Basics of Materials Engineering
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Lecture – 45
Fatigue Failure Theories (Fatigue strength correction factors)

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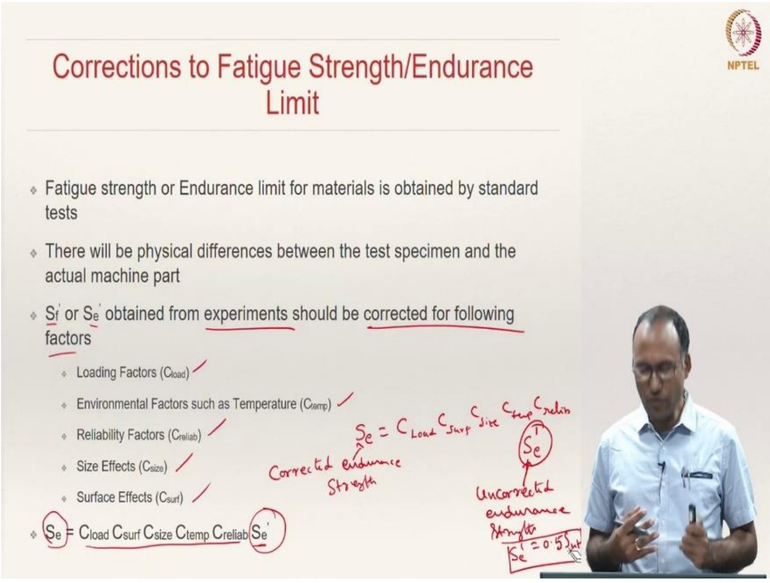

In the previous discussion when we have calculated the endurance strength of the material, we always referred to that as uncorrected endurance strength of the material, or uncorrected fatigue strength of the material, designated with $S_e{}'$ or $S_f{}'$. S_e' is the uncorrected endurance strength. Please note that this endurance strength data has been obtained by doing a standard test. For instance, if we are talking about fully reversal bending, we are actually talking about standard rotating beam bending test.

However, the specimen geometry and the conditions under which the original test data has been obtained are going to be different from the real-world applications. Hence, one needs to account for these variations between the test conditions and the real-life conditions. In order to do that, people have come up with certain correction factors based on the known differences between the experimental conditions and the real-life application situations.

In order to account for that, $S_e{}'$ or $S_f{}'$obtained from experiment should be corrected for following factors. What are those factors? A load factor, environmental factors such as temperature factor, reliability factor, size factor and surface factor. Once you have accounted for these factors, now the corrected endurance strength S_e is obtained by multiplying all these factors with the uncorrected endurance strength $S_e{}'$.

$$S_e = C_{\text{load}} C_{\text{surf}} C_{\text{size}} C_{\text{temp}} C_{\text{reliab}} S_e{}'$$

In the above expression, S_e' is the uncorrected endurance strength. We know that for steels,

$$S_e' = 0.5\, S_{ut}$$

In order to use this information of the endurance strength for real time applications, you need to account for these discrepancies between the conditions under which experimental data has been obtained vis-a-vis the conditions under which the actual component works.

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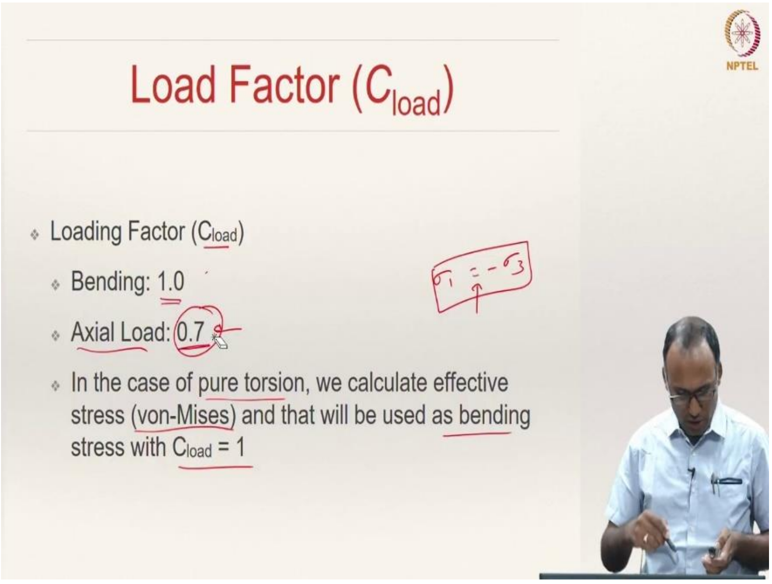

Let us go by each and every factor. The first factor that we are going to focus on is something called load factor, denoted by C_{load}. You know that the rotating beam bending test data is obtained by applying a bending load. If your real-world application is also subjected to the bending load, then you use load factor to be 1. But if the real-world application is using axial load factor, then you should use a load factor of 0.7.

These numbers are obtained by experience. People have done several experiments and they observed the behavior of the systems under different kinds of loading scenarios and then they have prescribed such a load factor which seems to work well with the real-life applications. There is no physical basis necessarily for choosing exactly 0.7. This 0.7 comes from the experience. Sometimes, it can be 0.68, 0.72, but 0.7 seems to match with the experimental data or the test data in the real-world applications.

Hence, when your load is different from bending load, and if it is axial load you take 0.7, if your component is subjected to pure torsion, what we do is, we calculate the von-Mises stress from pure torsion. When you have pure torsion, you know that your principal stresses $\sigma_1 = -\sigma_3$. From there, you calculate the equivalent stress and you use that stress as the bending stress; that is the load that you are going to consider being applied on the material.

