - d^4)}{32}$$

However, due to the presence of the transverse hole drilled into the system, the polar moment of inertia, J needs to be modified to obtain J_{net},

$$J_{\text{net}} = \frac{\pi A(D^4 - d^4)}{32}$$

The factor A for torsion and also K_{ts} have to be obtained from this table depending on a/D and d/D values.

K_{ts} is the shear stress concentration factor. Previously that is normal stress concentration factor; this is shear stress concentration factor, they are two different things. If the value of d/D value is not exactly 0.8 or 0.9, you need to interpolate between the values that you read from this table. That is what one needs to do when we are going to calculate the stress concentration factors for this scenario.

We found that for the hollow shaft scenario given here, in the case of bending, $\frac{d}{D} = \frac{34}{42} = 0.81$, and $\frac{a}{D} = \frac{6}{42} = 0.143$. These values are not directly available in the table. So, you need to interpolate from the available data. In this case we find that $A = 0.798$ and the bending stress concentration factor, $K_t = 2.366$.

Similarly, for a torsional load, we get, $A = 0.89$ and $K_{ts} = 1.75$. Before we apply the stress concentration factors, we need to find out the bending stress that is applied.

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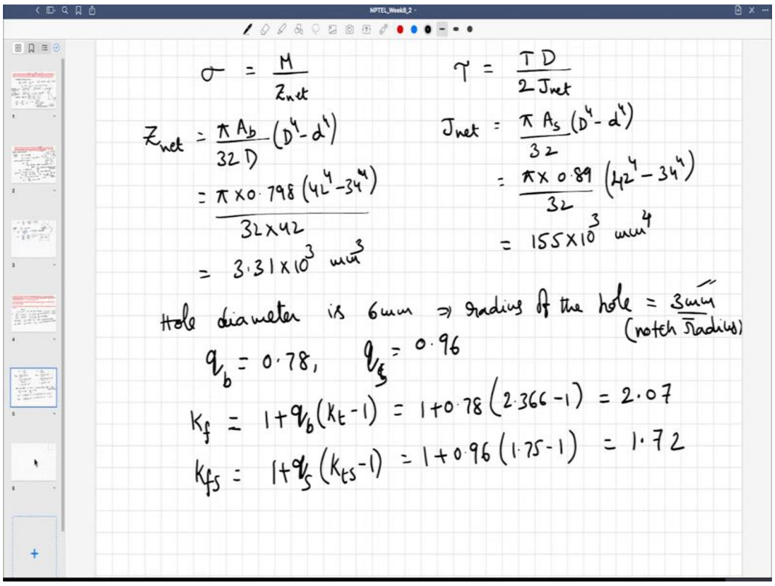

In order to calculate the bending stress, given by $\sigma = M/Z_{\text{net}}$, we need to find Z_{net}. In order to make it clear, let us denote the parameter A in the case of bending and shear as A_b and A_s, respectively.

$$Z_{\text{net}} = \frac{\pi A_b}{32D}(D^4 - d^4) = \frac{\pi \times 0.798(42^4 - 34^4)}{32 \times 42} = 3.31 \times 10^3 \text{ mm}^3$$

Similarly, for shear, we first need to calculate J_{net}.

$$J_{\text{net}} = \frac{\pi A_s}{32}(D^4 - d^4) = \frac{\pi \times 0.89(42^4 - 34^4)}{32} = 155 \times 10^3 \text{ mm}^4$$

Please keep that in mind that the units are different for Z_{net} and J_{net}.

The diameter of the hole that is drilled is 6 mm; that implies radius of the hole is equal to 3 mm, which is the notch radius. Why are we worried about it? From these tables, we only got the theoretical stress concentration factors, but for solving this problem, we need fatigue stress concentration factors.

In order to find the fatigue stress concentration factor, we need to find the notch sensitivity factor, q. So, q in bending corresponding to a hole radius 3 mm and the given diameters of the components, can be found from the tables. The notch sensitivity factor for bending, $q_b = 0.78$ and notch sensitivity factor for torsion in this particular scenario, $q_t = 0.96$.

When you do not write anything, that means, it is bending. The fatigue stress concentration factors in bending, K_f and shear, K_{fs} are calculated as,

$$K_f = 1 + q_b(K_t - 1) = 1 + 0.78(2.366 - 1) = 2.07$$

$$K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.96(1.75 - 1) = 1.72$$

Having calculated the fatigue stress concentration factors, you can now calculate the stresses.

