Basics of Materials Engineering
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Lecture - 18
Static Failure Theories (Maximum Shear Stress Theory)

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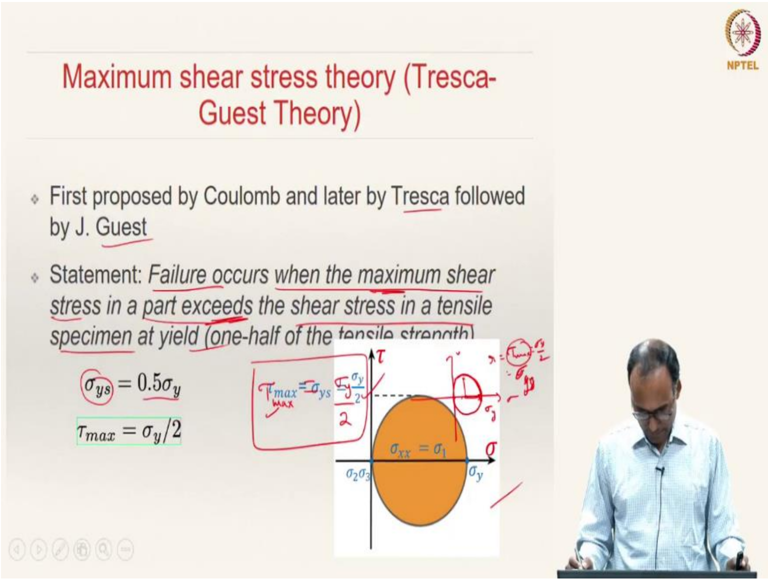

That is about distortion energy theory. In the next module, we will look at the maximum shear stress theory which is also called Tresca-Guest theory. It was first proposed by Coulomb, later by Tresca and followed by Guest. That is why it is famously known as Tresca-Guest theory.

The statement of maximum shear stress theory is as follows: The failure or plastic deformation occurs when the maximum shear stress in a part exceeds the shear stress in a tensile specimen at yield. How does the Mohr's circle for a uniaxial tension test look like? $\sigma_1 = \sigma_y$, the yield strength, and $\sigma_2 = \sigma_3 = 0$, and hence that would be the Mohr's circle.

The radius of Mohr circle that is equal to $\tau_{\max}$, because $\tau_{\max}$ corresponds to yielding and hence, we call it as shear yield strength $\sigma_{ys} = \frac{\sigma_y}{2}$. Whatever is the normal yield strength that you would have observed, one-half of that will be your shear yield strength. What we are saying is that when the maximum shear stress that is induced in a material exceeds the shear yield strength of a material under uniaxial tension test, failure occurs.

We have to calculate the maximum shear stress in the material and if that is equal to the shear yield strength of the material which is equal to $\frac{\sigma_y}{2}$, that is when failure happens. This is the statement of the maximum shear stress theory. For any given stress-state, all that you need to do is to just calculate the value of $\tau_{\max}$.

Basically, if you draw the Mohr's circle, get the radius of the Mohr circle in this case. So, you get the value of a maximum shear stress induced in the material. And if that is 50% of the uniaxial yield strength of the material, then the material fails; otherwise, if it is less than that, the material is safe. So, that is what I have shown in this figure here.

