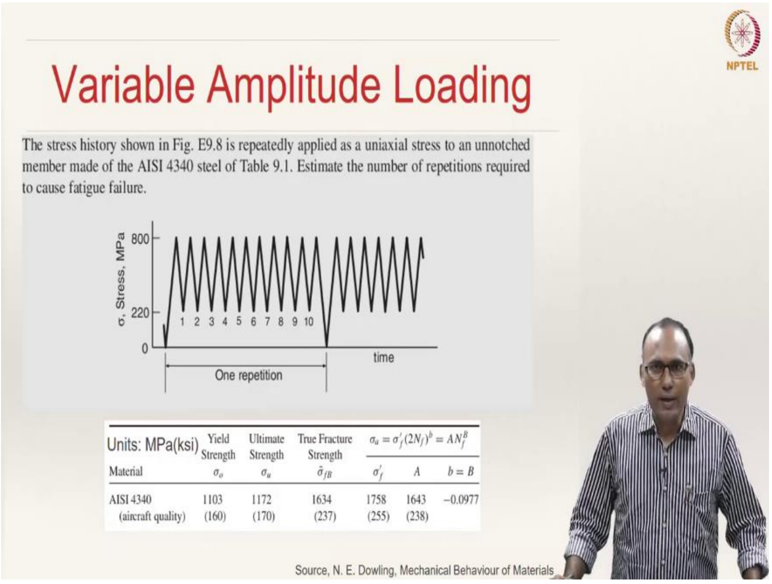
Basics of Materials Engineering
Prof. Ratna Kumar Annabattula
Department of Mechanical Engineering
Indian Institute of Technology, Madras

Week - 08
Lecture – 49
Fatigue Failure of Materials (Fatigue Stress Concentration Factor)

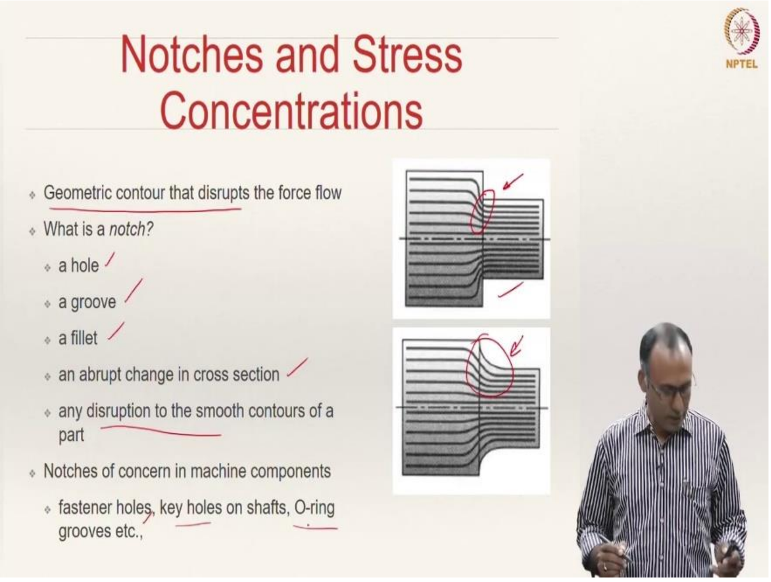
So, in the last class, we have looked at variable amplitude loading under multiaxial fatigue loading.

(Refer Slide Time: 00:20)



And so far, the material that we have covered as far as Fatigue Failure of Materials is concerned is under the assumption that there are no notches or stress risers. We have not focused on the fact that there may be some stress risers within the component.

