Basics of Materials Engineering
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Lecture – 20
Defects in Crystalline Materials - 5 (Different Stages of Slip in Single Crystal Materials)

Before we start our regular lecture, I wanted to bring another way to look at why dislocations prefer to move in close packed directions and close packed planes.

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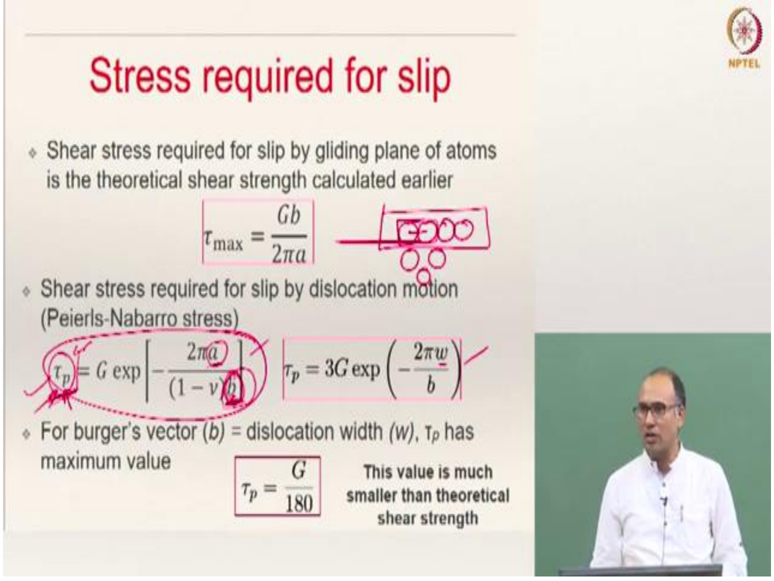

Stress required for slip

- Shear stress required for slip by gliding plane of atoms is the theoretical shear strength calculated earlier

$$\boxed{\tau_{\max} = \frac{Gb}{2\pi a}}$$

- Shear stress required for slip by dislocation motion (Peierls-Nabarro stress)

$$\boxed{\tau_p = G\exp\left[-\frac{2\pi a}{(1-\nu)b}\right]} \quad \boxed{\tau_p = 3G\exp\left(-\frac{2\pi w}{b}\right)}$$

- For burger's vector (b) = dislocation width (w), τ_p has maximum value

$$\boxed{\tau_p = \frac{G}{180}}$$

This value is much smaller than theoretical shear strength

Yesterday we have seen that physically why should they move, but we can also look at the mathematical expression and then get convinced. So, this is the expression for Peierl-Nabarro stress. In the previous class we have given this expression, but this is equivalent to this expression, where *w* is the dislocation core width.

Here, it is pretty clear a is the interplanar spacing and b is the interatomic spacing or Burgers vector -- in the direction of the slip whatever is the interatomic space. τ_p is the stress required to cause slip by dislocation motion.

That is what we have discussed in the last class. τ_p is the stress required to cause dislocation motion which happens to be much smaller than the shear stress required to cause slip by glide of a plane of atoms.

All the atoms in the plane should be moved at once vis-a-vis causing slip by moving the dislocation. This is the stress required to cause slip through dislocation motion.

This is your interatomic distance, right? If this becomes smaller, what happens to τ_p? For a given interplanar spacing, let us say a is given. In that interplanar spacing, you should choose the minimum b, such that this becomes minimum, because negative sign is present.

The minimum value of b will give rise to minimum value of the shear stress required to cause the slip, and that is the reason why the dislocations prefer to move in the close packed directions, because in the close packed directions, the interatomic spacing is minimal.

What do you mean by close packed direction? The neighboring atoms are touching each other. If it is not close packed, they will be something like that. The distance between two neighboring atoms will be minimal when this is close packed.

Why that should be minimum? If that is minimum then your τ_p will be less. If you increase the value of b, then τ_p increases.

What is the meaning of that? That means, the stress required to cause slip is going to increase. If you want to cause plastic deformation of slip with minimum stress the best bet for you is to choose a direction in which b is minimum, and that happens to be in the close packed direction. That is why dislocations prefer to move in close packed directions.

