

**Basics of Materials Engineering**  
**Prof. Ratna Kumar Annabattula**  
**Department of Mechanical Engineering**  
**Indian Institute of Technology, Madras**

**Lecture - 30**  
**Static Failure Theories (Introduction, Definition of Failure)**

Defects are part and parcel of the material system. We have discussed different kinds of defects (such as point defects, line defects, area defects, etc.) and the implication of these defects on the deformation behaviour of the material. Further, we discussed plastic deformations in both single crystals and polycrystalline materials and the key mechanical properties that are required to describe the deformation behavior (such as elastic stiffness in elastic regime, yield strength, ultimate strength, etc.). We also discussed the measurement of these properties and their meanings. Yield strength, for instance indicates the onset of plastic deformation and, therefore, dislocation motion.; the hardening behaviour arises from dislocations interacting with themselves or with the grain boundaries or point defects which impede dislocation motion; and so on.


So, if you know the mechanical properties, then you can now go about designing structures or components such that the stresses induced into the components, as a result of the external load that you are applying, are below the critical values. For instance, if you are interested in designing a component which should not break, then you should ensure that the stresses induced are below the ultimate strength. Similarly, if you want to restrict the deformation to the plastic regime, you should ensure that the stresses are below the yield point.

Using these mechanical properties, we should now be able to discuss various modes of failure. We should be able assess a component subjected to external loading on whether or not it is going to fail; whether it will be able to withstand loads or if it can resist plastic deformation and so on.

Firstly, we shall discuss static failure. Static loads refer to loads that do not change as a function of time. Dynamic loads involve time dependent forces and thus have separate


failure theories associated with them. Now, we must firstly define what we mean by failure.

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- ◊ Factor of Safety
- ◊ Stress Concentration at Notches
- ◊ Application of failure theories




We shall look at the stress tensor in detail and discuss the invariants of the stress tensor. Also, we shall discuss the various failure theories for both ductile and brittle materials with and without stress risers or stress concentrations.

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## Learning Objectives

1. Write invariants of a stress tensor
2. Deduce the relation between eigenvalues of a stress tensor and principal stresses
3. Write the stress concentration factors for specific types of loading and geometry
4. Write the names of static failure theories for ductile and brittle materials




Finally, we will talk about the application of failure theories to various design problems.

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## Learning Objectives


5. Derive the expression for von Mises and Tresca yield criterion and list the underlying assumptions
6. Plot the 2D yield surface for each failure theory mentioned in (5)
7. Design simple machine elements (bar, shaft and beam) under different loading conditions using different failure theories
8. Discuss relevant merits of each theory in the design





We will look at why ductile and brittle materials require different failure theories. Most importantly, we shall discuss the von Mises yield criterion and the Tresca yield criterion in detail. These are the two important failure theories for ductile materials.


We shall discuss these theories in the context of design of machine elements such as shafts, etc. Finally, we will discuss the relative merits and demerits

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## What is Failure?

- ❖ Does failure mean the ultimate fracture? 
- ❖ How does one define failure for ductile and brittle materials? 
- ❖ For engineering applications, what is considered as failure?
- ❖ Are material properties important?



What do we mean by failure? We need to have a common definition so that there is no ambiguity. Does failure mean ultimate strength, i.e. the breakage of the component? Do they mean the same thing for both brittle and ductile materials?

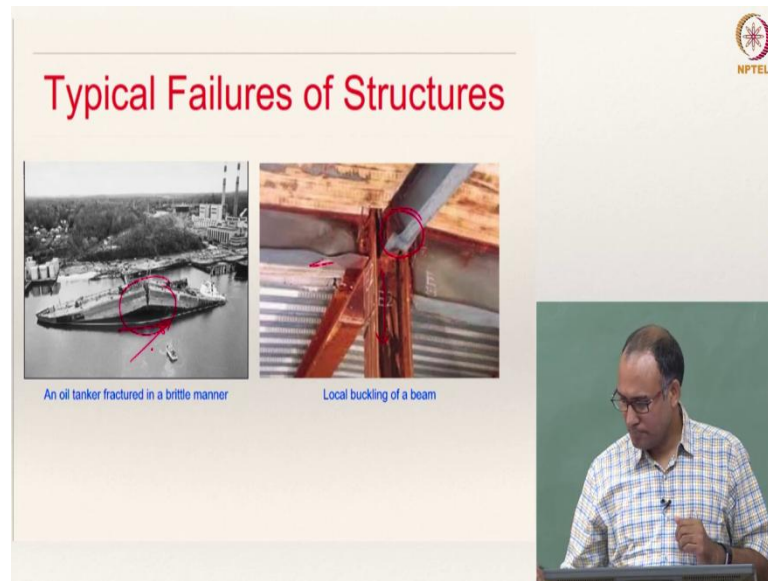
So, on a bicycle, we have a bell on the handle. If the arm of the bell is slightly bent, it is still usable; it may not be convenient ergonomically, but the function is not lost, so we may not consider it to have failed and continue using it. However, if the arm bends in a way such that you can no longer use it while cycling, we will consider replacing it. The failure here is defined by its failure to meet its intended functionality. So, for many machine components in mechanical engineering that we use, the intended functionality may not be lost completely. However, even if it is lost partially, you may consider it a failure of the component.

The length of cycle chains increases over extended periods of use. Once the chain exceeds a certain length, it causes frequent slipping of the chain off the sprockets. The permanent deformation causes the chain to stop functioning as intended. It is seen that for several machine components, permanent deformation causes failure. If a shaft supported between two bearings suffers a slight bend, you will have more extraneous vibrations than acceptable or intended.

When dealing with ductile materials, you are bound to see these permanent deformation and consequently, the functionality is compromised. Therefore, for ductile materials, typically, designs are such that a material is said to have failed if it deforms plastically. So, in general, the material is said to have failed if the stress induced is beyond the yield strength of the material.

