Basics of Materials Engineering
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Lecture - 23
Fatigue Failure of Materials (Introduction, Historical Events, S-N Diagram)

Until last class, we have looked at fracture mechanics and we have discussed what are the conditions under which actually a crack propagates and so on. Before that, we have looked at static failure theories wherein the load applied on the structure does not change with respect to time.

In this module, we will be looking at fatigue failure of materials, which basically means, the load changes as a function of time. We could also call this as dynamic failure theories, because the load is changing with respect to time.

We are primarily interested in the systems where load changes with respect to time in some periodic or cyclic manner. That means, there is a repeated load that is acting on the material and such a process is called fatigue. Hence, the theories that we are going to talk about are called fatigue failure theories and we will be spending some time understanding how fatigue failure happens and so on, in this module.

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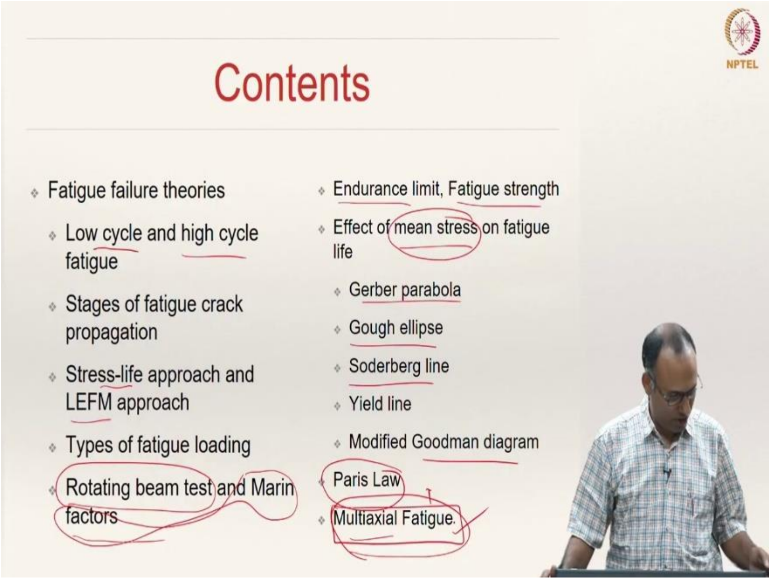

Let us look at the contents of this module. We will primarily introduce, what do we mean by fatigue failure and look at different failure theories, different kinds of fatigue loadings like low cycle fatigue and high cycle fatigue. Then, we will look at different stages of crack propagation under fatigue loading or we call stages of fatigue crack propagation.

And then we assess the failure of materials under repeated loading based on two approaches; one is stress-life approach, and LEFM approach or linear elastic fracture mechanics approach, where we do not consider any plasticity.

We will discuss different kinds of fatigue loading and spend some time on understanding the rotating beam bending test, a very important experiment that gives us an understanding about fatigue failure. We will talk about some factors called Marin factors, that need to be taken into account depending on various conditions.

We will discuss what is endurance limit and what do we mean by fatigue strength. Then, we will spend some time understanding the effect of mean stress; it will become clear when we get there.

Based on the effect of mean stress, we would define different kinds of failure theories, such as Goodman diagram, Soderberg line, Gough ellipse, Gerber parabola and so on.

All these are different kinds of failure theories that take into account the presence of mean stress on the material. Then, we will look at the crack life or fatigue crack propagation as derived by Paul Paris, famously known as Paris law. We will spend some time on understanding crack growth rate in some sense.

We will briefly introduce how do we go about dealing with multiaxial fatigue; because until just before this topic, we will be primarily focusing on uniaxial loading. That means, from the concept of stress tensor, we are looking at the existence of only one stress component such as σ_{xx}, σ_{yy} or σ_{zz}; only one component is present.

When you have multiaxial loading, you need to develop a separate theory for multiaxial fatigue. Similar to what we have done in the static failure theories, where we have discussed something called von-Mises theory, in which you defined equivalent stress, which is a one-dimensional equivalent of a 3D state of stress. We will have a similar discussion, when we study multiaxial fatigue.