Now, look at interplanar spacing. For τ_p to be minimum, -- with increase in a τ_p reduces. When does a increase? a is the interplanar spacing that will be maximum for close packed planes.

If you take two close packed planes, their interplanar spacing is going to be maximum. The higher value of interplanar spacing can be obtained between close packed planes and that leads to lower Peierls-Nabarro stress, and hence, that is also another way to say that dislocations prefer to move in close packed planes.

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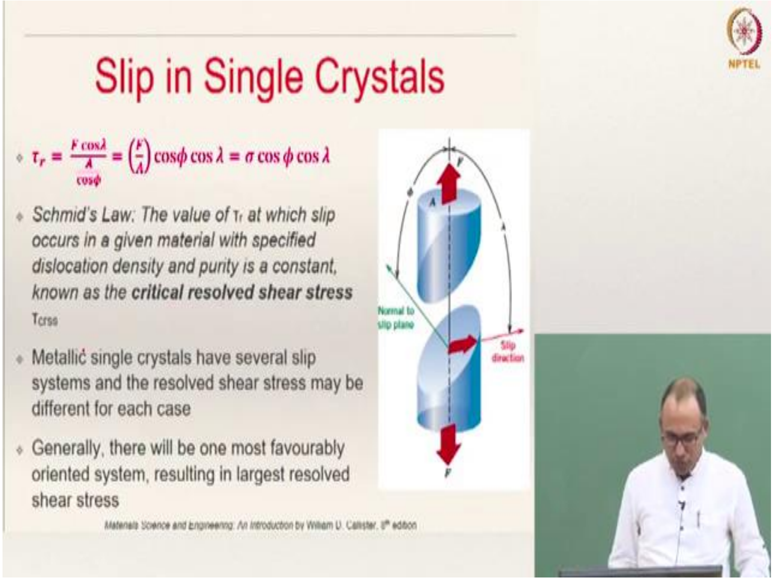

Slip in Single Crystals

- $\tau_r = \frac{F\cos\lambda}{\frac{A}{\cos\phi}} = \left(\frac{F}{A}\right)\cos\phi\cos\lambda = \sigma\cos\phi\cos\lambda$
- *Schmid's Law: The value of* τ_r *at which slip occurs in a given material with specified dislocation density and purity is a constant, known as the* ***critical resolved shear stress*** τ_{crss}
- Metallic single crystals have several slip systems and the resolved shear stress may be different for each case
- Generally, there will be one most favourably oriented system, resulting in largest resolved shear stress

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In the last class, we have derived the expression for resolved shear

$$\tau_r = \sigma\cos\phi\cos\lambda$$

where ϕ is the angle between the direction of the application of the load to the normal to the plane, on which you want to find the critical resolved shear stress. And λ is the angle between loading direction to the slip direction.

Once we have that information then you have this geometric factor $\cos\phi\cos\lambda$ which is called Schmid factor. For a given applied stress if you want to cause slip, you might want to have resolved shear stress to be maximum. How can you make your resolved shear stress to be maximum? By playing with ϕ and λ. For different orientations of slip plane that is described by ϕ you are going to get different resolved shear stress.

For a given ϕ different slip directions will give you different resolved shear stress. If you want to cause slip, all that we require is the resolved shear stress to be equal to the material property or critical resolved shear stress; when these two become equal that is when you initiate slip.

That means, slip can get initiated on any of the slip planes on any of the slip direction, along which you will have your resolved shear stress to be maximum. Typically, because with the same σ, one of the slip planes and one of the slip systems will result in maximum resolved shear stress.

When you have maximum resolved shear stress, that is going to be close to your critical resolved shear stress and naturally that is the slip system which will be subjected to initial slip.

