Basics of Materials Engineering
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Lecture – 26
Part - 2
Fatigue Failure of Materials (Effect of Mean Stress)

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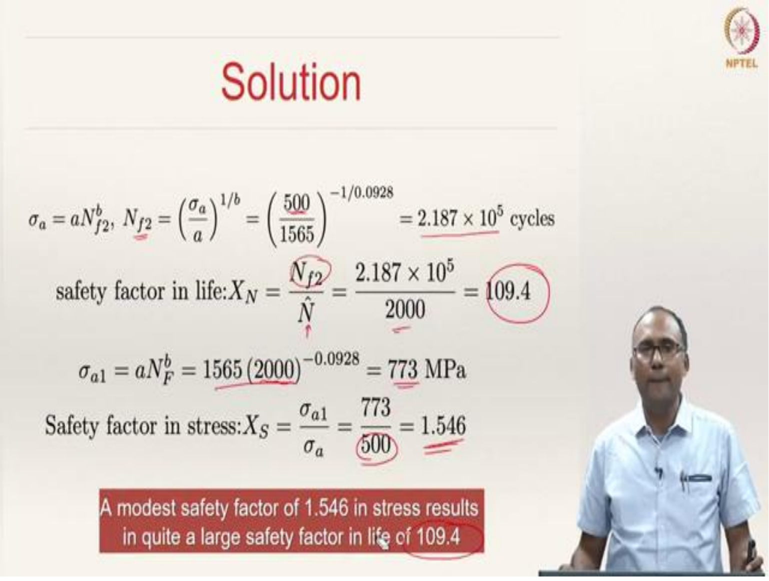

Until now, whatever discussion that we have done and the S-N diagram that we have discussed about, are taken under the assumption that the mean stress is 0; that means, fully reversed loading.

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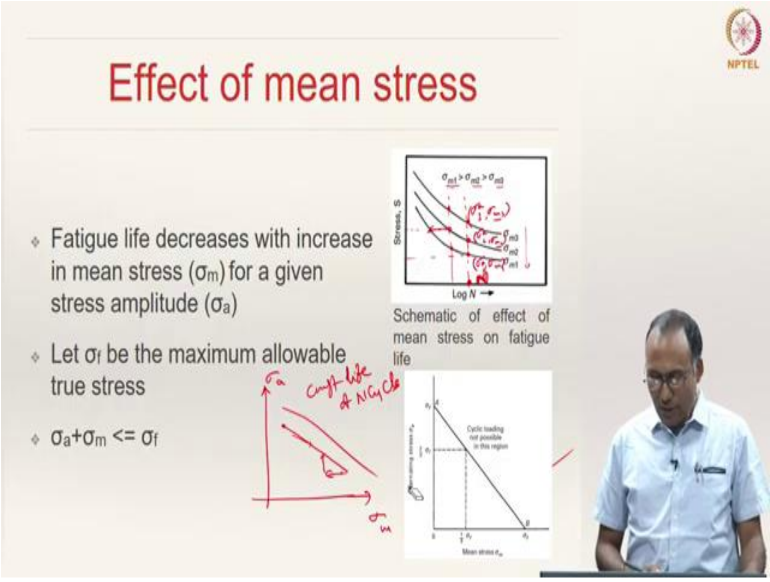

Now, we need to look into the effect of mean stress. What if a non-zero mean stress is applied to a component? Is it going to change the life of the component? Normally, the S-N diagram is obtained from the rotating beam bending test which is fully reversed bending.

But now, you can do experiments with some finite mean stress or positive mean stress. The mean stress is expected to be positive as a negative mean stress i.e., compressive mean stress, is not going to affect the crack propagation.

Here we are plotting the stress-life diagram. σ_{m1}, σ_{m2} and σ_{m3} are different mean stresses. What happens as you increase the mean stress for the same stress-life? Here, σ_{m3} is the lowest mean stress; that is the life that you got.