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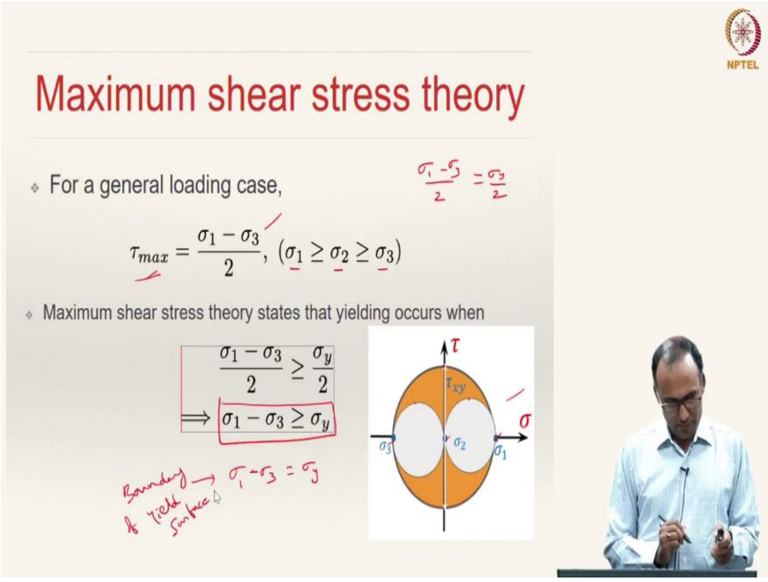

For a general loading case, you can say that $\sigma_1 > \sigma_2 > \sigma_3$; so that is a three-dimensional case. The maximum shear stress can be written as,

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2}$$

Here this is a shear stress, this is another shear stress, but this is the maximum shear stress. The maximum shear stress theory basically means, when $\tau_{\max}$ is equal to the shear yield strength given by $\frac{\sigma_y}{2}$, that is when failure happens.

$$\frac{\sigma_1 - \sigma_3}{2} \geq \frac{\sigma_y}{2}$$

$$\Rightarrow \sigma_1 - \sigma_3 \geq \sigma_y$$

$\sigma_1 - \sigma_3 = \sigma_y$ represents the boundary of the failure surface or yield surface in maximum shear stress theory.

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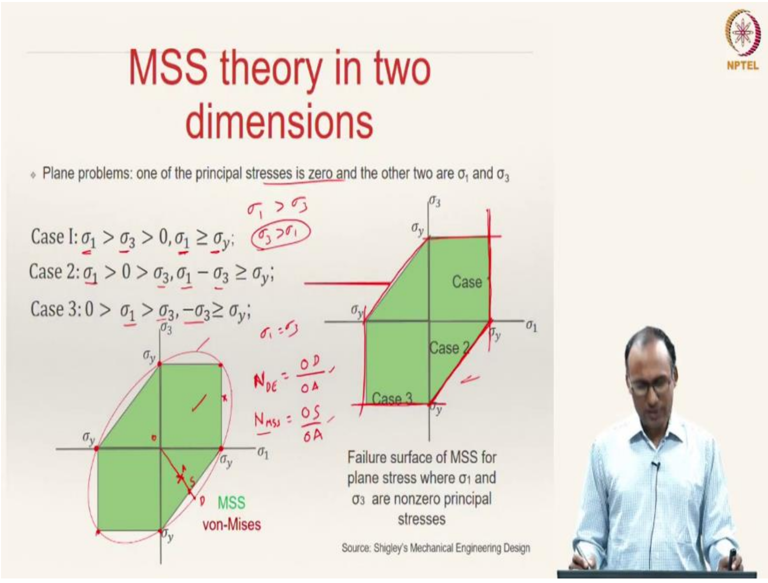

Let us look at how the failure surface looks like for a 2-D, i.e., a plane problem in which one of the principal stresses is 0 i.e., $\sigma_2 = 0$. Let us consider case 1 where the principal stresses $\sigma_1 > \sigma_3 > 0$. This line represents $\sigma_1 = \sigma_y$ which is basically the case 1. That is the first boundary of the failure surface.

In case 2, let us take $\sigma_1 > 0 > \sigma_3$, so that means, σ_1 is positive, σ_3 is negative which belongs to the fourth quadrant. We can say that this straight line represents $\sigma_1 - \sigma_3 = \sigma_y$.

In case 3, consider $0 > \sigma_1 > \sigma_3$. Both of them are negative which indicates the stresses are compression. Then, this boundary represents $-\sigma_3 \geq \sigma_y$. These are the boundaries if you assume $\sigma_1 > \sigma_3$.

