Fundamentals of Combustion for Propulsion

Prof. H S Mukunda

Department of Mechanical Engineering

Indian Institute of Technology, Madras

Lecture – 08

Premixed and diffusion flames: principal features and differences – Part 2

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An interlude…
Mathematics
of premixed flames

We will start with the one aspect which I seem to have ignored and we will do a brief interlude into what I call as mathematics of pre mixed flames. Why did I do this? In the 1970's there were lot of activity related to mathematics of premixed flames; and what you will find in standard books of Williams and others who have liked this business of the mathematics of pre mixed flames, created they are of that everybody should do something also has done I can how to you.

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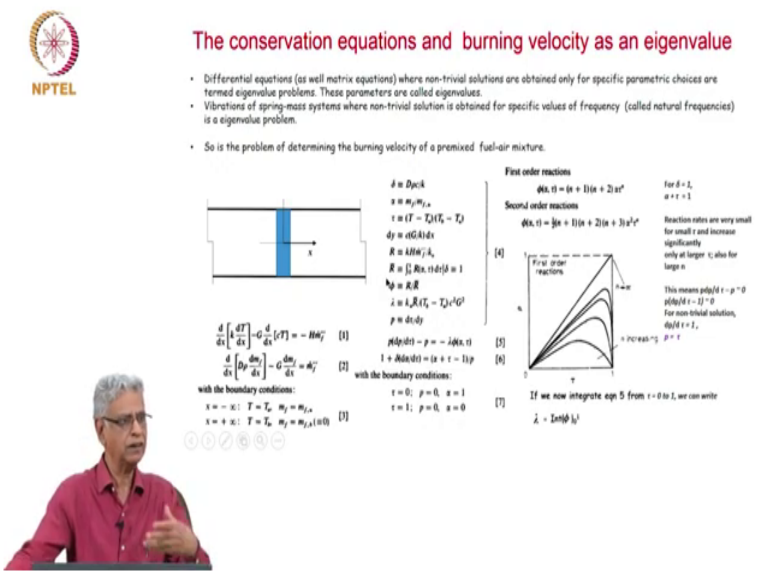

The conservation equations and burning velocity as an eigenvalue

- Differential equations (as well matrix equations) where non-trivial solutions are obtained only for specific parametric choices are termed eigenvalue problems. These parameters are called eigenvalues.
- Vibrations of spring-mass systems where non-trivial solution is obtained for specific values of frequency (called natural frequencies) is a eigenvalue problem.
- So is the problem of determining the burning velocity of a premixed fuel-air mixture.

$$\frac{d}{dx}\left[k\frac{dT}{dx}\right]-G\frac{d}{dx}[cT]=-H\dot{m}_f''' \quad [1]$$

$$\frac{d}{dx}\left[D\rho\frac{dm_f}{dx}\right]-G\frac{dm_f}{dx}=\dot{m}_f''' \quad [2]$$

with the boundary conditions:

$x=-\infty$: $T=T_u$, $m_f=m_{f,u}$

$x=+\infty$: $T=T_b$, $m_f=m_{f,b}(\equiv 0)$ [3]

$\delta \equiv D\rho c/k$

$\alpha \equiv m_f/m_{f,u}$

$\tau \equiv (T-T_u)/(T_b-T_u)$

$dy \equiv c(G/k)dx$

$R \equiv kH\dot{m}_f'''/k_u$

$\bar{R} \equiv \int_0^1 R(\alpha,\tau)d\tau|\delta \equiv 1$

$\phi \equiv R/\bar{R}$

$\lambda \equiv k_u\bar{R}(T_b-T_u)c^2G^2$

$p \equiv d\tau/dy$ [4]

$p(dp/d\tau)-p=-\lambda\phi(\alpha,\tau)$ [5]

$1+\delta(d\alpha/d\tau)=(\alpha+\tau-1)/p$ [6]

with the boundary conditions:

$\tau=0$; $p=0$, $\alpha=1$

$\tau=1$; $p=0$, $\alpha=0$ [7]

First order reactions

$\phi(\alpha,\tau)=(n+1)(n+2)\alpha\tau^n$

Second order reactions

$\phi(\alpha,\tau)=\frac{1}{2}(n+1)(n+2)(n+3)\alpha^2\tau^n$

For $\delta = 1$,
$\alpha + \tau = 1$

Reaction rates are very small for small τ and increase significantly only at larger τ; also for large n

This means $p\,dp/d\tau - p = 0$
$p(dp/d\tau - 1) = 0$
For non-trivial solution,
$dp/d\tau = 1$,
$p = \tau$

If we now integrate eqn 5 from $\tau = 0$ to 1, we can write

$\lambda =$ [illegible]

Show you the relationship to the simple expression that we have and also a certain question related to burning velocity; burning velocity is often treated as an eigenvalue. Are everybody is everybody familiar with the idea of eigenvalue or no?

Student: Yes.

Ok. If you take a spring mass system; you have something called a natural frequency. That natural frequency is an eigenvalue of that problem. You have the equations for vibration then you can solve the problem you will get somehow an equation which you will find non trivial solutions for specific values of a parameter.

Those specific values turn out to be frequency, that is eigenvalue. This eigenvalue is also there in matrix theory you can calculate eigenvalues and so on, they related to this aspect only ok.

Fame speed or the burning velocity is an eigenvalue of the conservation equations, it is probably not well recognized because you need not talk these days in that mode. It therefore, important for us to recognize the meaning of it just to elevate our elegance of understanding.

It may not add much value to reality, but our appreciation of the physics will improve. So, what is it that is being said you have a 1 dimensional flame of some thickness at the center of it, I will put a coordinate system 1 dimensional. So, only quarter x and the conservation equations essentially related to heat balance as well as the mass balance and u is written exactly like this conduction, convection and reaction with the h being the heat release for the mass fraction of the fuel you can write the same thing.

You can write it for oxidizer, you can write it for product they are all related by stoichiometric ratio. In this particular analysis which I have chosen, there is a parameter called D rho c p by k which is called essential Lewis number. I show you the results for Lewis number equal to 1 which is pretty good for methane for some other species like hydrogen which is different.

It is not crucial matter for discussion here, but since the analysis was done like that I will show you what it implies. What is shown here essentially is a set of transformations alpha is essentially the mass fraction at local location divided by mass fraction at unburnt state tau essentially dimensionless temperature T minus T u minus T b minus T u T b the adiabatic flame temperature.

Some coordinate d y is equal to G rho u rho u c p by k into d x. Reaction written like this phi is some R by R bar and here is one parameter called lambda. Lambda contains rho u square. This is in fact the parameter which is of importance.

We defined, a quantity called p d T equal to d T d y is such mathematical transformation. We noticed that in this the coordinate goes from minus infinity to infinity there is no specific location which has physical significance, I can locate it anywhere in this; but that is in this spectrum of equations.

That is the key to saying that flame period is an eigenvalue, but you will not be able to see it in this mode. So, we make a transformation like p equal to d tau by d y is equal to d T d d T d x essentially y; y is essentially related to x here and treated like this. Look at an equation of this format p d; p d tau minus p equal to minus lambda phi tau this is the diffusing term, this is the convection term, this is the reaction rate term.

Corresponding terms for mass diffusion because of non unity Lewis number and the reaction rate expression these were all at a time an things were being developed. Spaulding wrote down instead of a u by a r T b. He was more comfortable with an algebraic expression, but it makes no impact on the analysis it was not probably not required, it has done.

So, it is he took it like this; and if you take Lewis number equal to 1 you can show in this equation very simple analysis tells you alpha plus tau is equal to 1. So, dimensionless temperature and mass fraction ratio they are all related to 1.

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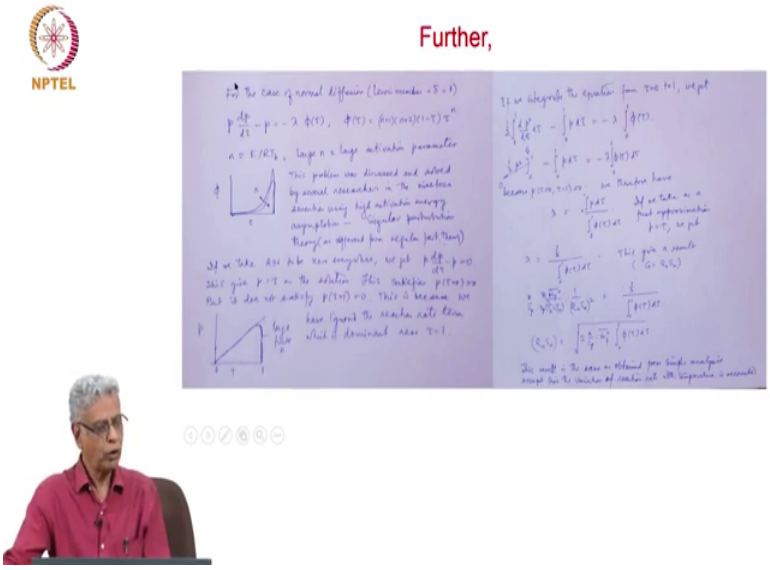

Further,

For the case of normal diffusion (Lewis number = δ = 1)

$p\frac{dp}{d\tau} - p = -\lambda\phi(\tau)$, $\phi(\tau) = (n+1)(n+2)(1-\tau)\tau^n$

$n \simeq E/RT_b$, large n ≡ large activation parameter

This problem was discussed and solved by several researchers in the nineteen seventies using high activation energy asymptotics — singular perturbation theory (as opposed from regular pert. theory)

If we take RHS to be zero everywhere, we get $p\frac{dp}{d\tau} - p = 0$. This gives $p = \tau$ as the solution. This satisfies $p(\tau=0)=0$ but it does not satisfy $p(\tau=1)=0$. This is because we have ignored the reaction rate term which is dominant near $\tau = 1$.

