Basics of Materials Engineering
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Lecture – 26
Part - 03
Fatigue Failure of Materials (Multiaxial Fatigue and Variable Amplitude Loading)

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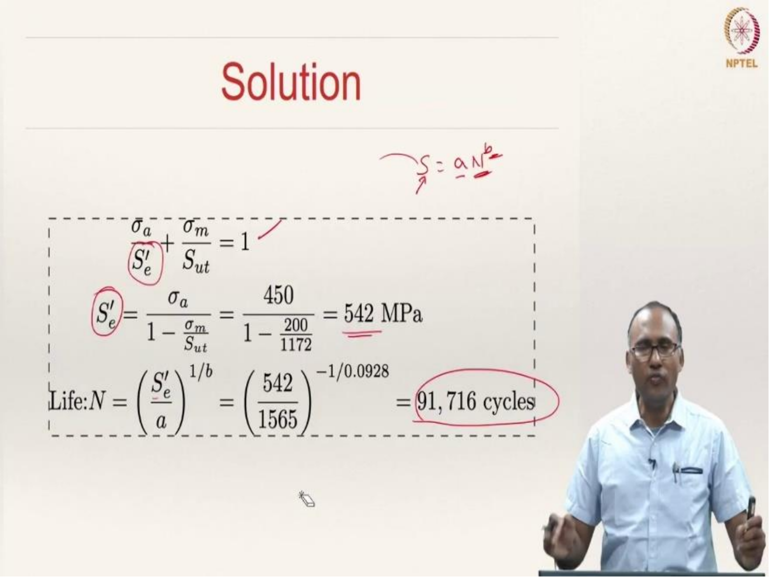

So far, whenever we are talking about the fatigue failure design, we are talking about mean stress and alternating component of the stress or stress amplitude, as if stress is a scalar quantity. But it is not a scalar quantity, right? You can talk about only one stress component when you have uniaxial loading.

When you have multi-axial loading, the stress is a tensor and hence you will have various components of the stress, and they may be changing as well. The load is cycling between two values; as a result, the stress is cycling between two values, but the stress what we are talking about is a uniaxial state of stress until now.

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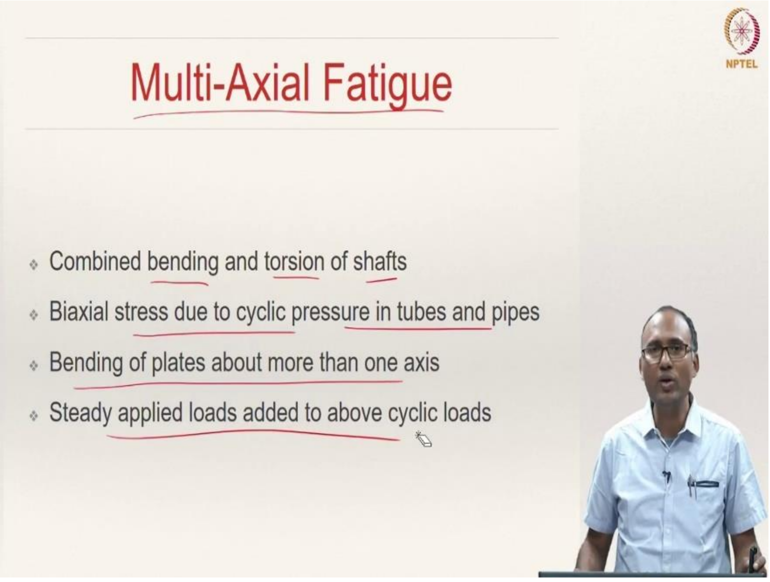

When you have multi-axial loading, you need to consider all the six stress components. Hence, we need to see how one would actually study the failure when the component is subjected to a multi-axial loading and that is what we designate as Multi-Axial Fatigue.

There are many improvised approaches, but in this class, we are focusing on a simplified approach based on the equivalent stress concept that we have discussed in the static failure theories. Typically, most of the shafts when we are designing, are subjected to bending and torsion.

Bending results in normal stresses and torsion results in shear stresses. As a result, you have a combined state of loading. Similarly, you will have biaxial stresses due to cyclic pressure in tubes and pipes. You can also have a plate bending about this axis and two transverse axes.

In addition to these cyclic loads, you can also have some steady load that is acting on the system. These are all going to contribute to multi-axial loading scenario. In such situations, how do we go about studying the system?