(Refer Slide Time: 00:38)



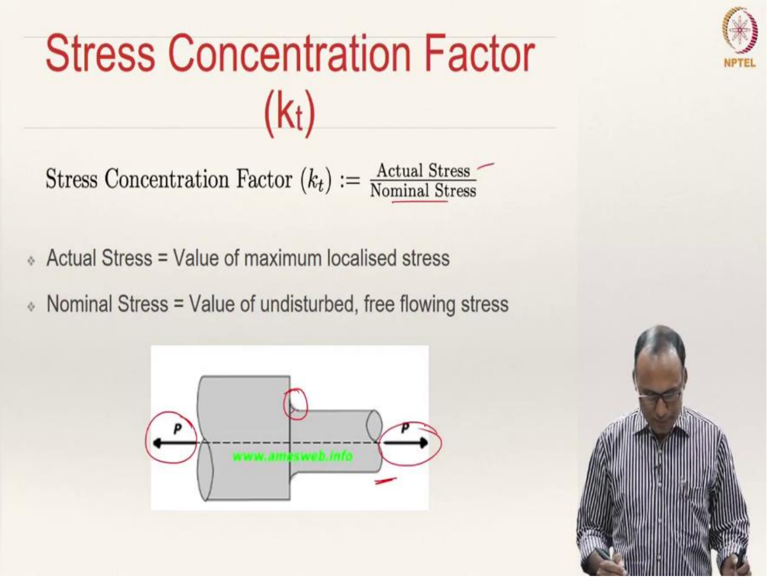
Today we will look at the situation when you have notches and the stress concentration zones within the material, and how do we go about dealing with them. What is a notch? We have defined theoretical stress concentration factor during static failure theories and there we have seen this figure already.

A notch or a stress concentration zone is actually a geometric contour that disrupts the force flow. As we have already mentioned in previous classes, you can take an analogy with the streamlines. During the fluid mechanics course, you probably heard of these streamlines. So, when you have a change of geometry, the streamlines get closer. You can draw a similar analogy between streamlines and the force flow lines.

This figure shows a sharp change in geometry. As a result, the distance between the force flow lines is much less compared to when you have a gradual change in geometry, wherein the distance between the streamlines is much less disrupted compared to here. This represents a very high stress concentration zone compared to this situation.

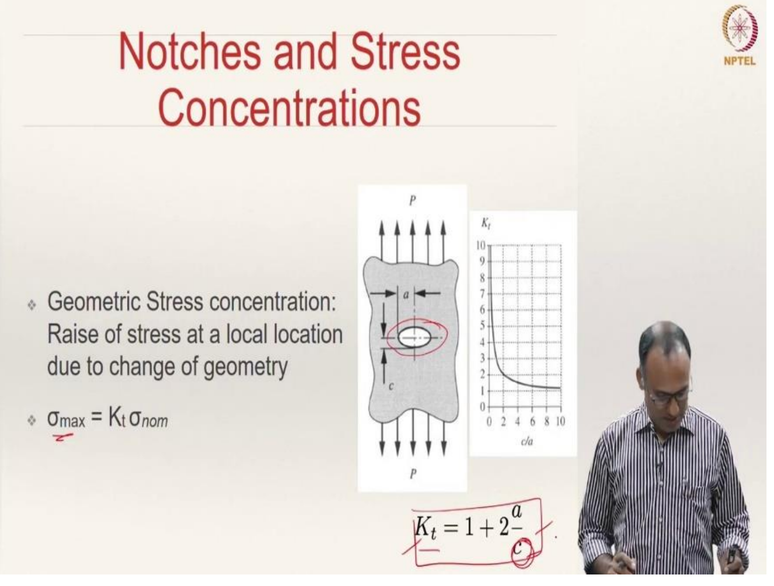
What are the different kinds of notches that we will come across in machine components? For instance, you may have a hole, groove, fillet, an abrupt change in cross-section, a keyhole and so on, or in general, any disruption to the smooth contours of a part can be considered as a notch or a stress riser. In machine components, the typical features that can be considered as notches and should be given proper attention to, are things like fastener holes, key holes on shafts, O-ring grooves, etc.

(Refer Slide Time: 02:56)



We have discussed the theoretical stress concentration factor already, which is defined as the actual stress prevalent at the position where you are interested divided by the nominal stress or the far-field stress. For instance, in this picture, this is the far-field load; divided by the cross-sectional area is your far-field stress -- that is your nominal stress. What is the actual stress because of the stress riser that is present here? The stress at this position divided by the far-field stress is the stress concentration factor.

(Refer Slide Time: 03:35)



That is what is called theoretical stress concentration factor or geometric stress concentration factor. We have seen for different kinds of geometry of the hole, shape and size, we have the formula for finding out the theoretical stress concentration factor.