As you are increasing the mean stress from σ_{m3} to σ_{m2}, the life reduced. Increasing the mean stress has a negative effect on the life of the component for a given stress amplitude.

The mean stress has an effect on the failure of the specimen. Hence, you need to understand and describe the fatigue failure as a function of mean stress of the component. Here this graph is drawn -- for instance if you take a constant life, so, let us say this is the life I am talking about.

At this life, this is the stress amplitude and mean stress combination (S_1, σ_{m1}), (S_2, σ_{m2}) and (S_3, σ_{m3}). From now on, I would represent stress amplitude not with S, but σ_a just to be consistent. So, this is $(\sigma_{a1}, \sigma_{m1})$, $(\sigma_{a2}, \sigma_{m2})$ and $(\sigma_{a3}, \sigma_{m3})$; all these points correspond to failure. At this, all these points correspond to failure, but at the same life. That means, the failure life is same number of cycles. So, that let us say this is N cycles.

If the material is failing at N cycles, if the mean stress is σ_{m1}, then the stress amplitude it can withstand isσ_{a1}. If the mean stress is σ_{m2}, the stress amplitude is σ_{a2}. If the mean stress is σ_{m3}, the stress amplitude is σ_{a3}; that means, for a constant life of N cycles, on the $x-$ axis if I plot σ_m, on the $y-$ axis if I plot σ_a, then I would have something like that.

As I am increasing σ_m, the stress amplitude that the material can withstand is also reducing, right? If I reduce σ_m, the stress amplitude that it can withstand for the same life increases. Similarly, if I would plot for a different life, lower life, then I would get a different curve and so on.

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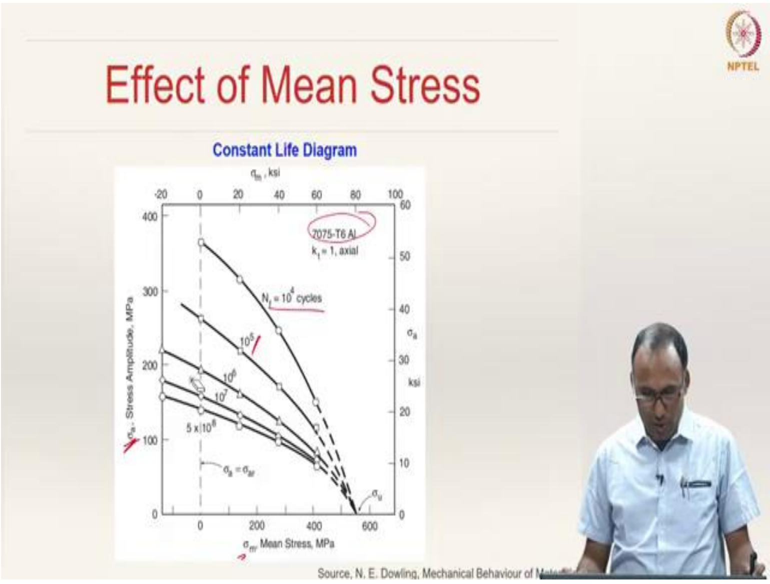

I can now draw the stress amplitude v/s mean stress. This is for an aluminum specimen, something called constant life diagram. This line corresponds to 10^4 cycles. This is the data corresponding to a life of 10^5 and 10^6 cycles, respectively.