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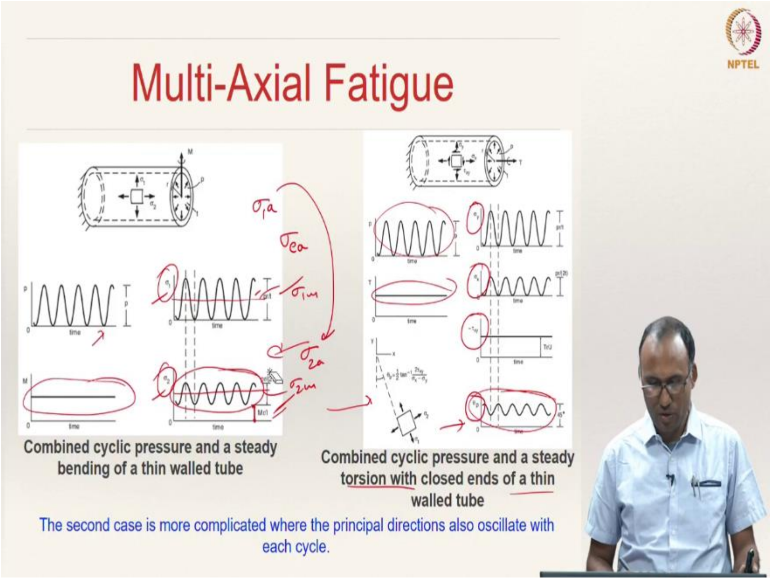

This is one example of a combined cyclic pressure and steady bending of a thin-walled tube. This is cyclic pressure and steady bending, i.e., bending moment is constant. As a result, you will have cyclic pressure loading. A steady bending load should be added to that, and as a result, the resultant system is subjected to such a variation.

So, you have some sort of an additional stress getting added because of the steady loading. In addition to that combined cyclic pressure and torsion with closed ends, then you will have much more complicated system wherein steady torsion and bending occur. The pressure results in normal stress, and together with that you will have an effective cycling loading.

These are two complex loading scenarios which will give you a multi-axial loading system. Here, you can see this is σ_1 and this is σ_2, here σ_y, σ_x and τ_{xy}, right? If you carefully look, this is where we have drawn θ_p which represents the direction of the principal stresses.

You can also clearly see that the principal stress direction is oscillating in this scenario. The direction of the principal components of the stress is changing as you are loading the system. Whereas, in this case, fortunately the direction of the principal stress is not changing.

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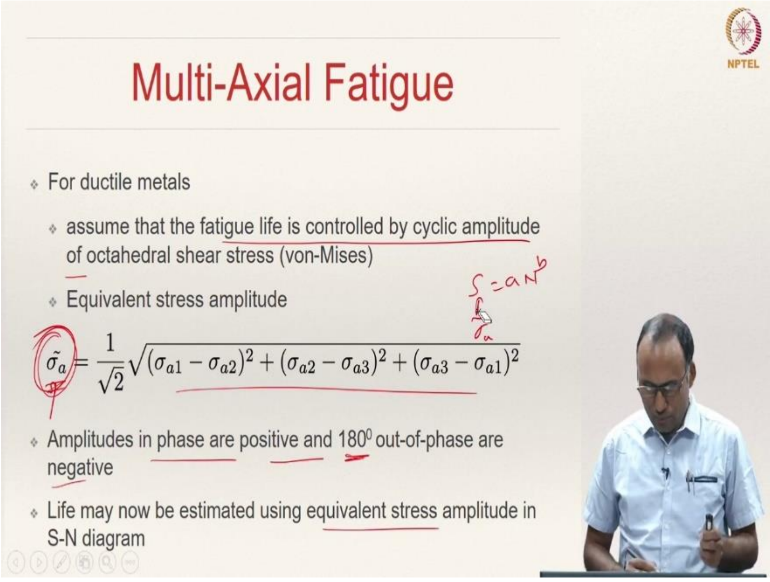

Here, we are going to prescribe a methodology to deal with multi-axial fatigue, wherein the direction of the principal stress remains constant, however, the other stress components are changing or fluctuating.

For ductile materials, you will assume that the fatigue life is controlled by cyclic amplitude of octahedral stress; that means, you look at the stress amplitude in all the components of the stresses. All those stresses are actually oscillating.

For instance, here σ_1 and σ_2 are oscillating. What you can do is, you can calculate the stress amplitude, i.e., σ_{1a} and σ_{2a} and then you calculate equivalent stress amplitude from σ_{1a} and σ_{2a}. Similarly, you have the mean stress σ_{1m} and σ_{2m}. From there, you have to calculate the equivalent mean stress.