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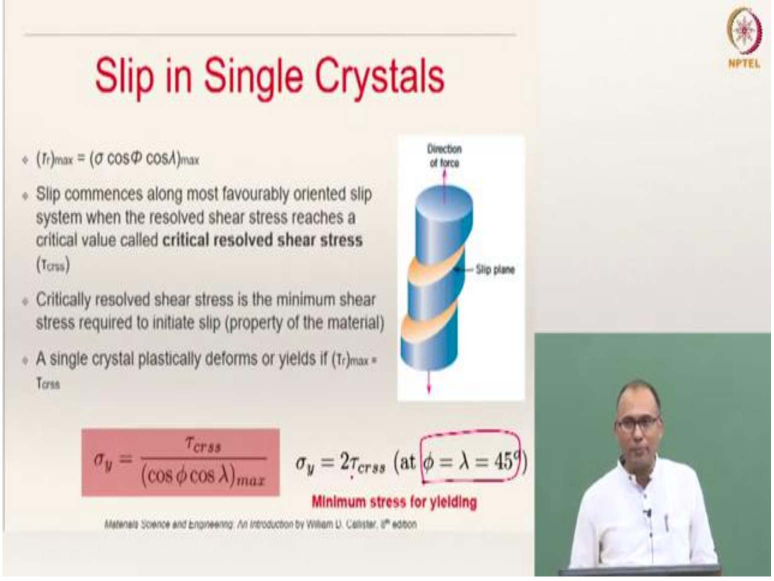

Slip in Single Crystals

- $(\tau_r)_{max} = (\sigma \cos\Phi \cos\Lambda)_{max}$
- Slip commences along most favourably oriented slip system when the resolved shear stress reaches a critical value called **critical resolved shear stress** (τ_{crss})
- Critically resolved shear stress is the minimum shear stress required to initiate slip (property of the material)
- A single crystal plastically deforms or yields if $(\tau_r)_{max} = \tau_{crss}$

$$\sigma_y = \frac{\tau_{crss}}{(\cos\phi\cos\lambda)_{max}}$$

$$\sigma_y = 2\tau_{crss} \text{ (at } \phi = \lambda = 45°)$$

Minimum stress for yielding

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$\cos\phi \cos\lambda$ is going to be maximum for $\phi = 45°$ and $\lambda = 45°$; then you will have your maximum resolved shear stress for a given σ.

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Slip in single crystal Zinc

Slip in a zinc single crystal. (From C. F. Elam, The Distortion of Metal Crystals, Oxford University Press, London, 1935.)

And that is reason you see the slip planes are oriented at 45° plane to the loading direction when the loading is uniaxial.

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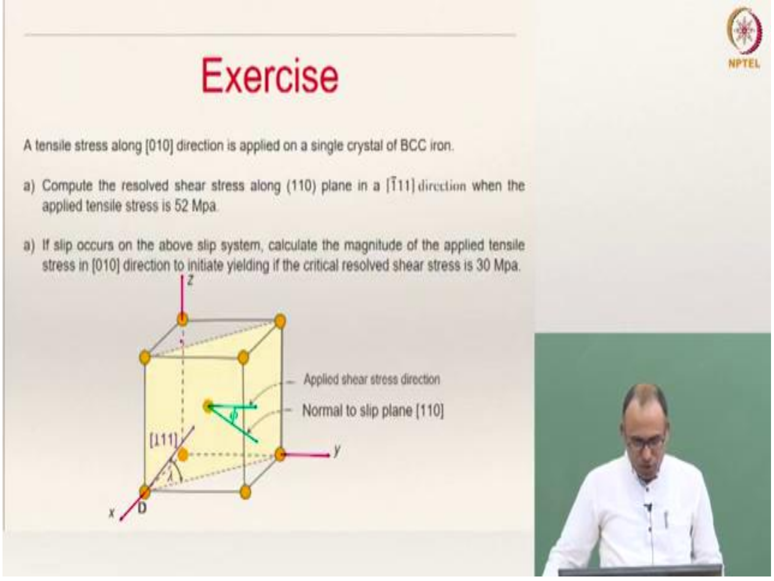

Exercise

A tensile stress along [010] direction is applied on a single crystal of BCC iron.

a) Compute the resolved shear stress along (110) plane in a $[\bar{1}11]$ direction when the applied tensile stress is 52 Mpa.

a) If slip occurs on the above slip system, calculate the magnitude of the applied tensile stress in [010] direction to initiate yielding if the critical resolved shear stress is 30 Mpa.

So, this is something that we have seen yesterday. We have solved this problem.

