Basics of Materials Engineering
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Lecture - 31
Static Failure Theories (General form of failure theory, Stress tensor, Principal stress)

I think in the last class we stopped here and we the question why do parts fail, enabled us to understand the mechanical behavior of materials and their lifecycle -- improvement of the lifecycle of the products in the long run.

I am assuming that all of you are familiar with the concept of Mohr circle, right? If you are not, you have seen this or you have heard of this in your strength of materials class or mechanics of materials class; please go back and refresh your memory.

Mohr circle is a very powerful way of representing stress state in a material graphically and we know how to represent the stress state at a point if the system is 2-dimensional. You can also represent a 3-dimensional state of stress at a point using Mohr circle. You probably have done for 2D state of stress, but you can also do the same thing for 3D state of stress.

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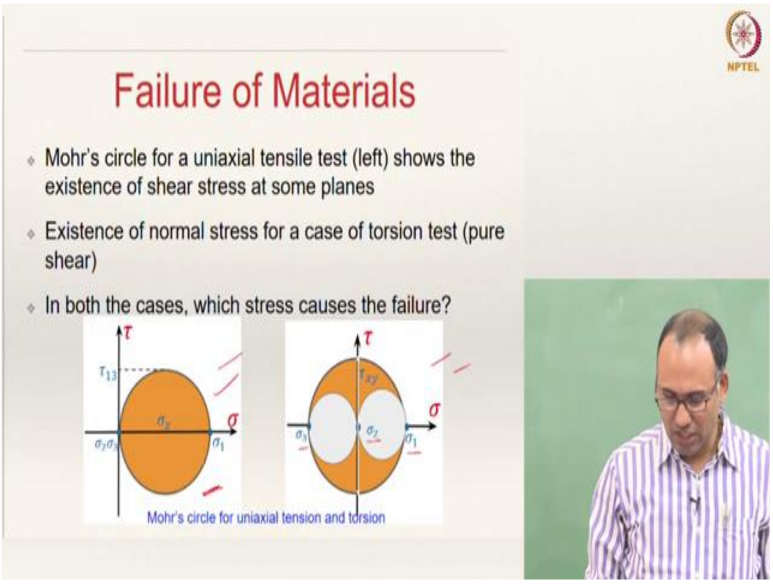

The figure here on the left-hand side is for a plane stress case and here you can say this it is a 2D state and you can actually say this as 3D, wherein you have 3 principal stresses σ_1, σ_2 and σ_3.

Here on the left-hand side, this is the Mohr circle for uniaxial tension test, right? When you are doing uniaxial tension test, there is only one normal stress. That is your principal stress. Why is that your principal stress? Because in that plane, there is no shear stress that you are applying; you are only pulling like that, isn't it?

The other 2 principal stresses will be 0 because it is a uniaxial loading scenario and if you would take that and then draw the Mohr circle, that is how it looks like. It clearly shows that although the applied stress is uniaxial in nature, there are planes along which you also may have shear stress.

Each and every point on the Mohr's circle boundary is representing the state of stress at some other planes. For instance, this plane is at an angle of 45° to the direction of application of the uniaxial load having a maximum shear stress.

You have observed that during a uniaxial tension test, the failure or yielding happens along 45° plane. That is because maximum shear stress occurs at 45° plane. What does that represent?

If you have a maximum shear stress, it is probable that along that plane you will have initiation of plastic deformation because plastic deformation is primarily due to the slip of atom planes which is caused by shear stress. Now on the right-hand side, you have Mohr's circle for the case of pure shear or torsion.

When you have pure shear, if you are applying torsional loading, then you know that the principal stress $\sigma_1 = -\sigma_3$. Let us say this is a plane problem, i.e., $\sigma_2 = 0$. $\sigma_1 = -\sigma_3$ and that is when you see that these are same, but in the opposite direction and then you can draw the Mohr circle.

In both the cases, whether you are applying uniaxial tension or shear, yielding happens only because of the shear stress. Here when we say failure, we mean plastic deformation i.e., yielding of the material.

That is something that I have mentioned. In the last class, we have discussed there are different ways that one can define failure and as far as ductile material design is concerned, we will restrict the definition of failure to the yielding of the material.

We really do not have to completely break the specimen because the moment the yielding happens, there is some permanent deformation set into the material and because of the permanent deformation the functionality for which it is designed might not be met, and that itself can be considered as failure.

Whenever we are designing components made of ductile materials, the failure criterion is based on the yield strength of the material. But, for brittle materials there is no possible yielding that can be observed and usually for brittle materials you will consider permanent fracture or breakage as the failure of the material.

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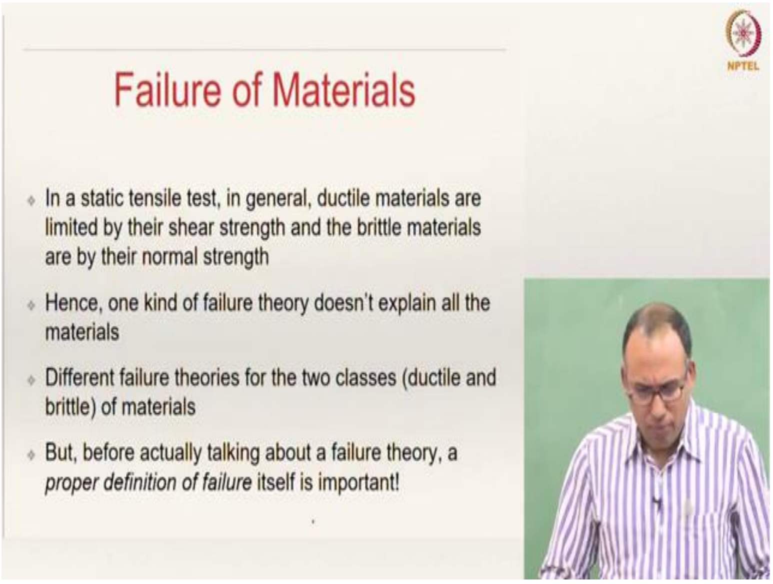

In a static tensile test, in general, ductile materials are limited by their shear strength and brittle materials are limited by their normal strength, because they fracture by normal strength.

