Basics of Materials Engineering
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Lecture - 50
Fatigue Failure of Materials (Fatigue Crack Growth, Paris' law)

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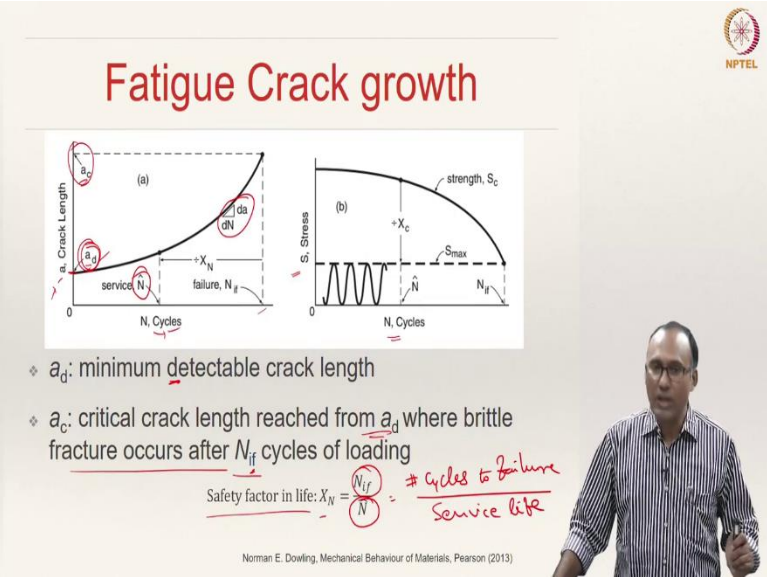

So far, we have looked at stress-life approach and now we need to look at LEFM approach for fatigue crack growth. So far, we have discussed how the fatigue crack grows and how a crack growth is happening in the material.

Let us now look at this figure. The first one actually shows the crack length as a function of number of cycles and the second one shows the stress as a function of number of cycles. So, here let us say a_d is the minimum detectable crack length distance; d stands for detectable.

In any material, if by doing an inspection, depending upon the resolution of the equipment that we are using, we will be able to find out a crack length of -- say for instance, 1 mm is the resolution, then you cannot detect a crack whose length is less than 1 mm.

Let us say that a_d is the minimum detectable crack length in the material and then, a_c is the critical length reached from a_d, where brittle fracture occurs after N_{if} cycles, f is failure cycles of loading. You do not have any crack which is larger than a_d in the material.

Let us say you have a crack whose initial length is a_d, then you are loading, you are applying fatigue loading on the material, after N_{if} cycles, the crack length reaches a critical value of a_c, that is when the material breaks as if it is a brittle fracture.

As we have seen in the case of the fracture mechanics module, when the crack length reaches critical crack length, that is when the material breaks as if it is a brittle material. The same thing we are doing here and so, here we are having a versus N and the slope of this line is $\frac{da}{dN}$ which is the crack growth rate; a is the crack length here and N is the number of cycles.

Let us say for this particular material, the maximum size of the crack to begin with is a_d and if this particular component is designed for service life of $\hat{N}$ cycles; that means, you are designing this to last for $\hat{N}$ cycles which are less than N_{if} cycles. Then, you can define, as we have already discussed, the factor of safety in life can be written as,

$$X_N = \frac{N_{if}}{\hat{N}}$$

This is something called factor of safety in life that we have already looked at, when we are talking about crack that is actually growing in size.