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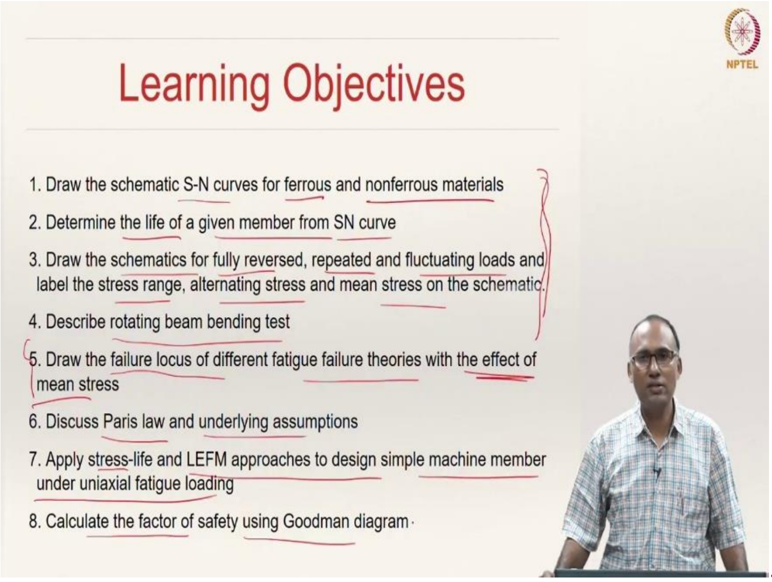

These are the learning objectives. We should be able to draw the schematic of S-N diagram or S-N curve for both ferrous as well as nonferrous metals. We will see what is the difference when we are going through the material. We should be able to determine the life of a given member from S-N curve.

With the knowledge of the S-N curve and the loading scenario, you should be able to predict the life of a component, that means how much time can this component used for, at that particular load level. Then, you should be able to draw the schematics for different kinds of fatigue loadings such as fully reversed, repeated and fluctuating loads and you should be able to label the stress range, alternating stress and mean stress on the schematic.

As I have already mentioned, one of the most important experiments in understanding fatigue life of materials is the rotating beam bending test and we will see how the test is conducted. Then, we will focus on drawing the failure locus for different fatigue failure theories with the effect of mean stress; that means until here we will assume that there is no mean stress.

We will first study what happens when the mean stress is zero, followed by the effect of a non-zero mean stress. We will try to draw the failure locus for each of these different failure theories. Then, we will discuss Paris law and the underlying assumptions. And then, apply stress-life and LEFM approaches to design simple machine members under uniaxial fatigue loading. We will also be able to calculate the factor of safety using Goodman diagram.

Previously, we have calculated the factor of safety using various theories such as von-Mises or distortional energy theory and maximum shear stress theory. You were able to identify the stress state, and then graphically we were able to find out the factor of safety. Similarly, on Goodman diagram, if you are able to identify the stress state, then you should be able to calculate the factor of safety.

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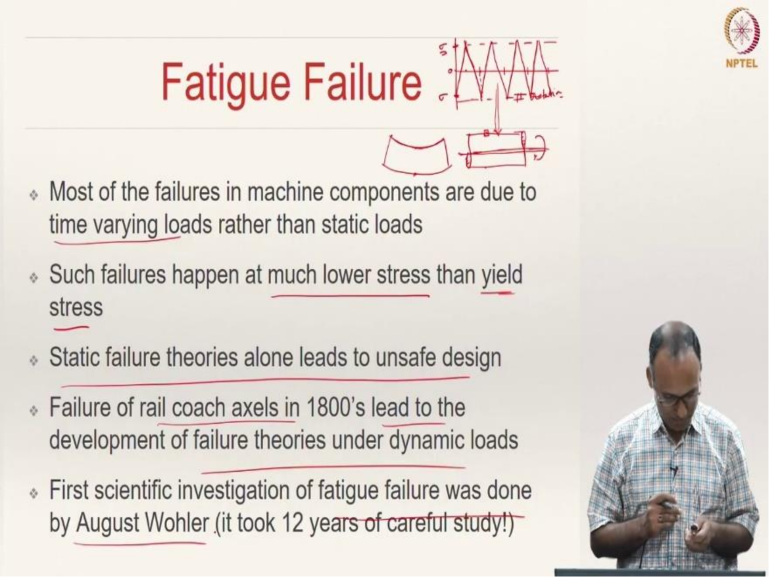

So, what is fatigue failure? Most of the failures that we see in machine components are due to time varying loads. Let us consider a shaft that is rotating about this axis and transmitting power. Let us focus our attention to this element and let us say there is a gear sitting on it. As a result, the shaft bends as it rotates.

When the shaft bends in this manner, there is going to be a compressive state of stress on this layer and tensile state of stress on that layer. What happens to the point which is on the top to begin with, by the time it makes half-rotation?

This point will move here. Initially the point which is at the top, because the load is not changing, and this point was initially in the state of compression; but when it makes one half a rotation, the state of stress changes from compression to tension.

The beam is symmetric about this axis and hence the magnitude of the tension and compression will be the same; that is something that we have leant in the strength of materials class. The same material point is changing its stress state from compressive to tensile in half a rotation and when it makes full rotation, it will go back to compression again.

