Basics of Materials Engineering
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Lecture - 50
Problems on Fatigue Failure - 2 (Effect of mean stress, Fatigue crack growth)

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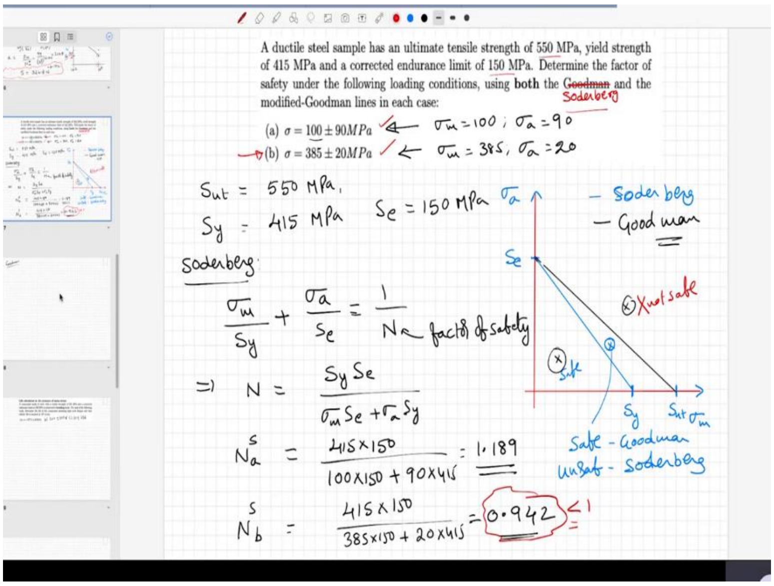

Welcome back. In this class, we will look at certain problems involving effect of mean stress and estimating the life in the case when you have mean stress acting on the specimen. This is the problem that we are going to solve. A ductile steel sample has an ultimate tensile strength of 550 MPa and yield strength of 415 MPa and a corrected endurance limit of 150 MPa. Determine the factor of safety under the following loading conditions using both the Soderberg and modified Goodman lines in each case. If you remember the discussion on the fatigue failure in the presence of mean stress, we have different failure theories as in the case of static failure theories. One of them was Soderberg and another one was Goodman and there was another one called Gerber's parabola, right?

In this graph, on the y-axis we plot the stress amplitude and on the x-axis, we plot the mean stress. Let us say on the x-axis, you identify this to be the yield strength S_y and that to be the ultimate strength S_{ut}. On the y-axis you have identified the corrected endurance strength S_e. This failure line, joining S_e with S_{ut}, is the Goodman line. The failure line joining S_e with S_y, is the Soderberg line. Let me draw the Soderberg line with blue color to distinguish these two criteria. So, the blue line is the Soderberg line. So, the Soderberg line between S e and S y. So, the blue line is Soderberg and the black one is Goodman or modified Goodman; these are the failure surfaces.

If any stress state, meaning a combination of mean stress and stress amplitude, is somewhere here, this is safe because it is within the failure domain.

If the stress state is outside that, then it is not safe. This is not safe and this is safe; both with respect to Goodman and Soderberg. However, if the stress state is somewhere here, it is safe according to Goodman and unsafe according to Soderberg.

The data that we have is the ultimate strength is 550 MPa and yield strength S_y is 415 MPa and endurance limit is 150 MPa. So, if you are looking at Soderberg criterion, the failure the line can be represented as,

$$\frac{\sigma_m}{S_y}+\frac{\sigma_a}{S_e}=\frac{1}{N^s}$$

where N^s is the factor of safety. We now need to find out the factor of safety, given by,

$$N^s=\frac{S_y S_e}{\sigma_m S_e+\sigma_a S_y}$$

We know that $S_y=450$ MPa, $S_e=150$ Mpa. We do not know σ_m and that is what we need to figure out. For the loading state 1, what is σ_m? So, this is 100 ± 90 MPa; that means, $\sigma_m=100$ MPa and $\sigma_a=90$ MPa, right? For the second problem, $\sigma_m=385$ MPa and $\sigma_a=20$ MPa.