The failure mechanism is different in both these materials and hence you cannot have same theory explaining the failure of both the materials. One failure theory cannot explain failure of all kinds of materials.

You need to have at least two classes of failure theories. One for the class of ductile materials and another for the class of brittle materials. That is the key thing to understand why we cannot have same failure theory for both ductile and brittle material because the mechanism themselves are different and the way that we are characterizing failure itself is different in both the materials.

Before we actually go about prescribing a failure theory, we need to understand what do we mean by failure. So, that is what something that we have discussed. What do we mean by failure in the cases of ductile and brittle materials?

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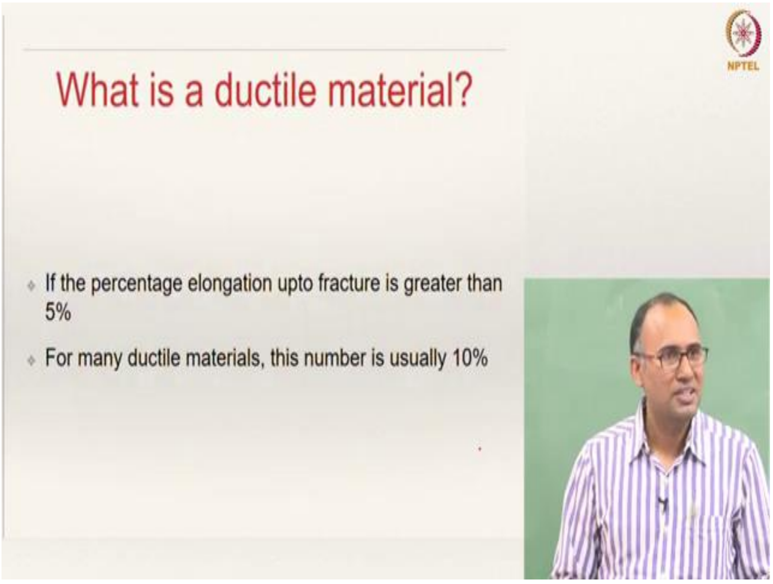

How do you consider a material to be ductile material? What is the limit? Can you say in black and white that this is a ductile material and this is a brittle material? Are there no materials which are somewhere in between? It is always possible, right? Hence, you need to have some quantification to classify a given material as a ductile material or a brittle material. So, the classification is basically percentage elongation.

If the percentage elongation up to fracture is greater than 5%, that is when you call something as a ductile material. Typically, most of the ductile materials will have almost 10%, but when you want to classify a material to be ductile, you will see the percentage elongation until fracture is more than 5 percent and that is when you say that this is what I am going to consider as a ductile material. If it is less than that, then you will consider that as a brittle material.

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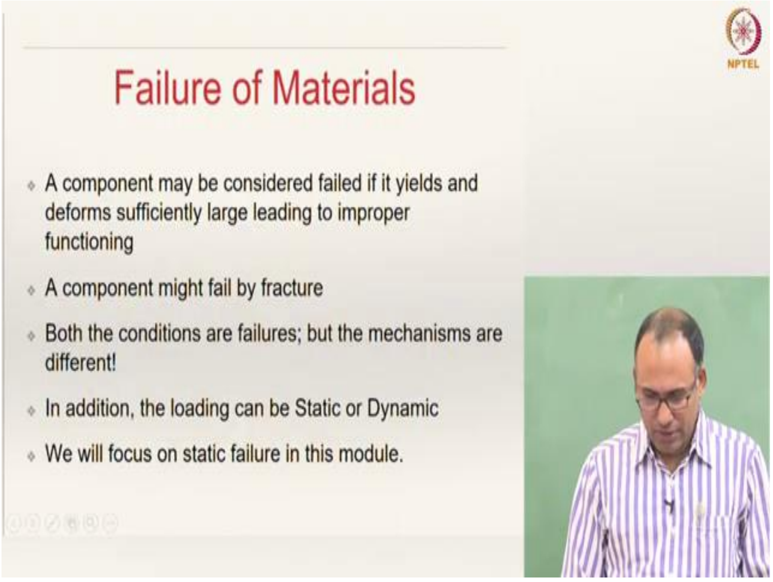

I think this is something that we already discussed. A component may be considered failed if it yields and deforms sufficiently large leading to improper functioning; that is the key.

The definition of failure for us as engineers is not necessarily breakage of the material, but the moment the component ceases to deliver the function for which it is designed; that itself is a consideration for failure of the component for us. Sometimes, the component might actually fail by fracture, it is possible for instance in brittle materials. So, both are considered as failures, but the mechanisms are different.

We are only talking about failure whether it is failing by slip or permanent deformation or breakage, but we have not paid attention to the kind of loading. Is the loading time independent or time dependent? As a function of time is the loading changing or not?