If we integrate the equation from τ=0 to 1, we get

$\frac{1}{2}\int_0^1 \frac{dp^2}{d\tau}d\tau - \int_0^1 p\,d\tau = -\lambda\int_0^1 \phi(\tau)$

$\left[\frac{p^2}{2}\right]_0^1 - \int_0^1 p\,d\tau = -\lambda\int_0^1 \phi(\tau)d\tau$

0 because $p(\tau=0, \tau=1) = 0$. We therefore have

$\lambda = \frac{\int p\,d\tau}{\int_0^1 \phi(\tau)d\tau}$. If we take as a first approximation $p = \tau$, we get

$\lambda = \frac{1/2}{\int_0^1 \phi(\tau)d\tau}$. This gives a result ($G = \rho_u S_u$)

[illegible] $= \frac{1/2}{\int_0^1 \phi(\tau)d\tau}$

$(\rho_u S_u) = \sqrt{2\,\frac{k}{c_p}\,[illegible]\int_0^1 \phi(\tau)d\tau}$

This result is the same as obtained from simple analysis except that the variation of reaction rate with temperature is accounted.

Now, I instead of showing here I will go to the next slide, where I put down these things because they are from your notes which I will had used for classes in way back in 80's for the case of normal diffusion; that means, Lewis number equal to one their same equation will become very simply this.

Alpha will become one minus tau therefore, this is the function of only tau there is no change. I am because these two are connected and n is equal to essentially you can show is proportional to E by R T b and because activation of these consider large for many fuel based fractions, which is a large activation parameter.

This problem was discussed and solved by several researchers in the 1970's using what is called high activation energy asymptotics or singular perturbation theory, and this is different from regular perturbation. I will not go to describe what is singular perturbation theory and so on. I will just show you what it implies in a moment.

Suppose, you take a large activation energy it turns out that the behavior of phi verses tau will be there is nearly 0 everywhere excepting towards tau equal to 1. Say n increases it; it goes like this reason is simple for any tau when n is very large you know tau power n is very small half to the power of 10 is very small.

So, we will discover that the phi is very small nearly where excepting near tau equal to 1, this is also principle of activation energy asymptotics where you say in a premix flame most reaction is concentrated at adiabatic condition flame temperature condition. That is the classical approach suppose I take the right hand side is nearly 0.

We where we get p d p by d tau minus p equal to 0? For which the solution is essentially p equal to tau what does this do you see here p versus tau p equal to tau it goes like this. You will see that it satisfies p equal to 0 at tau equal to 0, but it does not satisfy p equal to 0 at tau equal to 1 which is what is the required in the p tau is equal to 1 p equal to 0..

What is p? Essentially gradient of temperature, what this says is the gradient of temperature is 0 both at minus infinity and plus infinity which is classical there is nothing special about it you do not find infinite temperatures therefore, you must have gradients 0 at large distances that is all the message. This shows actually it is going to infinity here temperature is going to infinity here it cannot happen.

Now, that what asymptotic energy asymptotics asymptotic activation energy stuff we will do is to bring this thing down to 0 at tau equal to 1. And this occurs because we have ignored the reaction rate term and if you include it you will get it, but we do not go to a rigorous solution, but we will do take a shortcut. If you integrate this equation from tau equal to 0 to 1 is nothing but 0 to 1 half of d p square by d tau which is just take it inside and you get this expression minus p integral b d tau is equal to minus lambda integral of phi to 1.

P is d tau by d y it is equal to 0 both at tau is equal to 0 and 1 this goes off and you can write this simply as integral 0 to 1 p d tau by this expression. Now, p is equal to tau is true everywhere, an integral formalism it does not matter if you ignore some put p equal to tau you will get essentially half.

From this you get simply I mean it is like you know stir something your gravy just like that you will get rho u s u is equal to this expression which is identical to whatever we were discussing, root of k by c p into average reaction rate is in this is nothing but average reaction rate expression balanced by the actual variation of reaction rate.

So, the point I am making is; its possible to do a rigorous analysis and justify that, but you cannot invalidate the simple analysis because it is obtained by a simple averaging process this is the message which I wanted to communicate through this.

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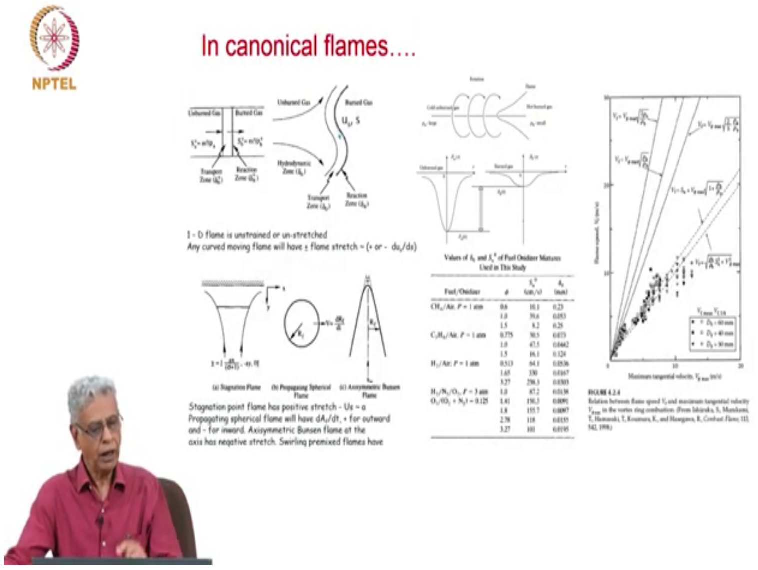

In canonical flames….

1 - D flame is unstrained or un-stretched

Any curved moving flame will have ± flame stretch ~ (+ or - du/ds)

(a) Stagnation Flame (b) Propagating Spherical Flame (c) Axisymmetric Bunsen Flame

Stagnation point flame has positive stretch - Us ~ a
Propagating spherical flame will have dA/dt, + for outward
and - for inward. Axisymmetric Bunsen flame at the
axis has negative stretch. Swirling premixed flames have

Values of δ_0 and S_u^0 of Fuel Oxidizer Mixtures Used in This Study

Fuel/Oxidizer	ϕ	S_u^0 (cm/s)	δ_0 (mm)
CH_4/Air, P = 1 atm	0.6	10.1	0.23
	1.0	39.6	0.053
	1.5	8.2	0.25
C_3H_8/Air, P = 1 atm	0.775	30.5	0.073
	1.0	47.5	0.0442
	1.5	16.1	0.124
H_2/Air, P = 1 atm	0.513	64.1	0.0536
	1.65	330	0.0167
	3.27	238.3	0.0303
$H_2/N_2/O_2$, P = 3 atm $O_2/(O_2 + N_2)$ = 0.125	1.0	87.2	0.0138
	1.41	150.3	0.0091
	1.8	155.7	0.0087
	2.78	118	0.0135
	3.27	101	0.0195

FIGURE 4.2.4
Relation between flame speed V_f and maximum tangential velocity $V_{\theta max}$ in the vortex ring combustion. (From Ishizuka, S., Murakami, T., Hamasaki, T., Koumura, K., and Hasegawa, R., *Combust. Flame* 113, 542, 1998.)

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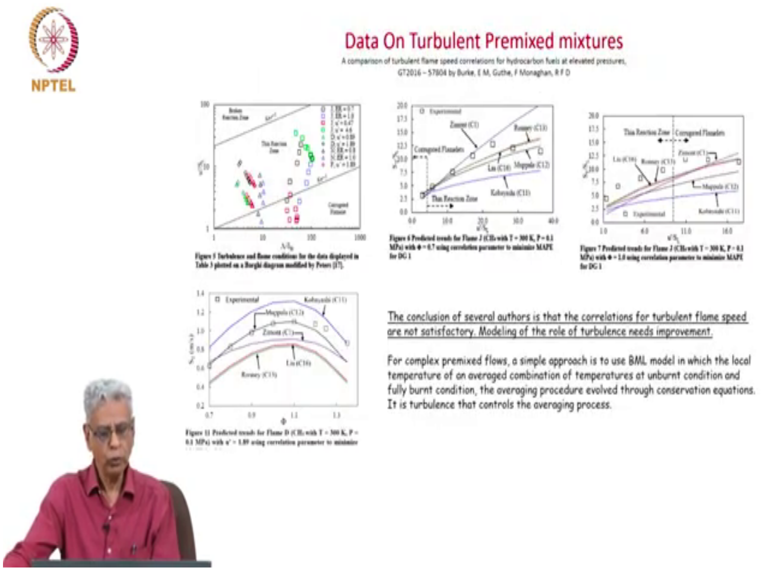

Data On Turbulent Premixed mixtures

A comparison of turbulent flame speed correlations for hydrocarbon fuels at elevated pressures, GT2016 – 57804 by Burke, E M, Guthe, F Monaghan, R F D

Figure 5 Turbulence and flame conditions for the data displayed in Table 3 plotted on a Borghi diagram modified by Peters [37].

Figure 6 Predicted trends for Flame J (CH₄ with T = 300 K, P = 0.1 MPa) with Φ = 0.7 using correlation parameter to minimize MAPE for DG 1

Figure 7 Predicted trends for Flame J (CH₄ with T = 300 K, P = 0.1 MPa) with Φ = 1.0 using correlation parameter to minimize MAPE for DG 1

Figure 11 Predicted trends for Flame D (CH₄ with T = 300 K, P = 0.1 MPa) with u' = 1.89 using correlation parameter to minimize

<u>The conclusion of several authors is that the correlations for turbulent flame speed are not satisfactory. Modeling of the role of turbulence needs improvement.</u>

For complex premixed flows, a simple approach is to use BML model in which the local temperature of an averaged combination of temperatures at unburnt condition and fully burnt condition, the averaging procedure evolved through conservation equations. It is turbulence that controls the averaging process.

We discussed this canonical flames; canonical flames and then we move to we discussed his as well.