First, let us look at case 1.

$$N_a^s=\frac{415\times 150}{100\times 150+90\times 415}=1.189$$

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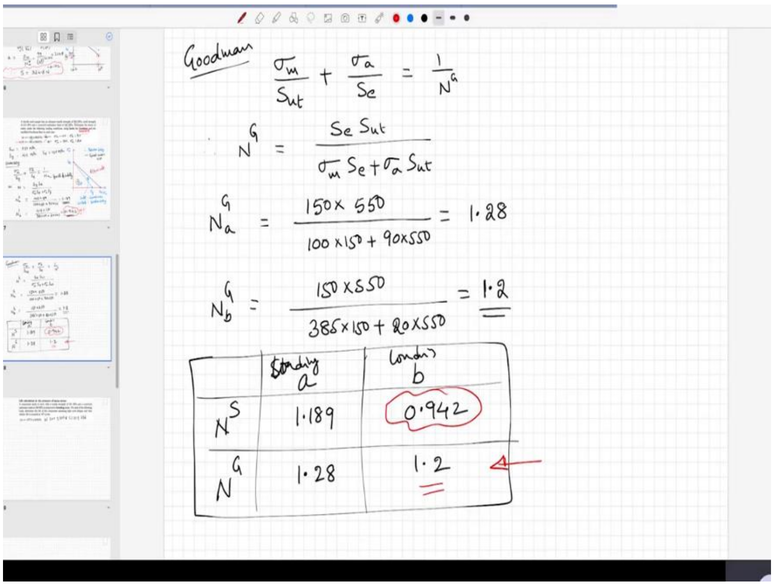

Let us also calculate factor of safety according to Soderberg at point b -- the numerator will not change because that is a material property.

$$N_b^s = \frac{415 \times 150}{385 \times 150 + 20 \times 415} = 0.942$$

The factor of safety here is less than 1; that means, according to Soderberg criterion the second stress state corresponds to failure because it is going out of the Soderberg line.

Let us now look at Goodman criterion given by,

$$\frac{\sigma_m}{S_{ut}} + \frac{\sigma_a}{S_e} = \frac{1}{N}$$

That is the representation of the boundary. Let us denote the factor of safety here as N^G. We have,

$$N^G = \frac{S_e S_{ut}}{\sigma_m S_e + \sigma_a S_{ut}}$$

S_e and S_{ut} will not change whether it is loading a or b.

$$N_a^G = \frac{150 \times 550}{100 \times 150 + 90 \times 550} = 1.28$$

$$N_b^G = \frac{150 \times 550}{385 \times 150 + 20 \times 550} = 1.2$$

On careful observation, we see that the factors of safety according to Soderberg criterion are always lower than the Goodman criterion.

According to Soderberg criterion, you would have said that the material has failed, but actually it did not according to Goodman criterion and we know that Goodman criterion is a better criterion for fatigue failure because the fracture actually means the ultimate strength and not really the yield strength. So, that is why the experimental results show a better match with the Goodman diagram compared to Soderberg diagram.

According to Goodman, both the loading scenarios are same, although the factors of safety are not really high i.e., 1.28 and 1.2, respectively. So, that is how we can actually calculate the factor of safety for a given scenario.