And if you are doing that, then you can still use $C_{\text{load}} = 1$, because you have converted the otherwise torsional loading into equivalent normal stress which can be correlated with the bending stress situation.

So, we know how to choose C_{load}. If it is bending, it is 1; if it is axial load, $C_{\text{load}} = 0.7$. That means, if you are applying an axial load, the endurance strength is going to be 0.7 times the bending load scenario; that means, endurance strength reduces. This is expected as we have already seen, when you are having an axial load scenario, the entire cross-section is subjected to the same amount of maximum stress.

There is a higher probability for greater number of cracks to experience higher stress, and hence it will fail much quicker. Hence, the endurance strength in the case of axial loading should be lower than the endurance strength in the case of bending. So, that is why the C_{load} comes down.

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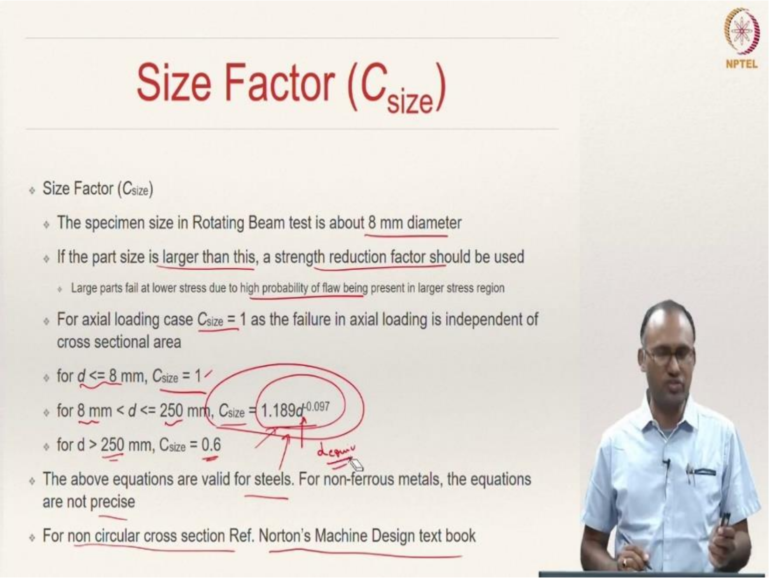

The next factor is the size factor, denoted by C_{size}. The reason why we have to include this size factor is because the rotating beam bending test uses a cylindrical specimen size of 8 mm diameter. However, the real-life scenario uses different sizes of the components, right? The component size need not be 8 mm all the time.

A larger size specimen is expected to have a greater number of cracks compared to the smaller size specimen and the failure phenomena is not going to be same as the 8 mm diameter size. Hence, you need to account for this difference in size when compared to rotating beam bending test.

If the part size is larger than this, a strength reduction factor should be used because large parts fail at lower stress due to high probability of flaws being present in larger stress region.

For an axial loading case, $C_{\text{size}} = 1$, as the failure in axial loading is independent of the cross-sectional area, because you are already taking into account of the effect of axial loading in C_{load}, and now independent of the cross-sectional area, everywhere you will have same stress. So that is already taken into account in the C_{load} scenario. Hence, when you have an axial load, $C_{\text{size}} = 1$. It need not be taken into consideration, only when you have bending or other loads you need to account for changes in C_{size}.

So, the prescription is for $d \leq 8$ mm, $C_{\text{size}} = 1$. If it is less than 8 mm, then it is not going to be worse and hence, to be conservative, you can take $C_{\text{size}} = 1$. For $8 \text{ mm} < \text{d} \leq 250 \text{ mm}$, C_{size} can be calculated using the empirical formula,

$$C_{\text{size}} = 1.189\, d^{-0.097}$$

This empirical formula is obtained by conducting several experiments, and seems to fit the data very well. Again, there need not be any physical basis for obtaining this empirical formula. It is obtained by fitting a curve to the experimental data.

If $d > 250$ mm, then $C_{\text{size}} = 0.6$, i.e., you will plateau the size factor at 0.6, and will not reduce it below that. Please note that this equation or these combinations are valid only for steels. For non-ferrous metals, the equations are not actually precise.

So, you need to look into the design data handbooks and see what is C_{size} for a different kind of a material, but for steels this is what is prescribed. In this course, we will primarily be using steels for design, and hence we are not presenting the details for other materials. But if you want the details of size factors, that should be available in the literature or design data handbooks.

There is an important assumption here, that the component is having a circular cross-section; because we are only talking about diameter. What happens when you have non-circular cross-section? Please refer to Norton's Machine Design textbook wherein how to deal with non-circular cross-section is presented in detail.

