

**Fundamentals of Combustion for Propulsion**  
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**Lecture - 33**  
**Additional Insights**

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**MILD combustor**

(Mild = Moderate, Intense or Low Dilution)

Question: Can one use a combination of premixed and diffusion burners to obtain low emissions of MILD combustor.

Let us first understand the way MILD combustion process differs from conventional combustion process.

In a conventional diffusion combustion process, the burning fuel jet (say in quiescent air) will have all the temperature values from ambient (say 300 K) to the flame temperature ( $\sim 2000$  K).

For turbulent flow conditions, there will be local density and pressure fluctuations that are not small. This gives rise to noise and NO<sub>x</sub> emissions that are not so small because they get generated in the high temperature flame zone, because...



One of our colleagues came up with a question after he saw my presentation. He asked, can one use combination of pre mixed and diffusion burners to obtain low emissions of MILD combustor. So, let us first understand the way MILD combustor, combustion process works and how it differs from the conventional combustion process. I want to understand people know about MILD combustion or no ok.

So, I spent some time showing what MILD combustion is. See, in a conventional diffusion combustion process, the burning fuel jet in the same quiescent air will have all the temperature

values from ambient to the flame temperature. You have a flame your ambient temperature, temperature inside goes to 2000 in odd K and therefore, we have all temperature present inside.

And if the flow were to be turbulent, there will be local density and pressure fluctuations that are not really small, that is why whenever you have a turbulent flame you will find a sound and that sound gives you healthy situation of the flame if you may want me to think so. But then this this is the one which gives rise to noise because noise is essentially because of pressure fluctuations.

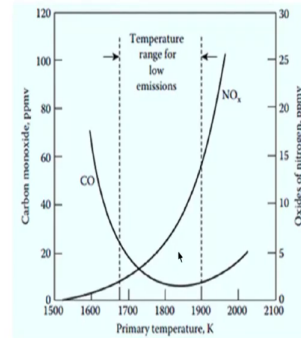
And NO<sub>x</sub> emissions are also not so small because, they get generated in the high temperature flame zone. When temperatures are high you know it is function of  $e$  to the power of my  $e$  minus  $e$  by  $r$   $t$   $b$  also related to  $p$  to the power of oxygen half and so on. So, you know that you will get higher NO<sub>x</sub> at higher temperatures.

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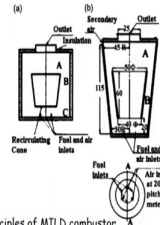
### Operating conditions to minimize emissions

As can be noted increase in temperature leads to higher NO<sub>x</sub> because Zeldovich mechanism is operating in this regime

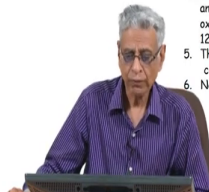
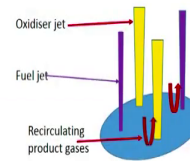
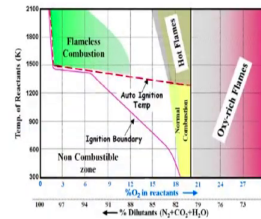


So in fact, to minimize the NO<sub>x</sub> emissions as well as co emissions the standard rule is that you must walk through a temperature range not far beyond 1900 K perhaps closer to 2700 K to make sure that both NO<sub>x</sub> and co are within limits. And as I mentioned a NO<sub>x</sub> is related to essentially Zeldovich mechanism which we discussed in the earlier lecture ok.

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- Principles of MILD combustor
1. Introduce fuel and oxidiser by keeping them separate.
  2. Bring up the combustion system to high temperature ( $> 1000$  K, say).
  3. Reduce the exit area to levels such that the flow recirculates within the system.
  4. The hot gases recirculate, dilute the "high speed" jets of fuel and air so that their temperature increases, but fuel and oxidiser quality comes down. When the temperature crosses  $1200$  K or so, a MILD combustion mode gets established.
  5. The temperature becomes quite uniform ( $\pm 100$  K or so) and so combustion becomes "flameless" and acoustic noise goes down.
  6. Noise goes down because there will be less pressure fluctuations.