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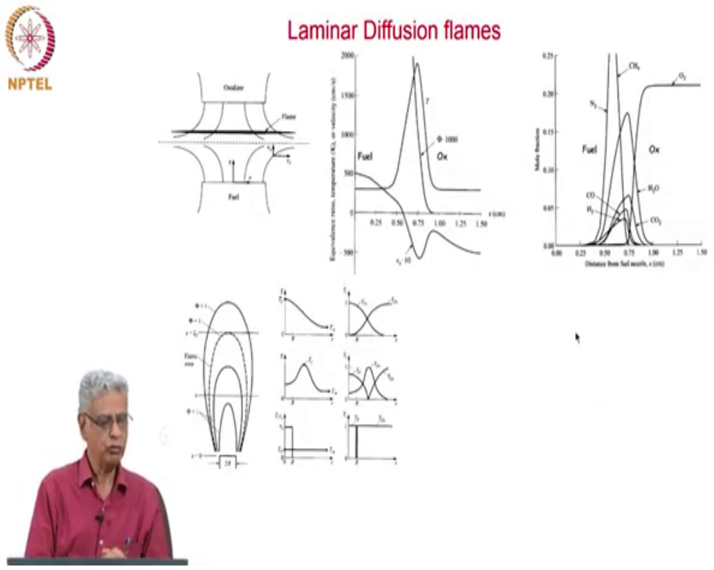

Laminar Diffusion flames

So, we move to laminar diffusion flames; diffusion flames occupy quite an important role in everything that is happening. So, we need to understand that you can create it by using a jet or by something like an opposed jet flame here. In fact, my PhD theses were roughly on this kind of problem then not relevant now but, because it has significance of that kind it has been enlarged into a very important tool for analysis subsequently by Peters, Williams and several other people.

Now, you have an oxidizer and fuel typically if you take hydrocarbons kind of substances, perhaps something which has the same density as air you take ethane its about same density as air 28 and 29. Methane is not necessarily right example if you want to do that you could mix methane with nitrogen, so that the density roughly matches.

If it density does not match the flames cannot become flat like this it will become curved because momentum balance will not take place that they are minor matter, but you have a flame here and usually it occurs in the oxidizer side not on the fuel side, because fuel aided mixture ratio is large 16 17 for stoichiometry. You know diffusion flame locates itself a stoichiometry it is a principle.

Now, so the fuel reaches through diffusion across this surface oxidizer comes to it by convection. A larger amount of oxidizer is required to sustain a flame that is all the logic. You then make measurements if you like you will discover fuel and temperature peaks here oxidizer is in this side, the temperature it goes like this; this is the stoichiometric ratio it is if you define this way it goes to 0 everywhere else goes to infinity here. This is the zone where reaction occurs, this is the velocity which goes across like this and the entire flame is located in this zone.

You can make measurements of composition which is has been done oxygen falls down to very low values methane falls down to low values, nitrogen I mean its plotted here in some format here you have CO H 2 CO 2 all of them form in this domain. So, you we will discover that the flame is centered around here the diffusion of most radicals occurs to both the sides that is responsible for the initiation of ignition some zones ok.

Yeah you will also discover that if you plotted in this manner you will find fuel dropped down at the flame, oxygen drop down from the other side product goes up like this peak temperature is obtained in this domain like somewhat like this ok. That is what laminar flame description is.

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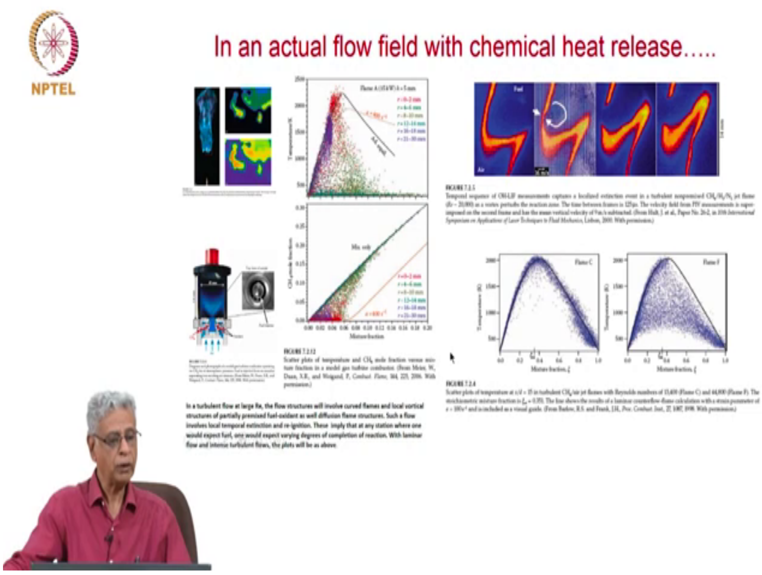
In an actual flow field with chemical heat release…..

In a turbulent flow at large Re, the flow structures will involve curved flames and local vortical structures of partially premixed fuel-oxidant as well diffusion flame structures. Such a flow involves local temporal extinction and re-ignition. These imply that at any station where one would expect fuel, one would expect varying degrees of completion of reaction. With laminar flow and intense turbulent flows, the plots will be as above.

Now, if you take a turbulent diffusion flame rate measurements; I let us say in any one particularly at several zones you measure the temperature local temperature and local fuel fraction which you can measure oxygen fraction you can measure; by measuring the fuel fraction you can see progress how much a fuel is consumed converted and that mixture fraction defined in a certain manner subsequently it will be done is a measure of how much it progress of the reaction ok.

And you will see that as the mixed mixture flow goes from 1 0 to point 2. We will discover it moves from fuel oxidizer or oxidizer to fuel as the case may be. And you will see that measurement of temperature show a lot of dispersion here in a turbulent flame, what this means is the flame is occasionally getting quenched and reignited on a continuous basis randomly. That is why you will find many data points here

If you take for example, in this particular case this shows you the measurement of the o h using laser induced fluorescence experiment measurements flame structure is like this and flame is very thin if you put a vortex here.

You will see a break up occurring here all these are structures which are obtained in an actual flame; in a diffusion flame and you measure the same thing in a strong flame you will discover most of them are distribute or points are distributed around the expectation of temperature versus mixture fraction in the theoretical line which is the black line.

But if you see that is closer to extinction we find data points randomly located at many many temperatures. What does this mean? And see in this case it means at 0.4 you would have to expect temperature of 2000 near stoichiometry. But you will discover here at 0.4. In fact, exactly at point 3 8 or something like that you will discover their points at showing many many temperatures what it means is even in that condition there have been local extinctions and reignition. So, this is the property in an actual diffusion flame in turbulent influence or turbulence.

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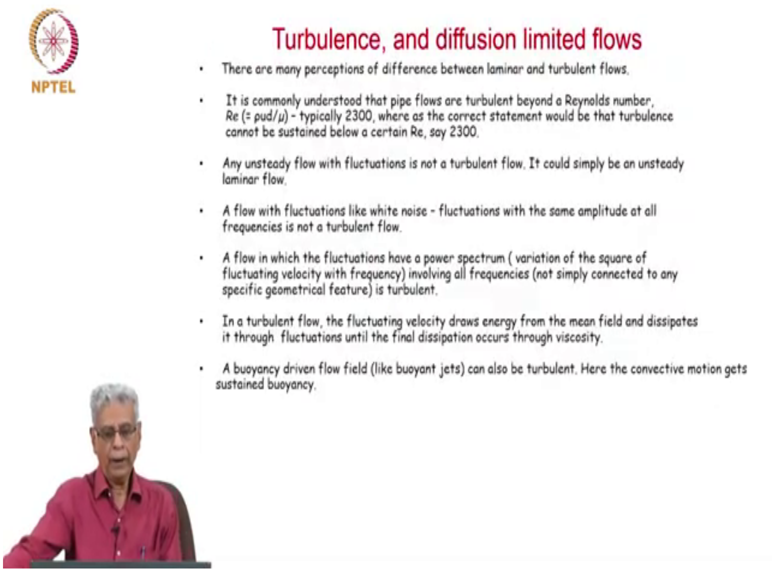

Turbulence, and diffusion limited flows

- There are many perceptions of difference between laminar and turbulent flows.
- It is commonly understood that pipe flows are turbulent beyond a Reynolds number, Re (= $\rho ud/\mu$) - typically 2300, where as the correct statement would be that turbulence cannot be sustained below a certain Re, say 2300.
- Any unsteady flow with fluctuations is not a turbulent flow. It could simply be an unsteady laminar flow.
- A flow with fluctuations like white noise - fluctuations with the same amplitude at all frequencies is not a turbulent flow.
- A flow in which the fluctuations have a power spectrum (variation of the square of fluctuating velocity with frequency) involving all frequencies (not simply connected to any specific geometrical feature) is turbulent.
- In a turbulent flow, the fluctuating velocity draws energy from the mean field and dissipates it through fluctuations until the final dissipation occurs through viscosity.
- A buoyancy driven flow field (like buoyant jets) can also be turbulent. Here the convective motion gets sustained buoyancy.

