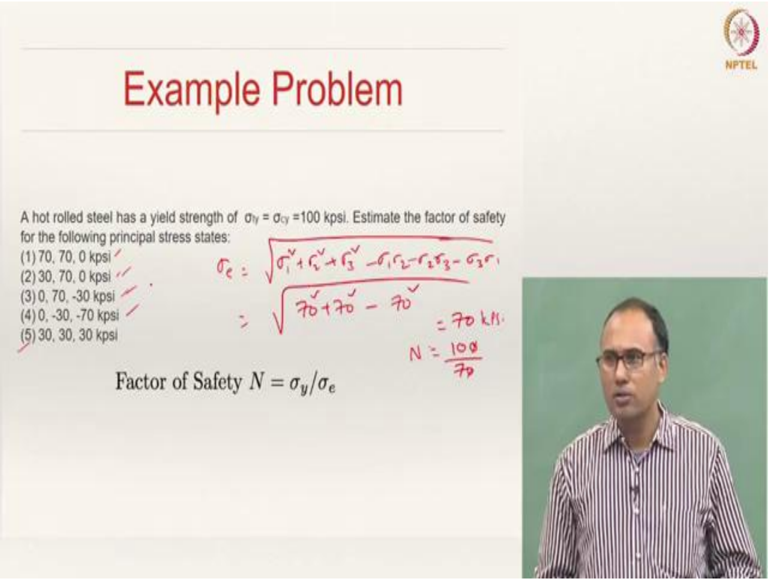
Basics of Materials Engineering
Prof. Ratna Kumar Annabattula
Department of Mechanical Engineering
Indian Institute of Technology, Madras

Lecture - 19
Static Failure Theories (Design Problems)



Let us now look at the calculation of factors of safety for different stress states. Consider the following problem

A hot rolled steel has a yields strength of $\sigma_{ty} = \sigma_{cy} = 100\ kpsi$. Estimate the factors of safety for the following stress states:

1. 70, 70, 0 kpsi
2. 30, 70, 0 kpsi
3. 0, 70, -30 kpsi
4. 0, -30, -70 kpsi
5. 30, 30, 30 kpsi

What will be the factor of safety for the stress state 5?

Equivalent stress will be 0. So, that means, you have a infinite factor of safety. It will never fail. When a material is subjected to hydrostatic state of stress, it will never fail; meaning, it will never yield. Hence, the factor of safety is infinity. The principal stresses are given and you can calculate the equivalent stress using:

$$\sigma_e = \sqrt{(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1)}$$

Also, the factor of safety is given as

$$Factor\ of\ safety, N = \frac{\sigma_y}{\sigma_e}$$

Therefore, the solution to the first will be

$$N = \frac{100}{\sqrt{70^2 + 70^2 - 70^2}} = 1.428$$

The other sections are left as exercises. When calculating the factor of safety, we must check at the point of maximum stress. There may be positions in the component where the stresses are at a safe level. However, there may be point subject to very high stresses and from the design point of view, we must consider the location where the stresses are maximum.

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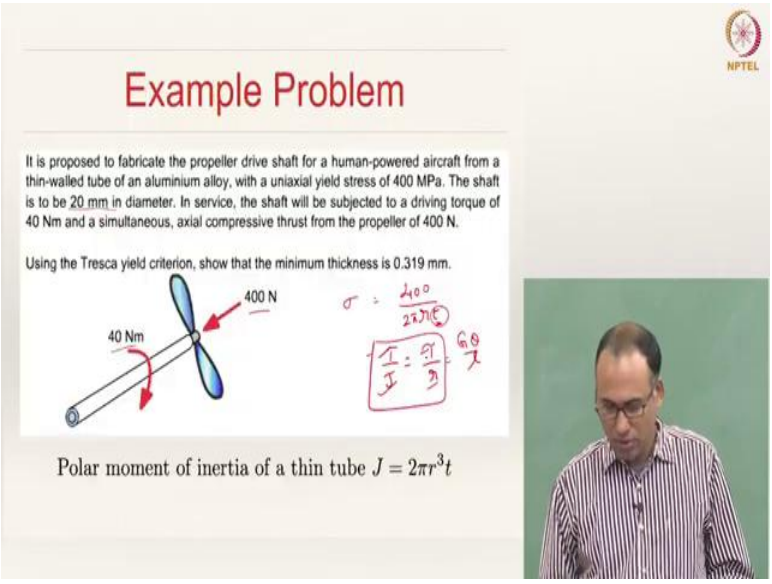