I will tell you about MILD combustor from one of our studies, what do you see here in this, you introduce fuel and oxidizer by keeping them separate you do not mix them at all. So, you have fuel inlets here oxidizer inlet here. Bring up the combustion system to high temperature, say more than  $1000$  K.

Now, as it is operating reduce the exit area to level such that the flow recirculates within the system. So, please do not worry about the presence of this it does not have to have this without that also it works so, the flow recirculates. The flow that goes out recirculates here because, you are in reducing the cross section here. So, this hot gases recirculate. Dilute the high speed jets of fuel and air so that, their temperature increases, but fuel and oxidizer quality comes down.

You see here, you have fuel jets and oxidizer jet I have shown this to be small because the flow rate of fuel is small compare to oxidizer and the hot gases which are around because of the high injection velocity of this oxidizer or fuel will the gases will get you know into the system and this causes mixing and as it goes up you will discover the oxidizer will get diluted. So, also fuel will get diluted. But then the heat that present there, that also mixes with this and gives you increase temperatures. And when the temperature crosses about 1200 K, you will find most reactions begin and this is called a MILD combustion this mode gets established.

The temperature becomes roughly uniform within about 100 to 100 K or so. And the combustion becomes virtually flameless and the acoustic noise goes down. I think experience will also there at IIT, Madras on this. The noise goes down because there will be less pressure fluctuations. The key is that not to have why temperature change in the combustion process. The trick is that they have introduced the burnt gases mixing with that so, no reaction can take place only mixing can take place increases the temperature reduces the fuel fraction and oxidiser fraction. They come up to a stage where at that temperature reactions go on and you will get the whole combustion process.

So, you will find that in the range of oxidizer refraction and temperature of reactants there is one normal combustion like 18 into 20 percent oxygen and if is you can also get hot flames if you use reactants which are high at temperature and this the mode where there is the flameless combustion. If you increase the oxygen fraction much more you have called oxy rich flames and this is one mode which is used in the industry particularly when you want to melt steel do things like that oxy combustion is used.

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## Comparison of different combustion strategies

**Table 1**  
Qualitative comparison of different combustor types.

	Combustor type			
	Lean premixed	Lean direct injection	RQL	Flameless-based
Combustion efficiency	High	High	High	High
Combustion instability	High	Low	Low	Low
Fuel flexibility	Moderate	High	High	Moderate
Integration into engine	Moderate	Moderate	Easy	Difficult
Mechanical complexity	Moderate	High	Moderate	Moderate
NOx emission	Low	Low	Moderate	Ultra-low
Operating range	Moderate	High	High	Low
Soot emission	Very Low	Low	Moderate	Low
Volume requirement	Moderate	Low	Low	High



If you look at the comparison of different strategies taken from literature. We will discover, you have 4 cases; lean pre mixed. Let me explain why this lean pre mixed is used, because related the question which was posed. I can go lean and mix it pre mix it so that the combustion process is clean, no diffusive combustion. So, there is one mode which is competing with MILD combustion in terms of pollution reduction emission reduction, but lean NOx. So, lean direct injection; this also possibility rich quench approach to the process of combustion. Then the flameless based which is what MILD combustion is about what is shown here in terms of the parameters. Combustion efficiency high, this is also high, high, high, all of them have high combustion efficiency.

If you are looking at combustion instability roar, whatever it call as a roar, it is high in lean pre mixed. We will see why it is low here, low here, low here, fuel flexibility; you want to use mostly gaseous fuels in all these studies. You, the question is, is it fuel flexible? Well, in this

case is stated to be moderate high, high and moderate. Mechanical complexity is moderate the lean direct injection is high because you have to raise the pressure or inject it is moderate basis.

NOx emission is low, low moderate and we stated to be ultra low. This taken from a literature and you have many other features are comparing them. The key issue is that the flameless based combustion system supposed to be ultra low compared to lean pre mixed. Well, this is a question which bothered me for last few days and when you look at literature you talk look at aircraft engines.