For instance, here you have an elliptic hole with semi-major axis a and semi-minor axis c, then the stress concentration factor is given by,

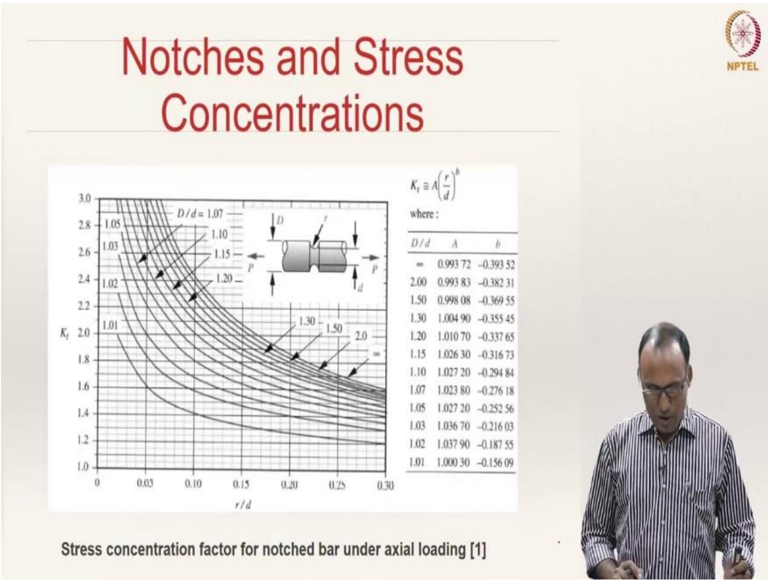
$$K_t = 1 + 2\frac{a}{c}$$

We are not deriving these equations or formula here; we are just taking them for granted. The derivation of this formula is something that you would come across when you do a course on elasticity.

As you reduce the value of c, the theoretical stress concentration factor increases, which means the maximum stress experienced by the component will increase as you reduce the size of c. As you keep reducing the size of c, the crack which is elliptic in nature would become a sharp crack.

A reduction of the value of c, actually means increasing the sharpness of the crack; as a result, you will have much higher disruption to the force flow. As a result, you will have high stress concentration in that regions.

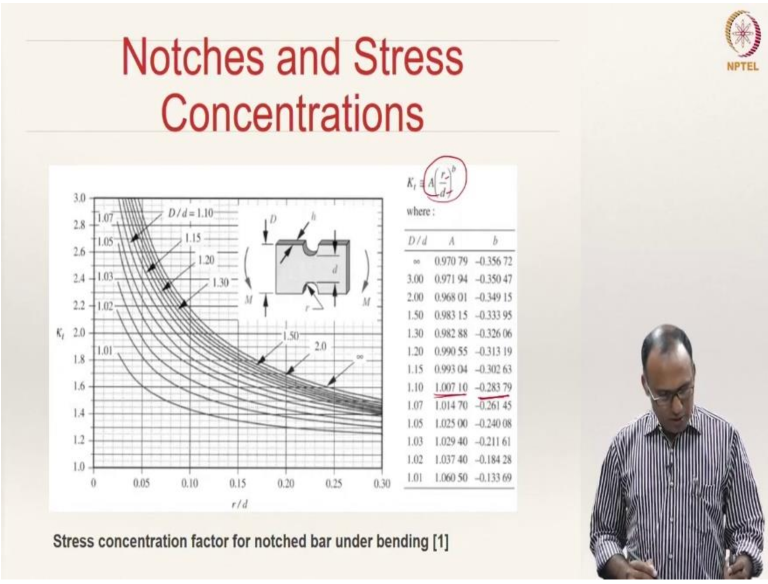
(Refer Slide Time: 05:05)



These are the different situations where depending upon the shape of the notch and also the type of the loading, you will have different kinds of stress concentration factors that people have actually derived these empirical formula or have done several experiments and came up with these charts.