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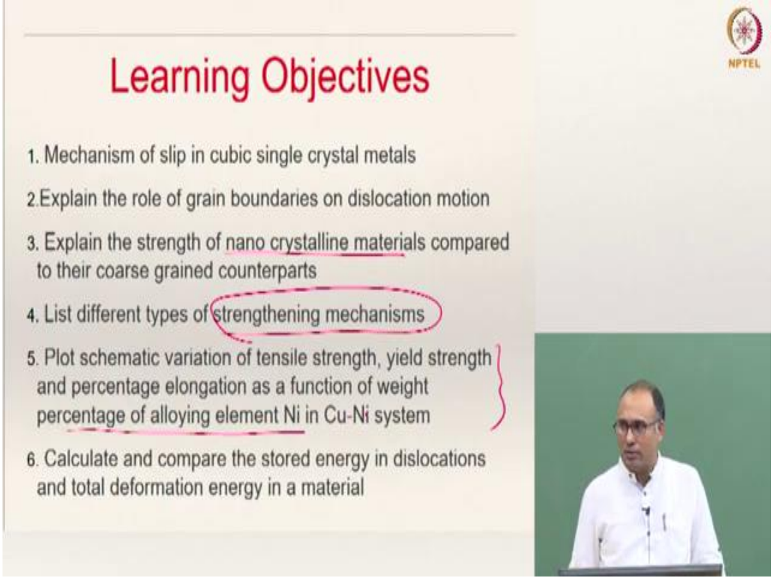

Learning Objectives

1. Mechanism of slip in cubic single crystal metals
2. Explain the role of grain boundaries on dislocation motion
3. Explain the strength of nano crystalline materials compared to their coarse grained counterparts
4. List different types of strengthening mechanisms
5. Plot schematic variation of tensile strength, yield strength and percentage elongation as a function of weight percentage of alloying element Ni in Cu-Ni system
6. Calculate and compare the stored energy in dislocations and total deformation energy in a material

We have understood the concept of resolved shear stress and how one would impart slip or in other words plastic deformation in a single crystal material by increasing the load or by playing with the geometry. You do not have to only increase the magnitude of the load, by changing the direction of the load, so that ϕ and λ are going to get varied, and that is how you can also impart slip with the same magnitude.

The next module that we are going to study is going to focus on the mechanism of slip in cubic single crystal material. How does plastic deformation actually take place in single crystal material? What are the different phases of plastic deformation, and what is the role of grain boundaries on dislocation motion? How can we explain the strength of nano crystalline materials compared to their coarse-grained counterparts? We will discuss these aspects.

And then, we will have to discuss about different kinds of strengthening mechanisms and why materials become harder or strengthened after plastic deformation. And, here this is something that we have also discussed, that the tensile strength, yield strength, and percentage of elongation of a solid solution change with the percentage of solute atoms. You can see how the percentage of alloying element of nickel in copper-nickel system is going to change the mechanical properties.

And then, calculate and compare the stored energy in dislocations and total deformation energy in a material. What is the meaning of that? That means, when you are applying a load, how much of the energy that you are putting in is actually going to dislocation and what fraction of that energy is going to be responsible or going to help the dislocation motion and what happens to the remaining fraction. So, that is something that is important to understand.

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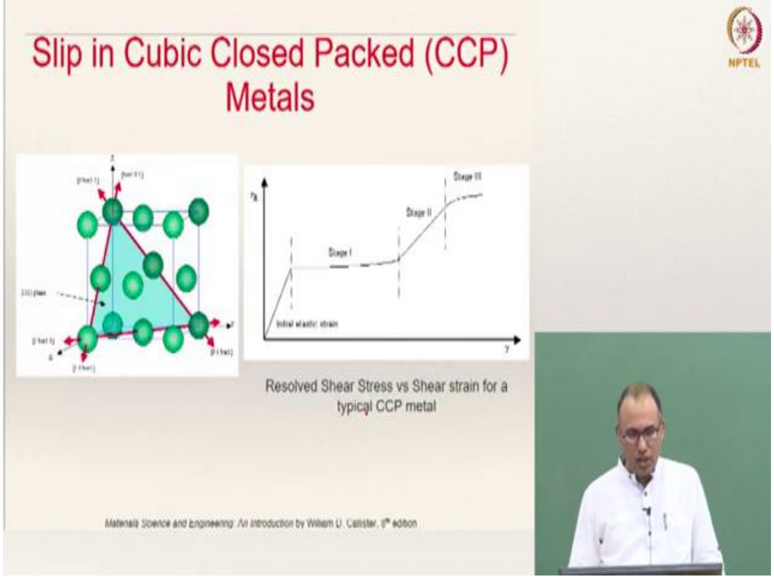
Slip in Cubic Closed Packed (CCP) Metals