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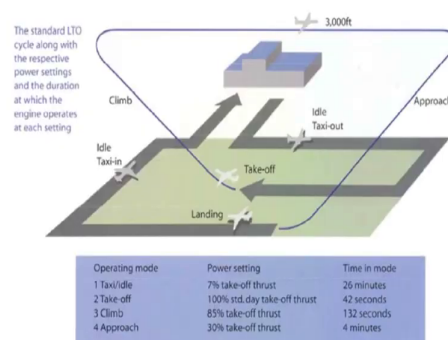
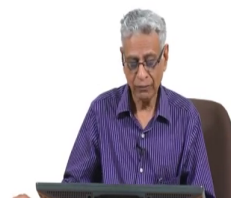


Fig. 3. Illustration of ICAO landing takeoff cycle (LTO cycle) [134].



You hardly find any literature of the kind which you think to compare the results. In aircraft engines, there is a requirement for limiting NOx and all the developments on the pre mixed combustion systems which we have taken place in the last 15 years is arising out of the need to

reduce the emissions both for aircraft and power generation packages using natural gas. And in the case of aircraft, you look jet fuel as the fuel required for the operations.

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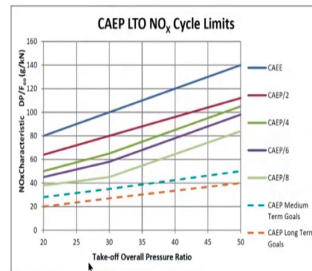
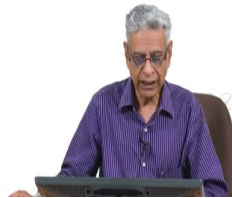


Figure 2-8: CAEP LTO Cycle Limits



A large amount of data on premixed systems is for aircraft and the combustion system design is centered around reducing the specific fuel consumption (and so, to operate in an overall lean mode). The NO<sub>x</sub> also depends on the operating pressure ratio. One must be careful in drawing comparisons with aircraft engine NO<sub>x</sub> emissions.

So, with this situation; if you look at the literature you will see that the NO<sub>x</sub> that is allowed far is a function of pressure and they always talk about little cycle which gives you NO<sub>x</sub> and on that measure one engine is qualified compared to the other. So, you will discover the large amount of data on pre mixed systems is for aircraft and the combustion system design is centred around reducing the specific fuel consumption, because you want to operate it in the lean mode and that is why it is so.

The NO<sub>x</sub> also depends on operating pressure ratio do see that. It is pretty linearly dependent on that. So, when you want to compare data, you must be very careful because the data related to aircraft is at pressure the data related to industrial combustion systems most likely



the time being pressure. And, very strong dependence on pressure is what you see here from 120 to almost like 20 20 atmosphere a very large reduction. So, when you go to one atmosphere it may be I will do the job as a lean premixed system may do the job that the hint that you get from this.

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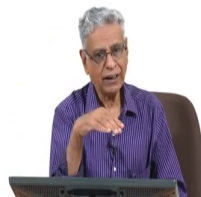


### In summary,

There is not enough data on MILD combustor NOx data with ambient pressure premixed combustors (I have not located them easily that I thought was possible!) to state that a simple ambient pressure "good" premixed combustion system maintaining the same exit temperature as a MILD combustor will perform in a way inferior to MILD combustor.

If one insists on safety of non-premixed arrangement providing industrial safety as the criterion for the choice of MILD combustor, it may be a reasonable choice.

However, I feel there is distinct case to create a safe "premixed" combustor which gives the same level of NOx as MILD combustor!



But I try to see if there are any data in in in pre mixed combustors to look at NOx. There is not enough data either MILD combustor in comparison with that of pre-mix systems. To say that there is firmly one or the other is correct and I thought it was very easy to locate it was really not possible right. It looks surprising that why is it you have premixed combustor and you do not measure NOx at various ratios of leanness or whatever, it does not seem to be available.

So, if one system the only argument why you MILD combustor seems to be superior is what I found in an argument is that the safety of a non pre mixed arrangement providing industrial safety as the criterion for the choice of MILD combustor, it may be a reasonable choice. I think there is a distinct case to create a safe pre mixed combustor which gives the same level of NO<sub>x</sub> as MILD combustor.