As a function of time, if the applied load does not change, then such a scenario is called static loading scenario. If the load changes as a function of time, then such a scenario is called dynamic loading. So, the failure phenomenon depends on whether you have your load changing as a function of time or not and hence you also need to pay attention to the nature of the loading, whether it is static or dynamic.

In this module i.e., the first module, we will be focusing on static failure theories; that means, there is no change of load as a function of time. As a function of time, if the load changes, then we will have to come up with a new way of deriving the failure theories and that we will do in the next module; that is when we will look at fatigue failure of materials. But, in this module we will restrict ourselves to static failure theories.

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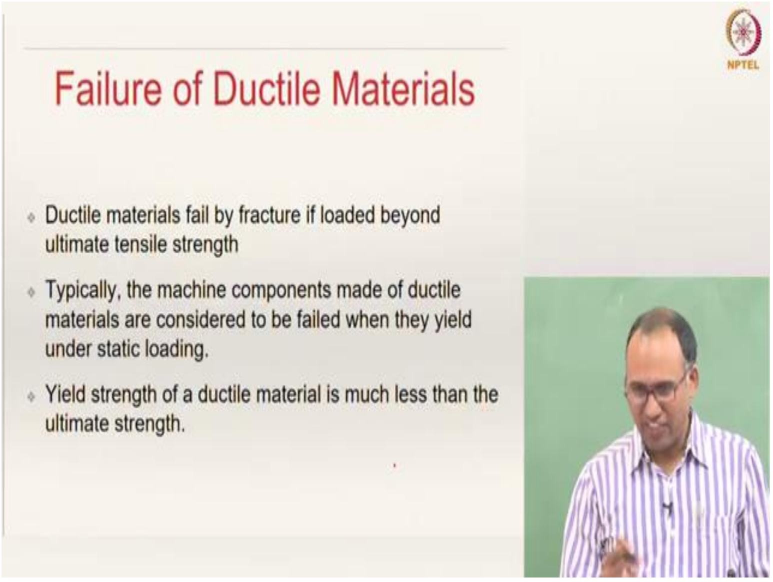

As we have already mentioned several times, ductile materials fail by fracture if loaded beyond ultimate tensile strength; until then they would not fail by fracture. But, when we are designing, we will not go up to there; we consider them to be failed when they yield under static loading. For ductile materials, it is important because the yield strength and ultimate strength are not the same.

For instance, the yield strength of mild steel is about 200 - 210 MPa, whereas ultimate strength can be up to 350 - 400 MPa. There is a significant difference between the yield strength and the ultimate strength. However, for brittle materials, it is not possible to distinguish these 2 things. That is why we cannot use yielding as a phenomenon, whereas for ductile materials, yielding as the failure criterion is preferable.

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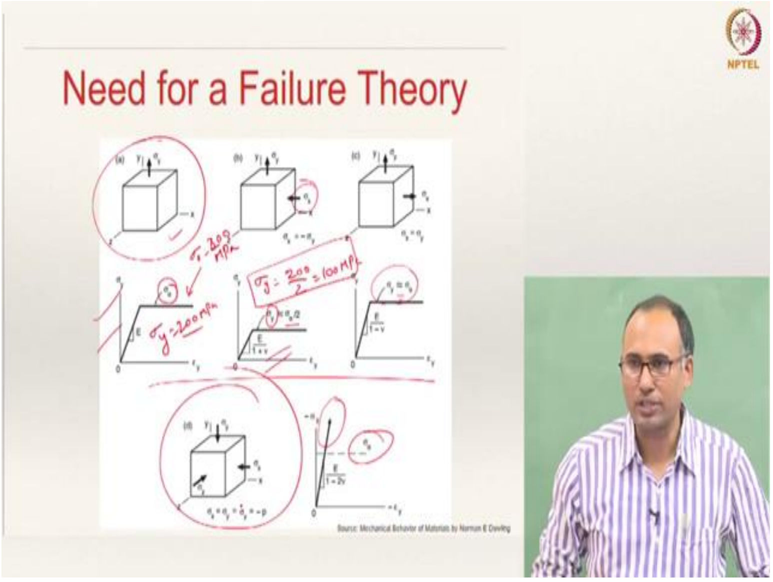

Now, we need to establish the need for failure theory. Why do we need to come up with a failure theory? So, here you have 3 scenarios. You see the first case. Here, we are considering a material which is elastic perfectly-plastic.

An elastic perfectly-plastic material will not have hardening; that is what we have discussed in the previous module when we have looked at the mechanical behavior of materials. This is a typical elastic perfectly-plastic material and now in the first figure -- this figure, we are applying uniaxial load and you observe that the material yields at σ_0.

σ_0 is the yield strength. Please note that σ_y is the stress applied along the Y direction, and not the yield strength. σ_0 is a material property.

When you are loading the specimen uniaxially, you observed that the material is yielding at $\sigma_y = \sigma_0$; that means, you need to apply σ_y until it becomes equal to σ_0, and that is when you see that there is a flow of the material; that means, that is where the plastic deformation has initiated.

Now, you do another experiment, wherein along with this load that is existing, laterally you apply another stress that is σ_x, but that is compressive. You are pulling the specimen in this direction and at the same time, you are applying a lateral normal stress, but that is in compression.