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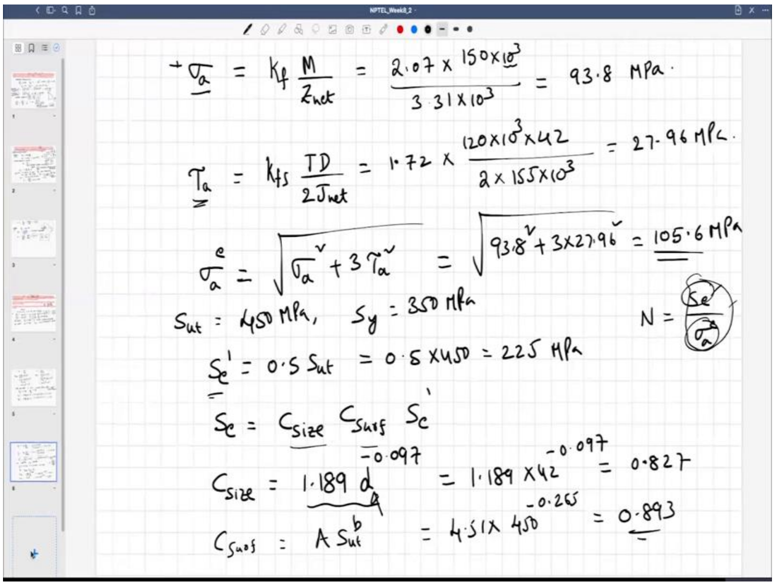

The bending stress amplitude is given by,

$$\sigma_a = \frac{K_f M}{Z_{net}} = \frac{2.07 \times 150 \times 10^3}{3.31 \times 10^3} = 93.8 \text{ MPa}$$

Please note that the length dimension for various quantities is taken in mm in the above expression and hence, the necessary conversions are made. In the bending scenario, there is no mean stress.

The stress amplitude in shear is given by,

$$\tau_a = \frac{K_{fs} T D}{2 J_{\text{net}}} = \frac{1.72 \times 120 \times 10^3 \times 42}{2 \times 155 \times 10^3} = 27.96 \text{ MPa}$$

Now, you have a multiaxial state of loading and in this case of multiaxial state of loading what you will do? Note that there is no mean stress here. So, you are only calculating equivalent stress amplitude given by,

$$\sigma_a^e = \sqrt{\sigma_a^2 + 3\tau_a^2} = \sqrt{93.8^2 + 3 \times 27.96^2} = 105.6 \text{ MPa}$$

In this problem, the material is specified as AISI 1018 steel. The ultimate strength of this material can be found in the tables. In exams these values will be given, but for practice, you should be able to look at the tables and obtain these numbers. The ultimate strength, $S_{ut} = 450$ MPa and yield strength is 350 MPa.

The factor of safety is endurance strength divided by alternating stress.

$$N = \frac{S_e}{\sigma_a^e}$$

You know σ_a^e, but you need to calculate the endurance strength. We know that $S_e' = 0.5S_{ut} = 225$ MPa. Please carefully look at the problem statement. The problem statement says that it is a cold drawn steel and the shaft diameter is 42 mm.

Hence, you need to account for correction factors as the shaft diameter is not 8 mm; if it is greater than 8 mm you need to account for the size correction factor, C_{size}. Similarly, depending upon the process, you also need to apply the surface finish correction factors.

Hence, you cannot right away use ${S_e}'$; you have to use S_e after applying the correction factors, namely the size correction factor and the surface finish correction factor. Since no other information is available, we assume the other correction factors to be 1. In the exam we will give you these correction factors or we will provide you the formulae required.

$$C_{\text{size}} = 1.189d^{-0.097} = 1.189 \times 42^{-0.097} = 0.827$$

In the above expression d does not necessarily mean the inner diameter of the shaft. In fact, the outer diameter value should be used. The surface correction factor is given as,

$$C_{\text{surf}} = AS_{ut}^b = 4.51 \times 450^{-0.265} = 0.893$$

In the above expression, the parameters A and B for the cold rolled sheet, when S_{ut} is given in MPa can be found from the tables - $A = 4.51$, and $B = -0.265$.

