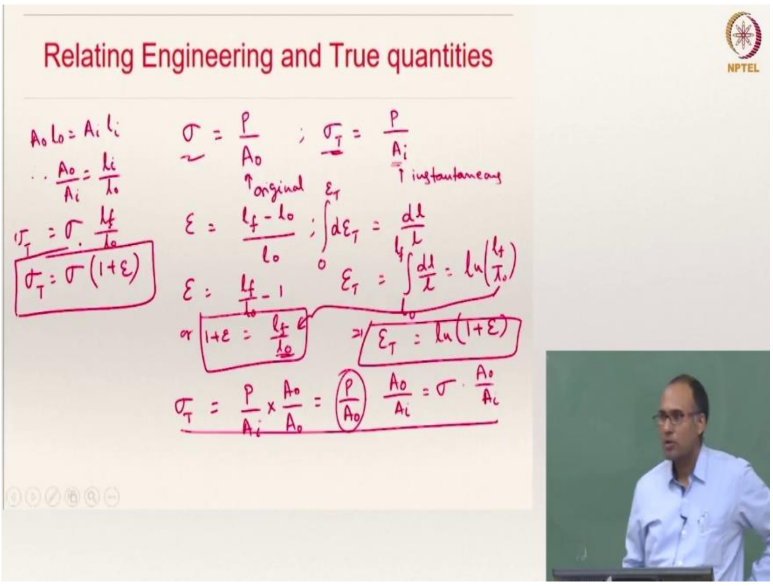
Basics of Materials Engineering

Prof. Ratna Kumar Annabattula
Department of Mechanical Engineering
Indian Institute of Technology, Madras

Lecture – 28
Mechanical Properties
(Tension Test-Elastic Deformation)

In the last class we have looked at the concept of engineering stress and engineering strain.

(Refer Slide Time: 00:25)



The engineering stress is given as

$$\sigma = \frac{P}{A_0}$$

where P is the load applied in the context of uniaxial tension test we discussed and A_0 is the original cross-sectional area. We have defined,

$$\sigma_T = \frac{P}{A_i},$$

where A_i is the instantaneous cross-sectional area. A_i and A_0 are going to be different as you continue the deformation.

That is the reason why the values that you would obtain for an engineering stress and a true stress will be different from each other. Strain is defined as change in length to the original length, i.e.,

$$\epsilon = \frac{l_f - l_0}{l_0}$$

We have defined,

$$d\epsilon_T = \frac{dl}{l}$$

If you want to find out epsilon T, you integrate it from 0 to whatever strain that you are interested in.

$$\epsilon_T = \int_{l_0}^{l_f} \frac{dl}{l} = \ln\left(\frac{l_f}{l_0}\right)$$

That is the definition of your true strain.

$$\epsilon = \frac{l_f}{l_0} - 1 \Rightarrow \frac{l_f}{l_0} = 1 + \epsilon$$

$$\Rightarrow \epsilon_T = \ln(1 + \epsilon)$$

That is how the true strain and engineering strain are related. Now, let us try to relate the true stress and engineering stress. We know the true stress,

$$\sigma_T = \frac{P}{A_i} = \frac{P}{A_0} \times \frac{A_0}{A_i}$$

$$\Rightarrow \sigma_T = \sigma \times \frac{A_0}{A_i}$$

$\sigma = \frac{P}{A_0}$ is the engineering stress. I am not writing anything as a subscript to σ when it is an engineering quantity. If it is a true quantity then I am writing subscript T; that is how you recognize. We have discussed about the volume constancy condition which

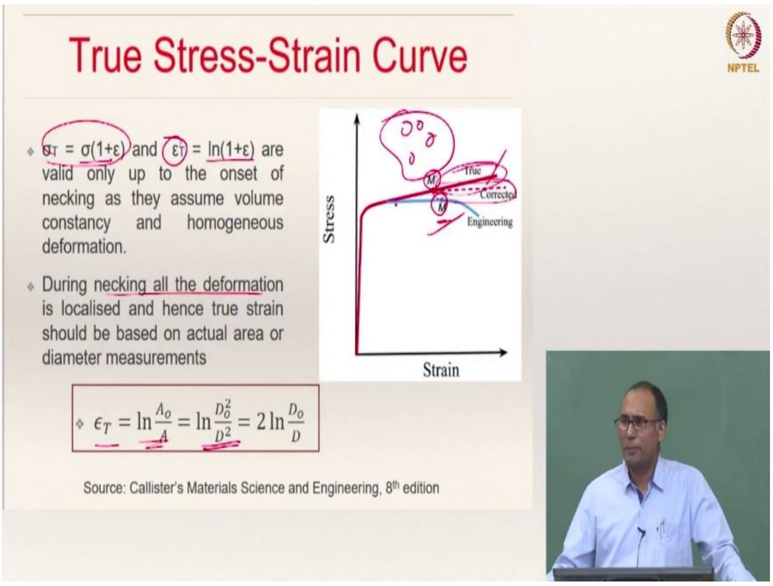
$$A_0 l_0 = A_i l_f \Rightarrow \frac{A_0}{A_i} = \frac{l_f}{l_0}$$