Then you see that the material starts yielding when you your applied stress is one-half of the yield strength in the previous case. So, let us say $\sigma_0 = 200$ MPa. In this case, $\sigma_y = 200$ MPa is the onset of yield. Here, I am applying σ_y in this direction, σ_x in this direction, but this is tensile in nature, this is compressive in nature; but the values are same, i.e., $|\sigma_x| = |\sigma_y|$.

Then what I observe is that, the failure or the yielding starts when $\sigma_y = \frac{200}{2} = 100$ MPa. You do not need to apply as much stress as you have applied in the case of uniaxial tension, only half of it will cause yielding.

You are applying this load and then you are pushing in this direction. When you are applying the load, because of the Poisson's effect, you have lateral contraction. Now, you are pushing it by an additional compressive force and hence we can say that we are helping the system to yield by pushing it with the external, and hence probably σ_y has come down.

If you change the sense of your lateral load, instead of compressing it, if you pull it in the lateral direction then what do you expect to happen? Do you expect your yield strength to increase? To be higher than this case?

Here it is reducing because you are pushing in the lateral direction. Now, instead of pushing, you are pulling in the lateral direction with the same value. Then, do you expect to have your strength to be same or less or more? Are you able to see what I am trying to convey?

Normally, we would expect it to increase because the lateral load is accelerating the process of yielding; that means, at a lower stress this should postpone the yield, but it turns out that if you do the experiment the yield strength is almost same as the uniaxial case. It is not significantly depending on the lateral load. If you do the experiment, that is what you will observe. If you have a non-uniaxial loading in the material, to define failure is not obvious.

Just by changing the direction of the loading, you are not able to make sense out of the nature of the failure i.e., what stress leads to failure and so on. That kind of an ambiguity is what brings in the need for defining a theory which can encompass all these variations in the loading scenarios, but still be able to give us a criterion for defining failure irrespective of the type of loading that you have in the system. That is why we need a failure theory. These 3 scenarios are similar and fourth case is even more interesting.

In the fourth scenario, you have a hydrostatic stress; which means the direction and magnitude of the normal stress in all the 3 directions is the same, i.e., $|\sigma_{xx}| = |\sigma_{yy}| = |\sigma_{zz}|$. Such a loading scenario is called as a hydrostatic state of loading.

This is typically what you can generate by keeping a specimen underwater. If you keep it underwater, then that specimen is usually subjected to a hydrostatic state of compression. Hypothetically, let us say this material has the same yield strength in tension and compression. Then, it should yield when $\sigma_y = \sigma_0$, shouldn't it?

But it turns out that if you are applying hydrostatic loading, the material will never yield. You can see the material is continuing.

The plastic deformation cannot be imparted only by applying hydrostatic loading and that is why you do not see the flow here. Here, this will never reach σ_0 although you keep on applying load. In principle, you can go to any value and the material will still not plastically deform.

Unless a failure theory takes into account this physical phenomenon, you will not be able to prescribe one theory which can describe the failure irrespective of the type of the load, right? That is what brings in the need for defining a failure theory.

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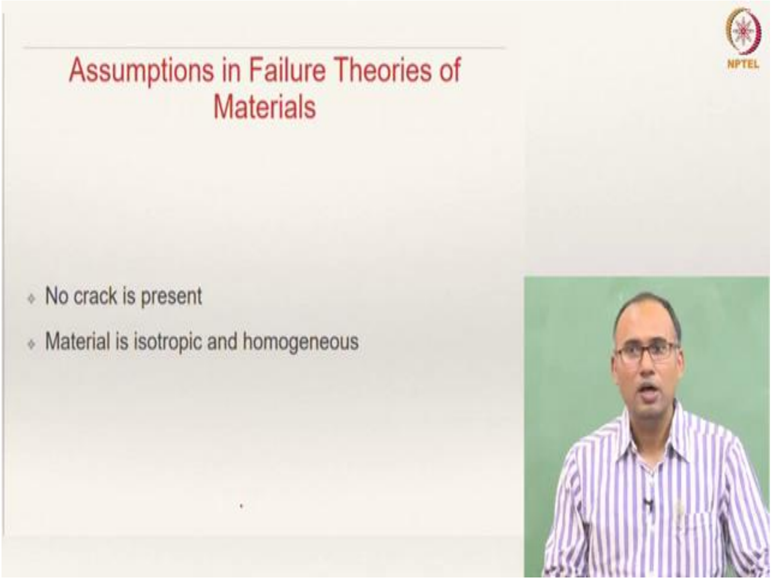

As far as this class is concerned, we have certain assumptions when we are defining failure theories. The first assumption as far as static failure theories are concerned is that there are no cracks present in the material; it is a very important assumption. The moment there is a crack, then you may have other issues. Another important assumption is the material is isotropic and homogeneous.

If the material is not homogeneous, the failure theory should be changing; you should be calculating the stress-state as a function of space, because the material itself is changing.

We are assuming the material to be isotropic; we are not talking about yield strength in a particular direction unlike for single crystal material. For a single crystal material, you cannot make such an assumption, right? Can you or can you not? Are single crystal materials isotropic?

Student: (Refer Time: 20:13).

Are you sure?

Student: (Refer Time: 20:19).

We have spent lot of time discussing.

Student: (Refer Time: 20:21).

Single crystal materials are anisotropic by nature. Polycrystalline materials are isotropic -- many of them, unless there is some additional processing that is done, but otherwise in general polycrystalline materials are isotropic, because?

Student: Average.