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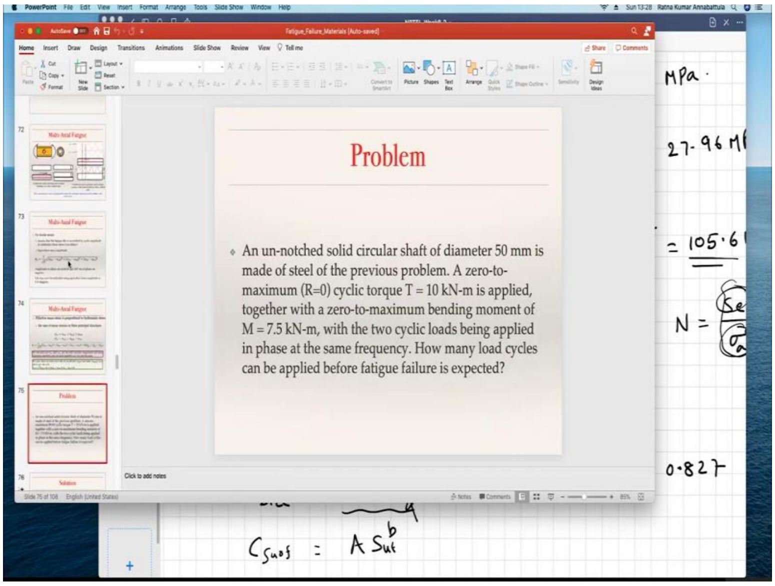

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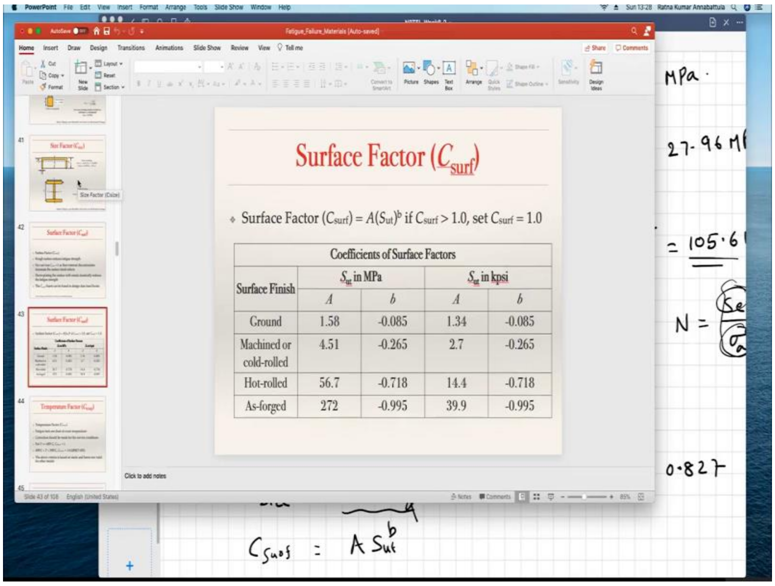

Substituting the values of C_{size} and C_{surf} in the expression below, the corrected endurance strength can be found.

$$S_e = C_{\text{size}} C_{\text{surf}} S_e = 0.827 \times 0.893 \times 225 = 166.2 \text{ MPa}$$

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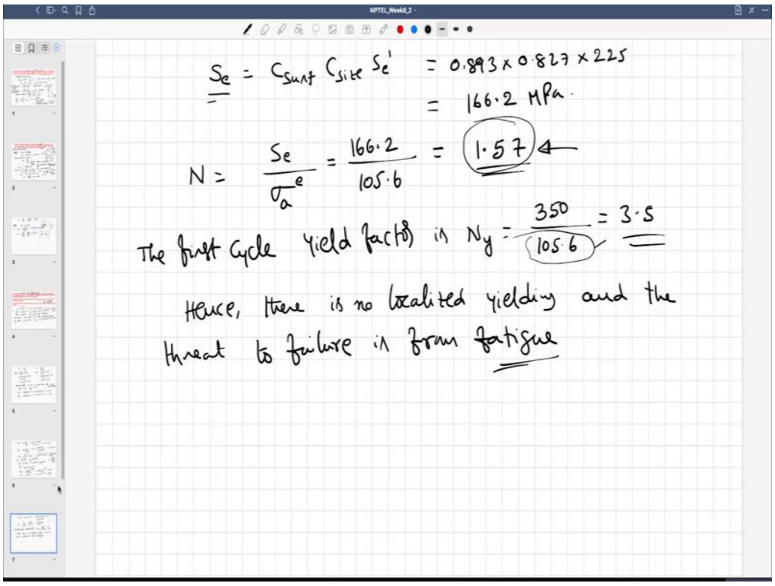

The factor of safety in fatigue is,

$$N = \frac{S_e}{\sigma_a^e} = \frac{166.2}{105.6} = 1.57$$

Now, you are also asked to find out the static factor of safety. The first cycle yield factor, N_y is given by,

$$N_y = \frac{\text{Yield strength}}{\sigma_a^e} = \frac{350}{105.6} = 3.5$$

Basically, this means that in the first cycle, the material is subjected to this particular stress amplitude 105.6 MPa and the yield strength of the material is 350 MPa. The specimen is not reaching the yield strength of the material in the first cycle and hence, there will not be localized yielding.