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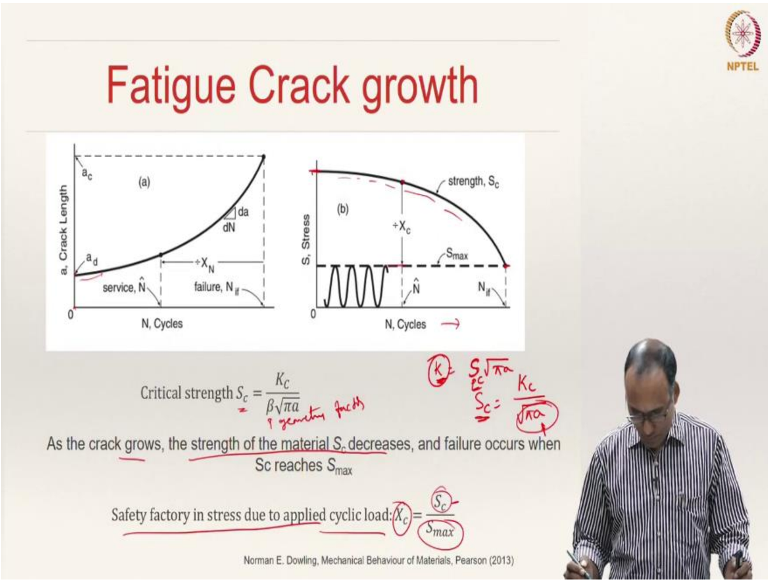

Initially at zero cycles, you have the initial crack length to be a_d, the minimum detectable crack length. At that time, let us assume that the strength of the material is S_c. Now, you can say that the critical strength of the material is equal to $\frac{K_c}{\beta\sqrt{\pi a}}$, where β is the geometry factor.

You remember we have written $K = \sigma\sqrt{\pi a}$, where σ is the stress. So, when this becomes your critical strength of the material, then we call it as σ_c. So, now, this σ is represented by S. So, K becomes fracture toughness then.

$$S_c = \frac{K_c}{\sqrt{\pi a}}$$

Now, as the crack grows, we know that the strength of the material S_c decreases, as we can see from here.

As the length of the crack is increasing, S_c reduces because the length of the crack is in the denominator. This graph here shows the reduction in strength as the number of cycles are increasing because as the number of cycles are increasing, the crack length is increasing.

If $\widehat{N}$ cycles are the prescribed life for this component, then the safety factor in stress due to applied cyclic load X_c is given by,

$$X_c = \frac{S_c}{S_{\max}}$$

$S_{\max}$ is the stress at which it fails and S_c is the residual strength in the material.

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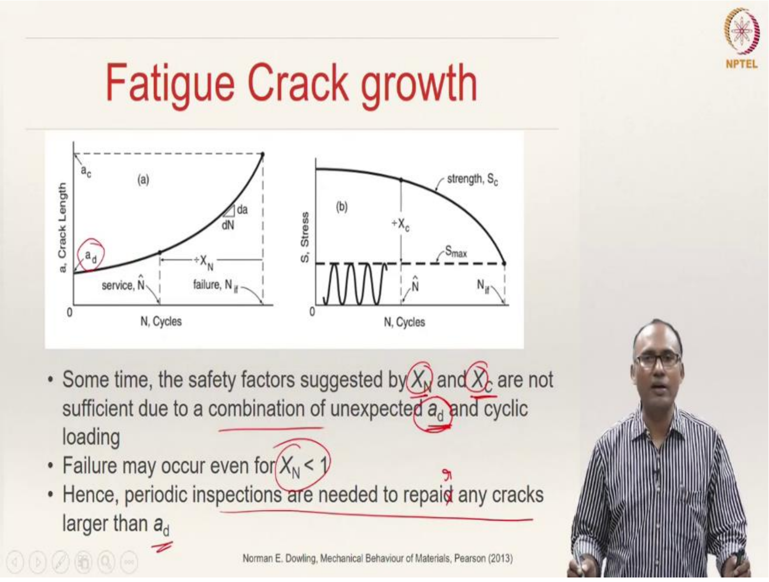

Sometimes however, the safety factor suggested by X_N and X_c are not sufficient due to a combination of unexpected a_d. Because you are saying that a_d can actually also suddenly change because of the cyclic loading nature and sudden unexpected loads that may be there or unexpected crack detection mechanisms that are available.

And hence, a_d that we have measured might as well be having some error associated with that and hence, X_N and X_c alone, sometimes the suggested values may not be sufficient for the design. Sometimes failure may occur even for $X_N < 1$, for that reason. Hence, it is extremely important to ensure that the detectable crack size is well within the safe limits and hence, periodic inspections are needed to repair any cracks that are larger than a_d.