$$\Rightarrow \sigma_T = \sigma \times \frac{l_f}{l_0}$$

$$\therefore\ \sigma_T = \sigma(1 + \epsilon)$$

That is how you can relate the true stress with the engineering stress and engineering strain right. Given the true quantities, you should be able to find out the engineering quantities and vice versa.

(Refer Slide Time: 04:49)



Typically, the engineering stress-strain curve which is obtained from your experiment, looks something like this. The blue color one is the engineering stress-strain curve. When you are drawing the true stress-strain curve, your area should be instantaneous area.

When you are doing the tension test, A_i is going to be larger than or smaller than A_0? A_i is going to be smaller because you are increasing the length; so, cross-sectional area has to reduce. Naturally, $\frac{P}{A_i}$ will be having a higher value than $\frac{P}{A_0}$, that is why that curve is up. You can see that the red one is your true stress-strain curve. For the time being forget about this dashed line, that is something called Bridgman correction; we will talk about it in a moment.

For time being, do not think about it, you are only going to focus on the solid blue line and solid red line. The solid red line is the true stress-strain curve. We have just shown the relations between the true stress and the engineering stress as well as the true strain and the engineering strain.

The peak in the engineering stress-strain curve is what we call ultimate strength point. The stress corresponding to that peak in the engineering stress-strain curve is what is called ultimate strength of the material.

That is where typically in a ductile material, the localized deformation takes place; that means, typically in a ductile material this phenomenon called necking happens. The local cross-section suddenly starts decreasing. This means all the deformation is localizing in that area, that is why it is called localized deformation and eventually failure happens there.

The mechanism responsible for necking in ductile materials is void coalescence in which voids will coalescence with each other, and as a result, you will have net local reduction in volume; we have assumed volume constancy until now. The moment necking starts, the volume constancy condition cannot hold good. Because microstructurally, the voids that are present in this ductile material are coalescing with each other, and as a result, the total volume of the material is getting changed. The volume constancy condition cannot hold good in the necking region.

So, during necking all the deformation is localized. For instance, if you think about the defining true strain as a function of lengths, it becomes little tricky because the deformation is localized. Hence, instead of writing the strain in terms of lengths, just before the onset of necking, you might want to write in terms of cross-sectional area.

We have defined the relation between A_i just before the onset of necking. This is not during necking, just before the onset of necking,

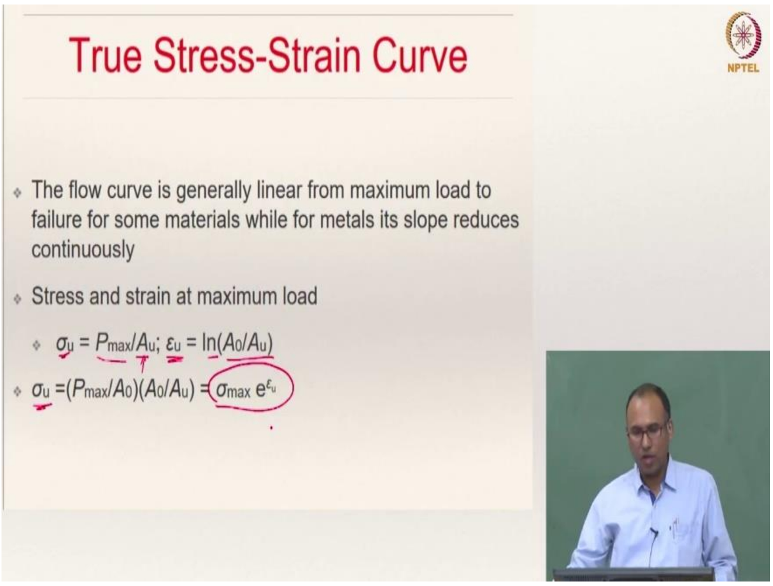
$$\epsilon_T = \ln\left(\frac{A_0}{A}\right) = \ln\left(\frac{D_0^2}{D^2}\right) = 2\ln\frac{D_0}{D}$$

This expression is only true for a cylindrical specimen. If it is some other specimen then $\frac{D_0}{D}$ does not work out, because you have to write appropriate cross-sectional area for that.