What happens if $\sigma_3 > \sigma_1$? If $\sigma_3 > \sigma_1$, then σ_3 would reach yield strength first. We have drawn these boundaries previously assuming $\sigma_1 > \sigma_3$ and the yielding happens when σ_1 reaches the yield strength first. When you assume $\sigma_3 > \sigma_1$, then you will get these three boundaries. The maximum shear stress theory in 2D looks like as a hexagon as shown here.

If you would plot both maximum shear stress theory and distortion energy theory/von-Mises theory those two theories look like this. The green area is the maximum shear stress theory, and the red curve represents the boundary of distortion energy theory. You can clearly see that when you are doing uniaxial tension test or compression test, both maximum shear stress theory and distortion energy theory give you the same factor of safety.

There are also two other places where they are same i.e., when $\sigma_1 = \sigma_3$. When $\sigma_1 = \sigma_3$, both the distortion energy theory and maximum shear stress theory will be the same. Suppose if your stress-state in a material is here, and if you are drawing a line, so let us say this is our *O*, this is our boundary and let us call this is the stress state *OA*, and this is *S*, and this is *D*.

The factor of safety based on distortion energy theory is equal to $\frac{OD}{OA}$, because *OA* is the stress-state position. According to maximum shear stress theory, the factor of safety is $\frac{OS}{OA}$. If you are talking about factor of safety here, the distortion energy theory is going to tell you that it is actually having higher factor of safety, whereas maximum shear stress theory gives you a lower factor of safety.

Also note that if your stress-state falls here, that means, according to maximum shear stress theory the material has already failed. Whereas, according to distortion energy theory, the material did not fail yet.

Both maximum shear stress theory and distortion energy theory seem to work very well for ductile materials, and they seem to represent the failure phenomena reasonably well when compared to the experiments.

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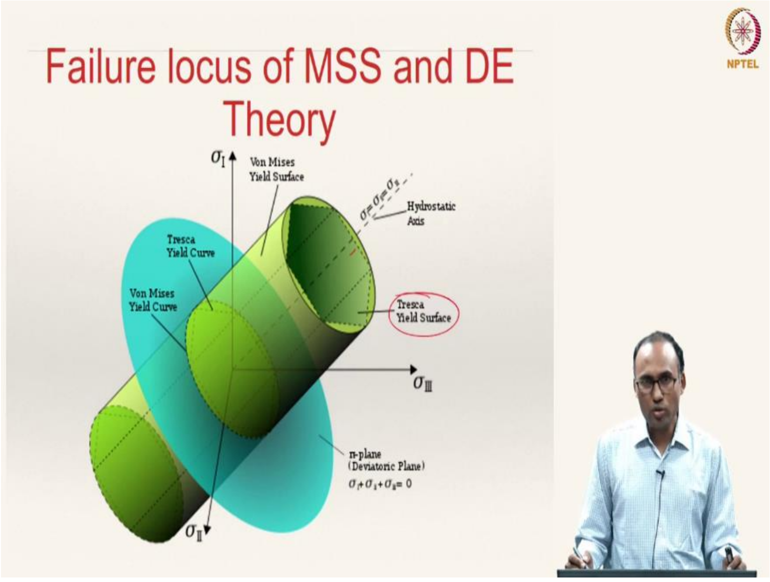

In 3D, the failure surface will be a hexagonal cylinder. As we can see here, the maximum shear stress theory or Tresca-Guest theory represents a hexagonal cylinder inclined at 45° to the $\sigma_1, \sigma_2, \sigma_3$ axes.