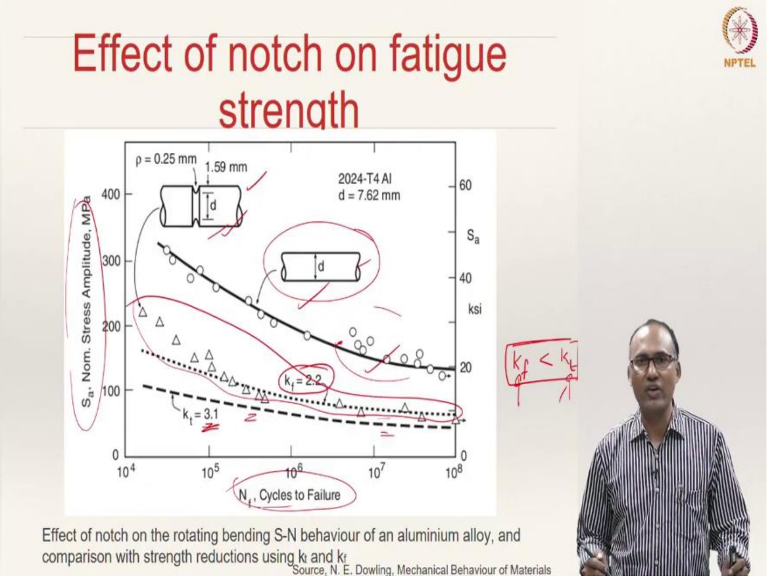
All that we need to do is, now you look at the appropriate chart, based on the type of the notch and the type of loading, and then identify the stress concentration factor, and use that to estimate the stresses in a particular region in your machine component.

(Refer Slide Time: 05:44)



This is for bending, where your cross-section is a rectangular cross-section. This graph here nicely summarizes the effect of stress concentration on the fatigue strength. The solid black line represents a fit to the fatigue strength life diagram for a un-notched aluminium specimen of 7.62 mm diameter.

(Refer Slide Time: 05:50)



You will not have a perfect line matching all these data points because of the variabilities in the crack structure in the material. Any experiment will have some error bar and fatigue failure also will have usually large error bars as we have seen in this figure.

This is the situation when there are no stress concentrations. Now, you have same shaft with a notch -- a groove in between. The radius of that groove is 0.25 mm. Then, you apply fatigue loading on this specimen, and try to develop the S-N curve. You can clearly see there is a sudden reduction in the fatigue strength of the material.

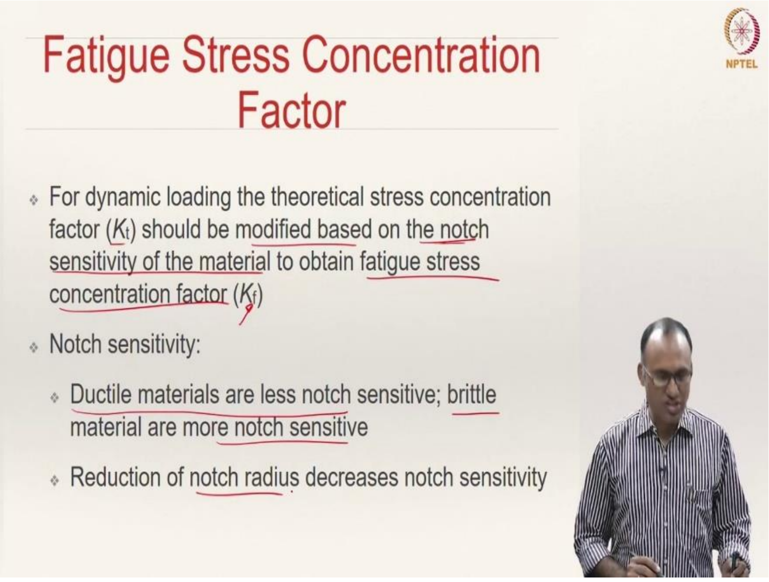
On the y-axis, we have stress amplitude and on the x-axis, we have number of cycles to failure. You can see that the fatigue strength of this material is much less than the one that which does not have a crack. However, if you would calculate the theoretical stress concentration factor of this material and if you simply multiply the theoretical stress concentration factor with the stress calculated from the far-field stress that is applied on this, the curve that you will get from this solid curve will be this dashed line, where $K_t = 3.1$ for this particular configuration.