The true strain in this case is calculated using local cross-sectional area variation i.e., $\ln\left(\frac{A_0}{A}\right)$. If it is a cylindrical specimen, you will be measuring the diameter changes during this process. Typically, what happens is, you have just now seen that your true stress-strain curve; if you see the corresponding point for ultimate strength point on engineering stress-strain curve is M; on the true stress-strain curve it is M′.

Typically, the true stress-strain curve will always be left to engineering stress-strain. And, then the flow curve beyond this point is found to be linear in this region. Many times, it is found to be linear, for some materials it may be non-linear too.

(Refer Slide Time: 10:21)



Now, if you look at the stress at the ultimate point, that means, where the necking is happening, that is the maximum load carrying capacity of the material. It cannot hold any stress beyond that value, after that actually failure is happening. That is why it is called ultimate strength of the material. The ultimate stress,

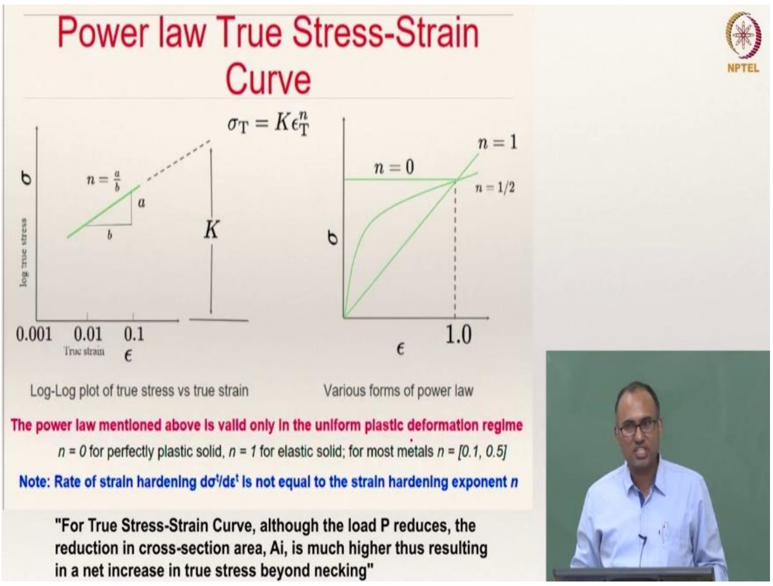
$$\sigma_u = \frac{P_{\max}}{A_u}$$

So, A_u is the area corresponding to the onset of necking.

And, then you can write $\epsilon_u = \ln\left(\frac{A_0}{A_u}\right)$; that means, corresponding to the arm set of necking which is the peak point. Then,

$$\sigma_u = \frac{P_{\max}}{A_0} \times \frac{A_0}{A_u} = \sigma_{\max} e^{\varepsilon_u}$$

(Refer Slide Time: 11:27)



When we have looked at uniaxial tension test in the last class and today's class, we have identified the initial elastic regime that could be a linear elastic regime or a non-linear elastic regime. If it is a linear elastic regime, then the slope of the line in that linear elastic regime is called your elastic modules or Young's modulus of elasticity.

If it is non-linear, the slope is not going to be constant and hence you need to take something called tangent modules; that means, local slope. That is what we have learned about the elastic regime and then this elastic regime is followed by a phenomenon called yielding; that means, the onset of plastic deformation. That is when the dislocations in the material start moving, until then they are not activated, right? That means, the stress is not large enough to actually cause the dislocation motion.

That is why the yield point represents the onset of plastic deformation; until yield point you will not have plastic deformation. That is again the property of the material. Depending upon the kind of material that you are using, your yield strength will change. Similarly, your elastic modulus will change from material to material. That is why these are material properties.

If you load the material beyond the yield point, you continue the plastic deformation, but the thing that you would observe is the material becomes hardened; that means, it becomes harder and harder for you to cause further deformation.

This regime is called hardening regime; from the yield point up to the necking is called hardening regime; necking represents the onset of localized deformation. Usually after necking if you are looking at engineering stress-strain curve, the stress-strain curve falls down, because the force required to cause further deformation will go down. Previously, until ultimate point, the additional force required to cause further deformation was increasing.

But, after necking the force required to cause further deformation goes down. That is the reason why your engineering stress-strain curve goes down. Because, $\frac{P}{A}$ in the engineering stress-strain curve, the area is original cross-sectional area which does not change, but P does change. The fact that P is reducing is the reason why beyond necking the stress-strain curve drops. But if you take true stress-strain curve, why is it not dropping?