The detectable crack size should not be greater than a_d. Several machine components, may start with an initial crack length a_d, but during the service, they may increase in size. As we have seen, as the number of cycles increase, the size of the crack increases; but it is possible that this size of the crack might not increase the way that we are expecting, it might increase much faster or at a much steeper rate.

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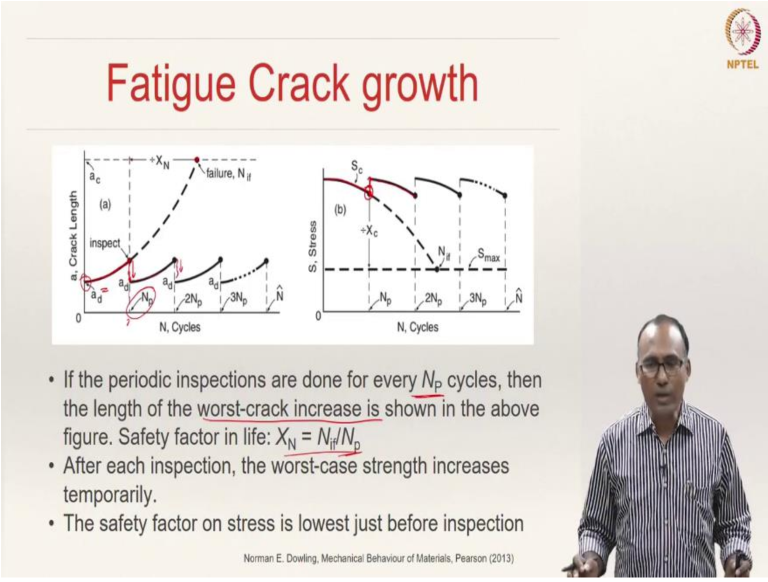

We need to do periodic inspections and ensure that the crack length at the time of putting the system into commission should be well within a_d. So, if the periodic inspections are done for every N_P cycles, let us assume, then, the length of the worst crack increase can be shown here.

Let us say failure happens at N_{if} cycles. Then, the initial crack which is a_d increases up to here; that means, the crack length has increased from a_d to some other value, greater than a_d. Then, since you have done an inspection after N_P cycles, then you would see that the crack length is larger than the detectable crack size or expected allowed crack size.

Then, you can repair the crack and reduce the crack size to a_d again and then, you further continue the loading and then, again inspect after N_P cycles from the first inspection and then, if there is any increase in crack which is expected to be there and then, you do the repair and so on. That means, the safety factor in life at this point of time is $\frac{N_{if}}{N_P}$, because we have done the inspection at N_P cycles. What happens after each inspection?

You are repairing the crack and during this process as the crack length is increasing, the strength of the material is decreasing. But at the end of N_P cycles, you are repairing the crack; as a result, the strength increases again and then, it continues again and so on.

And so, the safety factor on stress during this entire process is lowest just before the inspection. Once the inspection is done, the safety factor on stress has increased because you have done the repair to the crack, the crack length has actually reduced now from, say $a_d + \delta$ to a_d. So, here that means, during this process, we can monitor how the crack is growing.

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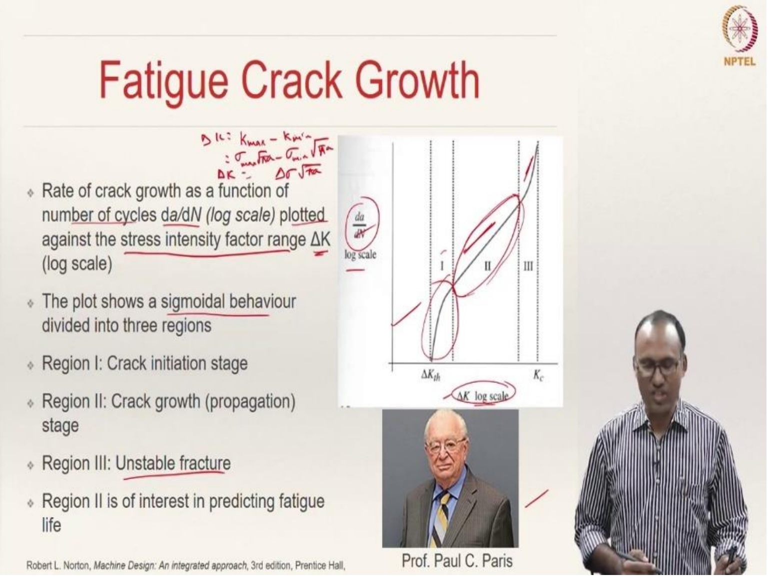