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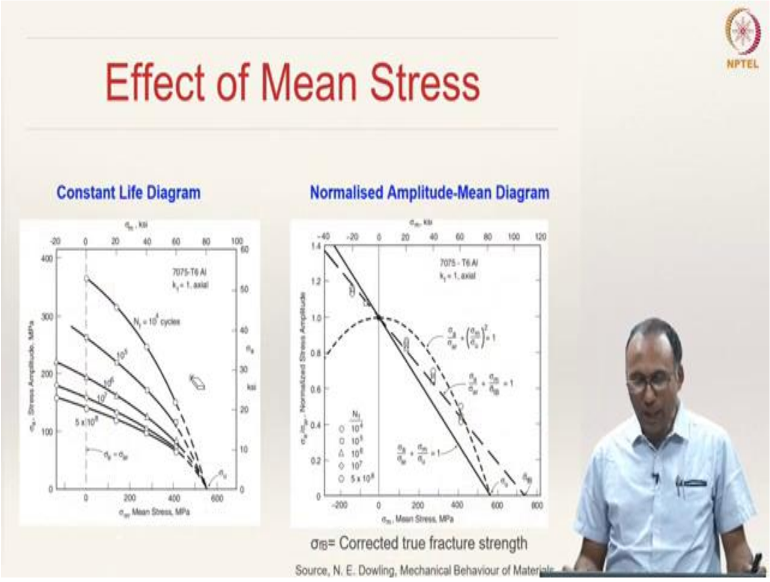

Now, I am normalizing the stress amplitude with say the ultimate strength of the material or something like that. In this class, we are only focusing on the positive mean stress. When I am normalizing this stress amplitude with ultimate strength of the material, that is 1. Then, you see that this data sort of represents a straight line.

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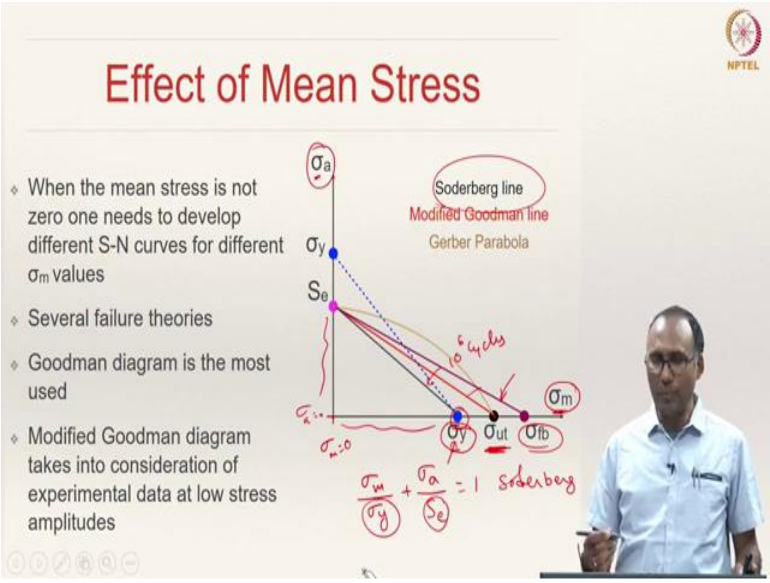

Let us now discuss different kinds of failure surfaces for a constant life. Let us say this is the mean stress and this is the stress amplitude. On $y-$axis, I have S_e which is the endurance strength and σ_y is the yield strength. This is the mean stress equal to 0.

When the mean stress happens to be 0, let us say this is the endurance strength, corresponding to 10^6 cycles. And this is the yield strength. If I draw a straight line between these two data points, that failure surface is called as the Soderberg line.

Instead of considering σ_y, we can also consider ultimate strength σ_{ut}. Why is that? When the stress amplitude is 0 i.e., $\sigma_a = 0$, it is static loading, right? Under static loading, the failure happens only at ultimate strength not at yield strength.

Soderberg initially gave this point to be σ_y, but later Goodman suggested to use σ_{ut} on the $x-$ axis. Hence, if you connect S_e and σ_{ut}, you have the modified Goodman line. Instead of taking them to be straight lines, you can also take it to be a parabola which seems to match better with the experimental data. Such a failure surface is referred to as Gerber's parabola.

If you have to take the corrected ultimate strength into consideration, then your failure surface would be that.

However, most of the times, modified Goodman criteria represented by this red curve seems to show a good match with the experimental data. Hence, we will use the modified Goodman diagram as a failure surface when mean stress has to be taken into account. In the previous figure, the normalization is done not with the ultimate strength, but with the endurance strength of the material or the fatigue strength.