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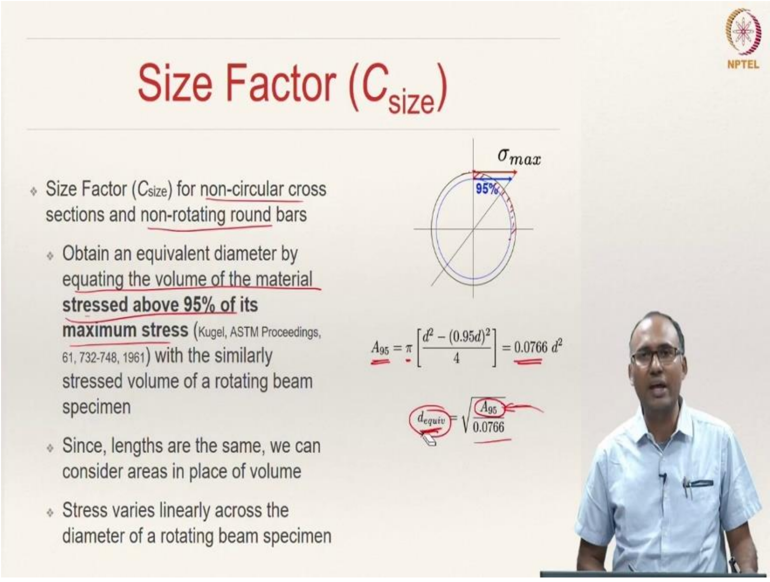

I will explain briefly how one should deal with non-circular cross-sections and also non-rotating round bars. If it is not rotating also, it is important because if it is not rotating then the material point that is under high stress region probably continues to be in high stress region and so on. So, it is also important to see what happens for the non-rotating bar and non-circular cross-sections.

This is the prescription. One needs to obtain an equivalent diameter by equating the volume of the material stressed above 95 percent of its maximum stress, with a similarly stressed volume of a rotating beam specimen. Suppose, if you take this cross-section of a circular rod, we know that the maximum stress is outermost layer, and 95 percent of the maximum stress is at certain distance, right? Because it is linear, you can say that it is $0.95d$.

This diameter will be $0.95d$. Now, we are looking at the area and since the out-of-plane distance is uniform, I am not considering length. Instead of considering volume, I am calculating the area; both are equivalent in this particular scenario.

What we need to obtain is the material stressed above 95 percent of its maximum stress. This is the maximum stress here, 0.95 times maximum stress. This region is the one that is subjected to stress above 95 percent of the maximum stress, right? A_{95} represents the area that is subjected to 95 percent or more of the maximum stress.

$$A_{95} = \pi\left[\frac{d^2 - (0.95d)^2}{4}\right] = 0.0766d^2$$

Now, you calculate the equivalent diameter d_{equiv} as,

$$d_{\text{equiv}} = \sqrt{\frac{A_{95}}{0.0766}}$$

If you have a different cross-section, you calculate the A_{95} of that cross section and plug in that value here, and then you calculate the equivalent diameter.

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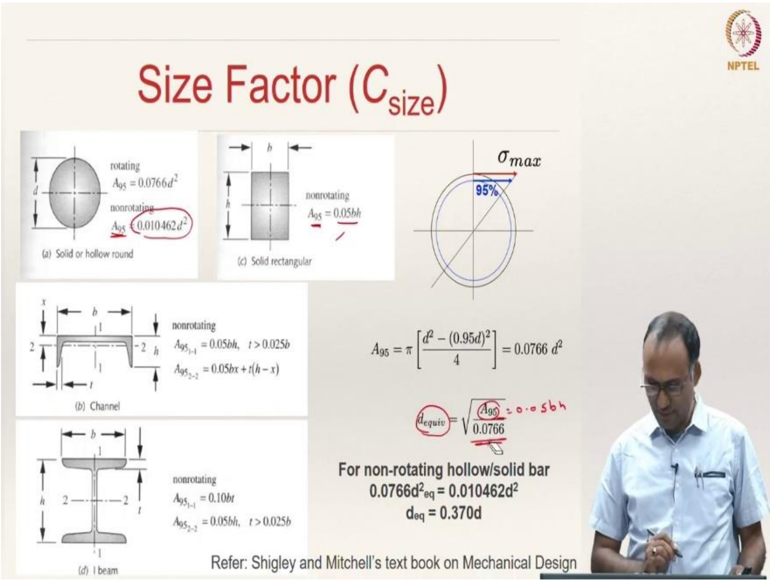

Let us see one quick example. For a rectangular cross-section, you know that $A_{95} = 0.05bh$. That is the area subjected to greater than 95 percent of the maximum stress. All that you need to do is, in order to calculate the equivalent stress, you plug in $A_{95} = 0.05bh$, and then calculate the equivalent diameter.

If the equivalent diameter is less than or equal to 8 mm, the size factor will be 1; if it is greater than 8 mm, then you will use this formula to calculate the size factor. d is replaced by d_{equiv} for non-circular cross-sections.

For different cross-sections, the A_{95} values are calculated in the literature, and they will be available in the design data handbooks. If the geometry is simple, then you can calculate that too. For non-rotating circular bars, $A_{95} = 0.010462\ d^2$. You need to substitute that here and then you will get d_{equiv}.

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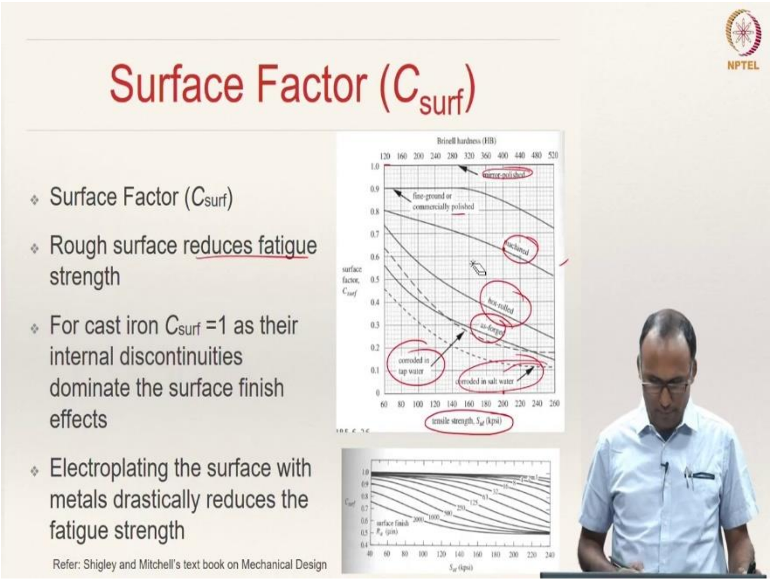

That is about size factor. Until now, we have looked at the load factor and size factor, and now let us discuss about the surface factor. The surface finish of a material is going to significantly influence the fatigue strength of the material. The rough surfaces reduce the fatigue strength, and smooth surfaces increase (correction: do not affect) the fatigue strength. Hence, you need to account for the surface finish.