Let us look at how will we go about it. The equivalent stress amplitude is calculated by using the von-Mises stress or equivalent stress formula that we have already discussed in the previous modules where we have discussed about distortion energy theory. The same formula you will use to calculate the equivalent stress amplitude. Now, this is a scalar quantity as if it is a one-dimensional situation.

While calculating the stress amplitudes, care should be taken. Amplitudes that are in phase are positive and those that are 180° out of phase are negative; that is the care that we need to take. Then, you can estimate the life using equivalent stress amplitude in S-N diagram.

In the S-N diagram, if there is no mean stress, instead of taking σ_{a1}, σ_{a2} and σ_{3a} separately, you will calculate the equivalent stress amplitude and use $S = aN^b$ and S is the equivalent σ_a.

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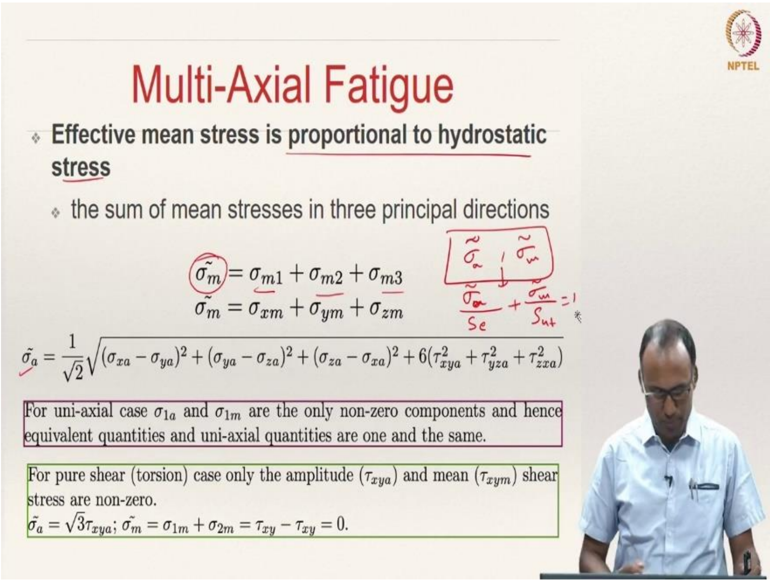

However, if we also have mean stress, its effect is to be considered. It is known that the effect of mean stress is proportional to hydrostatic stress. Hence, equivalent mean stress can be calculated as sum of the 3 mean stresses in the principal directions.

$$\widetilde{\sigma_m} = \sigma_{m1} + \sigma_{m2} + \sigma_{m3}$$

Thus, you can calculate the effective stress amplitude and effective mean stress. Once you have these two things, you can use Goodman criterion i.e.,

$$\frac{\widetilde{\sigma_a}}{S_e} + \frac{\widetilde{\sigma_m}}{S_{ut}} = 1$$

Then, you will be able to solve the problem the same way that we have done before.

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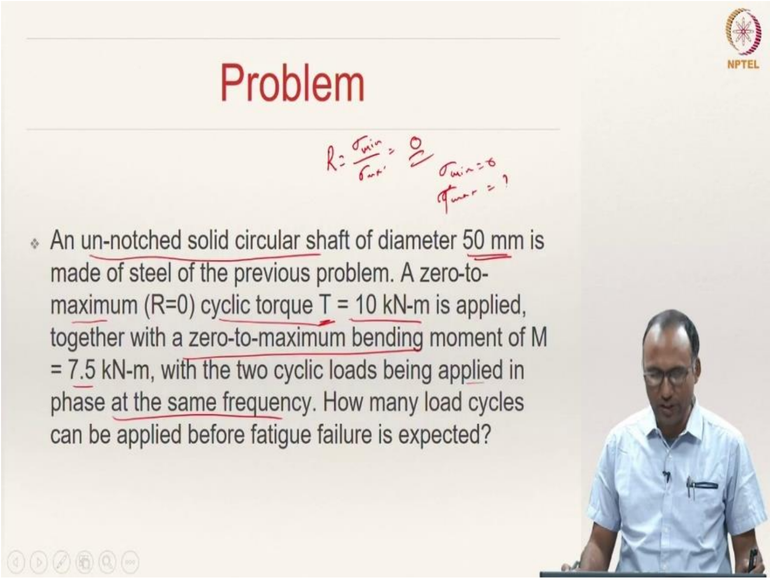

Here, we have a problem wherein an un-notched solid circular shaft of diameter 50 mm is made of steel. This is same as the previous problem. A zero-to-maximum cyclic torque, so that means, $\sigma_{min} = 0$, $\sigma_{max} = \tau_{max}$. Here we have torque, torque means you are cycling shear stresses.