However, when we do the experiments, it turns out that the fatigue strength of the material is actually a little bit higher than what you would have calculated using the theoretical stress concentration factor. If you would calculate the stress concentration factor directly from the geometry, you will get 3.1.

However, after measuring, if you fit this line, you would find out that the fatigue stress concentration factor represented by K_f is actually 2.2 which is less thanK_t; that is how one would observe.

Why is the fatigue stress concentration factor very different from theoretical stress concentration factor? And a further question would be: why is the fatigue stress concentration factor lower than the theoretical stress concentration factor?

(Refer Slide Time: 08:54)

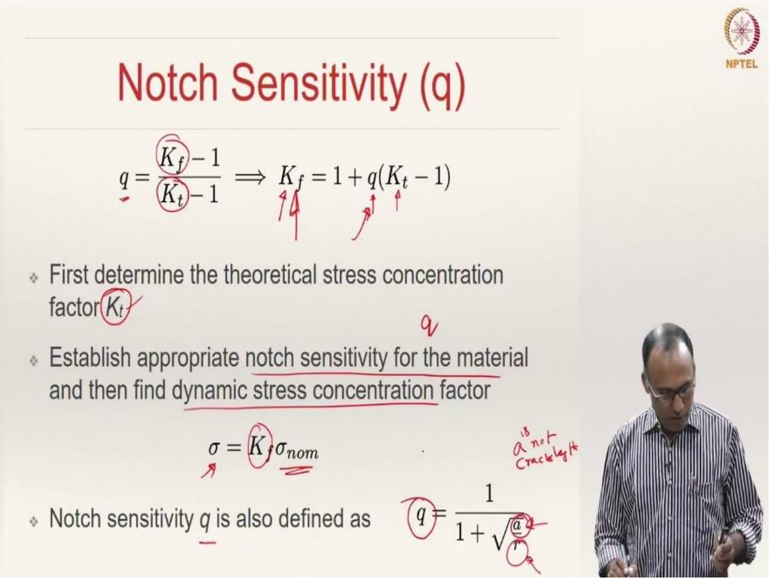


For dynamic loading, the theoretical stress concentration factor K_t should be modified based on the notch sensitivity of the material; there is something called notch sensitivity that needs to be accounted for when we are talking about stress concentration factor, and hence you cannot use theoretical stress concentration factor; you have to use a modified theoretical stress concentration factor for fatigue loading and that is called fatigue stress concentration factor represented by K_f.

Notch sensitivity is the sensitivity of the material to the geometry of the notch, particularly when you have fatigue loading. This sensitivity is material dependent. It is very less for ductile materials and very high for brittle materials.

So, brittle materials are more notch sensitive compared to ductile materials. When you reduce the notch radius, the notch sensitivity decreases, as the plastic deformation ahead of the crack tip increases; that will lead to making the material get into the ductile regime because you are introducing plastic deformation and hence they become less notch sensitive.

(Refer Slide Time: 10:16)



This is the correlation between the notch sensitivity q, fatigue stress concentration factor and theoretical stress concentration factor.

$$q = \frac{K_f - 1}{K_t - 1}$$

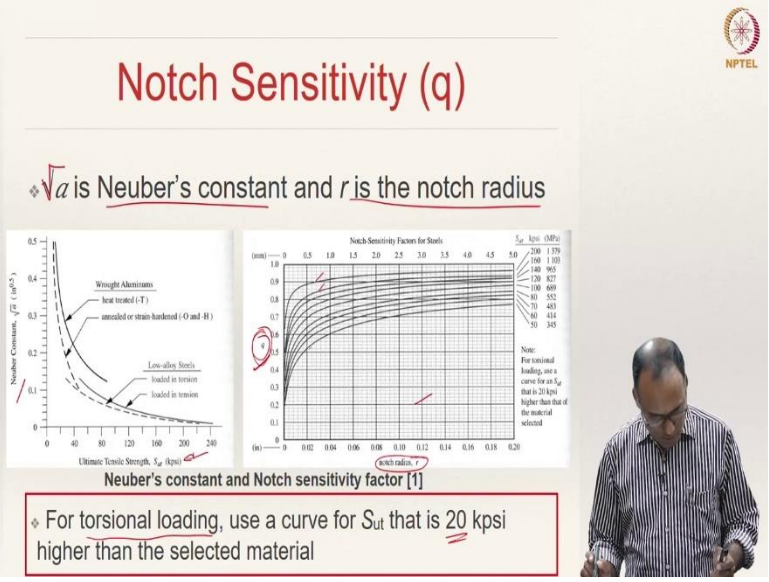
By knowing the notch sensitivity and theoretical stress concentration factor, one can obtain fatigue stress concentration factor.