In a way, I am actually disagreeing with the situation low for lean pre mixed and ultra low, you may ask me why. So, we can subject it to a little discussion. The argument that I made was when you look at a MILD combustor, you are pushing in some fuel and oxidiser in some ratio and you are taking it out of the exhauster. There are lot of dynamics taking place inside.

Let it take place it is not my problem to find out what happens inside. You look at a time temperature that comes out of the combustor which may be close to the pre mixed temperature, because it is well mixed is enough time for combustion to take place and what you get out this must be close to pre mixed flame temperature for that mixture ratio. And you take a pre mixed combustion system, you will doing the same. And to say that the conditions are very different there is no lateral change of a conditions when you look at a pre mixed combustion system.

There is a variation of the temperature going from the premixed state from the initial condition to the flame temperature. It is a short domain of the flame thickness. So, I cannot find enough arguments to say why a premixed combustor cannot perform as well as a MILD combustor and I do not find any data in literature that I can lay hands on the last few days to say one thing or the other.

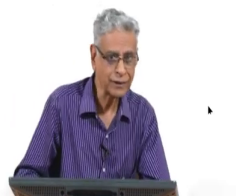
So, I leave this question at that point and there is some opportunity therefore, for somebody to do research to do one thing or the other. Because there is too much of hype around MILD combustors in some ways I am a part of that in the I was there in a lecture in in international symposium when the German person presented it so, it looked exciting. So, Sudarshan Kumar who is now a professor at IIT, Bombay. He did a PhD in this subject.

But I no longer I am able to defend that position at this moment, I am being honest with you and therefore, there is some case for doing a careful study on this subject ok.

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### Commonalities between erosive burning and hybrid rocket motor burn rate



Yeah. I will switch over to a question was asked by Kalyani, tell me something about he did not ask this question. She asked you talked about  $g$  parameter in erosive burning. How did you uncover that? What is the basis of that? Then I thought I will just present the paper and show what is what then I discovered that I can enhance the knowledge value little more by including what happens in hybrid rockets. So, I picked out this commonalities between erosive burning and hybrid rocket motor burn rate.

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Erosive burning set up

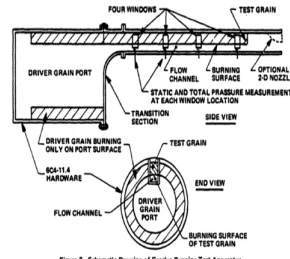
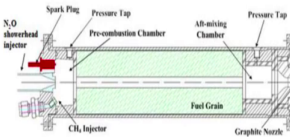


Figure 1. Schematic Drawing of Erosive Burning Test Apparatus.

Hybrid rocket motor

Burn rate of both is influenced by convective flow of gases




Hopefully I will answer the question. See in the case of erosive burning what do you have is a system here which produces hot gases and hot gases pass through this. You know how much is going through what, at what pressure and you measure the burn rate here and you know the flux at the burn rate and so, you get erosive burning component.

I may brought it up in an earlier presentation as well. In a hybrid rocket motor, what would you have? You would have a fuel block with a port then you have an oxidizer coming in in this case what is being shown is  $N_2O$ , nitrogen oxide. But this, the flow through this will create essentially a boundary layer next to the wall cause heat transfer cause fuel vaporization mixing combustion that is how it happens.

This behaviour and this behaviour are not very different, only difference here is even if the propellant gases are not going there if you ignite it will burn on it is own, that is the difference.

Here, if you do not have oxidizer the fuel would not burn on its own, but therefore, the burn rate of both these will be influenced by convective flow of gases.