So, a torque of 10 kN-m is applied together with a zero-to-maximum bending moment. What is the stress ratio R? $R = \frac{\sigma_{min}}{\sigma_{max}}$, this happens to be 0; zero-to-maximum bending moment of 7.5 kN-m, with the two cyclic loads being applied in phase at the same frequency. How many load cycles can be applied before fatigue failure can be expected?

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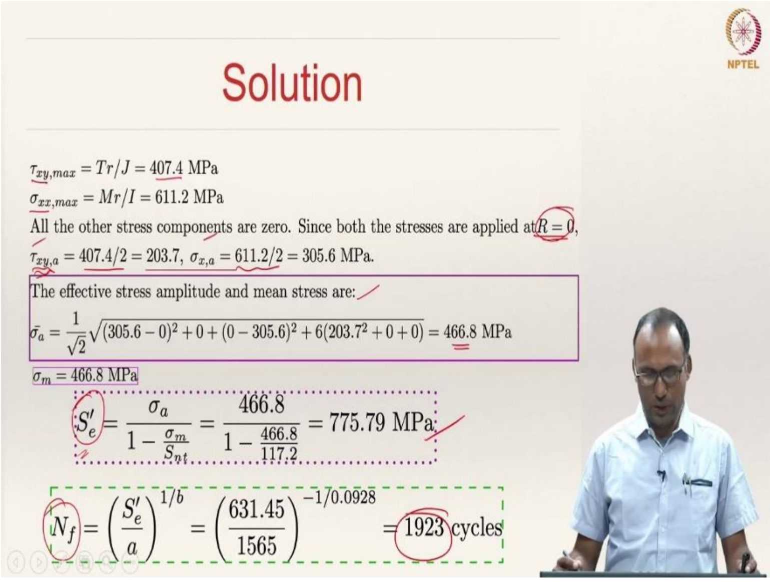

So, you calculate $\tau_{xy,max} = 407.4$ MPa because minimum is 0. We are only calculating the maximum values now, and this is from bending. All the other components are 0, because these are the only two things that are acting. Both the stresses are applied such that the stress ratio $R = 0$.

If the stress ratio is 0, the normal and shear stress amplitudes are given by,

$$\tau_{xy,a} = \frac{\tau_{xy,max}}{2} = \frac{407.4}{2} = 203.7 \text{ MPa}, \sigma_{x,a} = \frac{611.2}{2} = 305.6 \text{ MPa}$$

Then, you calculate the effective stress amplitude using the von-Mises stress formula by knowing normal stresses and shear stresses and that comes out to be 466.8 MPa. Having done that, the mean stress amplitude comes out to be 466.8 MPa, because $R = 0$.

When $R = 0$, your stress amplitude and mean stress are same, as we have already discussed when we were talking about types of fatigue loading. The uncorrected endurance strength S'_e can be calculated as,

$$S'_e = \frac{\sigma_a}{1 - \frac{\sigma_m}{S_{ut}}} = 775.79 \text{ MPa}$$

Corresponding to that, the number of cycles to failure, you will use $S = aN^b$, and then you find that we find that the number of cycles to failure is 1923 cycles, when we have a combined state of stress.

The procedure is exactly the same except that instead of having one stress amplitude, you have two stress amplitudes; one is in shear and one is in normal. All that we need to do is to calculate the effective stress amplitudes, and similarly effective mean stress.

Use that in your Goodman formula and then use the failure and from there you calculate the fatigue strength. From that fatigue strength, you calculate the life from the power law relation between stress amplitude and the number of cycles for failure.