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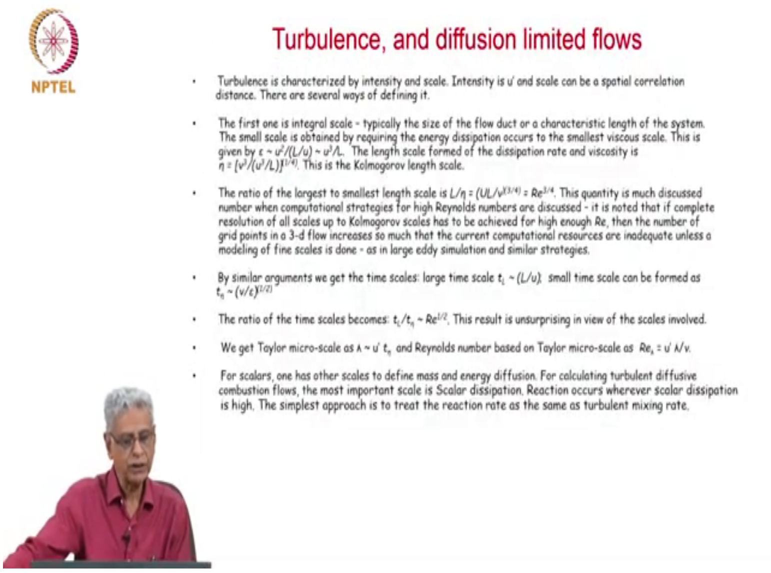

Turbulence, and diffusion limited flows

- Turbulence is characterized by intensity and scale. Intensity is u' and scale can be a spatial correlation distance. There are several ways of defining it.
- The first one is integral scale - typically the size of the flow duct or a characteristic length of the system. The small scale is obtained by requiring the energy dissipation occurs to the smallest viscous scale. This is given by $\varepsilon \sim u^2/(L/u) \sim u^3/L$. The length scale formed of the dissipation rate and viscosity is $\eta = [\nu^3/(u^3/L)]^{(1/4)}$. This is the Kolmogorov length scale.
- The ratio of the largest to smallest length scale is $L/\eta = (UL/\nu)^{(3/4)} = Re^{3/4}$. This quantity is much discussed number when computational strategies for high Reynolds numbers are discussed - it is noted that if complete resolution of all scales up to Kolmogorov scales has to be achieved for high enough Re, then the number of grid points in a 3-d flow increases so much that the current computational resources are inadequate unless a modeling of fine scales is done - as in large eddy simulation and similar strategies.
- By similar arguments we get the time scales: large time scale $t_L \sim (L/u)$; small time scale can be formed as $t_\eta \sim (\nu/\varepsilon)^{(1/2)}$
- The ratio of the time scales becomes: $t_L/t_\eta \sim Re^{1/2}$. This result is unsurprising in view of the scales involved.
- We get Taylor micro-scale as $\lambda \sim u'\, t_\eta$ and Reynolds number based on Taylor micro-scale as $Re_\lambda = u'\, \lambda/\nu$.
- For scalars, one has other scales to define mass and energy diffusion. For calculating turbulent diffusive combustion flows, the most important scale is Scalar dissipation. Reaction occurs wherever scalar dissipation is high. The simplest approach is to treat the reaction rate as the same as turbulent mixing rate.

So, this we discussed I will skip this; I will allow Varun to speak about it some length.

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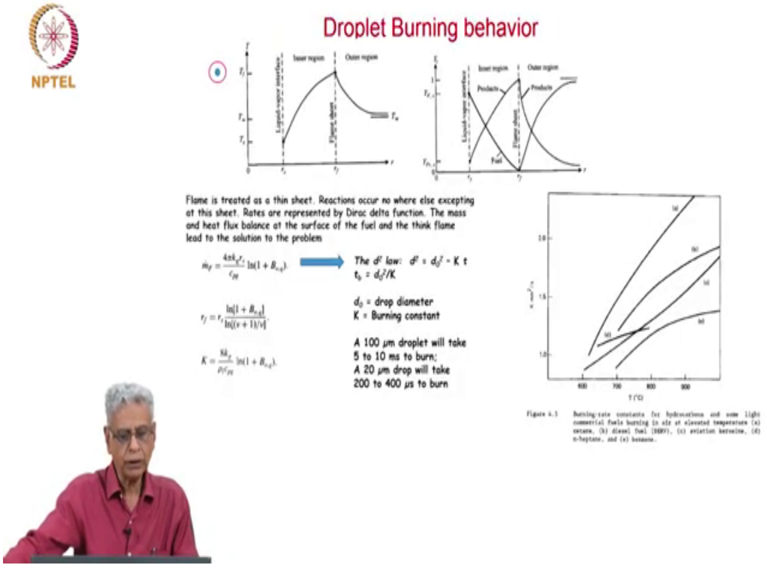

Droplet Burning behavior

Flame is treated as a thin sheet. Reactions occur no where else excepting at this sheet. Rates are represented by Dirac delta function. The mass and heat flux balance at the surface of the fuel and the think flame lead to the solution to the problem

$$\dot{m}_F = \frac{4\pi k_g r_s}{c_{pg}} \ln(1 + B_{o,q})$$

$$r_f = r_s \frac{\ln[1 + B_{o,q}]}{\ln[(\nu + 1)/\nu]}$$

$$K = \frac{8k_g}{\rho_l c_{pg}} \ln(1 + B_{o,q})$$

The d^2 law: $d^2 = d_0^2 - K t$

$t_b = d_0^2/K$

d_0 = drop diameter

K = Burning constant

A 100 μm droplet will take 5 to 10 ms to burn;

A 20 μm drop will take 200 to 400 μs to burn

I want to move to droplet combustion behavior you saw how it; it actually happens. I want to look at some features associated with it. There were droplet in the flame around, and you will see this is the liquid vapor interface at r s which is a droplet radius. The in the regions the temperature increases to the flame front and it goes down at to the infinity that is this is the r s, r f and infinity and this is the temperature profile flames can be treated as a thin sheet.

Typical what happened? Reactions occur nowhere accepting at this sheet. Well, you can technically treat it as the direct delta function analyzing we will not necessarily do it here. The mass and heat flux balance with the surface or the thin flame not thin flame lead to the solution like this m dot f is equal to 4 phi k r s by c p into log of 1 plus B and b is called transfer number; normally called Spalding's transfer number related to temperature ratios or enthalpy ratios.

This expression leads directly to the d squared law and in which the burn time is proportional to the droplet diameter square divided by the burning constant you will discover the flame radius also can be analyzed in terms of flame radius to droplet radius.

And you can get this result this analysis is pretty straight forward anybody can do it following the conservation equations. You can have it described in many books and its the burning constant is dependent on k by c p into 1 by density i by density and log of 1 plus b again a standard result. What does this tell us? That if you take simply hundred micron droplet.

It will take about 5 to 10 milliseconds to burn. A 20 micron droplet will take approximately 200 to 400 microseconds to burn. As you bring down the droplet radius the burn time decreases substantially depending on the surface area and it is the key takeaway feature that is why you must do droplet atomization is the logic.

There are also experimental data with many; many fuels n octane, diesel, aviation, kerosene all this meant for application related to air breaking engines you find this constant vary from 1 to 2 the burn rate constant. So, this one feature which you can get from fundamental experiments which will have value in what happens in actual systems.

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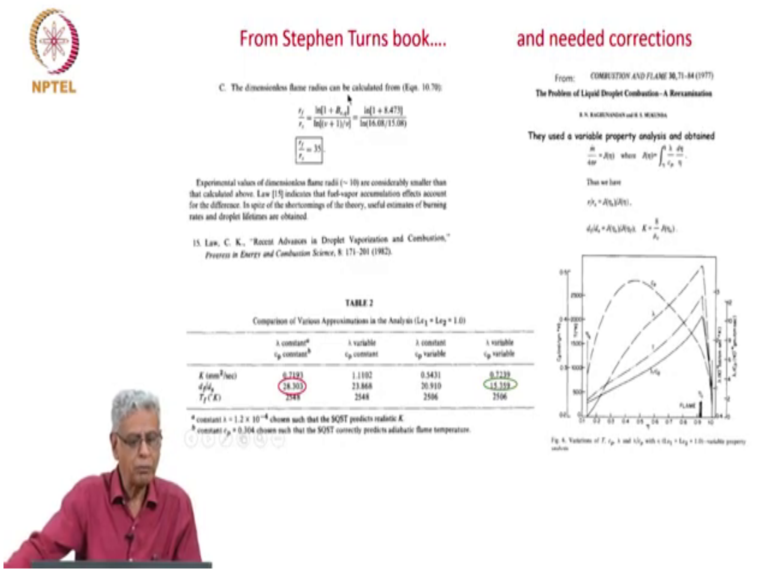

From Stephen Turns book....

C. The dimensionless flame radius can be calculated from (Eqn. 10.70):

$$\frac{r_f}{r_s} = \frac{\ln[1 + B_{o,q}]}{\ln[(\nu + 1)/\nu]} = \frac{\ln[1 + 8.473]}{\ln(16.08/15.08)}$$

$$\boxed{\frac{r_f}{r_s} = 35}$$

Experimental values of dimensionless flame radii (~ 10) are considerably smaller than that calculated above. Law [15] indicates that fuel-vapor accumulation effects account for the difference. In spite of the shortcomings of the theory, useful estimates of burning rates and droplet lifetimes are obtained.

15. Law, C. K., "Recent Advances in Droplet Vaporization and Combustion," *Progress in Energy and Combustion Science*, 8: 171-201 (1982).

TABLE 2

Comparison of Various Approximations in the Analysis ($Le_1 = Le_2 = 1.0$)

	λ constant[a] c_p constant[b]	λ variable c_p constant	λ constant c_p variable	λ variable c_p variable
K (mm^2/sec)	0.7193	1.1102	0.5431	0.7239
d_f/d_s	28.303	23.868	20.910	15.359
T_f (°K)	2548	2548	2506	2506

[a] constant $\lambda = 1.2 \times 10^{-4}$ chosen such that the SQST predicts realistic K

[b] constant $c_p = 0.304$ chosen such that the SQST correctly predicts adiabatic flame temperature.

and needed corrections

From: COMBUSTION AND FLAME 30, 71-84 (1977)

The Problem of Liquid Droplet Combustion–A Reexamination

B. N. RAGHUNANDAN and H. S. MUKUNDA

They used a variable property analysis and obtained

$$\frac{\dot m}{4\pi r} = J(\eta) \quad \text{where} \quad J(\eta) = \int_{\eta_s}^{\eta} \frac{\lambda}{c_p}\frac{d\eta}{\eta}.$$

Thus we have

$$r/r_s = J(\eta_s)/J(\eta),$$

$$d_f/d_s = J(\eta_s)/J(\eta_f), \quad K = \frac{8}{\rho_l} J(\eta_s).$$

Fig. 6. Variations of T, c_p, λ and λ/c_p with η ($Le_1 = Le_2 = 1.0$)–variable property analysis

I just want to show you know you must have seen Stephen Turns book, but if not seen you will see it sometime these how. I happen to read it recently; I mean for this course otherwise, I would not have read found something strange.