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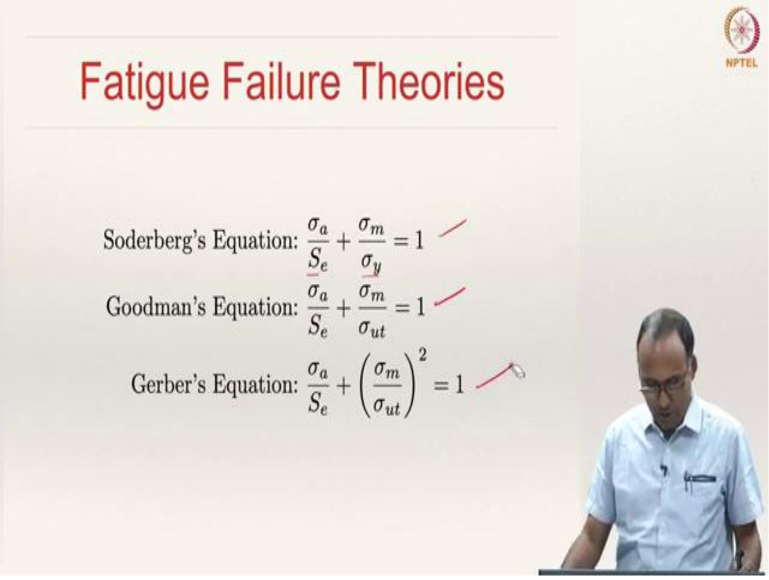

Let us go about writing the equations for the three failure criteria.

$$\text{Soderberg's Equation:}\ \ \frac{\sigma_a}{S_e} + \frac{\sigma_m}{\sigma_y} = 1$$

$$\text{Goodman's Equation:}\ \ \frac{\sigma_a}{S_e} + \frac{\sigma_m}{\sigma_{ut}} = 1$$

$$\text{Gerber's Equation:}\ \ \frac{\sigma_a}{S_e} + \left(\frac{\sigma_m}{\sigma_{ut}}\right)^2 = 1$$

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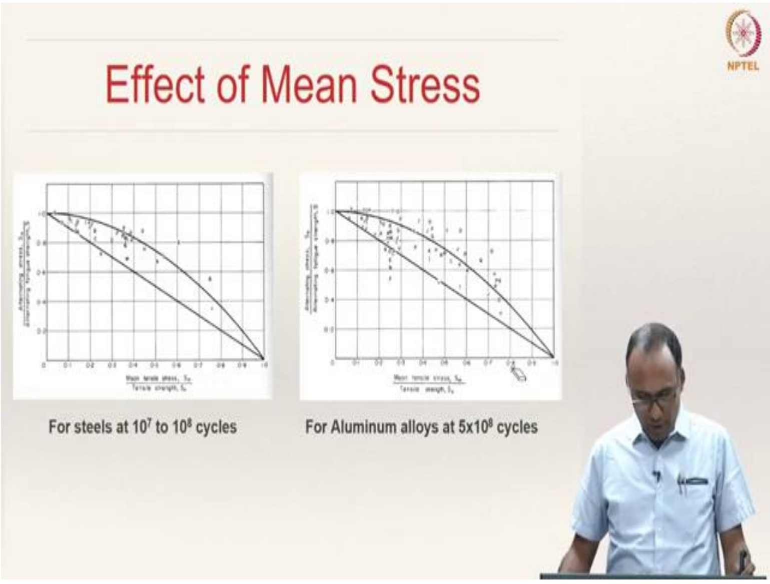

The mean stress is normalized with ultimate tensile strength and alternating stress is normalized with alternating fatigue strength as we have discussed already. That is why the x and y axis limits are up to 1.

Here, this is for aluminum alloys which corresponds to 5×10^8 cycles and this is for steels which corresponds to $10^7 - 10^8$ cycles.