If you see this graph here, the mirror polished surface -- extremely well-polished surface will have a surface factor of 1; that means, you do not actually have to account for reduction in the fatigue strength. Whereas, all other -- for instance fine ground or commercially polished, machined surface, hot rolled surface, as-forged surface, and sometimes you have a material that is corroded in water, or corroded in saltwater, all these things are going to contribute to surface factor. The surface factor is different for different materials, and known to be dependent on ultimate strength of the material.

Such graphs are available in the literature. By knowing the ultimate strength of the material. Suppose if you know that material is machined and if it is having an ultimate strength of 200 MPa, you refer to this graph and identify C_{surf} and so on.

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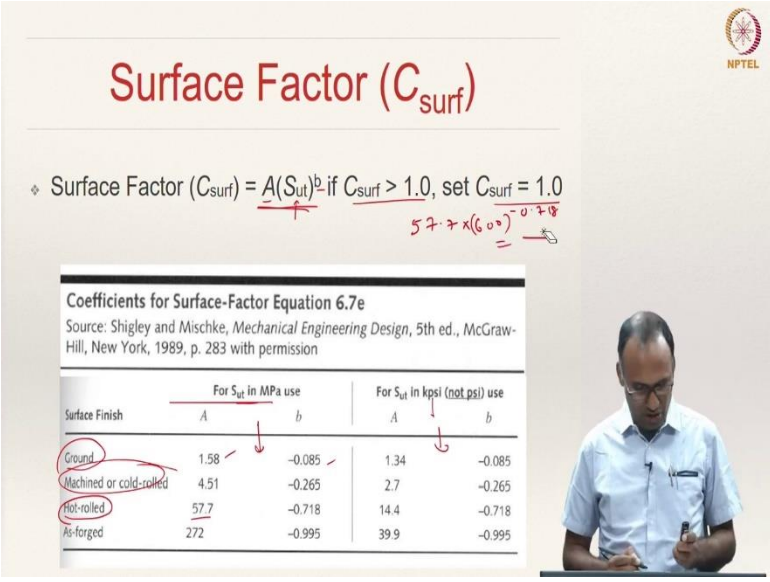

There is also an empirical relation available to compute C_{surf}.

$$C_{\text{surf}} = AS_{ut}^{b}$$

A and b are given here for different surface finishes. If the units are in MPa, use this A and b; if your units are in kpsi, use this A and b.

If by doing such a calculation, you get $C_{\text{surf}} > 1$, then you have to use $C_{\text{surf}} = 1$. You should not increase the endurance strength. Only if it is less than that, then you will take C_{surf} equal to whatever is obtained from this calculation.

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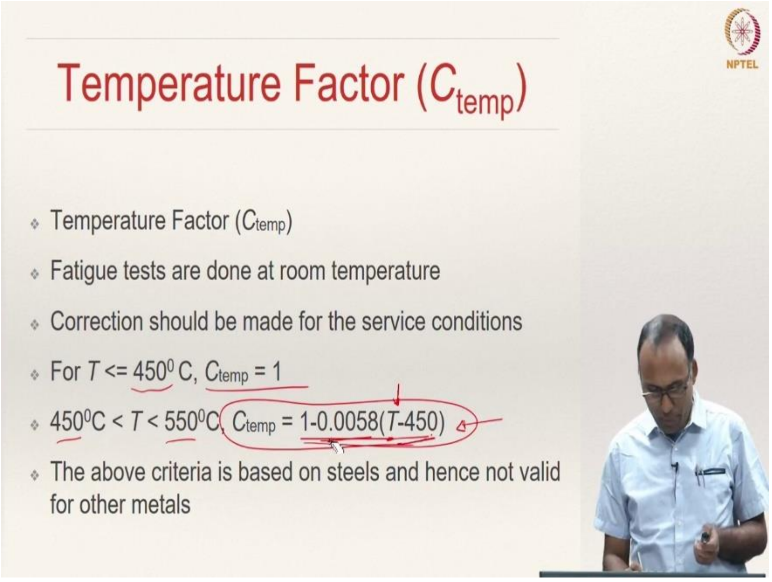

The next factor is the temperature factor, denoted by C_{temp}. Usually, the rotating beam bending tests are done at room temperature, and hence correction should be made for service conditions. We are following these procedures for steels, but for other materials, similar expressions should be available. For $T \leq 450°$ C, C_{temp}=1, i.e., it need not be changed.

$$450° \text{ C} < \text{T} < 550° \text{ C}, C_{\text{temp}} = 1 - 0.0058(T - 450)$$

Usually, the regular steels are not employed beyond 550° C. But if at all, you have some components working beyond 550° C, you need to look into the design data handbooks, and then appropriately identify what should be the empirical expression for C_{temp} as a function of temperature. Please note that the temperature here is in Celsius and not Kelvin; that is important.