The first thing that we need to do while doing the design based on fatigue for components which have stress risers or notches is to determine the theoretical stress concentration factor from the charts that are available or from the empirical formula. Then, you establish the notch sensitivity of the material and find the dynamic stress concentration factor or fatigue stress concentration factor.

Once you find K_t and q, then you get K_f. Once you have K_f, you multiply that with the nominal stress to get the actual stress experienced by the component at the position where you are actually calculating the stress concentration factor.

The notch sensitivity q can also be defined in terms of the geometry and another parameter called a. Please note that this a is not the crack length. We have been using a for crack length during the module on fracture mechanics, but here, this a is a constant that comes from material property or the ultimate strength of the material. So, your notch sensitivity is a function of the geometry as well as the material property.

(Refer Slide Time: 12:00)



$\sqrt{a}$ is called Neuber's constant and r is the notch radius. The Neuber's constant is dependent on ultimate strength of the material. For different materials, the Neuber's constant is different. You have graphs like that or tables, where from which you can directly calculate the Neuber's constant.

We can see that in a moment. We know that q is a function of Neuber's constant and the notch radius.

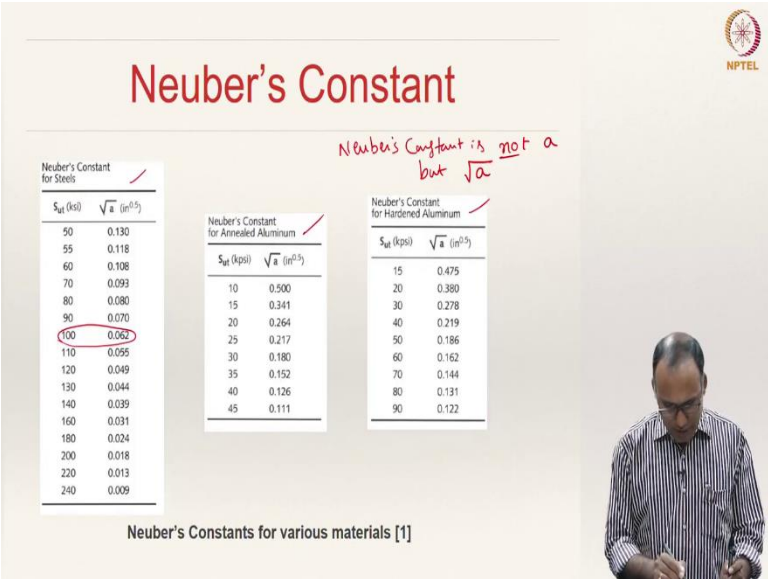
This graph here shows notch sensitivity q as a function of notch radius and the effect of material property is plugged in here through different curves. Each curve represents a particular ultimate tensile strength of the material. You can directly use this graph in order to calculate q for a given material, and knowing the geometry. Then, you will be able to calculate theoretical stress concentration factor from the type of loading and also the geometry.

Knowing these two parameters, you can calculate the fatigue stress concentration factor as,