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Lenoir and Robillard Law

$$r = \alpha p^n + \frac{\alpha G^{0.8}}{L^{0.2}} \exp\left(\frac{-\beta \rho_p r}{G}\right), \quad (1)$$

where  $L$  is the characteristic length argued to be the axial distance in the original work [3], but modified to be the port diameter by some later workers [5, 6]  $G$  is the mass flux, and  $\beta$  is a dimensionless constant. The constant  $\alpha$  depends on the density of the propellant  $\rho_p$ , the flame and surface temperature  $T_f$  and  $T_s$ , the gas phase viscosity  $\mu$ , and the specific heats in the condensed and gas phases  $c_p$  and  $c_s$ . Its functional form is

$$\alpha = \frac{0.0288 c_p \mu^{0.2} \text{Pr}^{-2/3} (T_f - T_s)}{\rho_p c_s (T_s - T_m)}, \quad (2)$$

$r_0 = \alpha p^n$  and defining  $\eta$  as  $r/r_0$  and  $g_0 = G/\rho_p r_0$ , to give

$$\eta = 1 + 0.0288 \frac{c_p}{c_s} \text{Re}_0^{-0.2} \text{Pr}^{-2/3} \tau_f g_0^{0.8} \times \exp\left(\frac{-\beta \eta}{g_0}\right), \quad (3)$$

where  $\text{Re}_0$  is a Reynolds number defined by  $\text{Re}_0 = \rho_p r_0 L / \mu_0$  and  $\tau_f = (T_f - T_s) / (T_s - T_m)$ . The ratio of the gas phase to condensed phase specific heats is about 1, and  $\tau_f$  can be estimated from the data on flame and surface temperatures for low- to high-energy propellants to be between 1.3 and 4.0. One can postulate the functional relationship in erosive burning as

$$\eta = \eta(g_0, \text{Re}_0, \tau_f, r_s), \quad (4)$$

$$\eta = 1 + \alpha' g_0^{0.8} \exp(-\beta \eta / g_0).$$

I switch to erosive burning, taking on from the paper and I also mentioned is Lenoir and Robillard law, which is like this. The burn rate this is the non erosive part which we have been looking at over a period of time and this is the contribution due to convective flux. The  $G$  the mass flux,  $\alpha$  is a constant and  $L$  is a length scale,  $\beta$  is the another constant  $\rho_p r$  which is the same thing as the burn rate here divided by  $G$ .

This part is related to what is called blowing effect. I must explain to you what blowing effect is. When you transfer heat from the boundary layer to the surface it vaporizes suppose the vaporization is much more, what happens is? It pushes away the boundary layer and because the it pushes away the bound layer the heat flux comes down this is called blowing effect.

So, you must account for this blowing effect this way of accounting for it by using by using analysis, but it is behaviour is expressed through this route I must tell you after all this there is still a most elegant piece of work the work of Lenoir and Robillard. Now this constant  $\alpha$  is expressed in his own paper in this manner.

So, until this there is nothing new that has been is done ok. What did I do is a few may ask me I said ok. Let define  $\eta$  is  $r$  by  $r$  naught and whenever there is an  $r$  naught we will multiply by  $\rho p$  because it is the mass flux which counts and never really reactive linear regression. So,  $\eta$  is equal to this, I have put together a quantity  $g$  naught which is here. Mass flux by  $\rho p r$  naught. See the this burn rate occurs changes because of pressure, mass flux also does something to burn rate. This ratio is important because if  $g$  is 0 depends only on pressure if  $G$  is present will be additional flux and therefore, burn rate.

So, this parameter therefore, is a natural choice. So,  $g$  naught to the power of 0.8 and you put together other things you will get a small change of the Reynolds number and this parameter due to blowing effect it is also present here. Then you follow through some analysis and you can express it all this into a constant and you can show that the  $\eta$  burn rate ratio is proportional to  $g$  naught to the power of 0.8 and a constant here is always present, that is all. Still two constants are present here.