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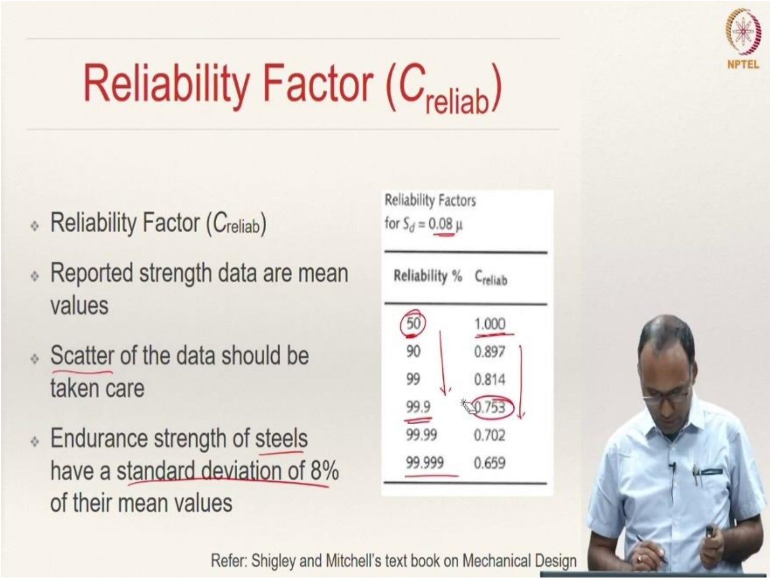

Let us now discuss about the reliability factors. We know that the reported strength data in the literature are not definitive as they have a range. There is a standard deviation or an error bar associated with the strength data i.e., there is always some scatter. You need to take care of the presence of the scatter through the reliability factor.

If the reliability is very low, then you need not worry. If 50 percent reliability is what you are expecting of your component's performance, then $C_{\text{reliab}} = 1$. If you increase the required reliability, i.e., if you want the component to be lasting, the prediction should be as accurate as possible. As the reliability increases, the reliability factor comes down and the endurance strength of the material should be lowered accordingly, so as to ensure higher confidence on the design.

This is the data given for reliability factors and please note that this is given for standard deviation of 0.08μ, as the endurance strength of steels has a standard deviation of 8 percent of their mean value. Based on that, these reliability factors have been obtained. As we have discussed, it is true only for steels; for other materials, this may be different.

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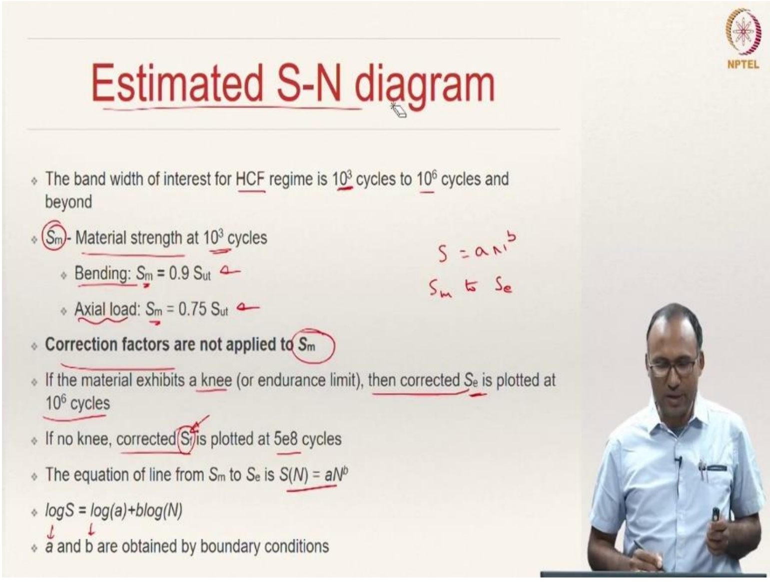

We have looked at five correction factors: load factor, size factor, surface factor, temperature factor and reliability factor. After calculating these five correction factors, you need to correct your endurance strength by using the formula,

$$S_e = C_{\text{load}} C_{\text{surf}} C_{\text{size}} C_{\text{temp}} C_{\text{reliab}} S_e{}'$$

When we are talking about the S-N diagram, we are actually talking about the HCF regime which is known to be active between 10^3 and 10^6 cycles primarily for steels, and for other materials it is beyond 10^6 cycles. The stress amplitude corresponding to 10^3 cycles is denoted by S_m which is nothing but material strength at10^3 cycles.

If the loading is bending, then $S_m = 0.9 S_{ut}$. If the loading is axial, then $S_m = 0.75\, S_{ut}$, very important to know. You should never apply correction factors to S_m. Correction factors are applied only to $S_e{}'$. If the material exhibits a knee or an endurance limit, then the corrected S_e is plotted at 10^6, not the uncorrected one.

If there is no knee, corrected S_f, which is the fatigue strength is plotted at 5×10^8 cycles. These are the guidelines. The equation of the line that we are talking about $S = aN^b$ is true from S_m to S_e or S_f. a and b are obtained by plugging in the boundary conditions. That is how you would plot an estimated S-N diagram.