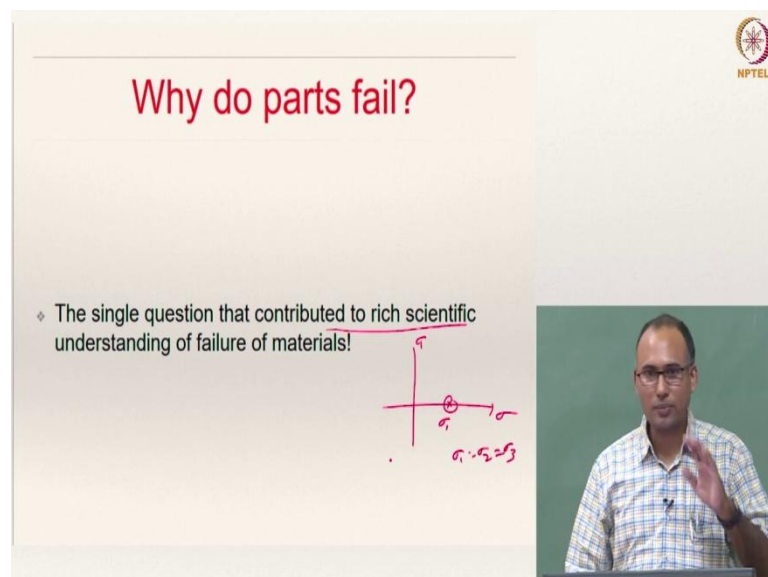
For brittle materials, it is extremely difficult to identify something called yielding. For brittle materials, they fail by breakage; there is no significant plastic deformation. Therefore, brittle material do not show a clear yield strength and usually they break due to fracture. Consequently, when you are designing a brittle material, you have to design for the ultimate strength.

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So, for instance, take these two examples. This is an oil tanker which has fractured in a brittle manner; it has broken into pieces. On the other hand, this is a beam and a column where the beam has buckled and reached permanent deformation. Buckling is another mode of failure. The oil tanker broke due to something called ductile brittle transition that we will talk about when we discuss fatigue failure of materials.

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So, why do parts fail? People did not know, until they started asking this question. So, the fact that people started asking this question, led to lot of interesting observations; it led to a lot of insight into the mechanical behaviour of materials. This was probably the single most important question that contributed to a rich scientific understanding of mechanical behaviour of materials..

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**Failure of Materials**

- ♦ Mohr's circle for a uniaxial tensile test (left) shows the existence of shear stress at some planes
- ♦ Existence of normal stress for a case of torsion test (pure shear)
- ♦ In both the cases, which stress causes the failure?

Mohr's circle for uniaxial tension and torsion

$\sigma_1 = \sigma_3$

$\sigma_1 = -\sigma_3$

$\sigma_1 = \sigma_2 = \sigma_3$

So, this is a Mohr's circle for uniaxial tension. Uniaxial tension has only one principal stress; other two principal stresses are 0. We can see from the Mohr's circle that even though we have applied a normal stress, there exist planes in the material where shear stresses occur. We know when we are talking about plastic deformation, plastic deformation happens due to slip under shear stress. So, if at all you are going to talk about plastic deformation, you have to talk about the shear.

So, unless there is shear, you cannot cause plastic deformation, because shear is what causes slip. Similarly, when torsion is applied to a shaft (pure shear), we will have normal stress along certain planes and we get the principal components as


$$\sigma_1 = -\sigma_3$$

For hydrostatic state of stress,

$$\sigma_1 = \sigma_2 = \sigma_3$$


The radius of Mohr's circle then reduces to 0. Therefore, the Mohr's circle reduces to a point on the  $\sigma$  axis. This implies that there exists no plane on which shear stresses exist.

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## Failure of Materials


- ❖ In a static tensile test, in general, ductile materials are limited by their shear strength and the brittle materials are by their normal strength
- ❖ Hence, one kind of failure theory doesn't explain all the materials
- ❖ Different failure theories for the two classes (ductile and brittle) of materials
- ❖ But, before actually talking about a failure theory, a proper definition of failure itself is important!



In a static tensile test, in general, ductile materials are limited by their shear strength because shear is what is causing yielding. Brittle materials are limited by their normal strength, because the fracture happens due to normal strength. So, the same kind of failure theory cannot be applied to both the materials, because both of them are failing due to two different phenomena. Different failure theories are therefore required, However, a proper definition of failure is firstly required.




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## What is a ductile material?

- ❖ If the percentage elongation upto fracture is greater than 5%
- ❖ For many ductile materials, this number is usually 10%




If the percentage elongation up to fracture is greater than 5 percent, then that is typically a ductile material.

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## Failure of Materials


- ❖ A component may be considered failed if it yields and deforms sufficiently large leading to improper functioning
- ❖ A component might fail by fracture
- ❖ Both the conditions are failures; but the mechanisms are different!
- ❖ Static or Dynamic!



As mentioned before, a component may be considered failed if it permanently deforms or when it fractures. Both are failures, but their mechanisms are different.




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## Failure of Ductile Materials

- ❖ Ductile materials fail by fracture if loaded beyond ultimate tensile strength
- ❖ Typically, the machine components made of ductile materials are considered to be failed when they yield under static loading.
- ❖ Yield strength of a ductile material is much less than the ultimate strength.



Typically, ductile materials are said to have failed when they begin to yield. So, when designing ductile materials, you really do not worry about the ultimate failure or fracture; you are only focused on whether they are yielding or not. Yield strength of a ductile material is much less than the ultimate strength. For a brittle material it is almost the same, but ductile material yield strength is much lower. That is the number that you need to take into account when you are designing a ductile material.