The rate of crack growth as a function of number of cycles denoted by $\frac{da}{dN}$, plotted against the stress intensity factor range $\Delta K = K_{\max} - K_{\min}$.

$$K_{\max} = \sigma_{\max}\sqrt{\pi a}$$

$$\Delta K = \Delta\sigma\sqrt{\pi a}$$

When you plot this on a log-log scale, a sigmoidal behavior is seen as shown here.

The region I here is what you call crack initiation stage and this is what is called crack propagation stage and then, this is unstable crack growth. Region II is crack propagation stage and region III is unstable fracture. As for us, region II is very important because here, the crack is propagating in a stable manner; that means, even if the crack propagates, it is not going to cause any catastrophic events in the components' service life.

If you know when stage II ends, then you can say that how many more cycles can I use or for how many more cycles can the component be used. This information is very important and one would be able to get by looking at the relation between crack growth rate i.e., $\frac{da}{dN}$ and the stress intensity factor range.

It is observed that $\frac{da}{dN}$ v/s ΔK, when plotted on a log-log scale, the region II shows a straight line behaviour and this is something that was observed by Paul Paris. He gave a relation to $\frac{da}{dN}$ the crack growth rate to stress intensity factor range in the region II, that we will see the formula how it looks like.

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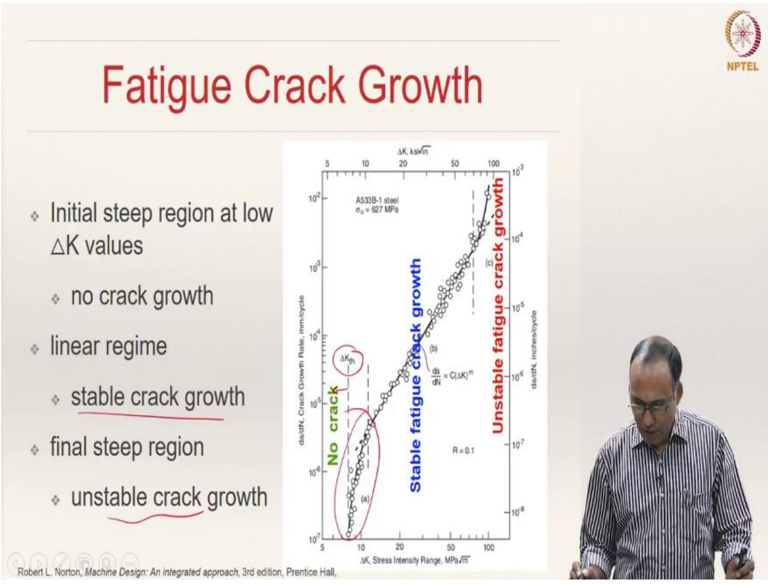

Let us look carefully at the three regions. The initial steep region at low ΔK values -- so, it gives you a threshold ΔK_{th}. In this region there is no crack growth and below ΔK_{th}, there is no problem for the material; the cracks do not grow at all.

In the region II which is a linear regime, is a stable crack growth regime and the final steep region is an unstable crack growth regime. This is below ΔK_{th}, i.e., no crack. This is stable fatigue crack growth rate and unstable crack growth.