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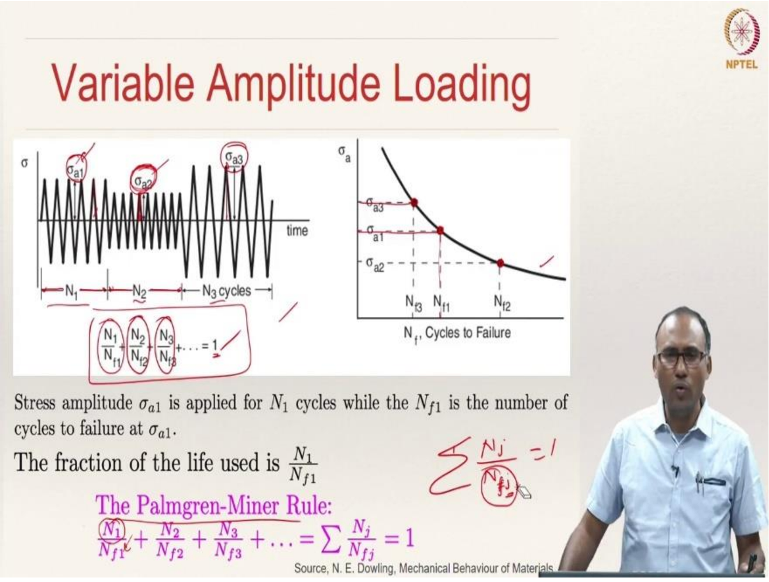

So, until now, the kind of problems that we have dealt with are having the same stress amplitude, i.e., the stress amplitude is uniform. However, in real systems, you may also have variable amplitude; that means, during this loading, the stress amplitude might change as shown in this figure here.

The stress amplitude for N_1 cycles is σ_{a1}, for N_2 cycles is σ_{a2} and for N_3 cycles is σ_{a3}. If such a cycling is applied on the material, how does one estimate the failure in such systems?

Let us look at the S-N diagram for this material. Then, let us say you are cycling the material at σ_{a1}, the intermediate stress amplitude. The maximum stress amplitude is σ_{a3}. The minimum stress amplitude is σ_{a2}, and hence this is expected to give longer life.

If you are running the machine at a stress amplitude σ_{a2}, you can expect a life of N_{f2}. However, please note that at that stress amplitude I have only run the machine for N_2 cycles.

If $N_2 < N_{f2}$, that means, the specimen still has some residual life. The remaining residual life can be written as $1 - \frac{N_2}{N_{f2}}$.

Similarly, if I am loading the specimen at σ_{a1}, at N_{f1} cycles, it will fail. However, I have run this machine only for N_1 cycles; that means, I have used only a partial life of the component at that stress amplitude. Similarly, at σ_{a3}, the failure occurs at N_{f3} cycles. However, I have run it only for N_3 cycles assuming $N_3 < N_{f3}$, then I have only consumed a part of the specimen's life.

As the total time should conserve, we can write,

$$\frac{N_1}{N_{f1}} + \frac{N_2}{N_{f2}} + \frac{N_3}{N_{f3}} = 1$$

This law is called Palmgren-Miner rule, wherein if you are having different stress amplitude and the number of cycles used refer that particular stress amplitude is N_1 while the failure cycles correspond to N_{f1} and so on, then you can write,

$$\sum \frac{N_j}{N_{fj}} = 1$$

The denominator is the number of cycles to failure at that particular stress amplitude. This is how one can deal with variable amplitude loading.

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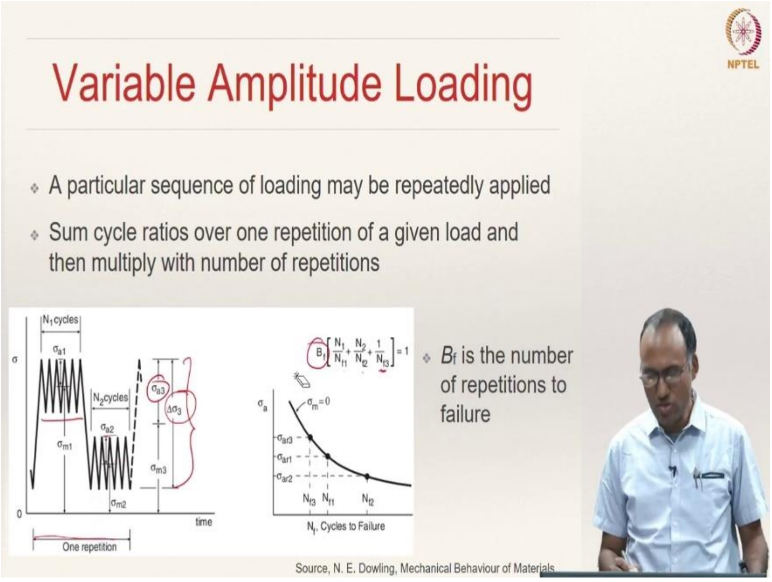

