


Basics of Materials Engineering
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Lecture – 19
Defects in Crystalline Material – 4
(Slip in Single Crystals and Resolved Shear Stress)

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Slip in Single Crystals

- ♦ Edge, Screw and mixed dislocations move in response to applied shear stress
- ♦ Even the applied stress is tensile, shear components exist at some alignments to applied stress direction called resolved shear stress
- ♦ Resolved shear stress depends on applied stress, orientation of slip plane and slip direction
- ♦ Slip occurs when the shear stress acting in the slip direction exceeds a critical value



Let us now try to understand a little bit more about slip in single crystals. We have seen that materials have line defects called dislocations, and that the motion of these dislocations is what we call plastic deformation in crystalline materials. When these dislocations move in a crystalline lattice, they prefer to move in certain planes and, in those planes, in certain directions. These are called slip planes and slip directions respectively and, together, they constitute a slip system. The slip systems are different for different crystal structures.

We have seen that FCC has 12 slip systems. Similarly, BCC in total, has 48 slip systems and HCP has 12. We also noted that FCC materials usually tend to be more ductile, in spite of BCC having a larger number of slip systems.


As we have discussed, we have three different kinds of dislocations: edge dislocation, screw dislocation, and mixed dislocations. Also, when you apply a shear stress on a crystal lattice, then the dislocations tend to move within the crystal lattice. Even if the

applied stress is tensile in nature, there are orientations along which you will have some shear. That is what you have learned when studying Mohr's circle. If the loading applied in a particular direction is known, then you can draw a Mohr circle and identify the state of stress in other planes. You can also identify planes which are under pure shear. The same concepts apply here as well.

So, even when the applied stress is tensile, shear components exist at some orientations or alignments. That shear stress is called resolved shear stress and slip happens due to this resolved shear stress. Note that the resolved shear stress depends on the applied shear stress, the orientation of the slip plane, and the slip direction to the applied loading.

In a particular orientation, if you want to calculate the resolved shear stress, it will depend on the magnitude of the load that you are applying and orientation of this load, with respect to the slip plane and the slip direction. Slip, then, occurs when the resolved shear stress, acting in the slip direction, exceeds a critical value. That means, it should be able to break the bond. In other words, the resolved shear stress should be able to overcome the material resistance (strength). If it does not exceed this value, slip will not occur; only elastic deformation will occur.

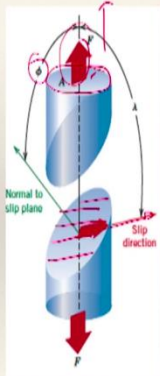
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Slip in Single Crystals

- ◆ ϕ is the angle between the normal to the slip plane and loading axis
- ◆ λ is the angle between slip direction and loading axis
- ◆ Area of the slip plane = $A/\cos \phi$
- ◆ Component of the force along slip direction = $F \cos \lambda$
- ◆ The resolved shear stress on the slip plane

$$\tau_r = \frac{\text{Resolved force acting on the slip plane}}{\text{Area of the slip plane}}$$





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So, let us assume that this is a single crystal material and you are applying a tensile load, F . Also, assume that you want to study the slip on this particular plane (as shown). Define ϕ to be the angle between the loading direction and the slip plane normal (of

whichever plane that you are interested in). Further, define λ to be the angle between the applied load and the slip. Let the cross sectional area along the loading direction be A .

The area of the slip plane will then be,

$$\text{Area of slip plane} = \frac{A}{\cos \phi}$$

Similarly, the component of that force acting on along the slip direction is $\frac{F \cos \lambda}{\cos \phi}$.