Averaging out the orientations of all the grains, eventually will result in isotropic behavior. Here, we are essentially talking about failures in a polycrystalline material in other words, because you are assuming isotropic behavior. But, in principal, you should also be able to derive these failure theories even for an anisotropic material. But in this class, we are restricting ourselves to isotropic materials.

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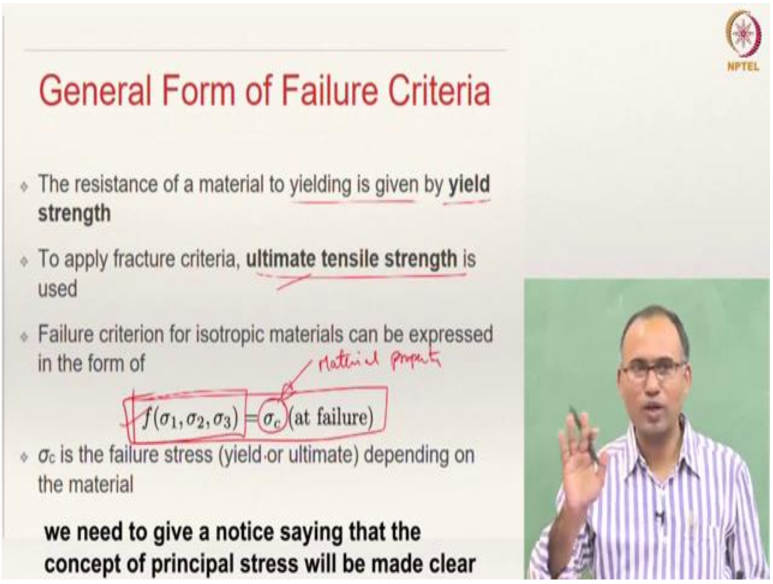

How does the general form of failure theory look like? What should be the functional form ok? Let us say the resistance of a material to yielding is called yield strength; that is something that we know, and that is for ductile materials.

If you want to design brittle materials and components made of brittle materials, then you need to consider ultimate strength as your failure criteria. So, the functional form of any failure theory for an isotropic material can be expressed as,

$$f(\sigma_1, \sigma_2, \sigma_3) = \sigma_c \text{ (at failure)}$$

where $\sigma_1, \sigma_2, \sigma_3$ are principal stresses at a point in a material; that is important.

You do not have same stresses everywhere, right? The stress-state in a material may change from position to position.

σ_c is a material property and for a ductile material, you will take that to be yield strength; for a brittle material you will take that to be ultimate tensile strength. So, you should come up with a functional form. What is that functional form is something that will depend on which theory you want.

When we are talking about different failure theories, all these different failure theories in the end, are different functional forms of the left-hand side. If you have one functional form, you can call it as maximum normal stress theory. If you have another functional form, you call it as Tresca theory. If you have another functional form, von Mises theory. It is as simple as that.

There is a reason why different theories have come into place because when there is no theory available, what would people do? They will try to come up with the simplest possible theory. Historically, we have been doing tension test and then we were able to see at some load it is failing.

What I can say is, when the maximum principal stress reaches the yield strength of the material, that is when failure happens; that is actually maximum principal stress theory or maximum normal stress theory. Like that different people have come up with different theories.

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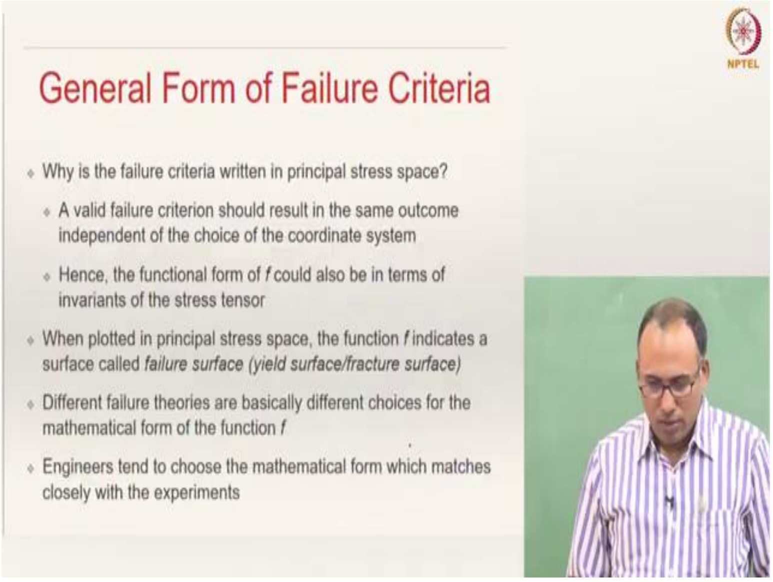

Usually, the failure criterion is written in principal stress space. You are familiar with the concept of principal stress? How many of you are not familiar with the concept of principal stress? Good, everybody is familiar.

So, if you know the stress-state at a point in a material, you can always find out a frame of reference which is oriented along the principal axis such that in that frame, your shear stresses will be zero; then you have only normal stresses. We will talk about it in a moment if you did not understand whatever I have said just now. You can express the stress-state at a point and then you can also find out the principal stresses; that means, in a particular orientation.

Typically, the functional form of the failure theory is written in the principal stress space. Why is it written? There are certain reasons. A good failure criterion should result in the same outcome independent of the choice of your coordinate system. The failure theory should not be depending on the choice of your coordinate system. It should be independent of your choice of coordinate system.

At a given point, if you know the stress-state, the principal stress-state is fixed, right. Hence, if you take the principal stress space, that automatically satisfies this criterion and hence failure criterions are usually written in principal stress space; but principal stress space is not the only choice.