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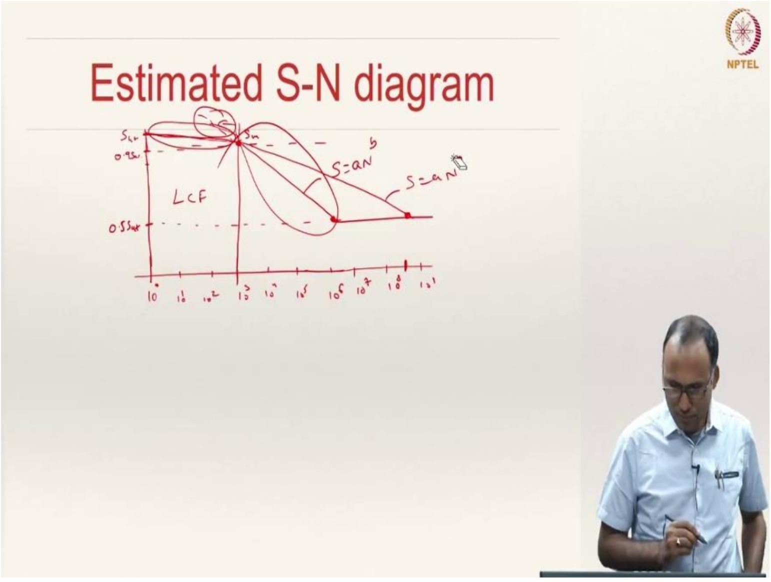

So, how do we do that? Let us say this is S_{ut}, this is $0.9S_{ut}$, and this is my 0.5 S_{ut}. As we have discussed, the HCF regime is from 10^3 cycles. That is the point S_m and S_e is here, and that is the straight line $S = aN^b$. The first point is here.

Even if you extend this line here, it is not valid beyond this point. From here to here, because the material has to fill in LCF regime, so that is this another line, but we are not concerned about this line; at this point of time, we are doing stress-life approach. Stress-life approach is only applicable in the HCF regime. So, we are only talking about this line. If it is a material which does not show endurance strength, what you need to do is -- let us say this is 5×10^8, then $S_e' = 0.5S_{ut}$will be here. Hence your graph $S = aN^b$ will be that.

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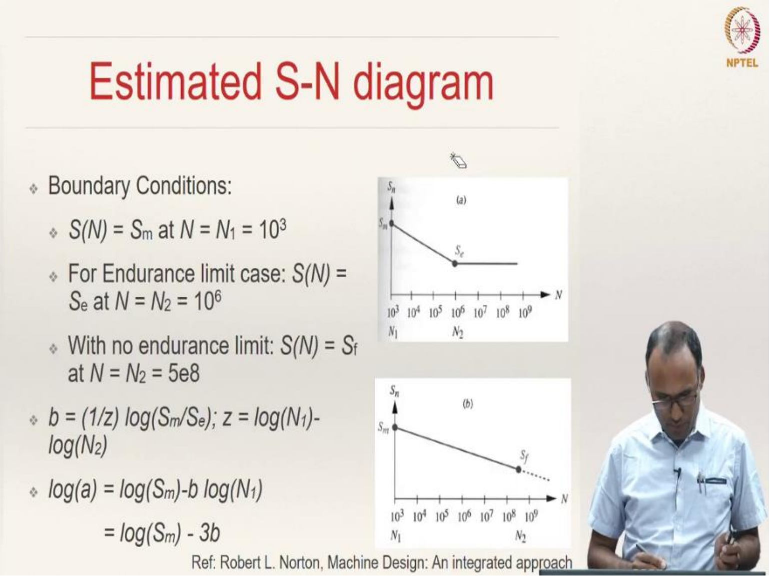

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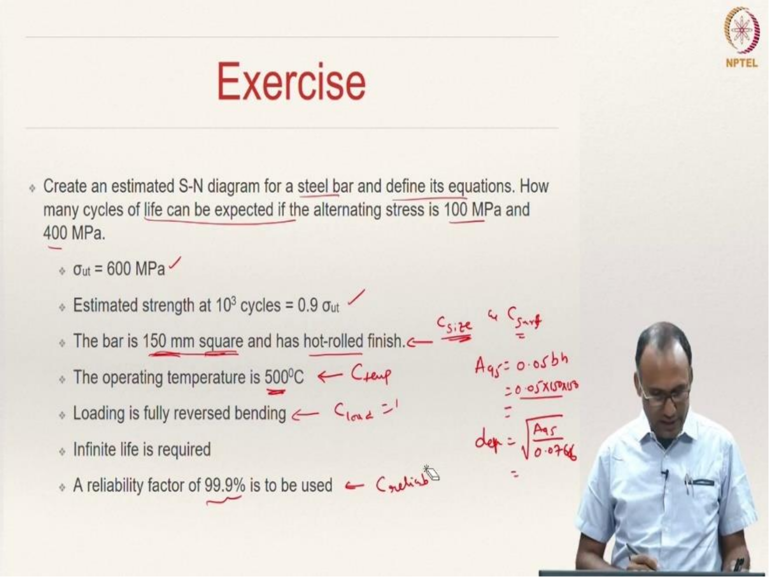

Let us do this exercise problem, then we will see how the S-N diagram can be created. **Create an estimated S-N diagram for a steel bar and define its equations. How many cycles of life can be expected if the alternative stress is 100 MPa and 400 MPa?** It is exactly the same problem, but now we have to incorporate the correction factors. Ultimate strength is 600 MPa, estimated strength at 10^3 cycles is $0.9S_{ut}$.

The bar is 150 mm, it is not 8 mm, and hence you need to account for size factor. It is a square shaped cross-section with dimensions $150\text{ mm} \times 150\text{ mm}$. It has a hot rolled finish. This statement tells you that you need to account for C_{size} and C_{surf}. The need for C_{size} is, it is not a circular bar first, it is a square bar. And right away you have to account for size factor.