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The slide is titled "Slip in Single Crystals" and features a diagram of a crystal under stress. The diagram shows a cylinder representing a crystal with a vertical force F applied. A slip plane is shown at an angle ϕ to the horizontal. The normal to the slip plane is shown at an angle λ to the vertical force. The resolved shear stress τ_r is shown acting on the slip plane. A dislocation is shown on the slip plane. The area of the slip plane is labeled $A / \cos \phi$. The slide also contains a list of points:

- ♦ $\tau_r = \frac{F \cos \lambda}{A \cos \phi} = \left(\frac{F}{A}\right) \cos \phi \cos \lambda = \sigma \cos \phi \cos \lambda$ (Schmid's Law)
- ♦ Schmid's Law: The value of (τ) at which slip occurs in a given material with specified dislocation density and purity is a constant, known as the critical resolved shear stress
- ♦ Metallic single crystals have several slip systems and the resolved shear stress may be different for each case
- ♦ Generally, there will be one most favourably oriented system, resulting in largest resolved shear stress

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Therefore, the resolved shear stress, τ_r , can be calculated as

$$\tau_r = \frac{F \cos \lambda}{\frac{A}{\cos \phi}} = \frac{F}{A} \cos \lambda \cos \phi = \sigma \cos \lambda \cos \phi$$

Now, we define a concept called Schmid's law. Schmid's law states that the value of τ_r , at which slip occurs in a given material with a specified dislocation density and purity, is a constant. This constant value is known as the critical resolved shear stress (τ_{crss}).


For any crystalline material, what is the critical resolved shear stress? That is the shear stress at which your slip in the material occurs, or initiates. It is a material property, like the yield strength of the material.

We know that metallic single crystals have several slip systems. The resolved shear stress may be different for each of these cases; not all the slip systems will have same resolved shear stress. Each slip system may have different resolved shear stress. For instance, in BCC, there are about 48 slip systems and each one of them will have different resolved shear stress, because the directions are different.

In general, what happens is you have several slip systems that are possible. Some of them are favourable and some of them are not so favourable. Typically, there will be one most favourably oriented slip system amongst all the slip. A slip system is called favourably oriented when the resolved shear stress on that slip system is maximum compared to all other slip systems. So, in principle, you can have several slip systems and all of them may be active, but one among them is most active.

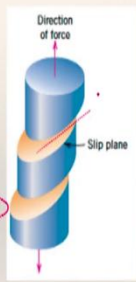
Note that the σ value is same on all the systems, but the quantity $(\cos \lambda \cos \phi)$ is the one that determines the magnitude of the resolved shear stress on that particular slip system. This factor is called Schmid's factor. The favourable slip system is the one which maximizes the Schmid factor, so that the resolved shear stress is maximum.

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Slip in Single Crystals

- ◊ $(\tau_r)_{max} = (\sigma \cos \phi \cos \lambda)_{max}$
- ◊ Slip commences along most favourably oriented slip system when the resolved shear stress reaches a critical value called critical resolved shear stress (τ_{crss})
- ◊ Critically resolved shear stress is the minimum shear stress required to initiate slip (property of the material)
- ◊ A single crystal plastically deforms or yields if $(\tau_r)_{max} = \tau_{crss}$




$$\sigma_y = \frac{\tau_{crss}}{(\cos \phi \cos \lambda)_{max}}$$

$$\sigma_y = 2\tau_{crss} \text{ (at } \phi = \lambda = 45^\circ \text{)}$$

Minimum stress for yielding

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The maximum value of τ_r is given as

$$(\tau_r)_{max} = \sigma (\cos \lambda \cos \phi)_{max}$$

We know that slip commences along most favourably oriented slip system when the resolved shear stress reaches a critical value called critical resolved shear stress and the critical resolved shear stress is the minimum shear stress required to initiate slip, and that is the property of the material.

So, a single crystal plastically deforms or yields when $(\tau_r)_{max} = \tau_{crss}$.

If you are doing a uniaxial tension experiment on a single crystal material, as opposed to a poly crystal material, then, the yield strength of the material will be

$$\sigma_y = \frac{\tau_{crss}}{(\cos \lambda \cos \phi)_{max}}$$

The maximum value of the Schmid factor will be when $\lambda = \phi = 45^\circ$, therefore the minimum value of σ_y will be $\sigma_y = 2 \tau_{crss}$.

Thus, there is a correlation between these two quantities.