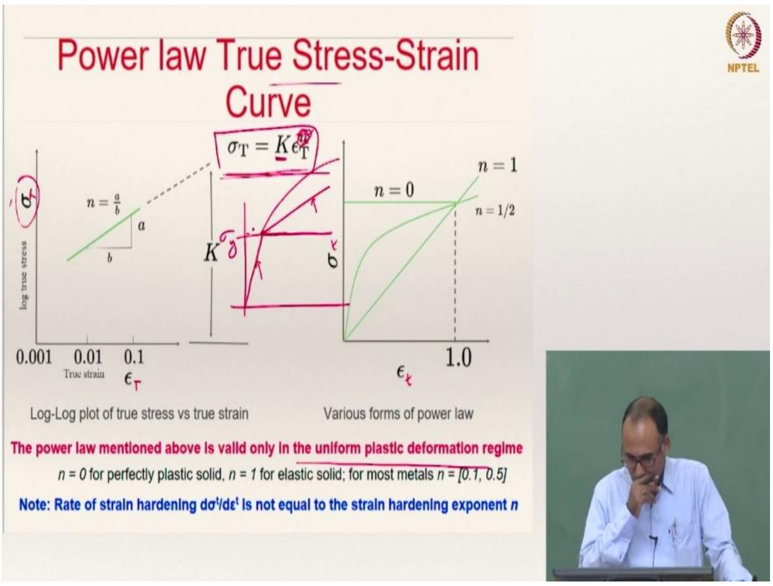
It is continuing to grow because, your cross-sectional area is actually reducing significantly as you continue the load. ~~Hence, although your requirement for P is also increasing, A is reducing; the total quantity is increasing~~ (Correction: Although P is decreasing, the cross-sectional area is reducing at a faster rate, and hence the true stress continues to increase). That continues to go up. Alright. Now, we have defined the first region which is called linear elastic regime. We have a formula for that, represented by Hooke's law.

It is named after Robert Hooke. He is an English engineer or a physicist, who understood the elastic deformation of the materials and did seminal experiments in understanding the mechanical behavior of the material.

$$\sigma = E\epsilon$$

This is the formula that we use to represent the linear elastic part of the system. How do we go about representing the hardening part? That means, from the yield point to the necking, what is the functional form that we use? There are several functional forms that are available.

(Refer Slide Time: 15:49)



To represent the true stress-strain curve, power law is one of the popular models, wherein σ_T is plotted on the y axis, because the *x* and *y* axes represent the true stress and true strain, respectively. If we plot on a logarithmic scale, you will have a straight line or in normal scale it will be looking something like this. Please note that this power law is from the yield point, not in the linear elastic regime. In the linear elastic regime, it is $E\epsilon$.

There are two constants to represent the behavior of the curve. If it is linear, $n = 1$, and it is called linear hardening. This is the linear elastic regime and let us say this is your yield strength σ_y. From here onwards, you can have no hardening at all i.e., when $n = 0,$ then $\sigma_T = K = \sigma_y,$ and this is what is called an elastic perfectly-plastic material. There is no hardening at all.

The material continues to deform at constant stress; that means, if you were to see that from the microstructural mechanism perspective, there was no resistance to dislocation motion whatsoever. Hence, at a constant stress, you are able to have deformation; that means, you are able to impart strain to the material. This is an idealization.

Typically, there are no materials which can actually show such a behavior, but it is an idealization. This is what is called *elastic perfectly-plastic* material. Several times, when you want to test the behavior of the materials in the plastic regime, when you are writing your own numerical codes, the simplest possible thing is to do this.

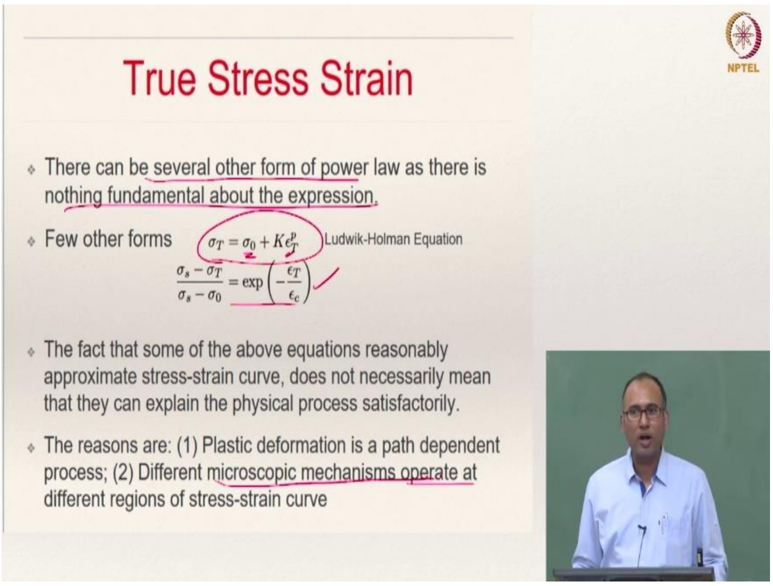
So, you will study what is the behavior of the system when the material is elastic perfectly plastic, because you do not have to worry about hardening, right? Because you do not know much about the hardening behavior, you try to see what happens if at all it is perfectly plastic, which is an ideal case. When $n = 1$, then this can be something like that. This is what is called linear hardening and you can also have non-linear hardening as something like that.