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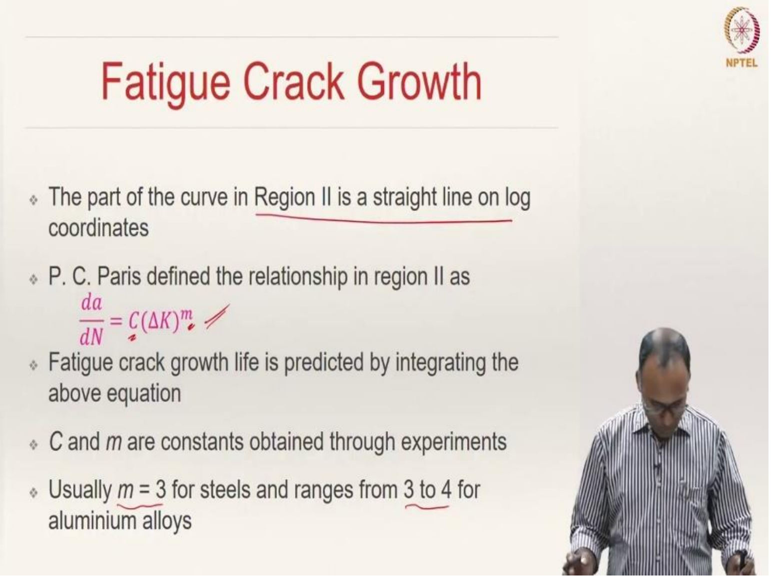

The part of the curve in Region II is a straight line on log coordinates. Paul Paris defined the relationship in region II as

$$\frac{da}{dN} = C(\Delta K)^m$$

C and m are material constants. Once you know the material constants and ΔK, integrate this equation in order to get fatigue crack growth life.

For steels $m = 3$ and for aluminum alloys, it ranges from $3 - 4$, in general. For other materials, if you want to find out m, you can refer to the design data handbooks.

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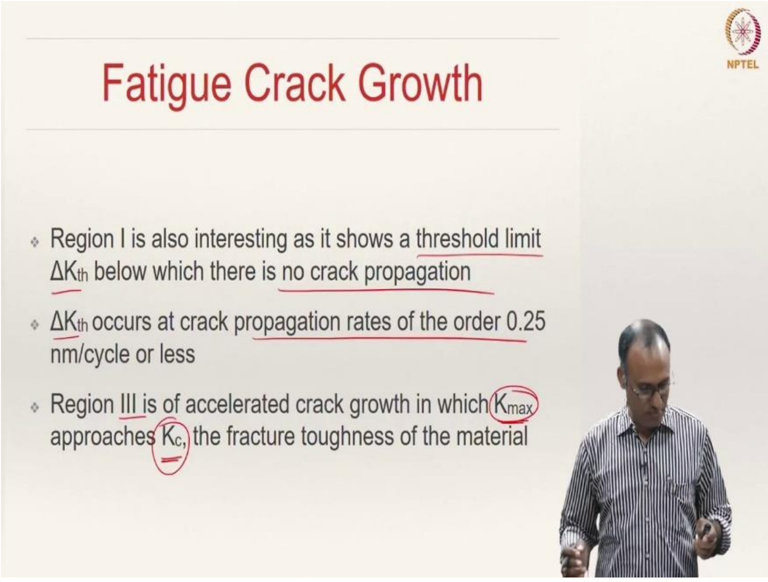

Region II is more interesting because that gives you information about fatigue crack growth in a stable manner. It allows us to calculate the remaining life of the component by knowing the initial crack length. However, region 1 is also interesting as it shows the threshold limit ΔK_{th} below which there is absolutely no crack propagation.

Usually, ΔK_{th} occurs at crack propagation rates of the order 0.25 nm/cycle i.e., extremely low crack propagation rates and hence, we can consider there is absolutely no crack growth.

Region III is called accelerated crack growth regime in which the $K_{\max}$ of the applied load approaches K_c, the fracture toughness of the material. When $K_{\max} = K_c$, that is when the material breaks suddenly as if it is a brittle fracture. And hence, that regime is characterized as unstable crack growth regime.