It is proposed to fabricate the propeller drive shaft for a human-powered aircradt from a thin walled tube of aluminium alloy, with uniaxial yield stress of 400 MPa. The shaft is to be 20 mm in diameter. In service, the shaft will be subjected to a driving torque of 40 Nm and a simultaneous axial compressive thrust from the propeller of 400 N. Using the Tresca yield criterion, show that the minimum thickness is 0.319 mm.

The polar moment of inertia of a thin tube $J = 2\pi r^3 t$

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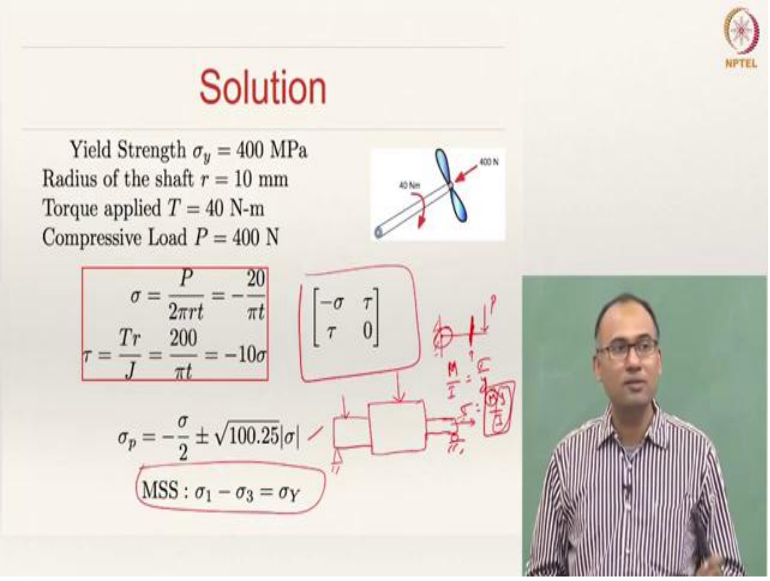

The stresses that are impresses on the shaft is pure shear and the compressive stress.

Yield strength , $\sigma_y = 400\ MPa$

Radius of the shaft $r = 10\ mm$

Torque applied $T = 400\ Nm$

Compressive load $P = 400\ N$

The compressive stress is given as,

$$\sigma = \frac{P}{2\pi r\, t} = -\frac{20}{\pi t}$$

The shear stress is given as

$$\tau = \frac{Tr}{J} = \frac{200}{\pi t} = -10\sigma$$

The stress tensor is given by

$$\begin{bmatrix} -\sigma & \tau \\ \tau & 0 \end{bmatrix}$$

We can find the principal stresses from this. We must then use the Tresca yield criterion and thus solve

$$\sigma_p = -\frac{\sigma}{2} \pm \sqrt{100.25|\sigma|}$$

We just need to check if the thickness given is safe. The remainder of the problem is left as an exercise.

This is what is meant by design – finding the working dimensions of the components that you want to use for a given loading scenario. For instance, what should the diameter of the shaft be such that it can withstand certain loads? So, all that we need to know is, what are the kinds of stresses that the component experiences. Once we know this information, then you can find the required dimensions.

Typically shafts will not be of uniform cross section and will be stepped shafts subjected to various loads such as weight of the gear, weight of the shaft, etc. You could also have axial loads in addition to radial forces. You may also have bending moment. One needs to consider all the combined loading scenarios. For a cantilever beam subjected to bending moment, it is maximum at the fixed point and therefore, you find the stresses at this point. The stress will be maximum at the fixed point and you can compute the factor of safety. However, if the cross sectional area of the beam is changing, the stress may not be maximum at the fixed point. While the bending moment may be maximum at this point, the stress may not be. One must take care to ensure that it is always the maximum stress that is used to calculate the factor of safety and during the design procedure.