If you were to draw number of rotations on the x-axis -- one rotation, two rotations, three rotations, four rotations and this is my half-rotation. This element initially is having say let us say some compressive stress, this is 0 and let us say the magnitude is σ and this is also σ, this is $-\sigma$, that is σ.

At the beginning when the shaft is at the first rotation, the stress is this and by the time it makes half a rotation, the stress state is going to positive. When it makes a full rotation, the stress state will become negative again and so on.

It continues to do so, because within one rotation, it has come back to its original configuration, but during this time the material point has changed its stress state. If you continue this again and again, the material point keeps on experiencing this variable stress that fluctuates between positive and negative values.

At a given point, the stress state is changing with respect to time; that means the load is getting repeated and such a process is called fatigue. We know most of the machine components are subjected to these kind of time varying loads, rather than static loads.

For instance, the sprocket teeth on the bicycle are subjected to fatigue failure; because at a point the number of sprockets, the teeth that are in contact with the chain are less than the total number of sprocket teeth. Hence, some of the teeth are having a finite amount of time in one particular rotation; the sprocket teeth will not be in contact for one entire rotation. During the time that it is not in contact, the force exerted on to the sprocket teeth is zero.

So, the load varies from some finite load to zero and so on, it is cycling between these two values. Similarly, you can take several examples and most of the systems are subjected to time varying loads. When such systems are subjected to time varying loads, it is observed that whenever they fail, they fail at much lower stress than the yield stress of the material. When we say fail, they are actually breaking.

It is not the ductile failure that we are talking about, we are talking about fracture like in a brittle manner. Although, there may be some local plasticity, we are talking about failure in a brittle manner. But the failure is actually happening much below the yield strength, which is not what we expect from the perspective of static failure. In the case of fatigue failure or when a machine member is subjected to time varying loads, we see that the materials are failing much below their yield strength.

People were puzzled by this strange phenomenon. We know that the material does not enter plasticity below the yield strength. But when the load is changing with respect to time, people have been observing that the material is failing below the yield stress. That means, you cannot explain this phenomenon based on the lessons that we have learnt until now on the static failure theories

Static failure theories alone lead to unsafe design, because people have been observing that the materials are failing when the service load is well below the yield strength of the material. In the history, there were several causalities just because the designs were not taking into account of the time varying nature of the load which led to premature failure of the components than what engineers thought will withstand.

For instance, the failure of rail coach axles in the 1800's led to the development of fatigue failure theories under dynamic loads. There were several rail road accidents around 1800's both in Britain and Germany and that led people pay attention to understand this particular phenomenon. That is what led to the development of failure theories under time varying loads.

The first ever detailed scientific investigation of fatigue failure was done by August Wohler in Germany, where he spent almost 20 years of his life understanding and doing several experiments and trying to understand what happens when you subject an axle to a repeated load and then how does the material fail. It took him 20 years to consolidate his experimental data and write one single paper; but that happens to be the seminal paper in the literature of fatigue failure. Till date, people refer to that work, because of the impact that work had on the design or the design philosophy of machine components that are subjected to time varying loads.

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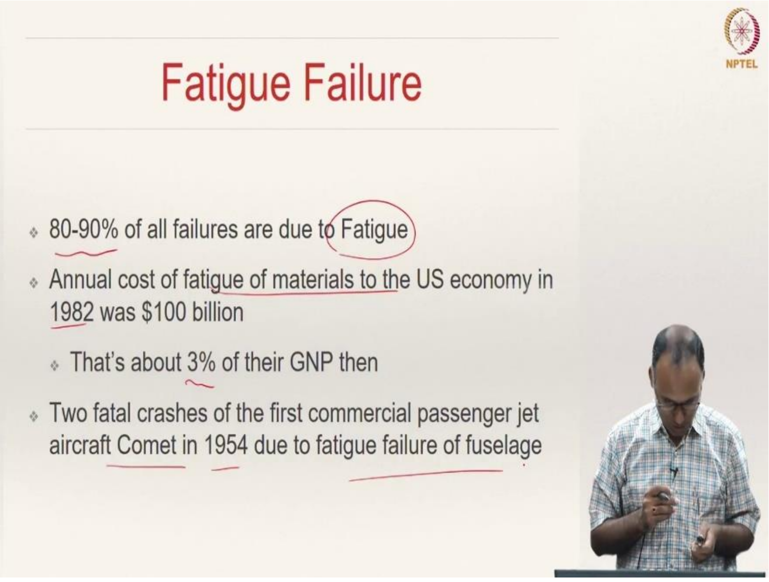

When you do the postmortem of several industrial failures, it turns out that about 80 to 90 percent of all failures are due to fatigue. Such is the importance of understanding this particular field of failure, the fatigue failure. The annual cost of fatigue of materials to the US economy in 1980s was about 100 billion dollars. That is about 3 percent of their GDP.