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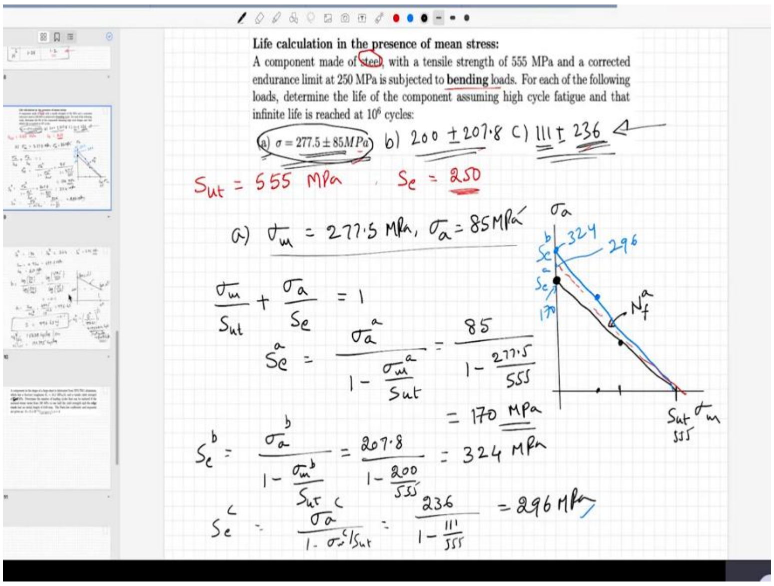

Let us look at another problem. How do we go about calculating the life in the presence of mean stress? Here we will again look at three different stress states and then we will make a comparison in the end. A component that is made of steel with tensile strength 555 MPa and a corrected endurance strength S_e of 250 MPa is subjected to bending loads.

For each of the following loads determine the life of the component assuming high cycle fatigue and that infinite life is reached at 10^6 cycles. So, you have a scenario wherein you know the endurance strength of the material, i.e., 250 MPa, S_{ut} is 555 MPa.

For each of these loading scenarios, what is the life of the component? That is what we need to figure out. If you take the case a, $\sigma_m = 277.5$ MPa and $\sigma_a = 85$ MPa. Let us now see the scenario of Goodman diagram. Since we are not given any criterion, we are using Goodman, because we know that is a better criterion. According to Goodman criteria,

$$\frac{\sigma_m}{S_{ut}} + \frac{\sigma_a}{S_e} = 1$$

Here, I am assuming factor of safety to be 1. Hence, I am not taking that factor of safety into account. Corresponding to this stress state, let us calculate S_e, given by,

$$S_e = \frac{\sigma_a}{1 - \frac{\sigma_m}{S_{ut}}}$$

For stress state a,

$$S_e^a = \frac{\sigma_a^a}{1 - \frac{\sigma_m^a}{S_{ut}}} = \frac{85}{1 - \frac{277.5}{555}} = 170 \text{ MPa}$$

For stress state b,

$$S_e^b = \frac{\sigma_a^b}{1 - \frac{\sigma_m^b}{S_{ut}}} = \frac{207.8}{1 - \frac{200}{555}} = 324 \text{ MPa}$$

For stress state c,

$$S_e^c = \frac{\sigma_a^c}{1 - \frac{\sigma_m^c}{S_{ut}}} = \frac{236}{1 - \frac{111}{555}} = 295 \text{ MPa}$$

For each of these stress states, we have calculated the endurance limit. S_e corresponding to stress state a is 170 MPa, stress state b is 324 MPa, and stress state c is 295 MPa. Rather than calling this S_e, I could actually calculate the failure strength. Normally, this is the stress state. So, what is σ_m?

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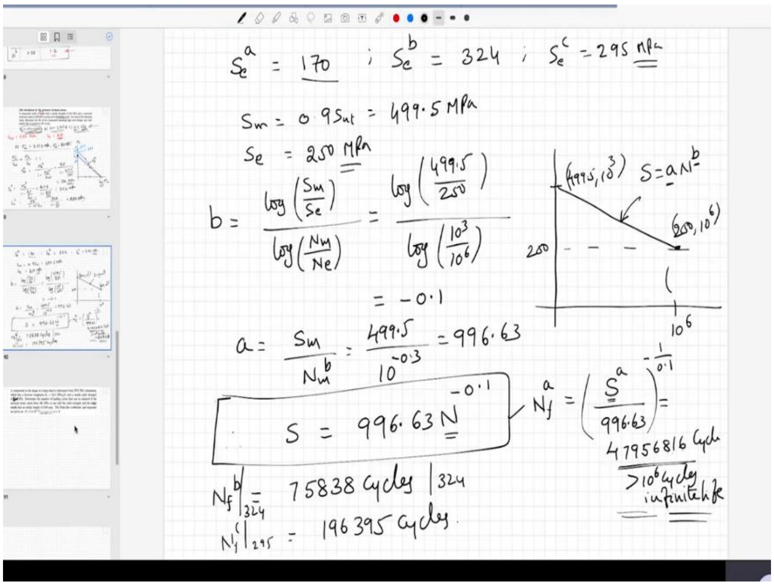