If it is a mirror surface, then the surface factor is 1. For all other finishes, you need to have surface factor. The operating temperature is $500°$ C. We have a guideline that if the operating temperature is less than $450°$ C, then you do not need to have a temperature factor, but if it is more than that, you need to consider C_{temp}, i.e., the temperature factor.

$C_{\text{load}} = 1$, because the loading is fully reversed bending which is exactly like the rotating beam bending test. And infinite life is required, that means, you are talking about the endurance strength. The reliability factor is 99.9 percent, it is not 50 percent. If it is 50 percent, reliability factor is 1. If it is more than that, then you need to calculate C_{reliab}. For a reliability of 99.9 percent, the reliability factor is 0.753.

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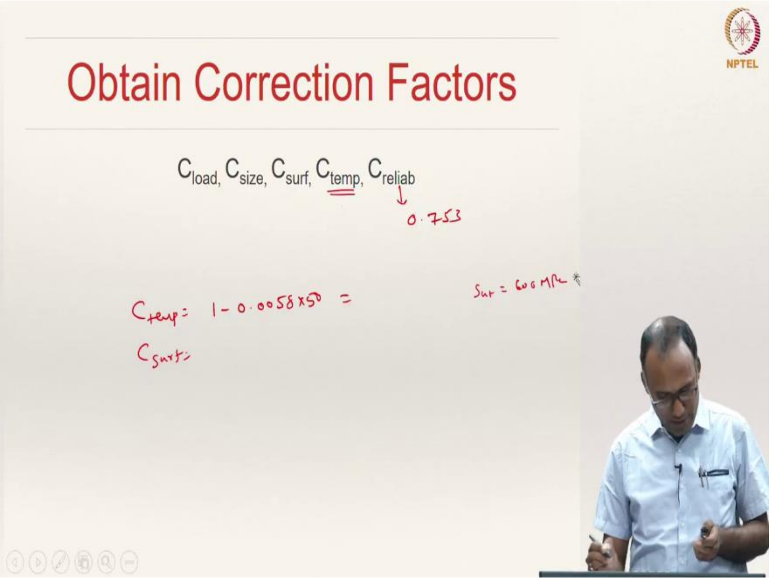

So, we can right away write $C_{\text{reliab}} = 0.753$. Since the working temperature is $500°$ C,

$$C_{\text{temp}} = 1 - 0.0058(T - 450) = 1 - 0.0058(500 - 450)$$

Given that $S_{ut} = 600$ MPa and the surface finish is hot rolled, we see that $A = 57.7$ and $b = -0.718$. So,

$$C_{\text{surf}} = AS_{ut}^{b} = 57.7\ (600)^{-0.718}$$

To calculate C_{size}, we need to know that, for a square cross section, $A_{95} = 0.05bh = 0.05 \times 150 \times 150$. Then,

$$d_{\text{equiv}} = \sqrt{\frac{A_{95}}{0.0766}}$$

From there, you know the formula for C_{size}, plug in that and then you will get the values.

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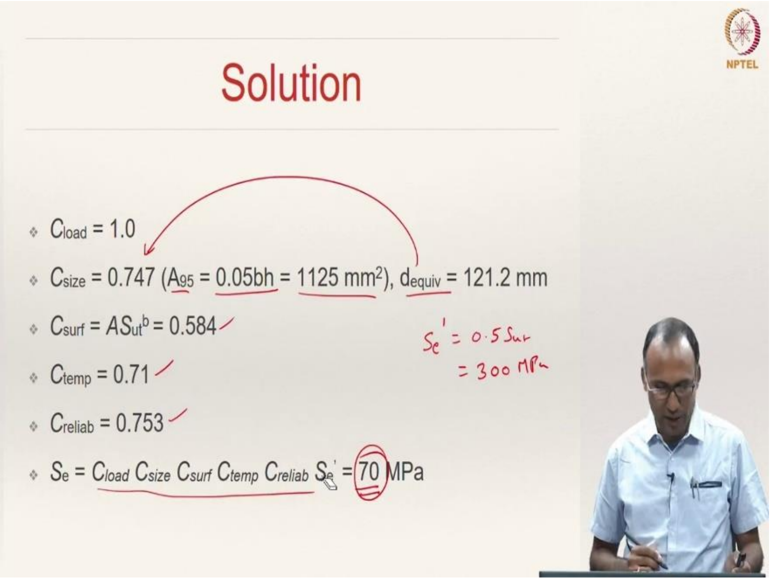

So, the values are calculated.

$$C_{\text{load}} = 1$$

$$C_{\text{size}} = 0.747\ (A_{95} = 0.05bh = 1125\ \text{mm}^2), d_{\text{equiv}} = 121.2\ \text{mm}$$

$$C_{\text{surf}} = AS_{ut}^{b} = 0.584$$

$$C_{\text{temp}} = 0.71$$

$$C_{reliab} = 0.753$$

$$S_e' = 0.5S_{ut} = 0.5 \times 600 = 300\ \text{MPa}$$

The corrected endurance strength can be found as,

$$S_e = C_{\text{load}} C_{\text{surf}} C_{\text{size}} C_{\text{temp}} C_{\text{reliab}} S_e' = 70 \text{ MPa}$$

By putting in all these factors, the endurance strength has reduced by a factor of more than 3. The corrected endurance strength comes out to be 70 MPa.