Depending upon the value of the exponent, you will have different responses. When $n = 0$ it is perfectly plastic, $n = 1$, it is linearly hardening material, $n = \frac{1}{2}$, then it is a non-linearly hardening material. The power law is only valid in the uniform plastic deformation regime. Until the onset of necking, there was no localization. The plastic deformation was uniform everywhere within the specimen.

The moment necking starts the localization kicks in, you cannot call that situation as uniform plastic deformation regime because all the plastic deformation is happening at one place.

The power law formulation is only applicable as long as your material undergoes uniform plastic deformation. n is called strain-hardening exponent and it is a material property; depending upon the kind of the material whether you have a linear hardening or non-linear hardening.

(Refer Slide Time: 19:56)



We have given one form that,

$$\sigma_T = K\epsilon_T^n$$

But there is nothing special about this form. In principle, you can write different kinds of forms that you want. People can come with different kinds of forms as long as they match with the experimental results. There can be several other forms of power law and there is nothing fundamental about the expression.

You do not have to have so much of attachment to this power law, the way that we have written. There are other forms, for instance this is another form

$$\sigma_T = \sigma_0 + K\epsilon_T^p$$

that includes the plastic strain; you have written in terms of plastic strain because until elastic regime, the plastic strain is 0. You have taken that into account and then you have written that, without actually worrying about where you are starting this.

There is another expression which is an exponential form written in this way.

$$\frac{\sigma_s - \sigma_T}{\sigma_s - \sigma_0} = \exp\left(-\frac{\epsilon_T}{\epsilon_c}\right)$$

All these forms are possible. Why do people have to come up with different forms of this expression? Because, somebody has given power law and they have done their experiment for a particular material. And, this set of materials does not seem to follow the power law.

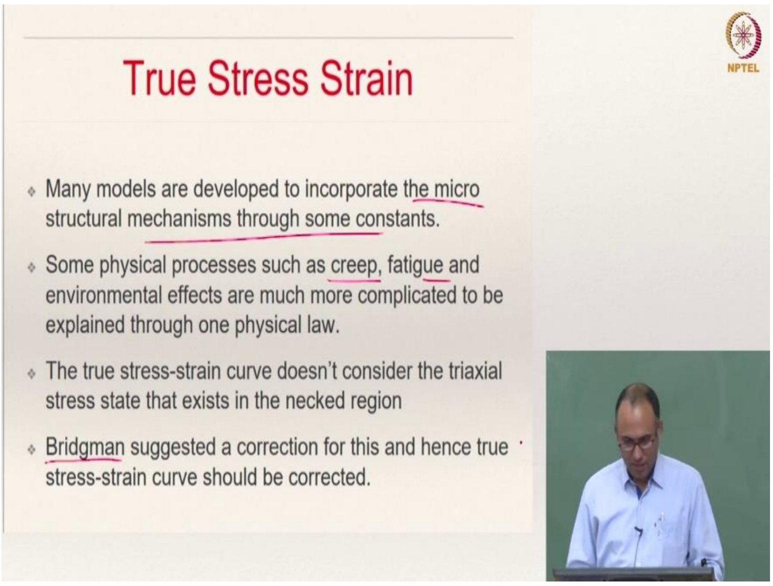
So, this is not a power law material, but it is some other material. The material is behaving the way that it wants, it is just us who are associating some mathematical form to it. Hence, we should not attach too much of value to the particular mathematical form that we are assigning to this behavior. No matter what mathematical form you assign, it may actually match very well with the experimental results.

But that does not mean that it can actually explain all the microscopic mechanisms at play in the material. You are just hypothesizing the macroscopic behavior of the material through a mathematical form. In no way can that represent all the microscopic mechanisms. After all, this is a law describing the macroscopic phenomena, where you do not have any information about the microscopic deformation mechanisms.

Hence, we should be very much careful when we are looking at these stress-strain relations in the plastic regime particularly, because there can be several other forms one can fit in. That form may work for a particular class of materials; the same form cannot possibly work for other class of materials. Please keep that thing in your mind. The reason why sometimes it may work and sometimes it may not work is primarily because the plastic deformation is a path dependent process. That is a very important thing to remember.