So, there will not be any localized plastic deformation and the threat to failure is from fatigue because the yielding factor of safety is much larger.

If at all the specimen fails, it will fail under fatigue before it reaches yielding - it will reach its life based on this loading. So, first it will fail in fatigue. Alright, that is for the first case of loading.

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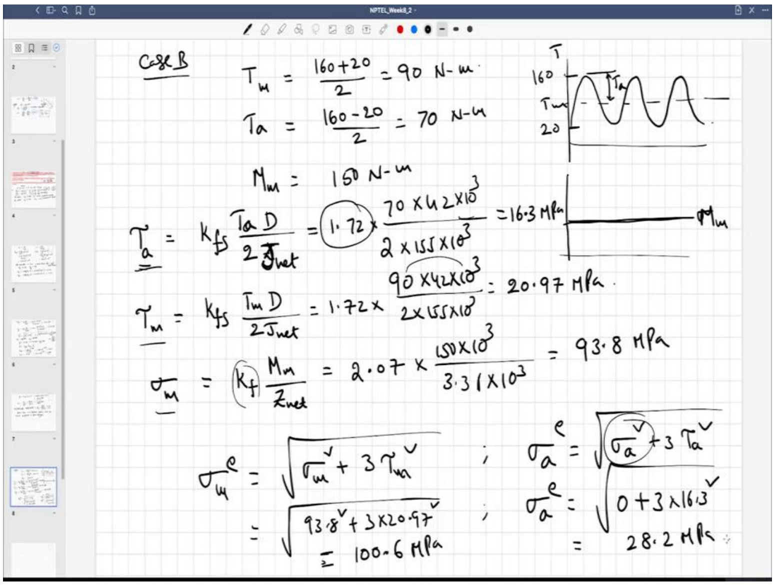

Let us now look at case B. In case B, the shaft is subjected to a pulsating torque fluctuating from $20 - 160$ N-m and a steady bending moment of 150 N-m. The mean torque and alternating torque are found as,

$$T_m = \frac{160 + 20}{2} = 90 \text{ Nm}$$

$$T_a = \frac{160 - 20}{2} = 70 \text{ Nm}$$

The bending moment is steady, i.e., it is not changing with time. The mean bending moment,

$$M_m = 150 \text{ Nm}$$

The shear stress amplitude is given by,

$$\tau_a = K_{fs}\frac{T_a D}{2\,J_{\text{net}}} = 1.72 \times \frac{70 \times 42 \times 10^3}{2 \times 155 \times 10^3} = 16.3 \text{ MPa}$$

The mean shear stress is given by,

$$\tau_m = K_{fs}\frac{T_m D}{2\,J_{\text{net}}} = 1.72 \times \frac{90 \times 42 \times 10^3}{2 \times 155 \times 10^3} = 20.97 \text{ MPa}$$

Since, the applied bending moment is steady, there is no alternating component to the normal stress; it is zero. The mean normal stress is given by,

$$\sigma_m = K_f \frac{M_m}{Z_{net}} = 2.07 \times \frac{150 \times 10^3}{3.31 \times 10^3} = 93.8 \text{ MPa}$$

Now, we have τ_m, τ_a and σ_m; it is a multiaxial state of loading and hence you need to calculate equivalent stress. The mean equivalent stress can be found as,

$$\sigma_m^e = \sqrt{\sigma_m^2 + 3\tau_m^2} = \sqrt{93.8^2 + 3 \times 20.97^2} = 100.6 \text{ MPa}$$

The alternating equivalent stress can be found as,

$$\sigma_a^e = \sqrt{\sigma_a^2 + 3\tau_a^2} = \sqrt{0^2 + 3 \times 16.3^2} = 28.2 \text{ MPa}$$

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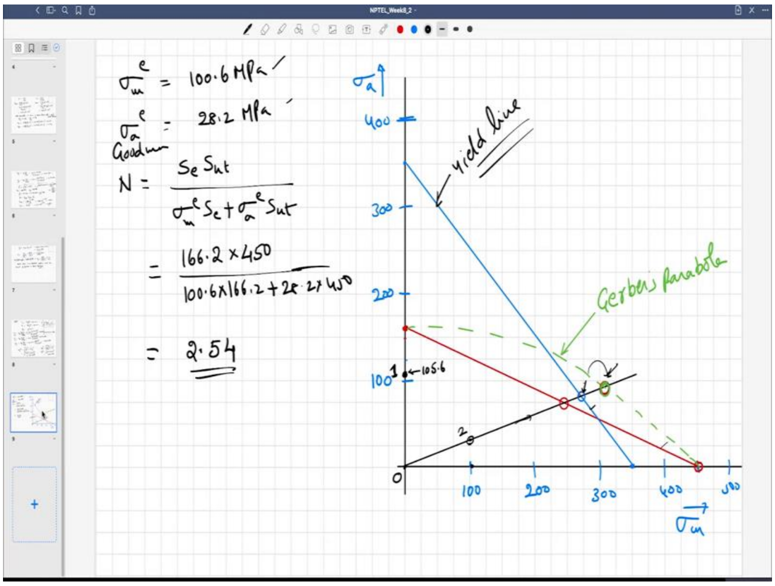