Resolved Shear Stress vs Shear strain for a typical CCP metal

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This is a typical shear stress-shear strain plot on a single crystal material. This is typically the kind of stress strain response of a single crystal FCC material. BCC and HCP may also show similar behavior, but some of the stages are not going to be as prominent as we see in FCC. So, we are restricting ourselves here, to the FCC material, but qualitatively the figure looks almost similar.

Here, you have FCC material and this plane shown in light green background, is your (111) plane and the three directions shown in the red color are your [110] direction; family of [110] direction.

The reason why we have shown them here is because in FCC material that is the close packed plane and those are the close packed directions. In a single crystal material, you have basically three stages of plastic deformation. Stage 1, stage 2, stage 3, preceded by the initial elastic deformation.

When you are applying a load on a crystal lattice, the first thing that you will do is actually to stretch the bonds. That stretching of the bonds is actually your elastic deformation. As long as you are not breaking the bonds, the moment you remove the load, the system comes back to its original configuration.

Only when you break the bond, then it will not be able to come back to its original configuration; that is what we are calling plastic deformation. Breakage of bond through dislocation motion is what we have seen as slip and that is nothing but plastic deformation.

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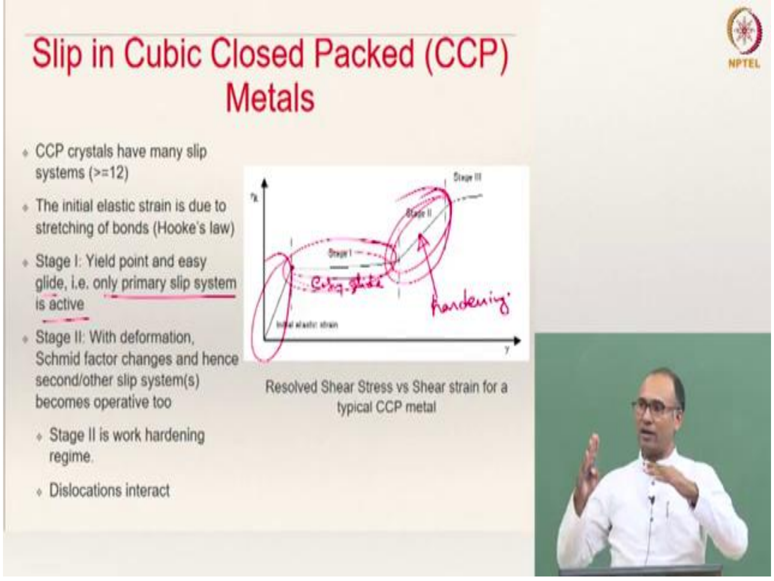

Slip in Cubic Closed Packed (CCP) Metals

- CCP crystals have many slip systems (>=12)
- The initial elastic strain is due to stretching of bonds (Hooke's law)
- Stage I: Yield point and easy glide, i.e. only primary slip system is active
- Stage II: With deformation, Schmid factor changes and hence second/other slip system(s) becomes operative too
  - Stage II is work hardening regime.
  - Dislocations interact

Resolved Shear Stress vs Shear strain for a typical CCP metal

We know that cubic crystals have many slip systems, typically greater than or equal to 12. In FCC you have 12, BCC 48 and HCP 12; however, in FCC, the 12 slip systems are primary slip systems. In BCC, you have 12, 12 and 24.

The first 12 that is (110) plane and [111] direction, this is the other way around compared to BCC. FCC, the close packed plane is (111) and close packed direction is [110]. In BCC, (110) is your close packed plane and [111] is our close packed direction.

That is your primary slip system (they are 12), and remaining 36 are secondary slip systems. Similarly, in HCP, the first three slip systems are the primary slip systems and remaining 9 are secondary slip system. Whereas, in FCC all the 12 are primary slip systems.