For instance, if you are modeling plastic deformation in a material, it is extremely important that you carefully follow the path dependency. Otherwise, you will be estimating your stress state in a completely wrong manner than what it is supposed to be. We have discussed that there are different microscopic mechanisms that can be operating at different regions of the stress-strain curve. So, you possibly cannot use one law to describe every part of the stress-strain.

(Refer Slide Time: 23:54)



As we have discussed, there are many models which are developed to incorporate the microstructural mechanisms through some constants like K and n. These constants sort of represent the microscopic mechanism in some average sense, but they cannot represent the microscopic mechanism at each and every point on the stress-strain curve. Because, the mechanism themselves might change when you go from one region to another region.

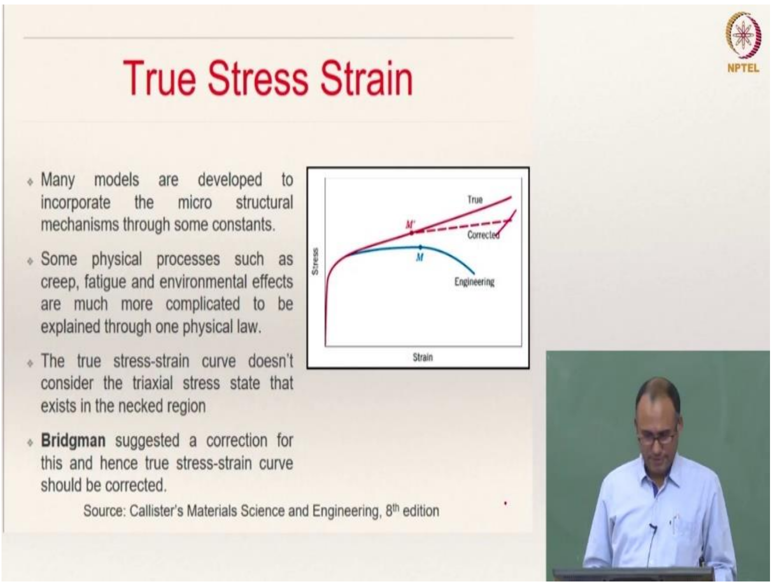
For instance, some physical processes like creep, fatigue and environmental degradation such as corrosion are much more complicated to be explained through simple laws that we have discussed. It is simply not possible. If you want to incorporate new mechanisms, you probably have to deal with more and more complex models to describe the mechanisms.

As we have discussed, the true stress-strain curve does not consider the triaxial stress state that exists in the necking region. There is local hydrostatic stress that is present during necking regime. That is not usually taken into consideration when you have drawn the true stress strain curve, but which has to be considered.

Bridgman did some experiments and he suggested that you need to take into consideration, the existence of the local hydrostatic state of stress during necking. We will talk about it when we are going to discuss failure theories, why one should be concerned about local hydrostatic stress in this situation. The solid red line does not entirely represent the true stress-strain curve.

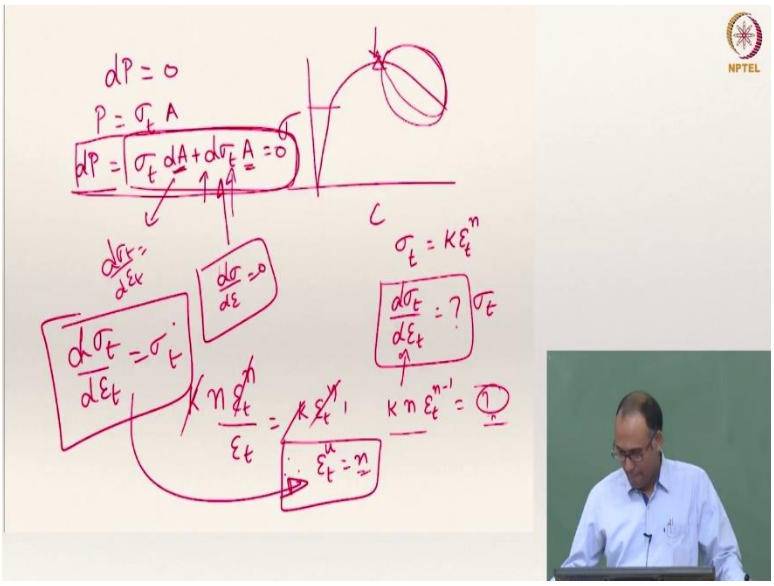
You need to account for some correction which has been suggested by Bridgman. The dashed line correctly represents the true stress-strain curve i.e., the dashed line is the corrected stress-strain curve.

(Refer Slide Time: 26:11)



Before going further, let me finish the condition for onset of necking.