What we now have is that $\sigma_m^e = 100.6$ MPa and $\sigma_a^e = 28.2$ MPa. We are asked to find the factor of safety. Due to the effect of mean stress, we need to use either Goodman criterion or Soderberg criterion or Gerber's parabola. According to Goodman,

$$N = \frac{S_e S_{ut}}{\sigma_m^e S_e + \sigma_a^e S_{ut}} = \frac{166.2 \times 450}{100.6 \times 166.2 + 28.2 \times 450} = 2.54$$

Let us do some analysis here so that we can comment on the static failure. Let us draw the $\sigma_m - \sigma_a$ plots to scale. σ_m is on the x-axis and σ_a is on the y-axis. Each division on the x and y axes in this plot corresponds to 25 units.

Now, how do we go about drawing the Goodman line? First let us identify S_e on the y-axis which is 166.2 MPa, and S_{ut} on the x-axis, which is 450 MPa. The Goodman line connects these two points.

Now, there is something called yield line. We have not paid attention to this when we were discussing in the class, but now I think this will be really helpful. Yield line is the line connecting the values equal to the yield strength both on mean stress line and stress amplitude line. What is the yield strength of the material?

The yield strength is 350 MPa. The yield line is shown in blue color here. Let us now look at the two states of stress. In case A, the material did not experience any mean stress, but only alternating stress.

The alternating stress in case A is 105.6 MPa and is marked as a black point on the y-axis and numbered as point 1. O1 will be your load line. If we are continuing in this load line, you are hitting the Goodman line first; that means, if at all it is failing under this loading it will only fail due to fatigue, i.e., fatigue failure will occur first.

In the second case, the mean stress is 100.6 MPa and the alternating stress is 28.2 MPa. This data point is numbered 2 on the graph and is also marked in black color. Let me draw a line joining those two points - that is my load line.

Now again if you actually move along this line, the failure due to fatigue happens here and here it is hitting the yield line. Even in this case, the material is expected to fail under fatigue. So, according to Goodman criterion the material is susceptible to failure under fatigue.

Let us draw the Gerber's parabola, which is another failure criterion with another color just to make it distinct. So, the Gerber's parabola would look something like this. If the failure criterion Gerber's parabola rather than Goodman, you would see that the failure is happening at this point instead.

So, along this line, but the failure according to Gerber's parabola happens at this point and let us say you do not have the Goodman criterion. Then this loading line is hitting the yield line first and then hitting the Gerber's parabola.

In this scenario, it is possible that the material might experience failure from notch yield; in the first cycle of loading, the material might actually experience notch yielding because first this yield line is hit rather than the Gerber's parabola.

However, if you are using Goodman criterion, you would see that it is hitting the Goodman line first and hence you would not expect notch yielding whereas, when you are talking about Gerber's criterion, you move would expect notch yield. We know that Gerber's parabola is a better approximation to the failure of the material compared to Goodman criterion. But Goodman criterion is simpler to apply and hence we are using Goodman's criterion.

That is how you can use this yield line to make comments on the static failure in scenarios where the material is subjected to dynamic load; that means, whether there is going to be a yielding or not in this scenario.

According to Goodman criterion, the failure may first come from fatigue. However, according to the Gerber's criterion, failure may first come from first cycle notch yielding because it is hitting the yield line first. If at all it goes along that direction, it will first rather show notch yielding rather than complete failure according to Gerber's criterion.

In this class, we have first looked at the multiaxial loading scenario without any notch. Then we have looked at the situation of one-dimensional loading, but in the presence of notch i.e., how do we take into effect of the notch or stress concentration factor.

Finally, we accounted for the presence of a notch in a multiaxial loading scenario. We are also able to add another complexity to explain about the notch yielding; whether the notch yielding will be prevalent in the material or not.

I hope these three problems have given you some understanding of how do we go about solving multiaxial fatigue problems and also how do we go taking into consideration of the presence of notches in the material.

With that, I will stop and thank you very much.