Other invariants of the stress tensor can also be possible candidates for writing a failure criterion. What do we mean by an invariant? We will come to it in a moment -- or maybe today or tomorrow.

So, the only requirement is that it should be independent of the choice of your coordinate frame of reference. That is the reason why principal frame of reference is considered, or you can also consider your invariants of the stress tensor. You can write it in both ways.

We will see there are failure theories which are written in terms of the invariants of the stress tensor. When you plot this function in principal stress space, it will be a surface -- 3-dimensional surface and that surface is called failure surface.

And if it is a ductile material, then it is called yield surface. If it is a brittle material, then we call that as fracture surface; but the function that you have plotted in principal stress space is basically the failure surface. As I have already mentioned, different failure theories are basically different choices for the mathematical form of your function f.

The moment you recognize that, you really do not have to worry about remembering different failure theories. You just remember the philosophy on which they have prescribed that failure theory. Then, your functional form comes out automatically; you will be able to derive the functional form.

Which functional form should one choose? The best way to do it is, we choose the whichever functional forms match with the experiments better.

Please understand that just because a particular failure theory is representing experiments perfectly well for a given material, it does not necessarily mean that that failure theory is going to be good for several other materials. This failure theory is only a mathematical abstraction or mathematical representation that we are seeking in order to represent the behavior.

But failure theory that you have prescribed might actually break down for some other cases. This is something that we should keep in mind. People have done several experiments and they have tested their experimental data against these failure theories, and then they have seen that there are certain failure theories which seem to match with a large fraction of experimental data, and hence they said this probably is the right failure theory to be used for this set of materials and so on.

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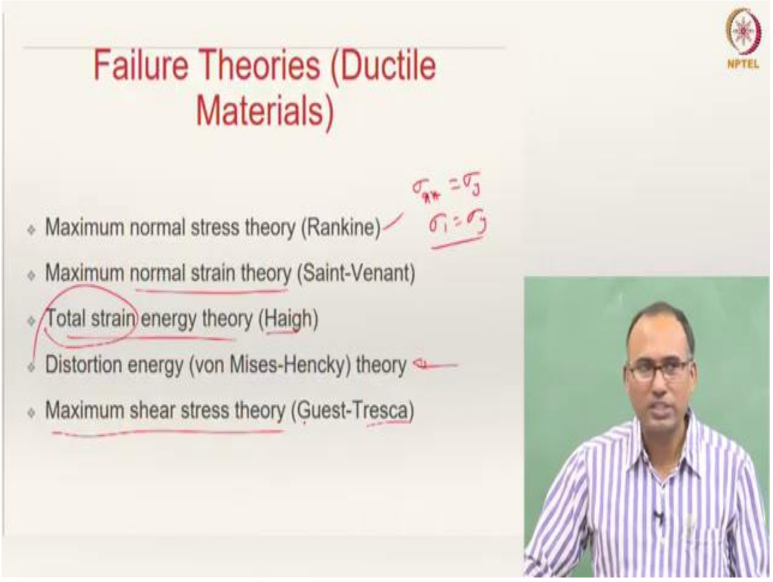

There are different failure theories as we have discussed. Maximum normal stress theory, also called Rankine theory is named after Rankine who prescribed this theory. This theory states that when the maximum normal stress is equal to yield strength of the material, that is when the material fails.

So when $\sigma_1 = \sigma_0$, that is when failure happens. Rather than defining the failure based on stress, Saint-Venant defined it based on strain and hence maximum normal strain theory is named after him.

Haigh came up with another theory called total strain energy theory. The total strain energy is theory deals with strain energy which is a scalar, whereas the maximum normal stress maximum normal strain theories deal with the tensorial quantities stress and strain, respectively. von Mises improved the total strain energy theory by considering only a part of the strain energy rather than taking into account the total strain energy to define failure. The von Mises theory is named after him.

Tresca came up with another theory for ductile materials. He took into account the shear stresses, as the plastic deformation in ductile materials occur due to slip of atom planes which is provided by the shear stress.

Hence, he defined the failure theory based on the shear stress and that is what is maximum shear stress theory. So, you have different kinds of failure theories and we need to look at how one would derive these failure theories. In this class, we will be focusing on these 2 failure theories i.e., *distortion energy theory* (von Mises-Hencky) and *maximum shear stress theory* (Tresca-Guest).

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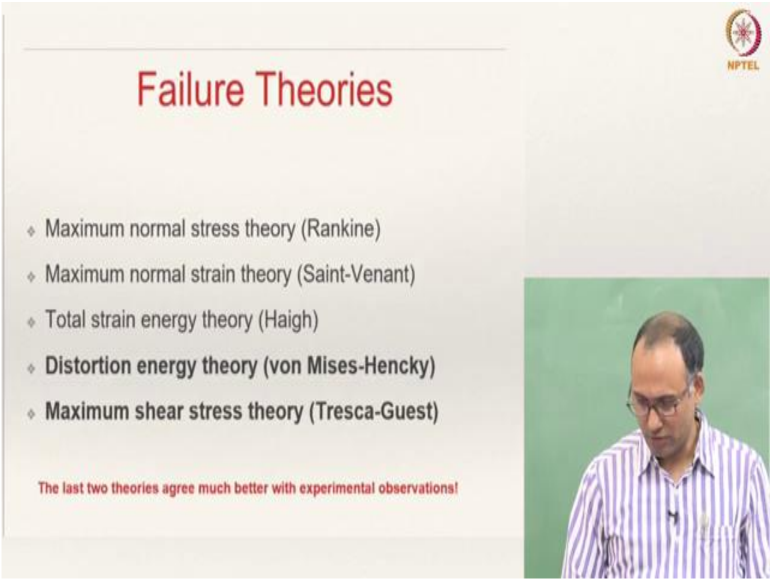