$$K_f = 1 + q(K_t - 1)$$

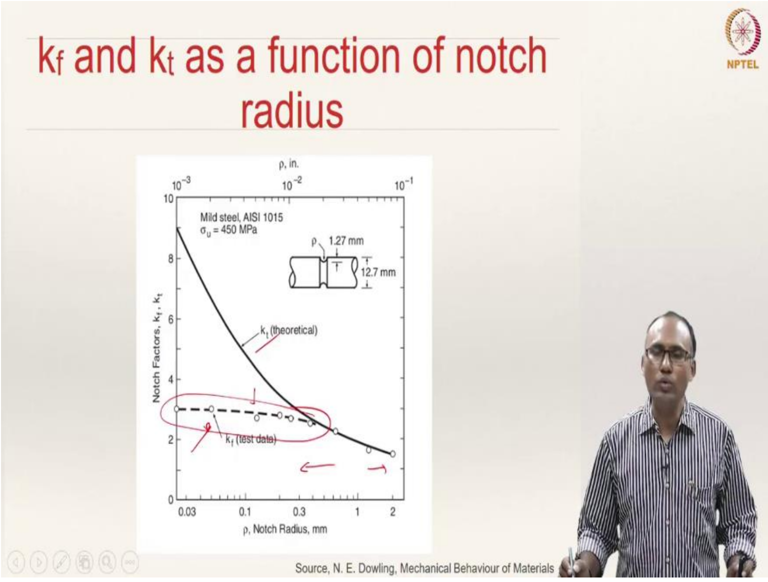
If the loading is torsional and if the ultimate tensile strength of the material is 100 kpsi, you need to add 20 kpsi to that and take that as your material to calculate q; that is only to be done when you have torsional loading.

(Refer Slide Time: 13:50)



These are the tables that are available for Neuber's constant for different materials. This is for steel, this is for annealed aluminium, this is for hardened aluminium, and then you can directly look at the table and then find out the Neuber's constant. Please note that Neuber's constant is not a, but $\sqrt{a}$.

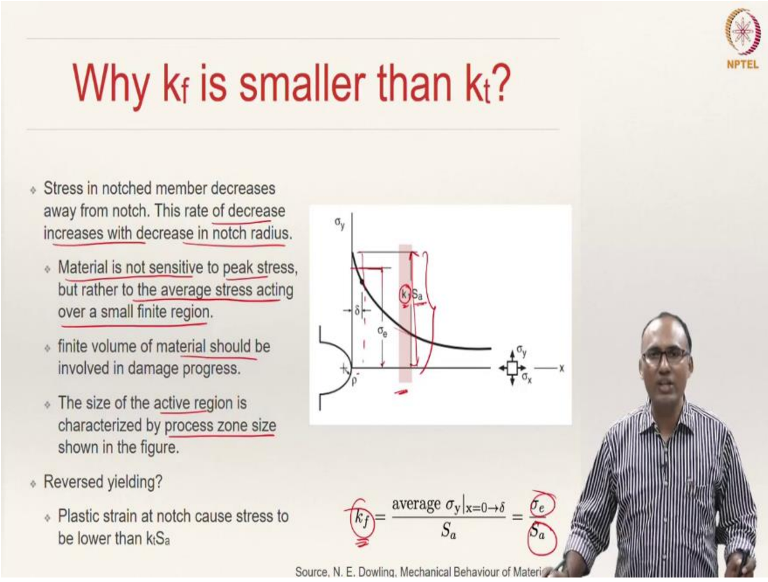
(Refer Slide Time: 14:23)



Here, we are looking at K_f and K_t as a function of notch radius. For large values of notch radius, K_f and K_t are almost the same. As you reduce the notch radius, K_f becomes much smaller than K_t. Hence, for a small notch radius, it is extremely important that you calculate the value of K_f.

As we have already discussed when ρ becomes smaller and smaller; that means, notch radius becomes smaller and smaller, then the materials notch sensitivity decreases; which means the locally the material is plastically deforming, because of the lower notch radius. As a result, it is becoming ductile and hence you need to account for this plastic deformation. That is taken care of in K_f, whereas K_t does not take that into account.

(Refer Slide Time: 15:27)



Now, the question is why is K_f smaller thanK_t. The stress in notched member decreases away from the notch as we can see clearly here. The rate of decrease increases with decrease in notch radius. As you reduce the notch radius, the rate of decrease increases, that means, there will be a steep decrease of stress as you go away from the notch tip. And the material is not sensitive to peak stress, but rather to the average stress acting over a small finite area.

Material failure under fatigue is not really dependent on the peak stress, but on a small average volume ahead of the crack tip. So, the finite volume of the material should be considered which gets involved in the damage process.