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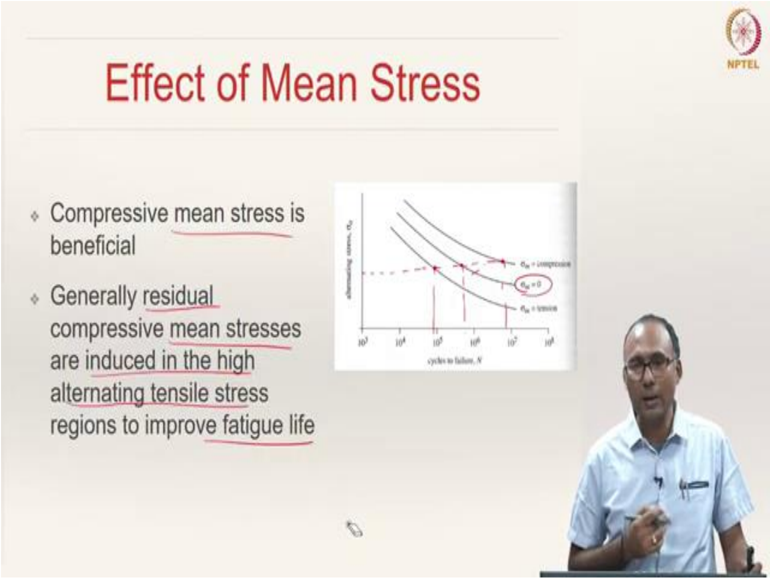

Another important question that we need to address is the effect of compressive mean stress. What happens when you have compressive means stress? It turns out that the compressive mean stresses are beneficial because when you have a local compressive state of stress, the cracks are actually going to close instead of opening up.

Only tensile stresses will be responsible for opening the cracks. Generally, the residual compressive mean stress is induced in the high alternating tensile stress regions to improve the fatigue life. This is done intentionally. To increase the fatigue life, you induce compressive mean stresses on the system in order to delay the fatigue failure.

For a given stress amplitude, if there is tensile mean stress, the component fails much early, zero mean stress a little later, compressive mean stress much later. This clearly shows an enhancement in the fatigue life for a given stress amplitude when the mean stress is compressive in nature.

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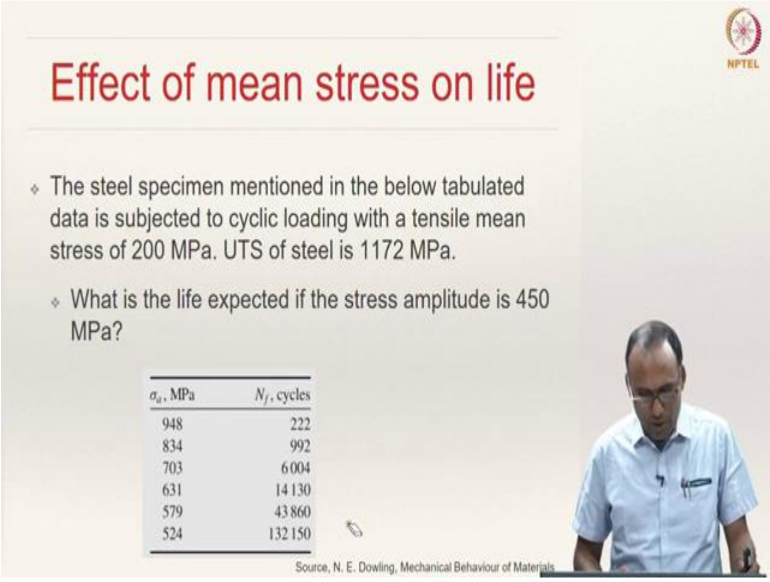

Let us now look at the effect of mean stress on life. A steel specimen, exactly the same steel specimen that we have discussed in the previous example is subjected to a tensile mean stress of 200 MPa and the ultimate strength of the material is 1172 MPa. What is the life expected if the stress amplitude is 450 MPa?