$S_{ut} = 555$ MPa, $\sigma_a = 277.5$ MPa, that is half way somewhere here and let us say this is the failure point. Then, you need to join these lines and then this corresponds to a particular number of cycles N_f^a. Similarly, if $\sigma_m = 200$ MPa, that is somewhere here, σ_a is here that is let us say point b 207.8 MPa, then that would be the corresponding failure line.

These points correspond to the endurance limit for the cases a, b and c. $S_e^c = 295$ MPa will be somewhere in between because this is 170 MPa and this is 324 MPa.

Corresponding to these stress amplitudes, what is the life for each of these loading states? Since the material or the component is made of steel, we can construct the S-N diagram by knowing $S_m = 0.9S_{ut} = 0.9 \times 555 = 499.5$ MPa, and $S_e = 250$ MPa for that particular scenario.

This is for 10^6 cycles. So, we have the two data points as $(499.5, 10^3)$ and $(250, 10^6)$. Now, you need to find out the material properties a and b. Note that a and b values themselves will not change depending upon the loading scenario, as they are material properties. We have,

$$b = \frac{\log\left(\frac{S_m}{S_e}\right)}{\log\left(\frac{N_m}{N_e}\right)} = \frac{\log\left(\frac{499.5}{250}\right)}{\log\left(\frac{10^3}{10^6}\right)} = -0.1$$

$$a = \frac{S_m}{N_m^b} = \frac{499.5}{10^{-0.3}} = 996.63$$

Having found the values of a and b, the stress-life relation for the system can be written as,

$$S = aN^b = 996.63N^{-0.1}$$

Corresponding to this particular loading scenario, the failure happens at 170 MPa, and the number of cycles to failure, N_f^a is given by,

$$N_f^a = \left(\frac{S^a}{996.63}\right)^{-\frac{1}{0.1}} = 47956816 \text{ cycles}$$

Since, $N_f^a > 10^6$ cycles, the specimen has infinite life in the first scenario.

In the second scenario, corresponding to $S_e = 324$ MPa, $N_f^b = 75838$ cycles, and corresponding to $S_e = 295$ MPa, $N_f^c = 196395$ cycles. Out of the three stress-states, the first stress-state gives infinite life, second stress state gives a life of 75800 cycles, and the third stress state gives a little bit more, i.e., 196395 cycles.

What we have done here is that you have a combined state of loading; that means, you have mean stress effect. In the presence of mean stress, how do we go about finding the life of the component.

The stress-life behavior is governed by the material property which is described by the equation, $S = aN^b$, where a and b are material constants, depending on which points you are connecting. From the mean stress and stress amplitude information, you need to calculate what is S_e i.e., failure corresponding to the loading and then you will be able to calculate the life of the component for different scenarios. That is about how we go about dealing with the effect of mean stress.

Let us look at one last problem for today's discussion. It is basically the crack growth rate depending on the Paris law. A similar problem we have probably solved during the regular lectures too, but I am trying to reiterate here.