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In the present case, there is also the effect of blowing. The combined effects can be set out in terms of a constant exponent on  $Re_0$  for simplicity. Following the classical understanding of the presence of threshold mass flux [1], we can write the erosive burning relationship as

$$\eta = 1 + K_1 (g^{0.8} - g_{th}^{0.8}) \mathcal{H}(g - g_{th}) \quad (10)$$

with

$$g = K_2 g_0 Re_0^m, \quad (11)$$

where  $\mathcal{H}$  is the Heaviside unit step function. The values of  $K_1$  and  $g_{th}$  are determined from actual data. Any convenient value can be chosen for  $K_2$ , as it only affects the scaling. (Equation 11 substituted into Eq. 10 gives a single constant  $K_1 K_2^{0.8}$ . For any arbitrary value of  $K_2$ ,  $K_1$  can be obtained by keeping  $K_1 K_2^{0.8}$  constant. A second constant is used only as a scaling factor for  $g$ , so that the range of magnitudes of  $g$  and  $g_0$  are the same.) The Reynolds number  $Re_0$  is based on the nonerosive mass burn rate and the port diameter. It must be remembered that the primary effect is expressed through the term  $g^{0.8}$ , which accounts for the mass flow effect and the Reynolds number correction is to account for the size effect.



So, having done this he said that time we had to make a choice between how to pick 2 constants and so on. I had done a project much earlier on erosive burning of homogeneous propellants and determine the constant alpha and beta. I must tell you I was quite unhappy with the numbers which we obtain will not very consistent. So, I for a variety of reasons I was displeased with the with the unsettled situation about erosive burning.

So, it was clear of course, to the burning rate ratio is crucial and so, is the mass flux. That is the time when you look at the data which was available which is available. And we know that  $g_{naught}$  is crucial. So, plot eta versus  $g_{naught}$  first let us see what happens.

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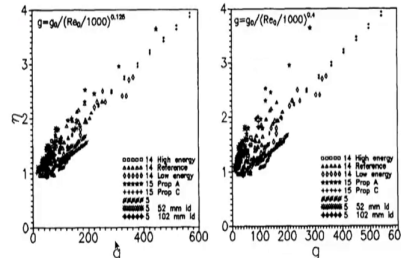


Fig. 6. Effect of Reynolds number index in correlating erosive burning data.

$$\eta = 1 + 0.023(g^{0.8} - g_{th}^{0.8})^2(g - g_{th}), \quad (12)$$

where  $g = g_0(Re_0/1000)^{-0.125}$  and  $g_{th} = 35.0$ .



There is a complex dependence because of, because of  $e$  to the power of minus beta eta naught and so on. Let us say at the moment there is some compensating features. Later sometimes a more helpful than what we think we can infer that is so complex, but let us assume it is not so complex and see what happens just a hypothesis. So, you plot eta versus  $g$  naught essentially you did this is actually in this particular it is just  $g$  naught to discover everything seems to be falling into this.

So, you do not need to complicate the you know expression with all this and include a coefficient and so on. And also that coefficient settles on it is own and so, that the lock which you see here comes out like you know when you cook you put various ingredients and stir and suddenly you will get the food available. So, it is something like that.



There is not too much that has to be done. Key thing is to plot the experimental data consistently and carefully in this mode ok. Now in one case you see this expression had some Reynolds number effect well say ok. Let us see maybe there is the Reynolds number effect. And sorry, you will see here I have plotted that with respect to a quantity with 0.125 and here 0.4.

Do you see is this cater seems to be much more here than here you know these 3 points fall away compare to whatever is here. And, there is no specificity should be only one number or the other see it is a scaling law experiment can be different. And, let it be settled by the experiment and not by me. So, that is how this 0.125 has chosen and you if you take the slope of this you will get this expression  $g$  equal to  $g_0$  into  $Re$  to the power of by thousands power of minus 0.125.

And some constant here that comes from the slope of this line and that there is a threshold flux beyond which is it begins to operate that is what this number is. So, you will discover that there is all constants are removed and the whole thing there is no constant which is free. It is just the law expressed by nature and so, you accept it that is how it has. So, it has been asked several times a question how do you know 35 is right.

See it is not me telling some 35 is right or 45 is right is what data is showing. You are welcome to choose at 38 and draw a line no issues about it. So, you and if you take 38 instead of 34. So, predictions will be slightly different, but not awfully different. It is erosive burning is something which influences the pressure time curve in the early part of the burn time.