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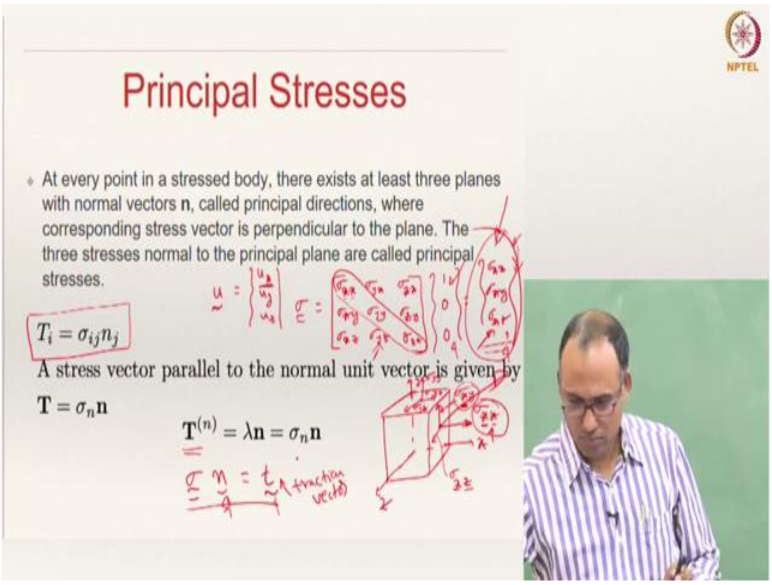

These 2 failure theories seem to agree with experimental data available in the literature much better than other failure theories. They actually have seen other failure theories and then they saw that there are some discrepancies and then they want to fix. So, understanding the discrepancies from these failure theories, these people have come up with better failure theories.

I have already defined what these failure theories are or you go back and read some textbooks and then you will be able to write the statement of the failure theory. Each failure theory will have a statement, right? What could be the statement of the maximum normal strain theory?

When the maximum normal strain in the material at a given point reaches the strain corresponding to the yield of the uniaxial tension test, that is when failure happens; that is how they have defined. Similarly, in the total strain energy theory, you measure the strain energy in the material; when the strain energy is equal to yield strain energy (which is computed as the area under the stress-strain curve up to the yield point), that is when the failure happens.

Now, we need to spend some time understanding principal stresses. Did we discuss the concept of stress tensor in this class? No? Okay. So, you know that stress is a second order tensor?

Student (Refer Time: 32:43).

Are you familiar with the concept of second order tensor? If you are not familiar, please let me know, I will spend some time.

Student: (Refer Time: 32:52).

No, ok alright. So, what is a scalar?

Student: (Refer Time: 33:03).

You must have studied vectors in your 12^{th} standard, right? What is a scalar? A physical quantity which has only magnitude. What is a vector?

Student: (Refer Time: 33:16).

A physical quantity which has magnitude and direction. A second order tensor is a quantity which has magnitude, direction and a plane of action. We will talk about it in a moment.

Let us talk about stress tensor. Suppose you take an infinitesimal element, let us say these are the x, y and z axes. Now, if I have to represent the stress state at a point. This is an infinitesimal volume; that means, it is pretty small. We are only showing it as an infinitesimal volume.

But it is actually stress state at a point. It is represented by normal stress described as σ_{xx} ok and this I am calling it as σ_{xy} and this I am calling it as σ_{xz}. Similarly, on this plane you have one normal stress that is σ_{yy} and that will be σ_{yx} and that will be σ_{yz}. Like that, you can show your stress state on all the faces. If you want to describe one element of the stress tensor, how many indices are you using?

Student: 2 indices.

2 indices. If you carefully observe, the first index represents the direction of the normal to the plane. What is the direction of the normal to the plane? x axis. So, the first index represents the direction of the normal and the second index represents?

Student: (Refer Time: 35:39).

The direction in which it is acting. This is called normal stress component because the direction of that particular stress component is in the direction of the normal itself; that is why it is called normal stress component and this is shear because it is in the plane. So, the first index represents the normal; that means, it is representing the plane. This is basically your plane of action and the second index is representing the direction in which it is acting.

Let us take velocity for instance. How many components will velocity vector have in 3D? 3 components. If this is my velocity or let us say displacement; $\boldsymbol{u}$ is my displacement vector. I can write $\boldsymbol{u} = \begin{Bmatrix} u_x \\ u_y \\ u_z \end{Bmatrix}$. What does this index represent? The direction. So, elements of the vector $\boldsymbol{u}$ are the projections of this vector on each of the coordinate frames.

That basically represents components of the displacement vector along x, y and z directions. But now if you come to the stress, it is a second order tensor. Every element in this tensor is also required to be presented with its plane of action; that is why you need to use 2 indices.

The first index represents the plane of action; second index represents the direction in which it is acting. In order to represent the stress in some mathematical form that is concise, we use matrix representation i.e.,

$$\sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

I put 2 tildes below to represent that it is a second order tensor. One tilde to represents a first order tensor. So, a vector is a first order tensor

Student: (Refer Time: 38:10).