So, I will bring to your attention those features and in 1 page is a dimensions frame radius can be calculated from the same expression which I mentioned here the same expression here and you will get a result like 35 this is correct what he says is the following experimental values of dimensionless flame radius; which are actually mentioned in the book 10.

That is smaller than what is calculated here, C.K. Law a very famous man; he written a book in combustion science you know indicates a fuel vapor accumulation effects account for this difference. It is not clear what he is meant this is his quotation from his book.

In spite of the short comings of the theory useful estimates of burning rates and droplet lifetimes can be are obtained saying that you can get some features properly, but whether this is correct or not is a subject of question. This is something which bothered there is a student of mine who called Professor Raghunandhan who is retired.

Now, we did; he did his work during his PhD time on this subject. Then it bothered us, and the result is the following that is nothing to do with all this it has got to do with the fact that when you look at the property variation inside the flame in a coordinate system eta, eta s is the surface eta equal to 1 is infinity. We will discover a behavior like this you will find on the fuel side and the oxidizer side the specific heat.

The specific heat the thermal conductivity and lambda by C p vary through the field very significantly and you can show that; that integral of that variation is responsible for r by r s and the net result is shown in this table where you treat lambda is constant C p is constant; you will get number of d f by d s 28.3 what he calls as 35, somewhere close to that. And you can take various approximations, but who treats lambdas variable as C p is variable you will get this as 15.

So, what is perceived as 35 is really and then you draw conclusions for make this patently wrong that is actually fifteen if yours, if you actually make a simple clear analysis using variable properties is the message that I am telling you, but also there is one more message you see this all this is 1982 summary paper by C.K. Law and look at our paper 1977.

Now, this is something is a message to you that; you may do interesting work, but be please ready it to be ignored in international literature. Now, if you ask me what does it matter it does not matter vary a whole large, but this is the usually the order of the world which is been published in combustion and flame in 1977.

It is not that Dr. C. K. Law does not know me or I do not know C. K. Law, we have met in conference and spoken yet you will discover they choose to ignore the work done in this country just other message you should carry with you and be ready with for that.

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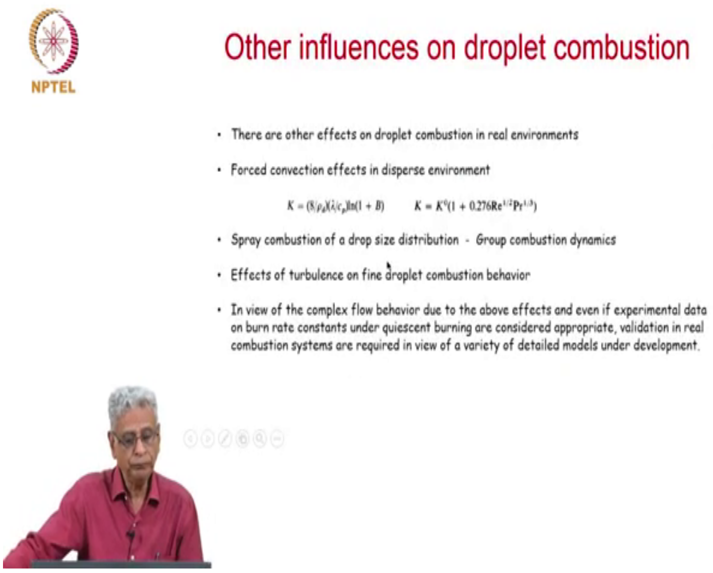

Other influences on droplet combustion

- There are other effects on droplet combustion in real environments
- Forced convection effects in disperse environment

$$K = (8/\rho_d)(\lambda/c_p)\ln(1+B) \qquad K = K^0(1 + 0.276\mathrm{Re}^{1/2}\mathrm{Pr}^{1/3})$$

- Spray combustion of a drop size distribution - Group combustion dynamics
- Effects of turbulence on fine droplet combustion behavior
- In view of the complex flow behavior due to the above effects and even if experimental data on burn rate constants under quiescent burning are considered appropriate, validation in real combustion systems are required in view of a variety of detailed models under development.

Let us look at other influences on droplet combustion the force convection effects in a dispersed environment. You know every drop moves in a fluid flow field with velocity and this is the statement which what any flow and with flow the Reynolds number effect is taken into account. A small change will occur because of the flow as droplet diameter comes now the Reynolds number effect also comes down.

So, in situations like for example, in a gas turbine engine, I recollect of a Lefebvre the famous man; gave a talk in gtra long ago. Maybe 76 77 I do not quite remember and he pose this question from gtra. What happened to the Reynolds efficiency and he gave a very correct and succinct answer. In the recirculation zone velocities are very small Reynolds number is very small.

So, even if you try take ignore that effect the errors that you create in the calculation of the flow field will not be significant that is the correct message. It is not that you should not take into account the effect of Reynolds number in a code you can take it.

But if you want to understand the behavior even if you choose to ignore that, the influence and the final result will not be varies very much this is the message that you get. So, this one point which I thought you should know. Also, there are other features it is not like a single droplet coming out its a spray and ignition occurs occasionally in a group of droplets in a spray.

So, there is something called group combustion dynamics in which lot of work has been done and I do not think it has got a great influence on the code in the development of the combustion processes in gas turbine engines, but it is a science which you should be familiar with sometimes even to ignore something you should know what it implies.

And there are also effects of turbulence and fine droplet combustion behavior very fine droplets and turbulence interact with each other. I did not something that we should keep in mind again there is a established body of literature, but I do not think the influence on the whole combustion behavior is very significant.

So, in view of the complex flow behavior due to the above effects and even if experimental data on burning constant under quiescent conditions are considered appropriate, validation and real combustion systems are required in view of variety of detailed models in under development.

You do this complex effects. So, you cannot ignore them you should factor them, but the validation comes only from a realistic comparison of performance otherwise it would no longer be appropriate.

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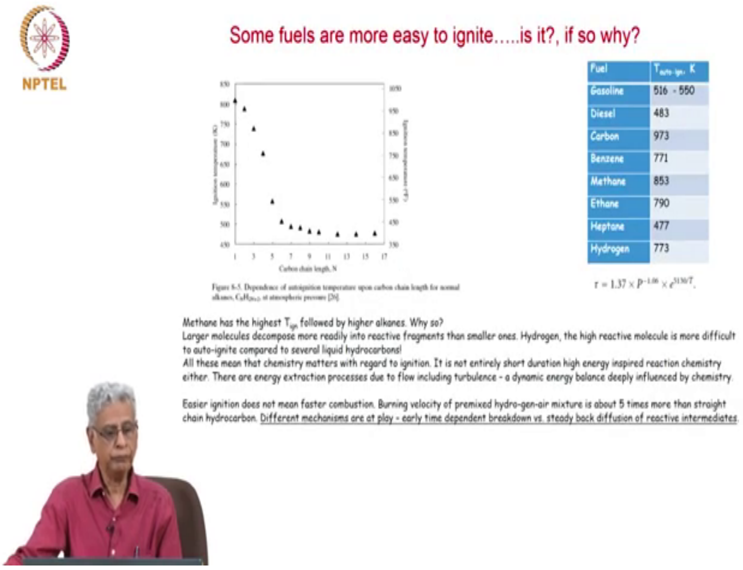

Some fuels are more easy to ignite…..is it?, if so why?

Figure 8-5. Dependence of autoignition temperature upon carbon chain length for normal alkanes, C_NH_{2N+2} at atmospheric pressure [26]

Fuel	$T_{auto-ign}$, K
Gasoline	516 - 550
Diesel	483
Carbon	973
Benzene	771
Methane	853
Ethane	790
Heptane	477
Hydrogen	773

$$\tau = 1.37 \times P^{-1.06} \times e^{5130/T}.$$

Methane has the highest T_{ign} followed by higher alkanes. Why so?
Larger molecules decompose more readily into reactive fragments than smaller ones. Hydrogen, the high reactive molecule is more difficult to auto-ignite compared to several liquid hydrocarbons!
All these mean that chemistry matters with regard to ignition. It is not entirely short duration high energy inspired reaction chemistry either. There are energy extraction processes due to flow including turbulence - a dynamic energy balance deeply influenced by chemistry.

Easier ignition does not mean faster combustion. Burning velocity of premixed hydro-gen-air mixture is about 5 times more than straight chain hydrocarbon. Different mechanisms are at play - early time dependent breakdown vs. steady back diffusion of reactive intermediates.

I want to post some subsidiary small questions at this time. Quite often, this said that some fuels are more easy to ignite and so we say, why is it so? You we will discover that the ignition temperature under the same set of conditions using different fuels.

They turned out to be as though with carbon chain length this is methane as you go to larger end you have n heptane, octane, movane etcetera; it will fall down. Bit strange, is not it? You think that the simpler molecule should ignite it simply, but is not true. Why is that? Methane has the highest ignition followed by higher alkenes and why so?

See larger molecules decompose more readily into reactive fragments than smaller ones sorry than smaller ones. See, if the molecule is small it requires greater energy to break it for example, o 2 is not as easy to break as CO 2. You have ability to have number of locations where collisions can become reactive and then break it.

Hydrogen the highest lactic molecule is more difficult to auto ignite compared to several liquid hydrocarbons. All these means that chemistry matters with respect to ignition ok, these are matters related to chemistry; is not entirely short duration high energy inspired reaction chemistry either. Their energy extraction process is due to flow including turbulence and a dynamic energy balance deeply influenced by chemistry which will control the ignition process.

Now, what I mean is sometimes ignition is treated as you know you hit a shock wave in a shock tube you create the fuel source, you heat it with shockwave and determine the ignition delay and treat that as a measure of the ignition. That is different from you putting a gas into a flow field and determining whether you know you put it in the ignition source by the flame will get where the flame will get sustained at ignition or the simply the flame is blown off which means it will not ignite. These are two different physical situations, unless you take that into account you will not be able to decide on the result very simply. Yeah?

Student: In the previous slide, we discussed about force convection effects on the droplets interaction in it.

Yeah.