(Refer Slide Time: 26:39)



In the engineering stress-strain curve, that is your ultimate tensile strength which indicates the onset of necking. This is the process during which necking evolves, but the onset happens here. Necking is not progressing here; it has just started. It is like this point where plastic deformation started, it did not progress. Similarly, necking started, it did not progress. This is where the necking is progressing.

From the load perspective, you can write this as $dP = 0$ since it is a peak point.

$$P = \sigma_T A$$

$$\Rightarrow dP = \sigma_T dA + d\sigma_T A = 0$$

If our hardening response is represented by

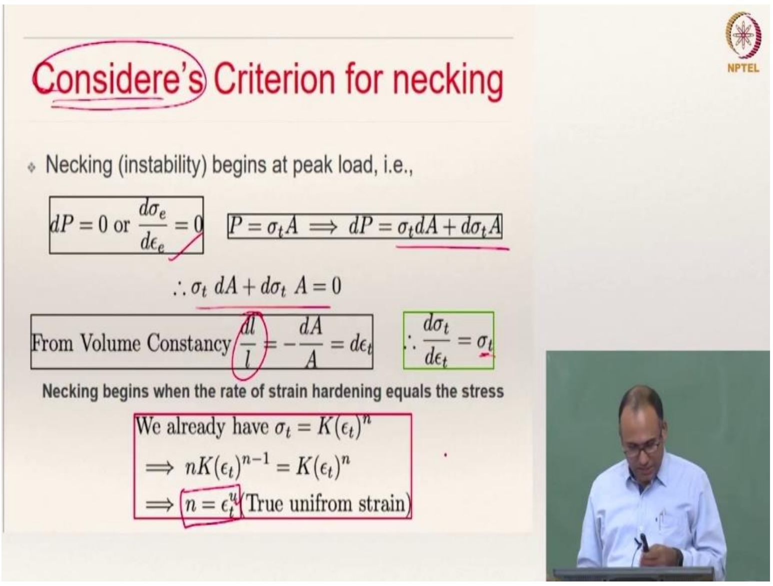
$$\sigma_T = K\epsilon_T^n$$

what is the condition for necking? What happens at the onset of necking to $\frac{d\sigma_T}{d\epsilon_T}$? Normally you say that $\frac{d\sigma}{d\epsilon} = 0$ at the onset of necking, right?

From the volume constancy, you can write $\frac{dA}{A}$ in terms of $\frac{dl}{l}$ and from there you can bring in the true stress strain value. Hence you can get $\frac{d\sigma_T}{d\epsilon_T}$ from here; what should that be equal to?

At the necking $\frac{d\sigma_T}{d\epsilon_T} \neq 0$, because there is no peak in true stress-strain curve, but $\frac{d\sigma}{d\epsilon} = 0$.

(Refer Slide Time: 30:24)



We have,

$$\sigma_T dA + d\sigma_T A = 0$$

From volume constancy,

$$\frac{dl}{l} = -\frac{dA}{A} = d\epsilon_T$$

$$\therefore \frac{d\sigma_T}{d\epsilon_T} = \sigma_T$$

So, necking begins when the variation of true stress with respect to true strain becomes equal to true stress. If you want to further work on it,

$$\frac{d\sigma_T}{d\epsilon_T} = \frac{d}{d\epsilon_T}(K\epsilon_T^n) = Kn\epsilon_T^{n-1}$$

$$\frac{d\sigma_T}{d\epsilon_T} = \sigma_T \;\Rightarrow\; Kn\epsilon_T^{n-1} = \;Kn\epsilon_T^n$$

$$\therefore \epsilon_T^u = n$$

In other words, when the true strain becomes equal to n, the hardening exponent, that is when necking begins. When you are writing from the perspective of true quantities, the necking begins when the strain rate of change of stress in true quantities becomes equal to true stress.

Necking begins when $\epsilon_T^u = n$, where u refers to the ultimate point.

Student: (Refer Time: 32:13).

This criterion for necking is called Considere's criterion. Considere has given this criterion in terms of true quantities.

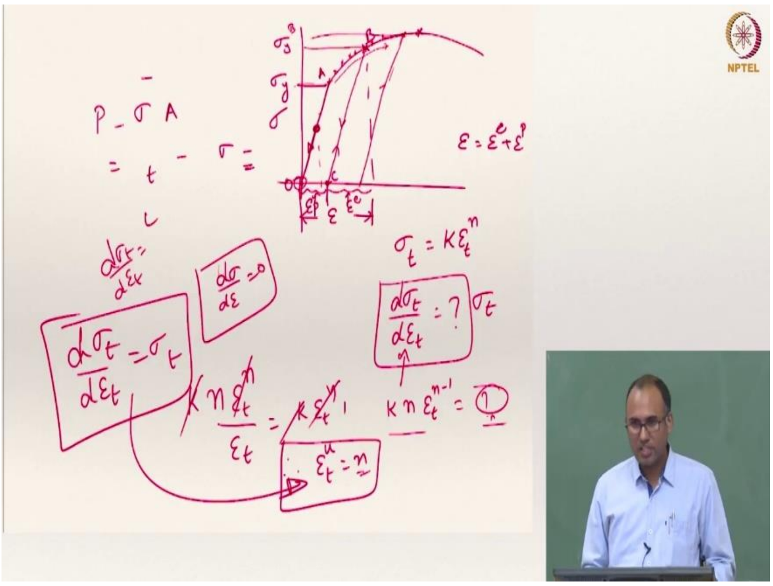
Student: (Refer Time: 33:00).