There are several countries which spend much less than 1 percent of their GDP on the entire science and technology budget. But the cost of fatigue failure in 80s alone was about 3 percent of the GDP of United States. You can imagine, if you neglect understanding of the failure of this kind of systems, what kind of a catastrophe that it can lead to the economy.

One of the very sad stories primarily about aircraft industry is the comet aircraft. There are two fatal crashes of this first commercial passenger jet in 1954 that happened due to fatigue failure of fuselage. There were some design issues, which led to such a fatigue failure and that led to bringing down the entire fleet of comet. As a result, they lost their market to their competitors elsewhere in the Europe and in the US.

The magnitude or the intensity of this fatigue failure is so high that it actually can bring down economies significantly, if you do not pay enough attention.

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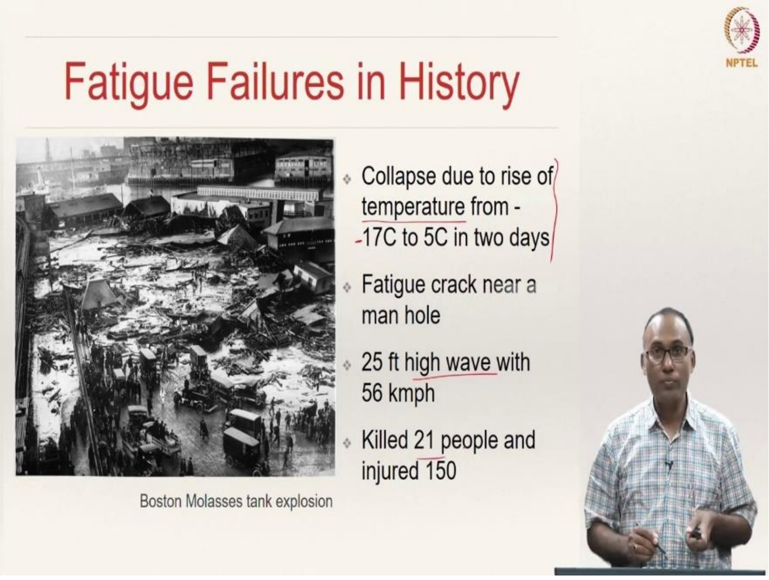

So, let us spend some time on fatigue failures in history, to put things in perspective and to understand the magnitude of such a failure. This is a famous Boston Molasses tank explosion. There was a molasses tank in the city of Boston. One early morning suddenly, this molasses tank exploded. If you see this picture, there was so much of wreckage around here and about 21 people were killed almost 150 people were injured in this accident.

Then, people looked at what could be the reason and they figured out that, the collapse happened due to rise of temperature from $-17°$ C to $5°$ C in two days. It is about a temperature rise of 22° C. This temperature gradient actually led to the failure.

How? In one of the manholes of the molasses tank, a fatigue crack developed because of the change in temperature. When there is a change in temperature, basically what we are talking about is the time varying load. So, the thermal stress is cyclic. Near a manhole, which is a stress concentration zone, a crack initiated and that crack became a critical crack during these two days of temperature variation and suddenly it broke due to brittle fracture and the molasses tank broke apart.

The molasses is a viscous liquid and when the tank broke, it led to a 25 feet high wave of molasses, going at 56 kilometers per hour. It is not like regular water, but it is a highly viscous liquid and once somebody gets trapped, it is extremely difficult for them to get out. Because of the high viscosity, you cannot even swim or move through. It led to killing of 21 people.

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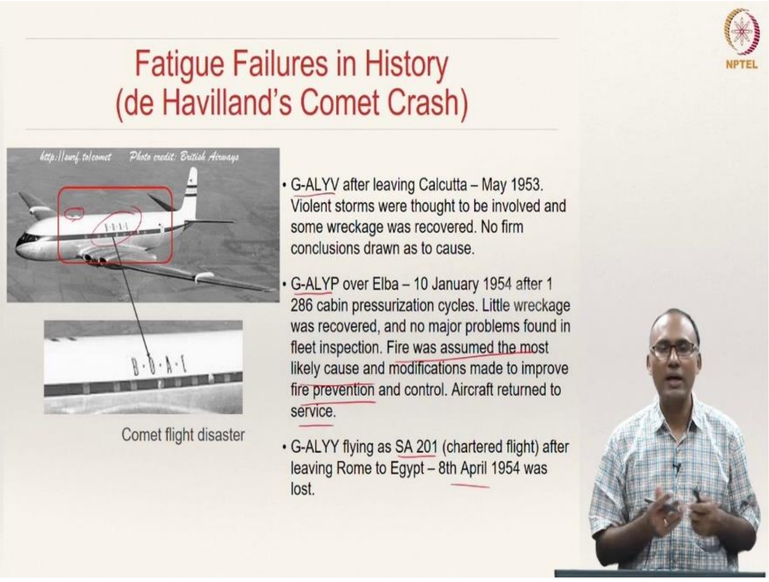