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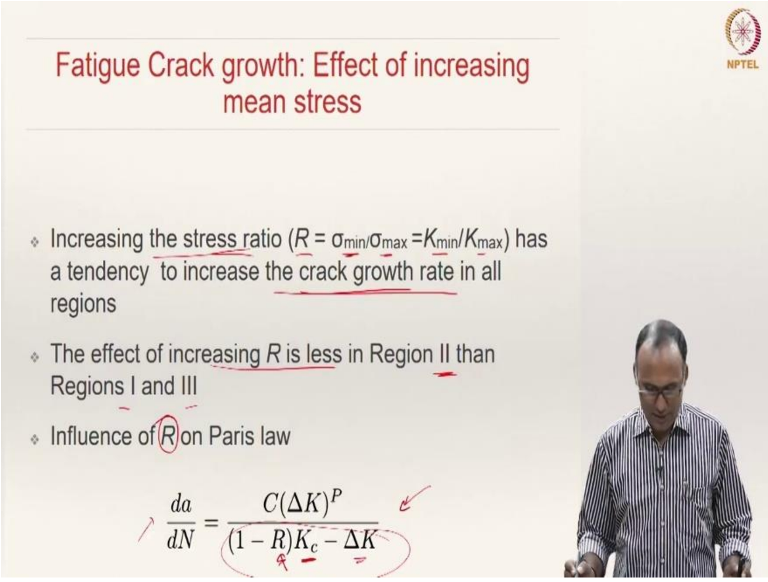

What is the effect of mean stress on the crack growth rate? Increase in the stress ratio, $R\left(=\frac{\sigma_{\min}}{\sigma_{\max}}=\frac{K_{\min}}{K_{\max}}\right)$ has a tendency to increase the crack growth rate in all the regions. However, the effect of increasing R is less in region II than region I and region III. So, how do we incorporate the effect of stress ratio on in on Paris law? One such way is shown below.

$$\frac{da}{dN} = \frac{C(\Delta K)^P}{(1-R)K_c - \Delta K}$$

Here, the effect of stress ratio has come in. What happened as you increase the stress ratio? As you increase the value of stress ratio, this quantity reduces right; as a result, $\frac{da}{dN}$ increases. As we have already seen, increasing the stress ratio has a tendency to increase crack growth rate.

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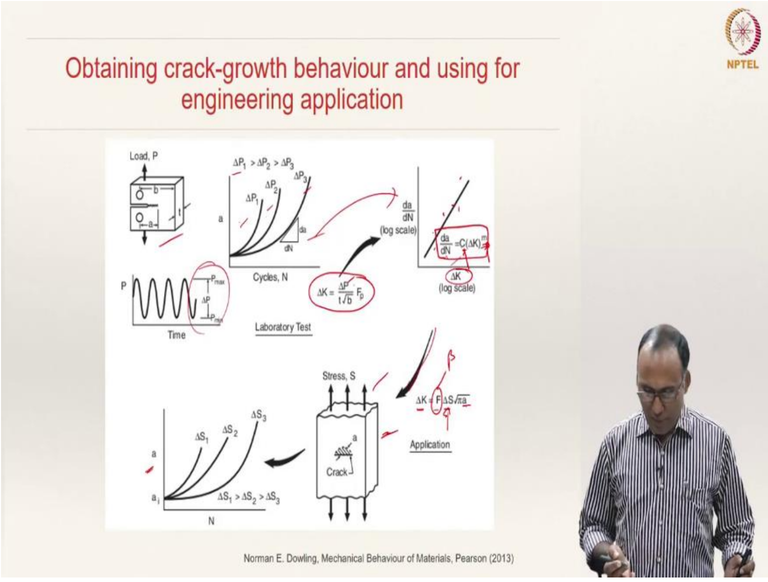

How do we go about using this in the real-life design? You do a laboratory test on a specimen of the kind of loading that you would expect in the real-life scenario on a laboratory specimen and then, do this for different $\Delta P'$s. $\Delta P = P_{\max} - P_{\min}$ as you have seen here.

P is the load applied and then, you calculate $\frac{da}{dN}$ as a function of different $\Delta P'$s and then, from this you plot $\frac{da}{dN}$ as a function of ΔK; ΔK is calculated from ΔP. So, for $\Delta P_1, \Delta P_2, \Delta P_3$ you will get different values.