Student: And this is quiescent.

Yeah.

Student: And the carelessness.

Yeah.

Student: When a droplet in the coherent zone get entangled with the local locally in the local Reynolds number may be smaller, but the effect of convection on the droplet; towards the droplet and the diffusion from the mass from the droplet will be having sufficient input on the flame induction, but how we can ignore this sir?.

No, nobody is saying ignore anything. I think you did not understand the message correctly. I was addressing a the situation in gas turbines. No, it is not now we will address every question, but you look at that question. There you have air coming in a swirl more and you have a spray. A spray means droplets until some zone high density and density comes down as you progress further.

Some bit of evaporation some bit of further atomization, secondary atomization will take place and we will get droplets. What is being said is the principal combustion occurs in a recirculation zone. The word recirculation means the flow velocities are small and also negative the flow will actually turn around.

And since velocities are small the Reynolds numbers are therefore, going to be very small and therefore, the effect due to the convection will not be very significant in this situation. Now, if you take the case of liquid droplet, gaseous oxygen is flowing a liquid droplet; the flow there is quite complex and the atomization processes is are not easy to describe ok.

So, I accepting that quite often you use a formula and pray that it works. It not too much more that you can do. And the only way it works is if you have some kind of model combustion chamber with which is similar to the one that you are looking you simulate the same set of conditions, you have measurements here or the progress of combustion, and that process of combustion is actually compared with experiments with predictions. You will not be able to say with confidence which is the model you have chosen was right.

You may have gotten a correct result to a mix up of errors, it is possible and it may be small it may not matter. So therefore, variety of such features occur in reality. Quite often you can ask a question what is the use of the theory? It gives you a general direction of how to go about, what may be happening and using that you may be able to make a change in the design in some formal method.

So, I think one should not scoff at the fact of pursuit of theory and one need not assume that every part of the theory should be very accurately represented. Ah But we should be aware which part is important and therefore, pay attention to it, not be guided by the fact that I have a theory which I have somewhere is written. So, I am using it; its correct because somebody has said it is correct let all that argument may not work.

So, I brought this to say that; sometimes you will find this process of scaling of ignition does not necessarily mean also for combustion and I am saying very specifically burning velocity of premixed hydrocarbon hydrogen air mixture is about 5 times more than straight chain hydrocarbons.

So, just because it is difficult to ignite it does not mean it. So, difficult to burn that different mechanisms at play we must remember that two. So, different mechanisms are at play early time dependent breakdown versus steady state back diffusion of reactive intermediates there are two different features. Yes, please go ahead.

Student: So, most of the times when we develop the answers we try to reorganize simply term kinetic mechanism we use; we use technology delay and auto emission characteristics as the one of the pure criteria of (Refer Time: 32:55) organization re parameters, but part of the mentioned in the (Refer Time: 33:00) are those parameters are like, we will compete which are (Refer Time: 33:05) by these parameters are the not valued or (Refer Time: 33:08).

Yeah, I mean to simply say that my kinetic parameters are optimized using auto ignition or ignition data, I want to use the same data for steady state propagation would not be correct, but you have predict state propagation information from pre mixed flame. Now, if it validates that it is fine, it is possible in some regimes that if you as from stoichiometrics; it may well be that the same kinetic mechanism is valid.

So, I am not trying to say that there is a total invalidity between the mechanisms at plane ignition and propagation, but to assume that it is the same is wrong; that you pay attention to in a given situation as to what happens in ignition what happens in flame propagation.

But if your; if your aim is to simulate a propagating flame like behavior you are better off in simulating propagating flames if it is pre mixed. if is diffusing you have to paid; you may not pay too much attention to chemistry. If it is related to burning velocity burning behavior is related to emissions you may have to pay attention to those aspects.

And most usually the turbulence interaction which matters and when turbulence takes over chemistry is brought down to that extent. Unless, you are looking at specific aspect like NO x or CO in which case the chemistry turbulence interaction is important and considerable work has been done with respect to those species.

You know, lot of effort has gone into modeling in turbulence chemistry interaction with respect to nox generation with respect to sibo generation. And, we have to look at those specific mechanisms and some of the advanced codes actually used those methodologies which have been developed to varying degrees of success.

And some of them have embedded in their course, they keep it in a protected form because they compared with the experiments in their test beds, comparisons look good, others should not capitalize on them is their property and public domain you may not get that information rest all these things will happen.

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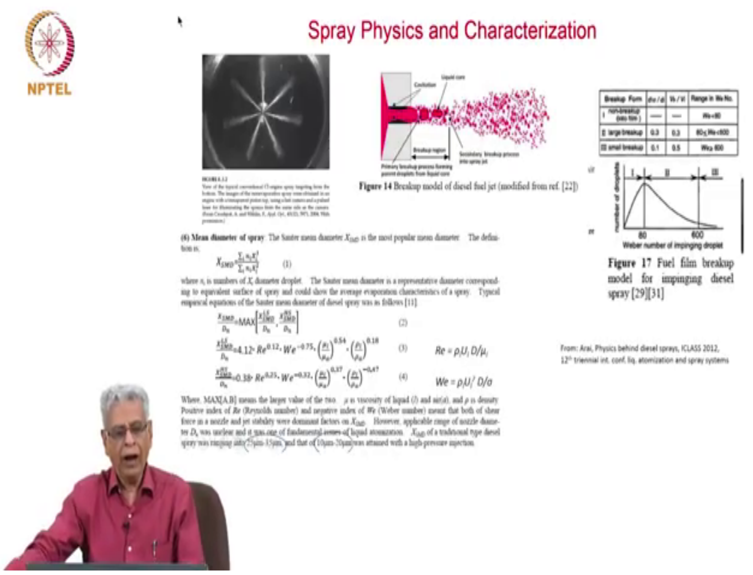

Spray Physics and Characterization

Figure 14 Breakup model of diesel fuel jet (modified from ref. [22])

Figure 17 Fuel film breakup model for impinging diesel spray [29][31]

$Re = \rho_l U_l D/\mu_l$

$We = \rho_l U_l^2 D/\sigma$

From: Arai, Physics behind diesel sprays, ICLASS 2012, 12th triennial int. conf. liq. atomization and spray systems

Oh I want to move to spray physics bit to understand a little bit more. This is diesel engine; you will see the spray coming off from these injectors and the spray physics involves and this diameter is about I told you 100 and 100 microns, 100 to 150 microns. It becomes droplets the droplet atomization process is controlled by number which is called Weber number as you will see here, which is essentially the dynamic pressure of the liquid divided by surface tension driven force say momentum ratio. So, if Weber number is large the shattering due to aerodynamics is very significant there are other mechanisms had play at lower speeds.

But, they are not quite relevant in the dynamics which occurs in engineering devices where you want to push lot of mass flow into smaller domain velocities are high and therefore, Weber numbers will turn out to be large. We will discover that many people have worked on this.

Diesel spray for example, you will discover a correlation which tells you about the shorter mean diameter. As a function as a ratio of the diameter of the nozzle, you know this particular correlation tells you is a maximum between two level; two values and two values are computed in some way..

And all properties are described here about them and measurements have shown that typically the numbers that you get is about 25 to 35 micron and in some cases 10 to 20 micron; are even I am emphasizing this.

Because of a general feature, you look at literature you look at actual experiments you will discover that the minimum shorter mean diameter obtained by best of the means of atomization take you to around 30 microns in gas turbine situations, 10 microns like the diesel are like situations executions in diesel injectors you must remember the orifice sizes are extremely small.

I told you extremely small and pressures are extremely high and therefore, they are able to get up to 10 to 20 micron that is the number which I mentioned here in gas turbine engines its a pressure drops are not that much.

In some case you use atomization because of air blast it is even lower and they want to use advanced methods of atomization that is called high shear injectors, Even then, the actual main drop size you get close to about 30 micron. It is not easy to break it much beyond that because the shear forces required to break it turn out to be very humongous that is the only conclusion that can be drawn from that ok. So, this is from a work of a Japanese guy he has done a very careful analysis of the drop size distribution.

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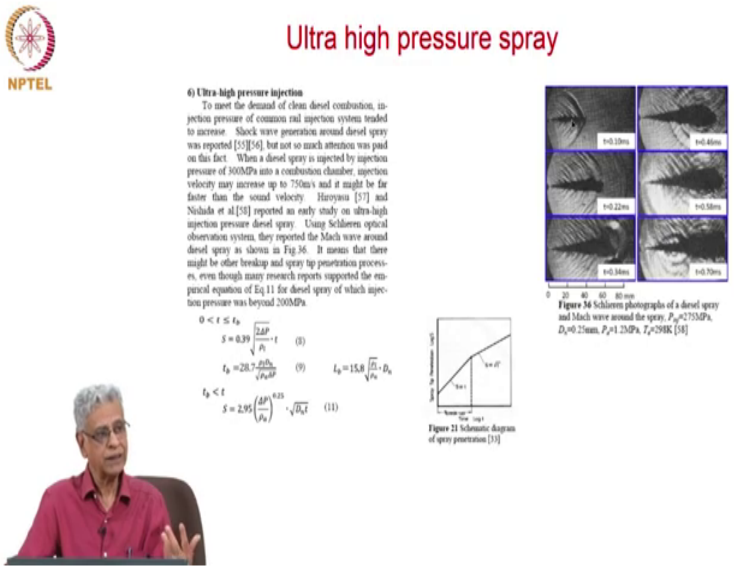

Ultra high pressure spray

6) Ultra-high pressure injection

To meet the demand of clean diesel combustion, injection pressure of common rail injection system tended to increase. Shock wave generation around diesel spray was reported [55][56], but not so much attention was paid on this fact. When a diesel spray is injected by injection pressure of 300MPa into a combustion chamber, injection velocity may increase up to 750m/s and it might be far faster than the sound velocity. Hiroyasu [57] and Nishida et al.[58] reported an early study on ultra-high injection pressure diesel spray. Using Schlieren optical observation system, they reported the Mach wave around diesel spray as shown in Fig.36. It means that there might be other breakup and spray tip penetration processes, even though many research reports supported the empirical equation of Eq.11 for diesel spray of which injection pressure was beyond 200MPa.