Another disaster, the comet crash which I have mentioned before, is one of the sad stories in the aircraft industry; there were a couple of accidents one after the other. The first aircraft was managed by BOAC (British oversees aircraft company). The first model of their aircraft called G-ALYV, left Kolkata in May 1953 and then midway, the flight got lost. There were violent storms during that time and hence it was thought the violent storm was the reason for the failure of this particular flight.

Later, people recovered some wreckage, but they could not draw firm conclusion. Almost 6 months or 7 months later, they had the next model G-ALYP and on 10th January 1954, it gave in. People assumed fire to be the most likely cause. They did not know why this aircraft has failed and they have done laboratory testing of several cabin pressurization cycles and then they thought fire was the most likely cause. They have made several modifications to improve the fire prevention and control.

The aircraft again came back to service in another four months. The next flight was a chartered flight flying South Africa airlines 201 from Rome to Egypt. On 8th April 1954, it was lost. It was again not clear, so people looked at the design.

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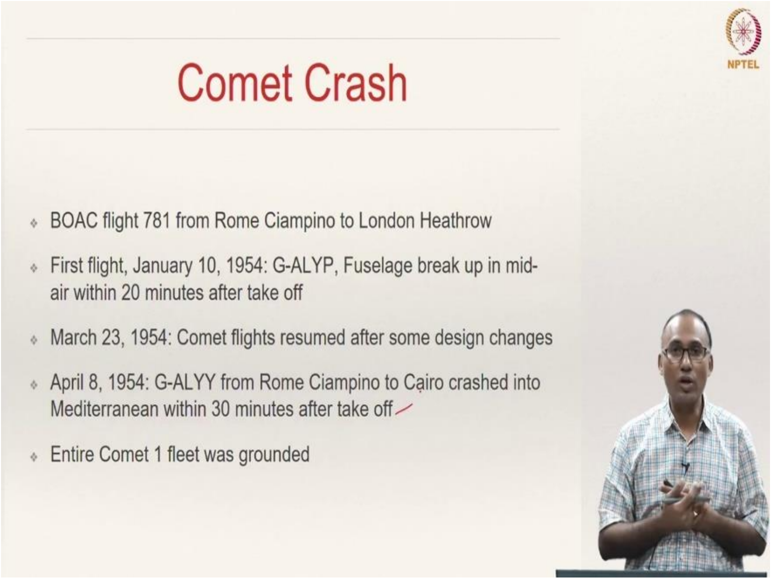

All these three consecutive failures, almost in the same year led to serious doubts about the design of this particular comet aircraft. After the final accident, the entire comet fleet was grounded, until they could actually do a thorough investigation.

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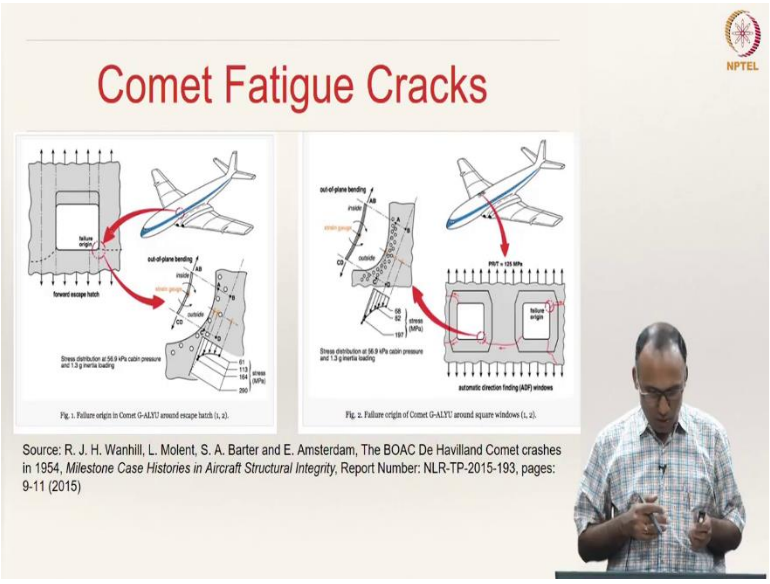

The windows of these aircrafts are not like todays aircraft windows; they were square shaped. On the roof, you have square shaped -- these are for different purpose, but the windows were also square shaped.