You will get several values you will fit a straight line on a log-log scale and then, from there, you can calculate C and m for that particular material and the geometric scenarios and use that information now, for real life applications wherein you know what is the stress.

$$\Delta K = F\Delta S\sqrt{\pi a}$$

F is like β, the geometric factor; ΔS is the stress range and a is the crack length. Use C and m that you have obtained from the laboratory test and then, use that in the real-life application and from there, you will be able to calculate the residual life or how much more crack length increase can be accommodated based on the loading scenario.

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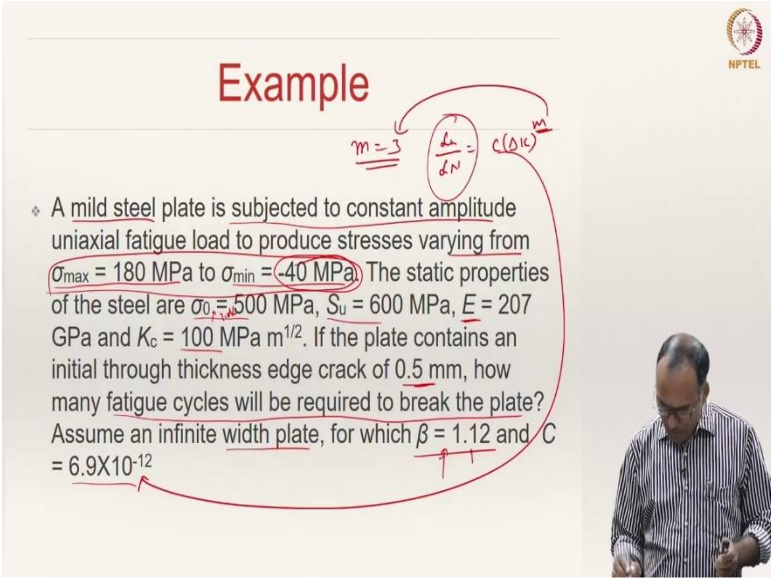

This is a simple example problem, wherein you have a mild steel plate which is subjected to constant amplitude uniaxial fatigue load to produce stresses varying from $\sigma_{\max} = 180$ MPa, $\sigma_{\min} = -40$ MPa. Please pay attention here, as $\sigma_{\min}$ is negative.

The static properties of the steel are σ_0 which is the yield strength, S_u is the ultimate strength, $E = 207$ GPa is the elastic modulus, $K_c = 100\ \text{MPa m}^{1/2}$ is the fracture toughness. If the plate contains an initial through thickness edge crack of 0.5mm i.e., the initial crack length is 0.5mm, how many fatigue cycles will be required to break the plate?

You know this is a material and you know the loading scenario and if you have an initial crack length of 0.5mm, the question now is: how many fatigue cycles can be will be required to break the plate? That means, how many fatigue cycles can this component be used for before it actually breaks in other words. You assume an infinite width of the plate and for which $\beta = 1.12$.

The reason the infinite width has been given is because the geometric factor comes into play and the geometric factor for $K = \beta\sigma\sqrt{\pi a}$, the $\beta = 1.12$ and $C = 6.9 \times 10^{-12}$.

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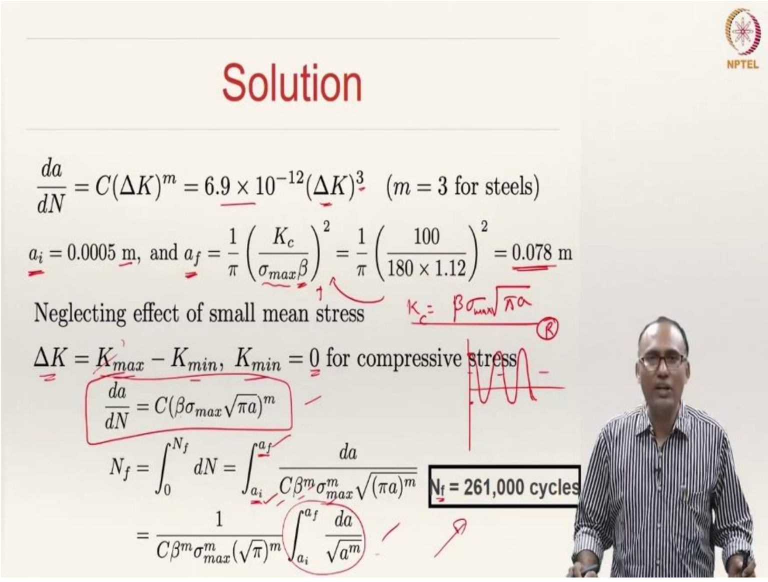