This data is corresponding to a tensile mean stress of 200 MPa; and if the alternating stress or stress amplitude is 450 MPa, what is the life expected?

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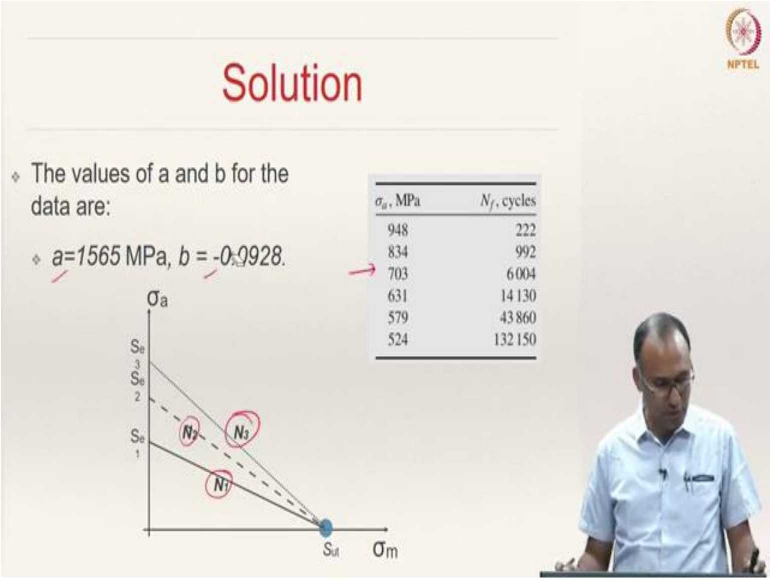

How do we go about doing that? Let us look at the schematic of Goodman diagram. This is S_{ut} and this is S_{e1}, S_{e2} and S_{e3}. S_{e1}, S_{e2} and S_{e3} correspond to N_1, N_2 and N_3 cycles, respectively. From this data, you calculated the values of a and b.

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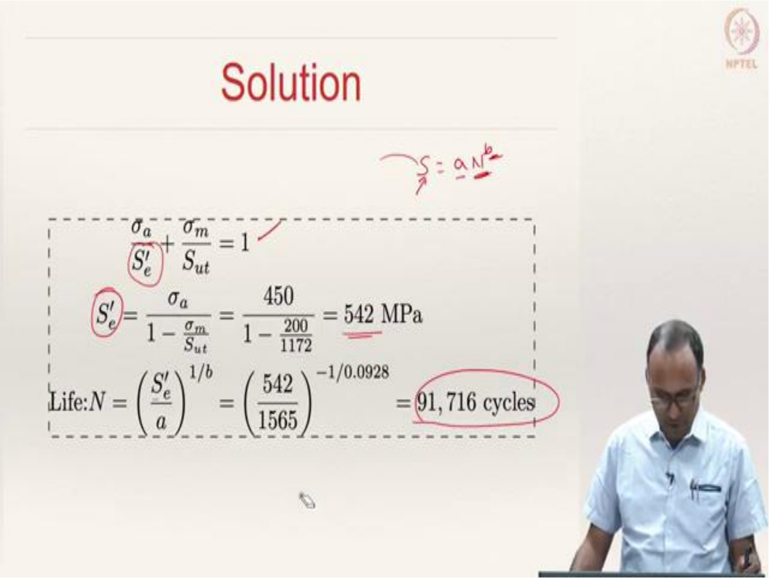

From the Goodman diagram, we have

$$\frac{\sigma_a}{S_e{}'} + \frac{\sigma_m}{S_{ut}} = 1$$

Here I am using the uncorrected endurance strength $S_e{}'$ because I do not have the information about corrected endurance strength. The uncorrected endurance strength turns out to be 542 MPa.

Once you know the uncorrected endurance strength, what you have is $S = aN^b$ and this is the uncorrected endurance strength which you need to plug in. Since a and b are known, you will be able to calculate N. That gives you a life of 91716 cycles. When there is no mean stress, you directly calculate what is the life of this component.