In the investigation, it was found that these window shapes were responsible for failure. The window being square shaped, becomes a very high stress concentration zone. As a result, the crack initiates there much easily and propagates quickly.

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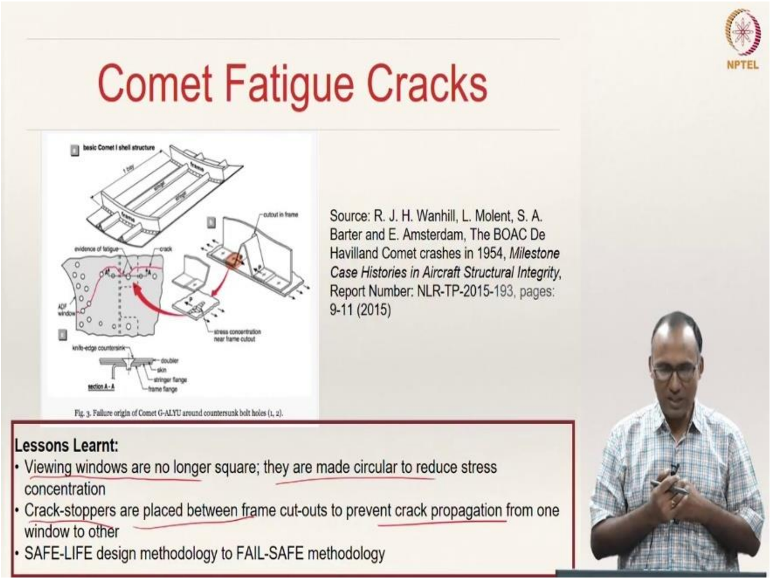

People realized that the primary reason for the failure is the stress concentration arising due to the shape of the window. Once you have a stress concentration, you initiate the crack much quickly at much lower stress. Once the crack gets initiated, it propagates; the moment it becomes a critical crack, it suddenly zips through, it propagates as if it is a brittle material.

There were a couple of lessons that were learnt from the comet aircraft crash incident. This is one single instance which has completely revolutionized or changed the way aircraft structure design is done today. There were several lessons that were learnt from comet aircraft crash. The viewing windows are no longer square. Today if you see, they are oval in shape, right? Almost in all the aircraft windows, there are no sharp corners. The corners are made circular to reduce the stress concentration.

Let us say you have a crack starting from one window corner and if you are able to arrest the crack going from one window to another window, you could safeguard the aircraft to some extent. Hence, the next important design modification was crack stoppers in the aircrafts. Today's aircrafts should have these crack stoppers and no window should be square or having sharp corners.

The crack stoppers are placed between frame cutouts to prevent crack propagation from one window to another window. The next most important design philosophy is that, until then people were using something called safe-life design. After the comet crash, all the design philosophy has changed from safe-life to fail-safe methodology.

What do you mean by safe-life methodology? If you are designing a component, you can be assured of certain number of cycles of life and after that you decommission the component from the service.

Suppose I am designing a component. I say that this component can last for hundred cycles. That means the safe-life is hundred cycles. After hundred cycles, I will simply decommission the component from the service and then use a new component; that is what is called safe-life approach.

However, for some reason, if you have a catastrophe; which means some unexpected loads are acting on the material and then what happens is that, whenever this material is subjected to unexpected loads, it breaks or it gives in catastrophically, which is not a good scenario for the systems.

The aircraft industry in particular, moved on from safe-life design to fail-safe design. In the case of fail-safe design, what we ensure is that, if at all the material fails, the failure does not lead to a catastrophe as much as possible, within the expected loads beyond the service load.

That means the failure is supposed to not be leading to a catastrophe. That is what is called fail-safe design and today's aircrafts are following the fail-safe design methodology.

So, you can see that one sad crash actually led to several new consequences and development of a complete set of new design methodologies for the entire industry. And that is a very important lesson that was learnt from the sad story of comet.

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The other sad accident in the history of German railways is Eschede derailment in Germany. German high-speed trains, ICEs (intercity expresses) are known to be very good high-speed trains in Europe and are efficient in their operation as well.