The initial elastic strain, as I mentioned, is due to stretching of bonds. It is described by Hooke’s law that we are all familiar with. And the stage 1 is also called easy glide.

This point is the yield point. What is happening? By now, your resolved shear stress has reached critical resolved shear stress on one of the slip systems and there is only one slip system. That is one active slip system. When we say active; that means, there is plastic deformation taking place on that slip system.

Only primary slip system is active in easy glide stage and up to yield point. Once you impart and that becomes easier almost at constant stress. It is not constant, it is slowly increasing, but almost at constant stress you will be able to impart large strain.

That is called easy glide because you are not required to put in additional effort. Once you are here at almost at the same stress level, you are able to cause plastic deformation; that is why it is called easy glide.

And the stage 2 is interesting. When you are causing plastic deformation, there is going to be some permanent deformation. As a result, you are actually changing the orientation of the crystal planes with respect to the loading direction.

Since you are changing the orientations, the slip system on which initial plastic deformation took place, it is possible that that is not going to be your favorable slip system anymore, some other slip system is coming into activity.

The new slip systems are getting activated. So, at the onset of stage 2 you do not have one slip system. There may be other slip systems getting activated which means, the dislocations are not moving in one plane, but in different planes.

Even within one plane you may be increasing the density of the dislocations, and as a result, what happens?

Imagine this example; we have discussed about the carpet example. When you create a hump, then you can easily move the hump toward the end to cause unit slip. Now, you create a hump here, imagine the carpet is big and some of your friends go to middle of the room and then they create a hump in other direction. You have a hump here, and there is a hump there, and both of you are moving like that.

What happens when they meet? You are actually able to cause plastic deformation with certain stress; when they meet what happens to the amount of load that you need to apply? The stress will be the same or you need to increase? Are you able to do this thought experiment? Will it increase or decrease?

Student: (Refer Time: 17:25)

Louder please. You can also try it with paper; you can create a hump here and then other side. And then move and then this is in one orientation, and that is in another orientation and they sort of entangle, right? Then they provide resistance to further motion.

That means, these dislocations are interacting and they hinder the dislocations. They do not stop, but they hinder the dislocation motion. You need to put in more effort to overcome that resistance and then you continue the dislocation motion.

By this time, you already have applied sufficiently large amount of stress; resolved shear stress, which means, it is very likely that several dislocations are in parallel planes or in other planes. The dislocation in one plane moving this way and the dislocation is moving and both of them are intersecting.

The dislocations on different planes can intersect, dislocations on the same plane can intersect and all of them going to hinder further slip and hence, in order to continue the plastic deformation, you need to apply higher and higher value of stress. That is what is called hardening.

That means, your material is becoming harder to cause plastic deformation. This response is called hardening response or it is also called work hardening. By doing work, the material is getting harder and harder.

This is primarily because the interaction of dislocations within the slip plane and within planes parallel to each other, and across the planes which are not parallel to each other.

You are going to have this sort of crazy interactions. The stage 3 of the plastic deformation in a single crystal material is typically the final stage. Here, the amount of resolved shear stress is so high that it can overcome all the obstacles.

The force that is applied or the resolved shear stress that is experienced by each and every dislocation is much larger or the dislocation locks experience much larger resolved shear stress so that, that will not be able to stop the plastic deformation,

That is usually the stage 3, and usually stage 3 is followed by fracture of the specimen. At the end of stage 3, you will have eventual fracture of the specimen. That is how the plastic deformation followed by fracture happens in a single crystal material. This is typical for cubic crystals, and this diagram is more relevant to FCC materials, although you will observe similar stage 1, stage 2, stage 3 in BCC and HCP materials.

The extent of stage 1 is going to be very small for BCC. Because there are so many slip systems in BCC, it is possible that the other slip systems are also getting activated. As a result, you will have stage 2 setting in much earlier.