The order is the number of indices that are required to represent all the elements of that particular physical quantity. If you carefully observe, all the diagonal elements will have same index. Both first and second index are same; that means, it is a normal component. All the off-diagonal components are your shear components. Now, if I multiply this stress tensor with, let us say 1 0 0. What is 1 0 0? The normal.

Student: (Refer Time: 38:51).

The unit vector in the x direction right. What will I get?

Student: Sigma xx.

$\sigma_{xx}, \sigma_{xy}, \sigma_{xz}$. This is a vector and this is what we call stress vector or traction vector. You probably heard of this traction vector.

What is this telling us? If you have a stress tensor and if you are operating -- using the stress tensor you are operating on a vector which is actually the normal to a plane, then what is it giving you? It is giving you the components of that stress tensor on that plane; that means, the stress vector on that plane. Is that clear? If I have a stress tensor $\boldsymbol{\sigma}$ and I am operating on a vector $\boldsymbol{n}$, then I will get some vector $\boldsymbol{t}$.

This vector $\boldsymbol{t}$ I am calling as traction vector. It represents the components of the stress tensor on this plane. This form of representation is called indicial notation, but do not worry about it. I do not want you to spend time on that, but all that I can write is that my traction vector on a plane with normal ***n*** can be obtained by operating this stress tensor on that normal vector. Is that clear? Is there any ambiguity there?

What is the direction of the traction vector? It can be in any direction. What happens if the direction of that vector is same as the normal to the plane? Is the question clear?

Student: Yes.

What if that direction of ***t*** and ***n*** or under what conditions these directions be the same?

Student: (Refer Time: 42:09).

When $\sigma_{xy} = \sigma_{xz} = 0$; when the shear stresses are 0, right? So, if my direction of the resultant traction vector is in the same direction as the normal, then on that plane I will not have shear stresses, right? What is such a plane called?

Principal stress plane, right? If that happens to be my principal stress plane, then I can write my ***t*** that is the traction is equal to?

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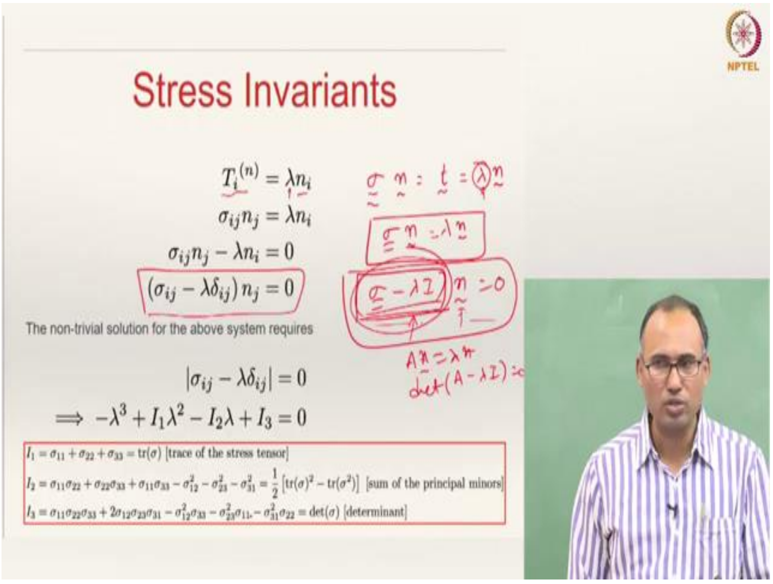

I can describe my traction direction to be same as the direction of the normal right. I can write this λ is a scalar.

$$\sigma_{ij} n_j = \lambda n_j$$

Are you familiar with this form?

Student: (Refer Time: 43:31).

It is an eigenvalue problem. So, what will be this magnitude by the way?

Student: (Refer Time: 43:39).

See if it is a unit vector in the same direction, then this will be your eigenvalue, but from the stress tensors perspective this is a normal stress component; that means, that is your principal stress, isn't it? That means, if you are given a stress tensor if you find the eigenvalues of that stress tensors, the eigenvalues correspond to principal stresses and the eigen vectors correspond to principal directions. So, that is how if you are given a stress tensor you can actually find out the eigenvalues and then they will be your principal stresses, right?

That is how we have derived this here; that is your eigenvalues problem. And then, you will say that this is your eigenvalue problem, then what should you solve? What should you do? If this your eigenvalue problems, your $\boldsymbol{n}$ can be $\mathbf{0}$? But it is not – it is a trivial solution. What should be a non-trival solution?

Student: (Refer Time: 44:47).

This should be 0 (Refer Time: 47:48).

Student: (Refer Time: 47:49).

$\boldsymbol{\sigma}-\lambda\boldsymbol{I}$.

Student: (Refer Time: 47:52)

But that is a matrix.

Student: (Refer Time: 44:54) once.

If that becomes 0, it is a redundant solution. This is possible for any generic $\boldsymbol{n}$ only when this matrix is singular, right? Because it is in the null space; that means, the determinant of this guy should be equal to 0. That is why we have to equate the determinant to be 0.

When you write $\boldsymbol{Ax} = \lambda\boldsymbol{x}$, several times we do not pay attention why should $|\boldsymbol{A} - \lambda\boldsymbol{I}| = 0$.

The reason why we need to do that is because for a non-trivial solution this should be singular and if that has to be singular then determinant of that matrix should be equal to 0, right? So, with that I will stop and then we will meet in tomorrow's class and build on, ok?