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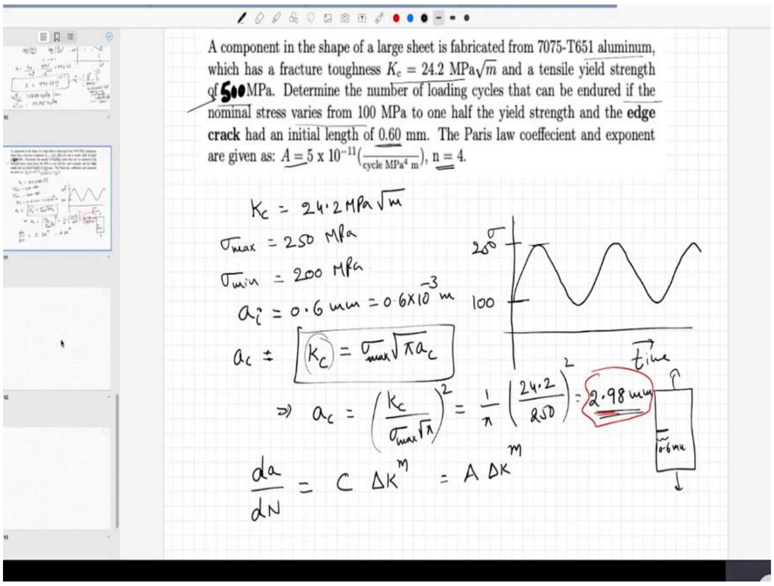

A component in the shape of a large sheet is fabricated from aluminum which has a fracture toughness of 24.2 MPa$\sqrt{\text{m}}$, and a tensile yield strength of 500 MPa. Determine the number of loading cycles that can be endured if the nominal stress varies from 100 MPa to one half of the yield strength.

So, you have a scenario wherein the fracture toughness of the material is given as 24.2 MPa$\sqrt{\text{m}}$. The loading as seen from the stress vs time graph, cycles from 100 MPa to one half of the yield strength, i.e., 250 MPa.

I have chosen it to be sinusoidal, but it does not have to be. What is important here is that $\sigma_{\max} = 250$ MPa, and $\sigma_{\min} = 200$ MPa. We are required to determine the number of cycles that can be endured when the stress cycles between $100 - 250$ MPa, if the initial crack length is 0.60 mm, i.e., there is an edge crack in the specimen.

Let us say you have this specimen with an edge crack of length 0.6 mm which is subjected to some sort of a bending load or which is continuously loaded between $100 - 250$ MPa. How much is the life that is left until it actually becomes a critical crack and then the specimen breaks?

This is something that we have discussed extensively that even if you have some crack to begin with it, you may still be able to use the specimen; that is precisely what we are trying to address through this problem. How many cycles of this loading can be applied on the specimen before it fractures? Of course, we are assuming that everything is linear elastic; there is no plasticity whatsoever.

The initial crack length is $0.6\ \text{mm} = 0.6 \times 10^{-3}$ m. What would be the crack length at which it actually fails? The final crack length is the critical crack length at which it; that can be found using the relation, $K_\text{c} = \sigma\sqrt{\pi a_\text{c}}$

When length of the crack becomes a_c , that is when, for a given σ, if that becomes equal to fracture toughness, the system collapses. In this system the maximum stress that the material is experiencing is 250 MPa. The critical crack length can be found as,

$$a_\text{c} = \left(\frac{K_\text{c}}{\sigma_\text{max}\sqrt{\pi}}\right)^2 = \frac{1}{\pi}\left(\frac{24.2}{250}\right)^2 = 2.98 \times 10^{-3}\text{m} = 2.98\ \text{mm}$$

The loading can be applied on the specimen until those many cycles where the initial crack length of 0.6 mm reaches a critical length of 2.98 mm. The crack is gradually growing from $0.6 - 2.98$ mm. Until then the structure is safe; the moment it becomes 2.98 mm, that is when the failure happens. So, how do we calculate this? We know that crack growth rate by Paris law is given by,