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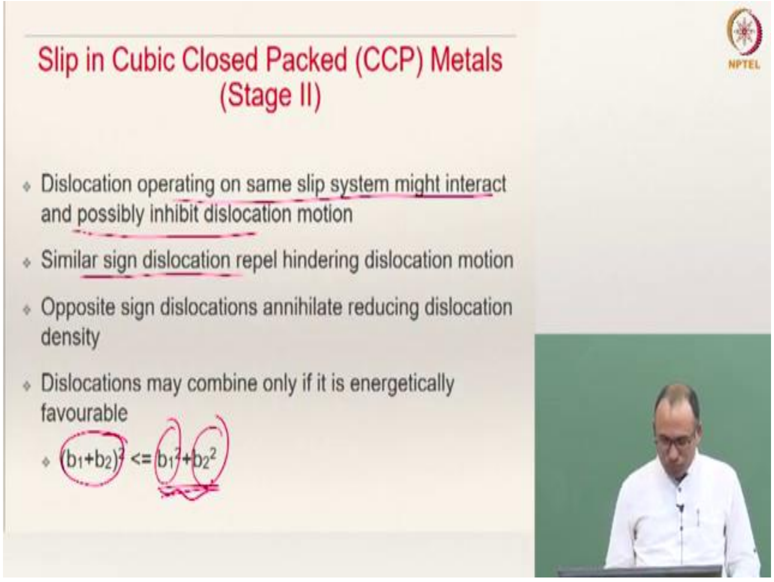

Slip in Cubic Closed Packed (CCP) Metals (Stage II)

- Dislocation operating on same slip system might interact and possibly inhibit dislocation motion
- Similar sign dislocation repel hindering dislocation motion
- Opposite sign dislocations annihilate reducing dislocation density
- Dislocations may combine only if it is energetically favourable
  - $(b_1+b_2)^2 <= b_1^2+b_2^2$

How does the dislocation interaction happen? We said the stage 2 or the work hardening stage is due to interaction between the dislocations, right? So, they interact and then they stop each other or they enhance the dislocation motion, all sorts of things can happen.

Typically, the dislocations operating on same slip system might interact as I mentioned and they possibly inhibit dislocation motion. How do they inhibit dislocation motion? For instance, similar sign dislocations, what do they do? If they are same sign, what happens if you apply further slip? They cannot move, they cannot come closer because the stress fields around them will force them to repel each other.

What happens if opposite sign dislocations are coming together? They annihilate, they become perfect and then what happens? There is no dislocation at all. So, there is no plastic deformation, you cannot cause slip; you are actually going from an imperfect crystal to a perfect crystal. How do the dislocation combine? How do they decide whether they want to combine or not? Will they always combine or will they not? Let us see. Whether the two dislocations have to combine or do not choose to combine, it should be depending on the energetics.

Consider two dislocations each of burgers vector $\mathbf{b_1}$ and $\mathbf{b_2}$. The energy associated with the first dislocation is proportional to b_1^2, and the second dislocation is b_2^2. The total energy is this, when they are combining the resultant dislocation will be $\mathbf{b_1} + \mathbf{b_2}$. So, the energy of the resultant dislocation will be $(\mathrm{b}_1 + \mathrm{b}_2)^2$. If this energy happens to be less than or equal to this energy, only then the dislocations combine, otherwise they don't; only then they will interact and they will form a new dislocation, otherwise they will not interact.

When the dislocations are of opposite sign, this quantity is always going to be smaller than this quantity. That is the reason why when two dislocations are of the opposite sign, they nicely combine, but if the dislocations are of the same sign, then this quantity is going to be larger than this quantity and hence they have no option but to repel.

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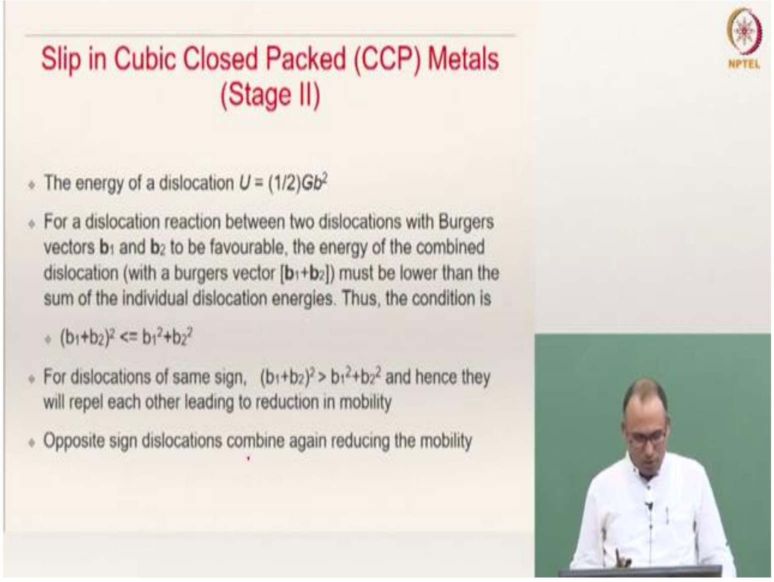