The size of the active region is characterized by the process zone as shown here. And then, the plastic strain at a notch causes the stress to be lower than $k_t \times S_a$. Normally, if the theoretical stress concentration factor is k_t and stress amplitude is S_a, then $k_t \times S_a$ is the actual stress amplitude experienced by the material.

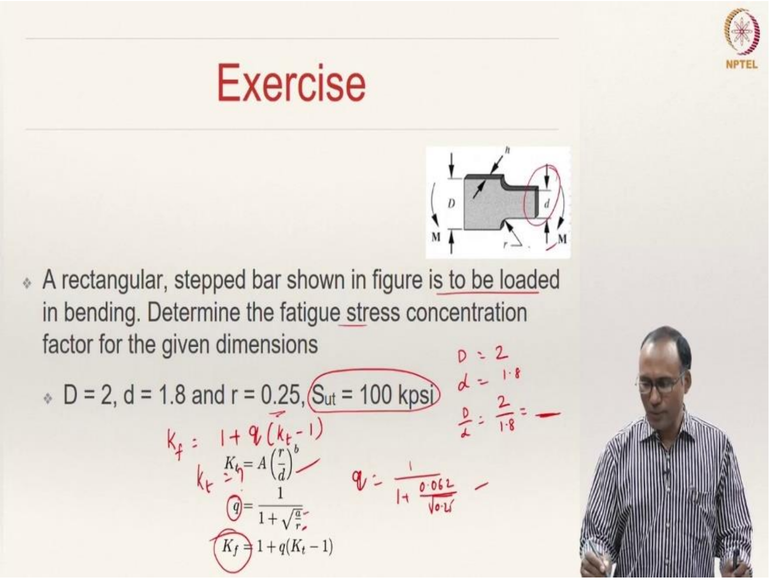
But because of the plastic deformation, this is not what is experienced by the material - a little less than that; k_f now is to be defined as,

$$k_f = \frac{\text{average } \sigma_y|_{x=0\to\delta}}{S_a} = \frac{\sigma_e}{S_a}$$

So, x from 0 to δ, you have to take the average of this region and that turns out to be $\frac{\sigma_e}{S_a}$; that will give you your fatigue stress concentration factor. Obviously, that is going to be smaller than k_t because you are only talking about this height compared to that height.

It is coming primarily from the fact that the damage happens over a small region, and you need to consider the small volume of the material in which the damage is taking place. As a result, you will have k_f being less than the value of theoretical stress concentration factor.

(Refer Slide Time: 17:43)



This is an exercise problem that you are required to try solving on your own, wherein you have a rectangular stepped bar as shown in figure, is to be loaded in bending. The kind of loading is given. Determine the fatigue stress concentration factor. In order to calculate fatigue stress concentration factor, $K_f = 1 + q(K_t - 1)$, right? So, now, first thing that we need to find out is K_t. How do we go about finding K_t? You go back and so it is a rectangular cross-section in bending and D and d are given, r is also given; all are in consistent units.

Let us go back and see one of the graphs. Here, we have got,

$$r = \frac{D}{d} = \frac{2}{1.8}$$

You know the values of a and b. After finding out these values, let us say it is 1.1 approximately; after finding out those two values, then you will plug that in this equation; a and b are known. Now, you know what is r and what is d in consistent units and then plug that in and you get your K_t. Now, we need to calculate q. q depends on a and r; r is known. For $S_{ut} = 100$ kpsi, we need to find out what is q.

We know the notch radius is equal to 0.25 units, now let us calculate Neuber's constant. For $S_{ut} = 100$ kpsi, Neuber's radius is this, and use $\sqrt{a} = 0.062$. q can be found using,

$$q = \frac{1}{1 + \sqrt{\frac{a}{r}}} = \frac{1}{1 + \frac{0.062}{\sqrt{0.25}}}$$

After finding q and K_t, you can calculate K_f, right? So, it is basically going back to lookup tables and then finding out the appropriate parameters and so on. The most important thing is that you should know what is the kind of loading that needs to considered.

Here it is bending. So, you need to look for the table that has bending deformation, and also you should ensure that your cross-sectional properties are rectangular. Let me check whether the one that we have looked at is rectangular or not. So, this is rectangular cross-section, and h is given. And based on that, you will calculate what will be the theoretical stress concentration factor.