$$\frac{\text{d}a}{\text{d}N} = A\Delta K^m$$

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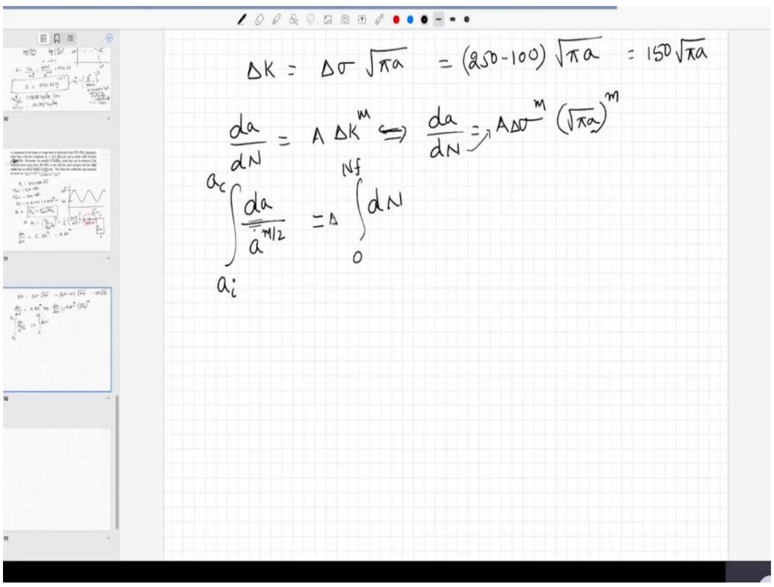

$$\Delta K = \Delta\sigma\sqrt{\pi a} = (250 - 100)\sqrt{\pi a} = 150\sqrt{\pi a}$$

$$\frac{\mathrm{d}a}{\mathrm{d}N} = A\Delta K^m = A\Delta\sigma^m\left(\sqrt{\pi a}\right)^m$$

Rearranging the above equation,

$$\int_0^{N_f} \mathrm{d}N = \int_{a_\mathrm{i}}^{a_\mathrm{c}} \frac{\mathrm{d}a}{A\Delta\sigma^m \pi^{\frac{m}{2}} a^{\frac{m}{2}}}$$

In this problem, we have $m = 4$, $\Delta\sigma = 150$, and $A = 5 \times 10^{-11}$ in the consistent units.

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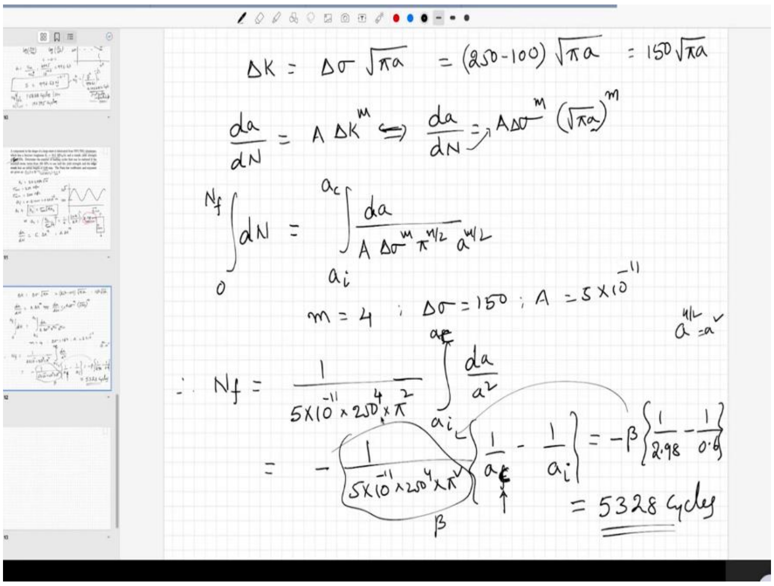

$$\therefore N_f = \frac{1}{5 \times 10^{-11} \times 250^4 \times \pi^2} \int_{a_\mathrm{i}}^{a_\mathrm{c}} \frac{\mathrm{d}a}{a^2} = -\beta\left\{\frac{1}{2.98} - \frac{1}{0.6}\right\}$$

You can calculate the value of β and then you find out $N_f = 5328$ cycles; that means, even if you have an initial crack of length 0.6 mm, you will still be able to retain the functionality until the cycle number of cycles are about 5328; until then the specimen will be safe.

So, with that I will stop here and in the next class, we will look at how the fatigue problems are to be solved in the presence of notches.