Slip in Cubic Closed Packed (CCP) Metals (Stage II)

- The energy of a dislocation $U = (1/2)Gb^2$
- For a dislocation reaction between two dislocations with Burgers vectors $\mathbf{b}_1$ and $\mathbf{b}_2$ to be favourable, the energy of the combined dislocation (with a burgers vector $[\mathbf{b}_1+\mathbf{b}_2]$) must be lower than the sum of the individual dislocation energies. Thus, the condition is
  - $(b_1+b_2)^2 <= b_1^2+b_2^2$
- For dislocations of same sign, $(b_1+b_2)^2 > b_1^2+b_2^2$ and hence they will repel each other leading to reduction in mobility
- Opposite sign dislocations combine again reducing the mobility

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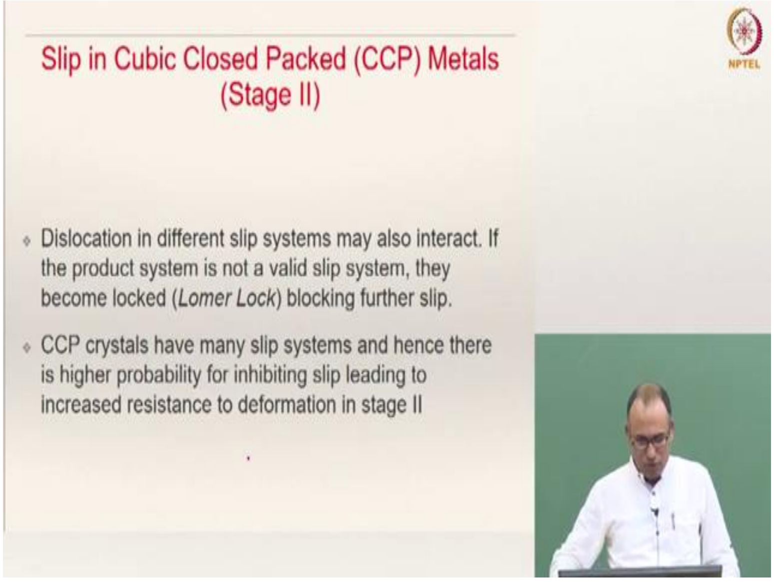

Slip in Cubic Closed Packed (CCP) Metals (Stage II)

- Dislocation in different slip systems may also interact. If the product system is not a valid slip system, they become locked (*Lomer Lock*) blocking further slip.
- CCP crystals have many slip systems and hence there is higher probability for inhibiting slip leading to increased resistance to deformation in stage II

That is about dislocations in the same slip systems, you can also have dislocations on different slip systems interacting.

If this is one slip system and this is another slip system, and their dislocations are interacting, if they have to continue to interact, they should be working on a resulting slip system and that resulting slip system maybe an invalid slip system. What is the meaning of invalid slip system?

Such a slip system cannot exist in this material, both from close packed density and close packed directions perspective. That is not going to impart further dislocation motion by working in that slip system; that means, the dislocations now don't know what to do. And then they get locked, right? Like in the carpet example one hump going from this side, another hump coming across not in the same plane, but in a different direction they get locked. And if you want to overcome what you need to do? You need to apply higher stress to break open the lock.

Until you break open the lock, you cannot continue further plastic deformation, right? Such a lock is called Lomer Lock and you need to apply higher stress in order to break open that lock to cause further dislocation motion. Cubic crystals have many slip systems as we have discussed and hence, typically they have very high probability of inhibiting slip, leading to the stage 2 deformation. That is why the stage 2 is characteristic of cubic single crystals, wherein, the dislocations interact with each other and cause work hardening.