No, because from the engineering stress-strain curve, necking begins at the peak, right? That is equivalent to $\frac{d\sigma_e}{d\epsilon_e} = 0$.

Student: (Refer Time: 33:18).

Alright, before I move on, I would like to spend a couple of minutes on giving little bit more understanding.

(Refer Slide Time: 33:34)



Let us say you have this stress-strain curve, for time-being let us draw engineering stress-strain curve. Now, I am doing this test and this is my σ_y. Let us say I have loaded this specimen up to here and then I decided to unload. What happens? I have loaded up to this point, now I decided to unload; I will come back along this, ok? So, I will be able to come back here.

What is the amount of plastic strain at this point? There is no plastic deformation and hence it should be 0. Let us say I have loaded up to this point. Now, I have not gone to the region of necking, I have only loaded up to this point. If I then unload, what will happen? What will be the path? Parallel to?

Student: (Refer Time: 34:45)

Straight line parallel to true?

Student: (Refer Time: 34:48).

This line? Okay, what did you say?

Student: (Refer Time: 34:54).

Hooke's region, okay. So, you will unload along a line whose slope is equal to your elastic line. Why is that? Why should it be parallel to that line?

Student: (Refer Time: 35:12).

No, you said you have to unload like that; that these two should be parallel. Why should it be parallel to that line? Why should it only unload along that line?

Student: (Refer Time: 35:23).

Because, when you are unloading, you are not causing plastic deformation, your plastic deformation is over. All the unloading is only elastic, and elastic behavior is described by elastic stiffness. That is the reason why it has to only go along that path, it cannot choose any other path. Please pay attention here, it is a very important concept. Now, when you have loaded the guy up to here, this is the total strain.

Let us say O, A, B and let us say this is C. And, now I have unloaded to zero stress. I could actually go in the opposite direction, but I am not talking about it. What happens then? Because, of the fact that I have gone into plastic regime, I am not able to get to this point from where I have started, right? What is the amount of strain that I have recovered?

This is the total strain, but this much strain is recovered, right? And this much strain is retained. So, what should I call this component and what should I call this component? Which one is plastic, first component or second? This is your plastic strain, and this is your elastic strain. Your total strain is given by the sum of elastic and plastic strains.

Now, if I start to reload the sample which is unloaded up to here, how will it start loading? Along the same line, and it will go up to? What will be the yield strength of this material? Will it be this or that? What happened to the material? You have got the same material.

Student: (Refer Time: 37:34).

Student: (Refer Time: 37:35).

Did you change the material? It was the same specimen, but now if you do this experiment you are observing that the material has the different yield strength. How is this possible?

Student: (Refer Time: 37:48).

Student: (Refer Time: 37:52).

The most important thing to realize is that when we are moving along each and every point on that line, you are actually transforming your material. It is not the same material anymore. It may have the same elastic modulus, but it definitely does not have same yield strength; its yield strength has changed. You have changed the material by doing plastic deformation; by doing plastic deformation you are hardening the material.

If you are given a material which is loaded to plastic regime and unloaded and if you are given that specimen; and, if you are trying to find out the yield strength of the material, you will find that to be more than what it would have been for a virgin material. So, by plastic deformation, you can harden the material or strengthen the material. So, if you take that specimen, is it going to be more brittle compared to the virgin material?

You will find that this is getting more and more brittle because, it is actually going towards its failure point. So, the important thing to realize is that when you are doing plastic deformation to a material, from each and every point you are actually transforming your material; it is not the same material anymore.

And then, you further load it here and unload it here again. Let us say this is P, then again, your yield strength is going to increase, right? If you unload and reload, then your yield strength has increased. If you have loaded just before the necking, you will see that the material is suddenly failing without giving any sign.

A material which is ductile to begin with, could actually be hardened significantly just by plastic deformation. So, that is a very important thing to realize when you are looking at mechanical deformation of materials.

So, with that I will stop and in tomorrow's class, we will discuss about the measurement of remaining mechanical properties.