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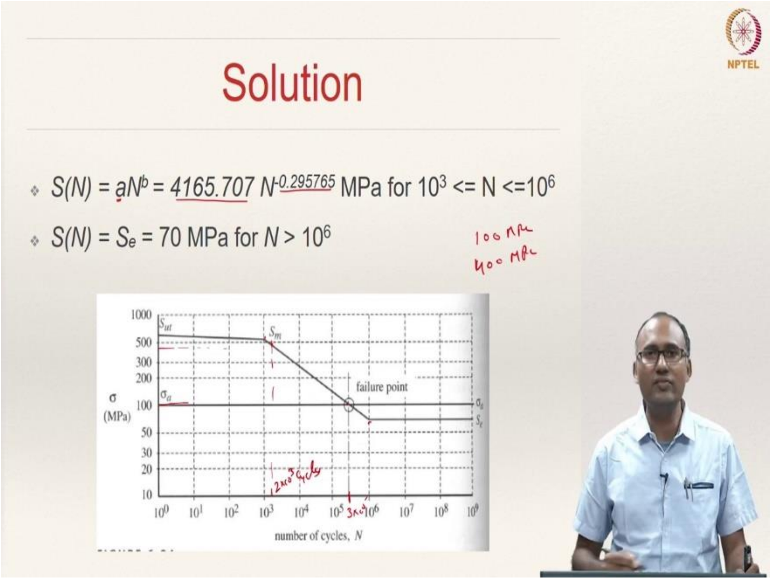

Now, you see what happens; this is the figure. This is S_m and this is S_{ut}. Now, if you take this data and a comes out be this, b comes out to be that, and then you can draw this line. Recall that the corrected endurance strength is 70 MPa. What are the two conditions that are asked in the question? What happens when the stress amplitude is equal to 100 MPa and what happens when the stress amplitude is equal to 400 MPa?

You can clearly see that at 100 MPa, it fails. It gives you a life of say 3×10^5 cycles, whereas for 400 MPa, somewhere here, it will give you a life of about 2×10^3 cycles; that is corresponding to 200 MPa. Please note a major difference. Previously, the same problem has been done without applying correction factors. I will go back and show you.

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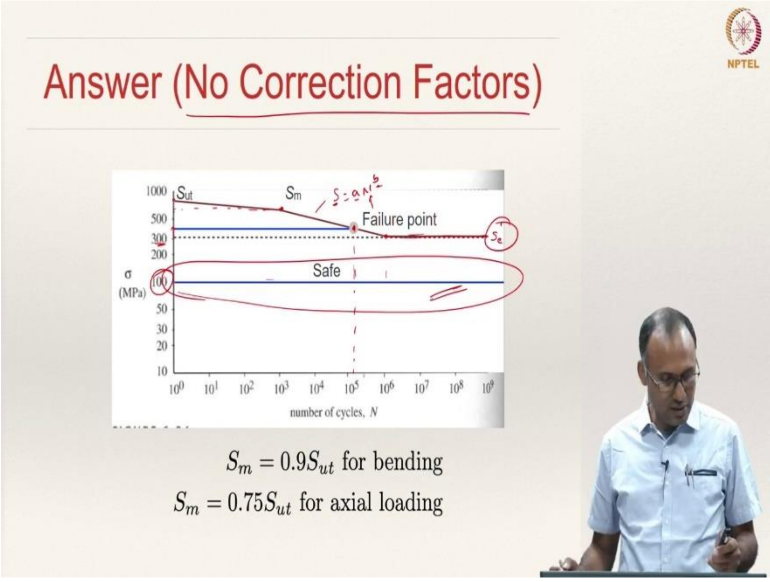

Without applying any correction factors this is exactly the same problem. For 100 MPa stress amplitude, that specimen turned out to be perfectly safe, right? However, after applying the correction factors, you have clearly seen that the material is not safe anymore even at 100 MPa stress amplitude.

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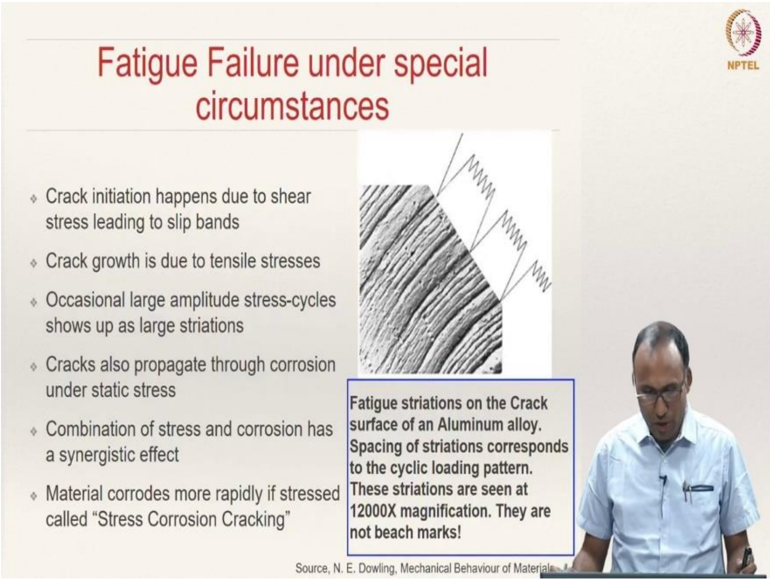

This clearly demonstrates that the correction factors are an important ingredient that needs to be considered while designing components for fatigue life.

With that, I will stop here and then we will resume from here in the next class. Thank you.