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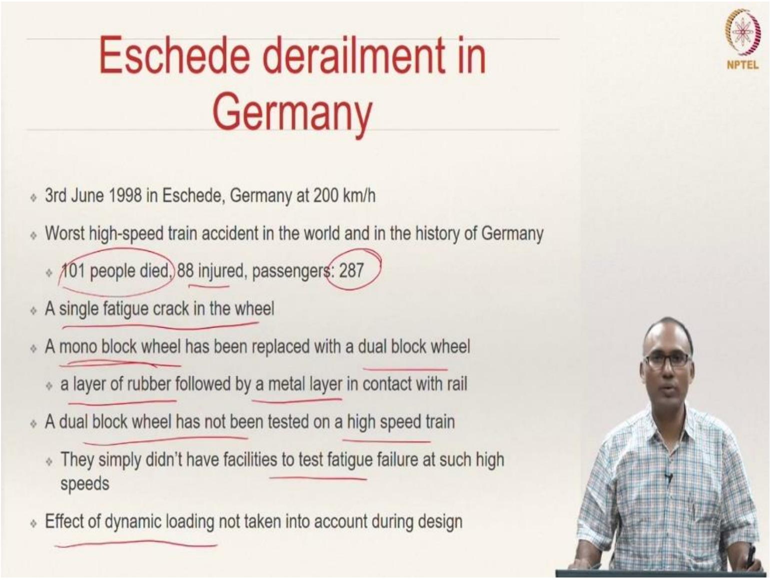

However, in 1998 on 3rd June, the German railway or Deutsche Bahn had a very sad accident at Eschede, famously known as Eschede derailment. This ICE was going at 200 kilometers per hour and during this time -- this is possibly the worst high speed train accident in the world and also in the history of Germany, as this led to the death of almost 100 people on board and 88 people injured in a train carrying about 287 people.

And what was the reason? The reason was a single fatigue crack in the wheel; there was a single fatigue crack in the wheel and that is what led to the catastrophic failure of this particular train.

Why does this happen? Until then the German railway or Deutsche Bahn was using a mono block wheel for their rails. Recently, what they have done; then they have changed this mono block wheel to a dual block wheel. How does this dual block wheel look like? They had a layer of rubber followed by a metal layer in contact with the rail.

The first layer will be a metal and below that it is a rubber layer and again you will have rail block. They have done this design change to the wheels to enhance the shock characteristics or vibration characteristics and improve the passenger comfort.

However, when they have done this design modification, firstly, the dual block wheel has not been tested on high-speed train. At that time, they did not have any facility to test fatigue failure at such high speeds. They did not have the means to determine the circumstances under which it could fail.

They have not taken into account the dynamic loading during the design of this dual block wheel, and that led to the failure at the dual block section i.e., the intersection between the metal and the rubber interface, which caused breakage of the outer metal layer. Once it failed, it went and hit one of the rail compartments and there was an instability that was created, and around the same time it was going under a bridge. The train hit the bridge and led to a catastrophic failure.

The point of discussing these accidents is to give you an understanding of the impact of neglecting the dynamic nature of the loads on machine components and that can actually lead to severe consequence.

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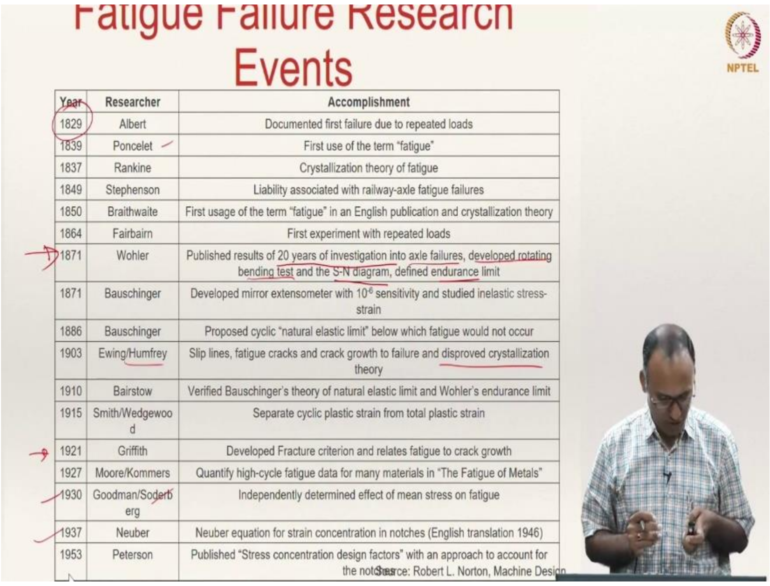

This table gives an overview of fatigue failure research events in the literature around the world. The first ever failure due to repeated loads was reported in 1829. Several scientists Poncelet, Rankine among others have extensively studied fatigue failure, and the first ever usage of the term fatigue in English, used by Poncelet in 1830s.

Then, people developed different theories. Initially, there was a theory called crystallization theory for fatigue failure, which was later disproved by Ewing and Humfrey who attributed the fatigue failure to events like slip lines, fatigue cracks and crack growth, which was capable of disproving the crystallization theory hypothesized by Rankine and others before that.