$0 < t \le t_b$

$$S = 0.39\sqrt{\frac{2\Delta P}{\rho_l}}\cdot t \qquad (8)$$

$$t_b = 28.7\frac{\rho_l D_n}{\sqrt{\rho_a \Delta P}} \qquad (9) \qquad L_b = 15.8\sqrt{\frac{\rho_l}{\rho_a}}\cdot D_n$$

$t_b < t$

$$S = 2.95\left(\frac{\Delta P}{\rho_a}\right)^{0.25}\cdot\sqrt{D_n t} \qquad (11)$$

Figure 21 Schematic diagram of spray penetration [33]

Figure 36 Schlieren photographs of a diesel spray and Mach wave around the spray, P_{inj}=275MPa, D_n=0.25mm, P_a=1.2MPa, T_a=298K [58]

In fact, current the engines diesel engines they hope to use ultra high pressure spray and the reason why I want to bring to your attention is in these cases the velocities are so large that you actually get shockwaves often the spray distribution and that dynamics controls the atomization and of course, combustion process.

So, you will discover that you know Modern day engines may really have more complex physics in the diesel engine which is going to be developed more times.

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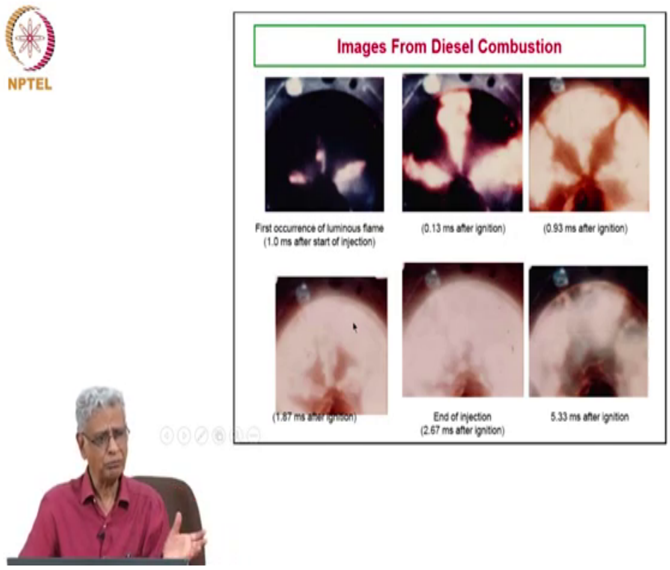

Images From Diesel Combustion

First occurrence of luminous flame
(1.0 ms after start of injection)

(0.13 ms after ignition)

(0.93 ms after ignition)

(1.87 ms after ignition)

End of injection
(2.67 ms after ignition)

5.33 ms after ignition

And typical combustion distribution you will see in each of these sprays is approximately one one and half millisecond the whole combustion is over. Because, it does not have much time in a periodic combustion process which occurs in a diesel engine.

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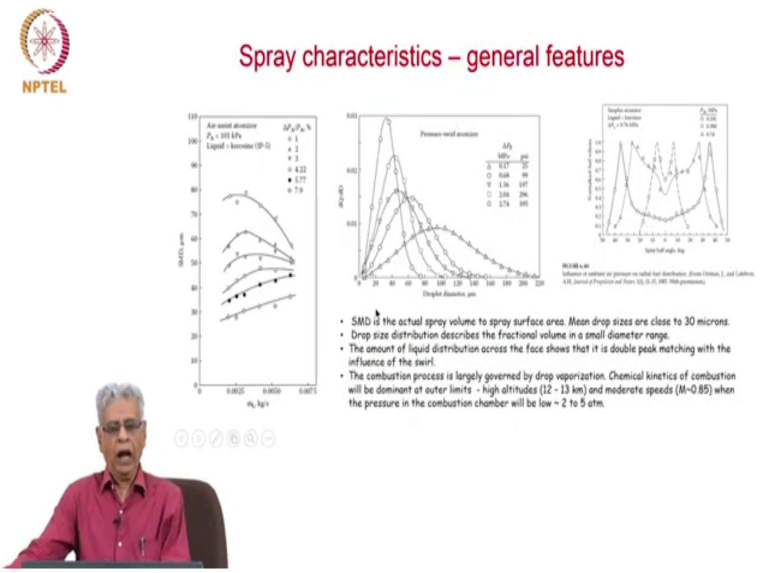

Spray characteristics – general features

- SMD is the actual spray volume to spray surface area. Mean drop sizes are close to 30 microns.
- Drop size distribution describes the fractional volume in a small diameter range.
- The amount of liquid distribution across the face shows that it is double peak matching with the influence of the swirl.
- The combustion process is largely governed by drop vaporization. Chemical kinetics of combustion will be dominant at outer limits - high altitudes (12 - 13 km) and moderate speeds (M~0.85) when the pressure in the combustion chamber will be low ~ 2 to 5 atm.

If you look at the spray characteristics relevant to gas turbine engines you look at air blast atomizer there are many parameters which control the atomization of something like kerosene.

For example and the pressure as pressure increases combustion chamber pressure increases it inhibits atomization as pressure drop increases improves the atomization that behavior is captured in this and individual injectors the flow rate is very small 2.5 gram per second is something you should keep in mind.

Even in the gas turbine engine, ccf 6 or whatever you will have about 20 orifices around and each one pushes off something in the range of 2 to 3 gram per second individuals ones, at in an orifice of about one and half mm or half an mm one and half mm size that is the kind of orifice which you will get unlike in rocket engines.

Where the orifices are much larger 2.5 mm in something in that range and then you take f one engine which actually used for half millimeter. It is a very large diameter there are various reasons for it, but the point to notice in a gas turbine engine the numbers are not vastly different from what you can see.

And the number distribution distribution of the of the drop sizes, but fraction of the drops having a given drop diameter is shown here as a function of various parameters here pressure you will discover that the high pressures increase; delta p increases you will find atomization is much better and as I told; told you s m d will turn out to be in this case closed to about 30 30 micron or so.

So, you will discover that pressure drop across the injector is crucial parameter which controls the atomization rocket engines do not have that freedom, then atmospheres eight atmosphere something in the drain not much more in gas turbine engines you can go to much larger numbers.

You can go up to about 20 30 atmospheres as well in pressure atomization. In the air blast atomizer it will turn out to be much lower, and if you look at the spray what matters also is how much they must distributed over the spray. And that is shown here we will see when the spray comes out at an angle large amount of mass comes out in this domain and combustion process is controlled by the availability fuel in a specific zone.

So, the flame structure is also will be such that will find peak temperatures in these domains. So, the I am summarizing it to say that the sorter mean diameters is the actual; is the actual spray volume to the spray surface area mean drop sizes are close to 30 micron and this; as I told you keep this general fact in mind. Drop size distribution describe the fractional volume in a small diameter range the amount of liquid distribution across the face shows that is double peak matching with the influence of the swirl.

And the combustion process is largely governed by droplet vaporization chemical kinetics of the combustion will be dominant only the outer limits of high altitudes and moderate speeds this offer is important in gas turbines in steady combustion not greatly important. For example, it way it reflects is that the combustion efficiency of a gas turbine at ground level operation will be close to about 97 percent, 95 percent in that range.

But the same engine when works at high altitudes 10 at 10 kilometers altitude. You will and mark number of 1.5 minus 0.75 or whatever in that domain the chamber pressures are lower, reaction rates are lower and therefore they compete with diffusion rates because you think the diffusion rates are low reaction rates are fast no as pressure goes down reaction rates are going down.

Therefore they compete with each other and therefore, they influence on it they discover for example, that the combustion efficiency goes down to 87 percent or 90 percent. They are unhappy no, no it must become ninety four its true, but that requires atomization much better degree of atomization over the range. Because, you have no way by which you can change the reaction rates that altitudes are fixed. So, these are the issues which we need to keep in mind as I told you that combustion chamber pressure in that zone will be lower.

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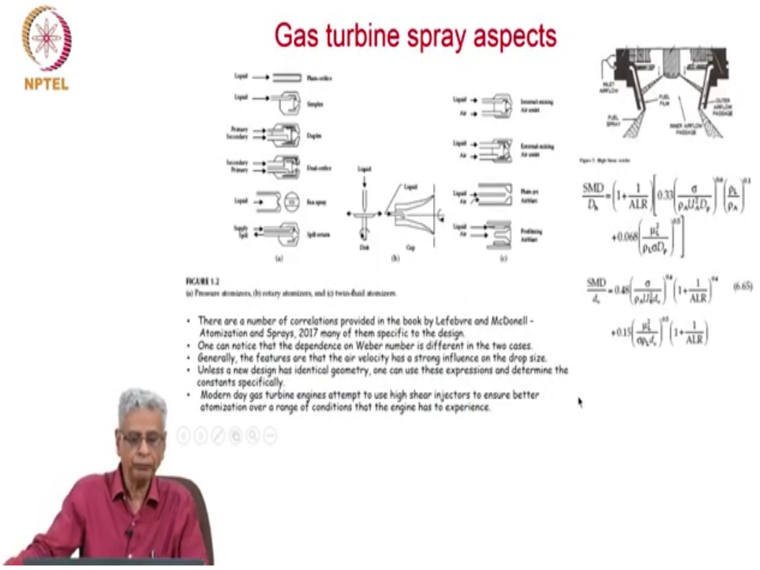

Gas turbine spray aspects

FIGURE 1.2
(a) Pressure atomizers, (b) rotary atomizers, and (c) twin-fluid atomizers.

$$\frac{SMD}{D_h}=\left(1+\frac{1}{ALR}\right)\left[0.33\left(\frac{\sigma}{\rho_A U_A^2 D_p}\right)^{0.6}\left(\frac{\rho_L}{\rho_A}\right)^{0.1}+0.068\left(\frac{\mu_L^2}{\rho_L \sigma D_p}\right)^{0.5}\right]$$

$$\frac{SMD}{d_o}=0.48\left(\frac{\sigma}{\rho_A U_R^2 d_o}\right)^{0.4}\left(1+\frac{1}{ALR}\right)^{0.4}+0.15\left(\frac{\mu_L^2}{\sigma \rho_L d_o}\right)^{0.5}\left(1+\frac{1}{ALR}\right) \quad (6.65)$$

- There are a number of correlations provided in the book by Lefebvre and McDonell - Atomization and Sprays, 2017 many of them specific to the design.
- One can notice that the dependence on Weber number is different in the two cases.
- Generally, the features are that the air velocity has a strong influence on the drop size.
- Unless a new design has identical geometry, one can use these expressions and determine the constants specifically.
- Modern day gas turbine engines attempt to use high shear injectors to ensure better atomization over a range of conditions that the engine has to experience.