However, if let us say a particular sequence of loading is repeated, for instance, in this case you apply σ_{a1} for N_1 cycles, σ_{a2} for N_2 cycles, and then you will repeat the same thing; this is 1 repetition. The 1 repetition is with $\Delta\sigma_3$. Eventually, we have N_1 cycles of σ_{a1}, N_2 cycles of σ_{a2}, and one cycle of σ_{a3}. That particular loading scenario is repeating several times. The failure surface is represented as,

$$B_f\left[\frac{N_1}{N_{f1}}+\frac{N_2}{N_{f2}}+\frac{1}{N_{f3}}\right]=1$$

In the above equation, B_f is the number of repetitions to failure.

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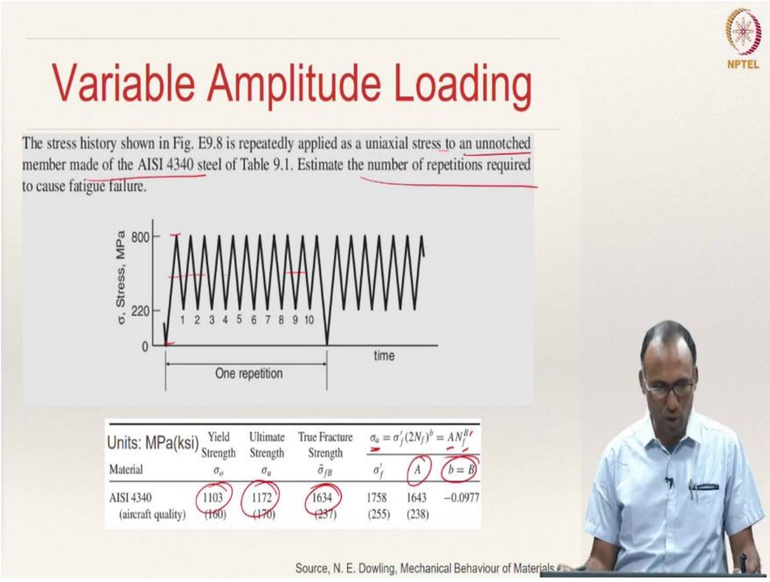

You can now solve this problem. The stress history shown is repeatedly applied as a uniaxial stress to an un-notched member of AISI 4340 steel. Estimate the number of repetitions required to cause fatigue failure. So, this is one stress amplitude, and this one is another one.

The yield strength, ultimate strength and true fracture strength of this material are given. The values of A and B in the equation $\sigma_a = AN^B$.

Based on that, one would be able to calculate the number of repetitions allowed. This is one repetition. One can calculate the number of such repetitions allowed, using the methodology that is described already.

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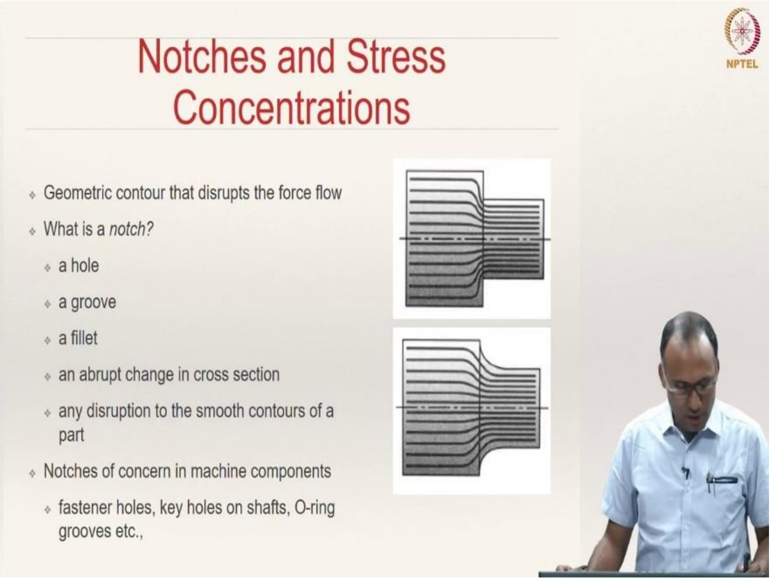

With that, we will stop here. In the next class, we will see the effect of notches. So far, we have not talked about notches. What happens when you have a notch or stress riser? How is the fatigue life influenced by the presence of the stress riser alone, like in the same way that we have done for static failure theories? With that, I will close today's class, and we will meet in the next class.

Thank you very much.