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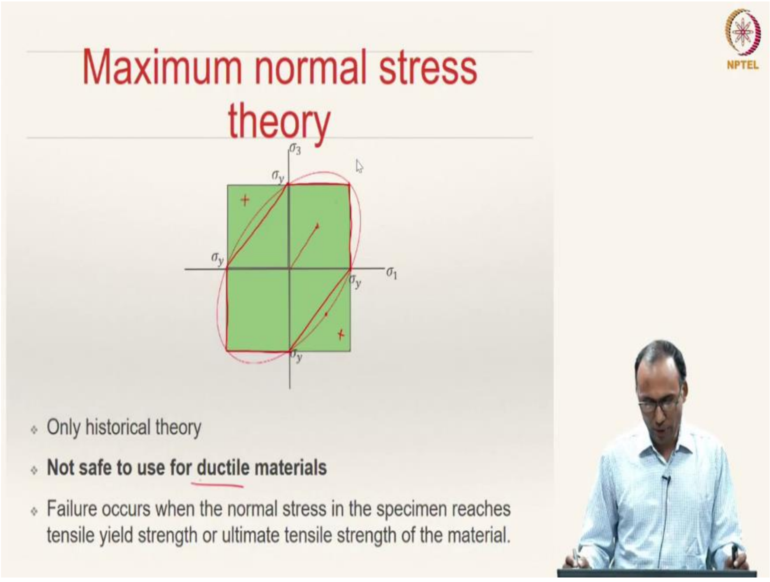

And then, we will look at the maximum normal stress theory and how does it look like compared to other theories. It is only a historical theory. It is not safe to use for ductile materials; only for understanding. You can clearly see that if you are using the maximum normal stress theory, it says that this is still safe, whereas distortion energy theory would have predicted it failed, particularly in these second and fourth quadrants.

Hence, maximum normal stress theory should not be used for ductile materials. This red line represents the distortion energy theory. The boundary that I am drawing with red color is our maximum shear stress theory, ok? We can see how one would also calculate the factors of safety graphically.

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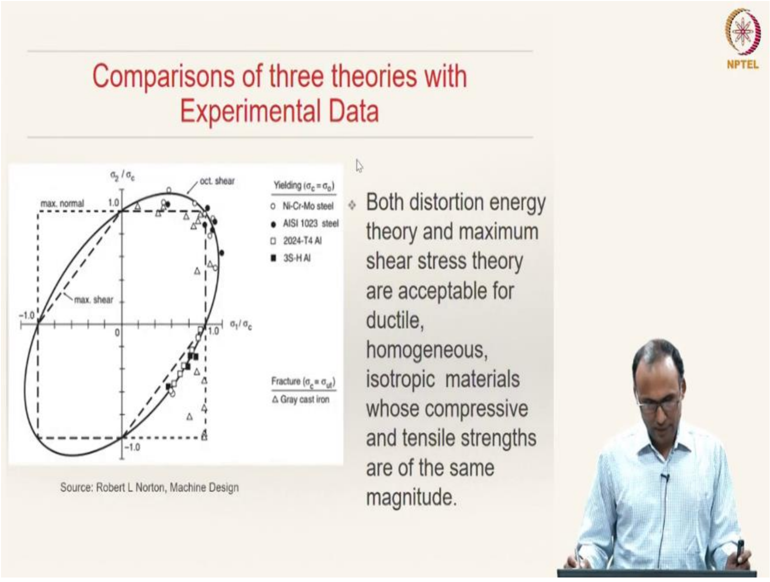

Here, in this graph, we are showing the relative merits of the distortion energy theory and maximum shear stress theory over the maximum normal stress theory for the experimental data of several materials.

The data matches very well except for the open triangles, that correspond to gray cast iron which seems to match very well with the normal stress theory. That is because gray cast iron is a brittle material and that is the reason why we say that the distortion energy theory and maximum shear stress theory cannot be used for brittle materials.

However, for materials which show yielding like nickel chromium, molybdenum, steels and aluminum alloys which are represented by open circles, close circles, and open squares and closed squares, we can see that the data matches very well with both distortion energy theory and maximum shear stress theory. That is the reason why these two theories are still used extensively for failure characterization when the material is subjected to static loading.

In the next class, we will solve one or two problems based on the two theories that we have discussed, and then move on to the failure theories of brittle materials.

Okay, thank you very much.