As I mentioned Lefebvre has written a whole lot on this subject, there are variety of gas turbine spray injectors designs which are described. It is not my aim to look at each one of them just to point out to you that there variety of designs.

More modern design is shown here, where you want to get is called high shear injector. It allows you to get a final drop size distribution or a range of conditions and again curiously the book on atomization by Lefebvre McDonell provides information on the shorter mean diameter there are literally tens of correlations.

I do not want to say hundreds of correlation and the dependence on sorry the dependence on various parameters you will see Weber number here and another number called Ohnesorge number is related to the viscosity and the surface tension being together here all is presented here, but there is other. So, many correlations you will be left in distress.

So, the only way you can do as a development engineer is perhaps to make measurements at specific conditions look at the performance of the system and see that set those set of numbers which are valid for yours. That is why many of these engine companies; engine houses that they are called they have their own measurements their own correlations.

They do not tell you what they do; they allow Lefebvre McDonell to publish a whole lot whole range, but they do not say what they are using these are protector property because they are putting effort to make those measurements and make sure that measurements are consistent with the experience come on full scale engines. So, that is crucial I bring out the fact that in these correlations the dependence and Weber number itself is very sometimes different 0.6, 0.4 and so on so forth, to depend on them big dicey. So, one needs to put in the effort and make calibration internally to make sure that these results are consistent this is the message you get from the study of this class of systems.

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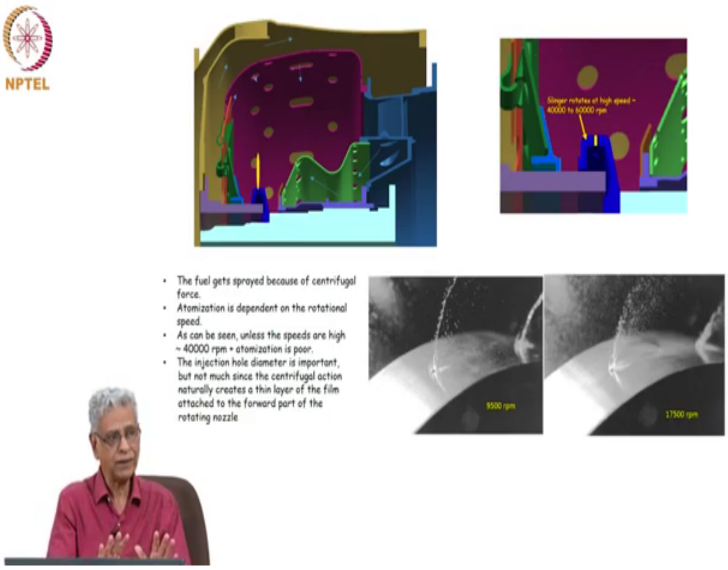

- The fuel gets sprayed because of centrifugal force.
- Atomization is dependent on the rotational speed.
- As can be seen, unless the speeds are high ~ 40000 rpm + atomization is poor.
- The injection hole diameter is important, but not much since the centrifugal action naturally creates a thin layer of the film attached to the forward part of the rotating nozzle

There is one more way of atomization this another design of a combustion system used in an engine called slinger combustor. One of the engines which g t r e developing has its class of designs what does it do here there is a shaft in which there are holes; the shaft is run at the normal speed of gas turbine speed in this case maybe 50000 rpm or close to that.

Because of the high speed the liquid comes through this up to that point and gets sprayed by centrifugal action and because the velocities are large it is equal to that a spray and then atomization occurs and if you see here atomization at some speed I have taken it from some other literature.

You will see that as speed increases the degree of atomization is substantive and if you recognize that the engine which g t r is developing as an r p m close to 50000 and you can say the atomization must be reasonably good and unless that happens you will never be able to get good performance. There is another method of introduction of the fuel into the combustion system using essentially the centrifugal action.

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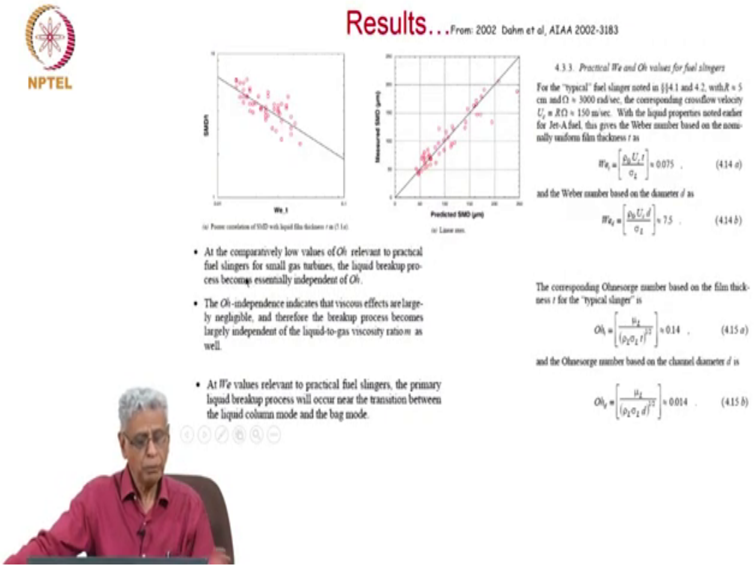

Results…From: 2002 Dahm et al, AIAA 2002-3183

- At the comparatively low values of *Oh* relevant to practical fuel slingers for small gas turbines, the liquid breakup process becomes essentially independent of *Oh*.
- The *Oh*-independence indicates that viscous effects are largely negligible, and therefore the breakup process becomes largely independent of the liquid-to-gas viscosity ratio *m* as well.
- At *We* values relevant to practical fuel slingers, the primary liquid breakup process will occur near the transition between the liquid column mode and the bag mode.

4.3.3. *Practical We and Oh values for fuel slingers*

For the "typical" fuel slinger noted in §§4.1 and 4.2, with $R \approx 5$ cm and $\Omega \approx 3000$ rad/sec, the corresponding crossflow velocity $U_c \equiv R\Omega \approx 150$ m/sec. With the liquid properties noted earlier for Jet-A fuel, this gives the Weber number based on the nominally uniform film thickness t as

$$We_t \equiv \left[\frac{\rho_0 U_c t}{\sigma_l}\right] \approx 0.075 \quad (4.14\,a)$$

and the Weber number based on the diameter d as

$$We_d \equiv \left[\frac{\rho_0 U_c d}{\sigma_l}\right] \approx 7.5 \quad (4.14\,b)$$

The corresponding Ohnesorge number based on the film thickness t for the "typical slinger" is

$$Oh_t \equiv \left[\frac{\mu_l}{(\rho_l \sigma_l t)^{1/2}}\right] \approx 0.14 \quad (4.15\,a)$$

and the Ohnesorge number based on the channel diameter d is

$$Oh_d \equiv \left[\frac{\mu_l}{(\rho_l \sigma_l d)^{1/2}}\right] \approx 0.014 \quad (4.15\,b)$$

Well people has studied this behavior; this well very interestingly we know the usually all correlations depend on the diameter injector hole. In reality because of the centrifugal force the liquid attaches itself to the surface which is moving forward and says they forms a thin layer it is not taking all of the that is really important in extracting the fluid.

And so the correlations two take the shorter mean dimension as the ratio of the thickness which is come obtained actually from an experiment itself and they seem to show within the spectrum of noise a little linear relationship with Weber number with increase in Weber number s m d goes down this all the message you get from here well.

There is something to do with Ohnesorge number as I mentioned a little earlier. It has certain descript wave if the things behave has very little dependence on this Ohnesorge number which is the ratio of the viscosity to surface to surface tension forces ok.

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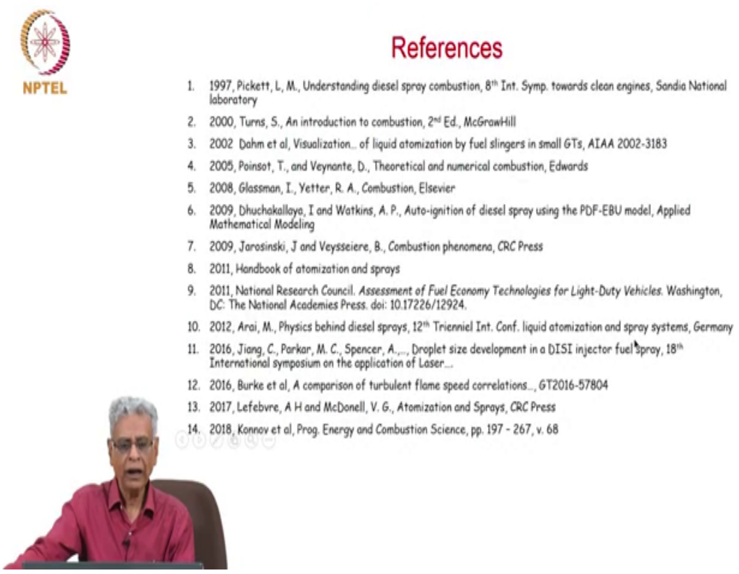

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I think I have said what I need to say and many of the aspects some of them cannot coordinated it in a sequential manner, but many aspects which contribute to the behavior some commonalities and some differences this is what I have described in this ok.