Probably the most important event in the fatigue failure research happened around this time; when August Wohler from Germany has published the results of his 20 years of investigation into axle failures. He also developed a special test called rotating beam bending test.

He is the one who gave the concept of S-N diagram and defined something called endurance limit. The year 1871 is probably one of the most important years for the researchers of fatigue failure. The understanding of fatigue failure improved significantly after Wohler's 20 years of results were published.

And then until the introduction of the fracture mechanics formally by Griffith, there were several theories that were developed, but Griffiths theory on fracture mechanics really pushed forward the understanding of failure of materials under repeated loading or fatigue loading.

And then scientists like Goodman, Neuber, Soderberg and Peterson have pushed the field much further and today we can say that we understand the fatigue failure primarily in metals or crystalline materials to a reasonable extent. That is why we are able to design structures, such that they can withstand the known dynamic loads to certain extent with reasonable factors of safety.

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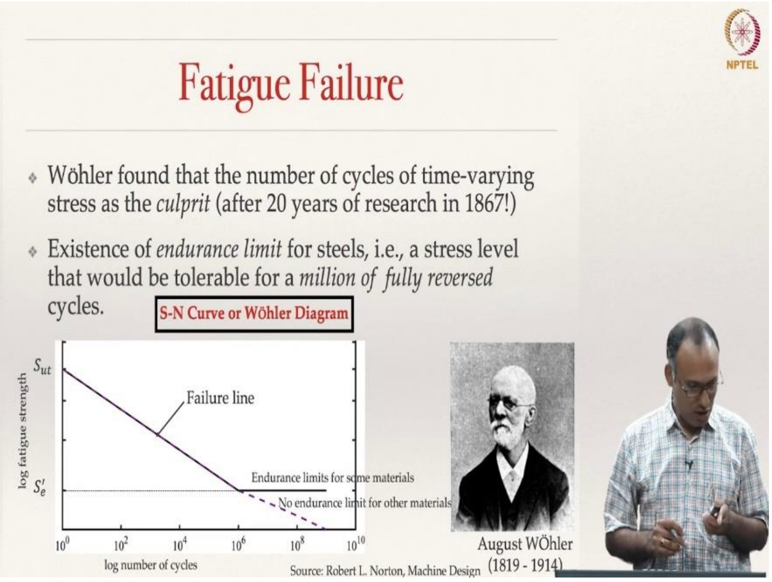

This is August Wohler. He is the one who first published detailed results of his work that he carried out over 20 years. His seminal contribution was, he found that the number of cycles of time varying stress is the culprit for the fatigue failure; the number of cycles of the time varying stress is the reason for fatigue failure.

And he also has shown the existence of something called endurance limit for steels. The endurance limit is the stress level that would be tolerable for a million fully reversed cycles.

We will talk about what do we mean by fully reversed cycle. The idea proposed by Wohler is that, if the stress level is below a certain value, then the material will withstand one million cycles. The stress value below which you will have life beyond one million cycles is called endurance limit. In some textbooks, the endurance limit is referred to as endurance strength of the material. This is typical for steels; primarily his work was on rail axles and hence he gave this conclusively for steels.

The diagram shown here is a schematic of S-N diagram. What do we mean by S-N diagram? S is the stress amplitude; we will talk about what do we mean by stress amplitude and the x-axis you have number of cycles. Please note that, here you see a straight line; but it is on a log-log scale and hence it is not really a straight line, it is a power law.

And then you see that, let us say 10^0; that means one cycle. When we are talking about fatigue, we are talking about crack propagation; that means breakage. Hence, we cannot consider yield strength of the material anymore; we should be considering ultimate strength of the material.

When you have one cycle, that means there is no repeated loading and hence we can think that situation has the case of static loading, right? And now, that means, the material fails at S_{ut}. If the stress amplitude is somewhere in between, the failure happens somewhere between 100 and 1000 cycles, say 900 cycles. At this stress amplitude, I can use my component for 900 cycles; after 900 cycles it will break.

This is what you call life and this is what you call stress. So, S-N diagram is nothing, but stress-life diagram or it is also called Wohler diagram. If the stress amplitude is less than the endurance limit S_e', then the material will definitely give one million cycles and actually gives beyond one million cycles; we call this system as having infinite life. That means, it never fails; because beyond that we say that it is constant.

For steels, the endurance limit exists, for other materials you may not have endurance limit. That means they there is no stress limit below which they do not fail.

So, with that, I will stop today and then we will look at the different kinds of fatigue loadings in tomorrow's class, ok?

Thank you.