Hence, if you apply a load in the normal direction, i.e. along the axis of the cylindrical specimen, then your slip tends to happen at 45° . Also, in a single crystal material, you can see the slip bands on the surface.

In summary, if you take a single crystal material and if you are apply any uniaxial load on the single crystal material, then you can derive something called resolved shear stress. When the resolved shear stress becomes equal to a property of the material called critical resolved shear stress; that is when slip initiates.

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Exercise

A tensile stress along $[010]$ direction is applied on a single crystal of BCC iron.

a) Compute the resolved shear stress along (110) plane in a $[\bar{1}11]$ direction when the applied tensile stress is 52 MPa.

a) If slip occurs on the above slip system, calculate the magnitude of the applied tensile stress in $[010]$ direction to initiate yielding if the critical resolved shear stress is 30 MPa.

"phi is the angle between force and slip plane normal and lambda is the angle between force and slip direction."

$\tau_r = \sigma \cos \phi \cos \lambda$

52 MPa angle between F & slip direction

lambda force & slip plane normal

Let us now solve a problem. You have a single crystal material which is BCC iron and on that you are applying a tensile stress along $[010]$ direction.

Student: (Refer Time: 15:42).

That is your $[010]$ direction (refer to video). Now, we need to compute the resolved shear stress along $[110]$ plane, the one shown in yellow, in $[\bar{1}11]$ direction.

Student: (Refer Time: 16:23).

First, we need to calculate Schmid's factor when the applied stress, σ , is 52 MPa. So, we have

$$\tau_r = \sigma \cos \lambda \cos \phi$$

So, what is the direction of the force?

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Exercise

A tensile stress along $[010]$ direction is applied on a single crystal of BCC iron.

- Compute the resolved shear stress along (110) plane in a $[\bar{1}11]$ direction when the applied tensile stress is 52 MPa.
- If slip occurs on the above slip system, calculate the magnitude of the applied tensile stress in $[010]$ direction to initiate yielding if the critical resolved shear stress is 30 MPa.

$\tau_R = \sigma \cos \phi \cos \lambda$
 52 MPa angle between F & $[110]$ direction
 $\lambda = \text{force on slip plane normal}$
 $[010] \rightarrow \phi$
 $[110] \rightarrow \lambda$
 $[111]$

The angle between $[010]$ and $[110]$ will give you ϕ and the angle between $[010]$ and $[\bar{1}11]$ will give you λ . The calculation of these is left as an exercise.

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Exercise

ϕ is the angle between $[010]$ and $[110]$
 λ is the angle between $[010]$ and $[\bar{1}11]$

$$\tau_R = \sigma \cos \phi \cos \lambda = 52 (\cos 45^\circ) (\cos 54.7^\circ) = 21.3 \text{ MPa}$$

$$\sigma_y = \frac{30}{(\cos 45^\circ) (\cos 54.7^\circ)} = 73.4 \text{ MPa}$$

$$\tau_r = \sigma \cos \lambda \cos \phi = 52 \cos 45^\circ \cos 54.7^\circ = 21.3 \text{ MPa}$$

And, now, the second question is if slip occurs on the above slip systems, calculate the magnitude of the applied tensile stress in [010] direction to initiate the yielding, if the resolved shear stress is 30 MPa.

We note that the 52 MPa applied in the previous problem resulted in a resolved shear stress of 23.1 MPa, which is lesser than the critical value of 30 MPa. Thus, it will not yield for that stress value. Now, if you have to impart slip, then what should be the stress that you need to apply? The calculations are pretty straightforward and we find that we need to apply 73.4 MPa in that direction.

Now, if you change your loading direction, you will observe a different behaviour. The material will not yield at the same value of the stress. Since, you are changing the loading direction, the resolved stress will be different and yield may occur earlier or later.

Thus, if you change the loading direction, you would observe different values of yield strength for your material. That means, yield strength is direction dependent. It is not isotropic anymore. The material property is not isotropic anymore; it is anisotropic - it depends on the direction of the application of your load. So, if you see FCC and BCC material (or single crystal materials), they have directional dependent properties.