We know that this is mild steel and for steel, $m = 3$ in the formula $\frac{da}{dN} = C(\Delta K)^m$. C is now given here in consistent units.

How do we go about calculating ΔK? The initial crack length, $a = 0.5$ mm. So, I am writing it in meters divided by 1000 and the final crack length is equal to; so, final crack length means crack length at which the fracture happens. Fracture happens when the stress intensity factor reaches the fracture toughness.

The final crack length is given by,

$$a_f = \frac{1}{\pi}\left(\frac{K_c}{\beta\sigma_{\max}}\right)^2 = 0.078 \text{ m}$$

This is coming from $K = \beta\sigma\sqrt{\pi a}$; when $K = K_c$, then this should become $\sigma_{\max}$.

Using that that equation, we have figured out that the final crack length should be 0.078 mm; that means, the material breaks when the crack length reaches from 0.5 mm to 0.078 meters or 0.0005 meters to 0.078 meters.

Note that the stress cycling is happening between 180 MPa to −40 MPa. So, there is some mean stress in the system, i.e., R is coming into picture. However, we will neglect that part in this scenario.

However, note that $K_{\min} = -40$ MPa; but when you have compressive stress the crack growth will not happen and hence, whenever you have a compressive stress, you should not take that into account and hence, you can take $K_{\min} = 0$. The compressive stresses will actually be beneficial.

But here, we are not the benefit into account, we are simply saying that since $K_{\min}$ is negative, we will not take that into account and then, we will say $K_{\min} = 0$. So, $\sigma_{\min} = 0$. Then,

$$\Delta K = K_{\max} - K_{\min} = K_{\max} = \beta \sigma_{\max} \sqrt{\pi a}$$

We have,

$$\frac{da}{dN} = C(\Delta K)^m = C\left(\beta \sigma_{\max} \sqrt{\pi a}\right)^m$$

Now, you integrate this equation and when you are integrating, dN should be integrated from 0 to N_f; whereas, initial crack length is a_i and final crack length is a_f.

We know a_i, a_f and you can integrate this expression; C is known, β is known, m is known, $\sigma_{\max}$ is known. So, that becomes something like that. And then, once you integrate this, you can find the definite integral and all the values are known and then, plug that in and then, you will be able to calculate $N_f = 261000$ cycles.

That means, when you have initial crack length as 0.5 mm for this given system, you can run the equipment for 261,000 cycles provided in between you are not doing any repairing. If at all you are doing periodic inspection, then you are actually repairing this system; that means, the crack length is brought back to a_d, the detectable crack size, then, your life might be increased further.

Here we are not assuming any intermediate inspection and repair of the specimen or the component. So, this is how one would be able to calculate the remaining life of a component by knowing what is the initial crack size and other properties of the material together with the type of loading that can be applied to the system.

So, with that, we have understood how do we go about calculating the remaining life from the fatigue crack growth point of view; that means, you have an initial crack and the you are letting the crack to grow and what is the limit up to which you can actually continue to use the system, right? So, with that, we sort of close this module on fatigue failure theories.

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These are the textbooks that I have used extensively during this module. Robert L. Norton's Machine Design, An integrated approach and George Dieter's Mechanical Metallurgy to discuss some of the microstructural details of the fatigue crack growth. And Mechanical Behavior of Materials by Meyers and Chawla is another interesting textbook and Norman E. Dowling's Mechanical Behavior of Materials.

So, with that, we will close this module on fatigue failure of materials and then we will meet with a new module in the next class.

Thank you.