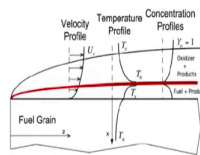
So, you must remember the port in a high density propellant grain the port is smallest in the beginning. So, the fluxes are must higher in the beginning. So, the influence of erosive burning is much more in the early part, that is why I said the pressure time curve will show a longer tail only in the beginning. Afterwards you will find the influence of it in the pressure time curve, but is not there much beyond.

So, it is that you accept this and you I told you the story of what happened with non axis symmetric grains and so on and that is what it is.

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### Basic physics and some thermo-chemistry



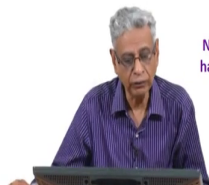
- In a "strong" diffusion flame, chemistry is very fast compared to diffusion. Hence, the flame is thin.
- Therefore, the burn rate is controlled by diffusion rate which is affected by boundary layer next to the regressing fuel surface
- To obtain the dependence of burn rate flux on flow velocity and thermochemical parameters, we use Reynolds Analogy

The heat flux balance at the surface gives  $\rho_f \Delta H_f = q'' = k \frac{dT}{dy} \Big|_0 \sim k \frac{dT}{dU} \frac{dU}{dy} \sim k(T_f - T_\infty)/U \cdot 1/\mu \cdot \mu \frac{dU}{dy} \sim (k/c_p \mu) c_p (T_f - T_\infty)/U \cdot \tau_w \sim (k/c_p \mu) c_p (T_f - T_\infty)/U \cdot (cf/2) \rho U^2 \sim (k/c_p \mu) c_p (T_f - T_\infty) (c_p/2) \rho U$   
 $[\rho_f f = \text{fuel/propellant mass flux from the surface; } \Delta H_f = \text{Heat of phase change; } [c_p/c_{p0}] = \text{Blocking effect}]$

From this, we get:  $\rho_f f \sim (k/c_p \mu) [c_p (T_f - T_\infty)/\Delta H_f] (c_p/c_{p0}) (c_p/2) \rho U \sim (1/Pr) B [\ln(1+B)/B] \rho U [0.054 (\rho U)^{0.2}] \sim C_g \ln(1+B) G^{0.8}$   
 where  $B = [c_p (T_f - T_\infty)/\Delta H_f] \cdot G = \rho U$ ;  $G = \rho U$ ;  $\ln(1+B)$  is sometimes treated as  $B^{0.25}$  in the range of  $B$ 's relevant

$$\text{Thus } \rho_f f \approx C_g B^{0.25} G^{0.8}$$

Note that the burn rate enhancement in the case of solid propellants and burn rate itself for hybrids have similar dependence on mass flux ( ) or Reynolds number



But, if you look at the comparison of the physics here and that in you know in hybrid rocket. In hybrid rocket you have a fuel grain you have a boundary layer, you have a velocity profile as you know your temperature profile which is like this flame being where it is, concentration profile, diffusion flame therefore, oxidizer in the outer part and fuel in products of the inner part, oxidizer and products in the outer part, fuel and products are the inner part and this is a diffusion flame.

And in a strong diffusion flame; chemistry is very fast compared to diffusion that is why it is thin arguments like you have heard earlier. Therefore, the burn rate is controlled by diffusion rate which is affected by boundary layer next to the regressing fuel surface to obtain the

dependence of the burn rate flux and flow velocity in thermo chemical parameters you use what is called Reynolds analogy. And then, if you look at the heat balance to the surface  $\rho_p r \Delta H_s$ . That the flux coming from the surface is equal to the heat flux coming from the gas phase  $k_d T_d y$ . I can please follow this algebra a little more carefully.

I can express this as  $k_d T_d$  by  $dU$  into  $dU dy$  and so,  $dT_d$ , I will express as  $d(T_f - T_s)$ ,  $dU$  goes from 0 to  $U$  therefore,  $k$  into  $(T_f - T_s)$  by  $U$ . I will add a  $1/\mu$  and add a  $\mu$  here and there is a  $dU dy$  ok,  $\mu du dy$  is essentially shear stress. So, I also rearrange this, I will add a  $k$  by  $c_p$  and add a  $c_p$  here. So, I get a  $k$  by  $c_p \mu$  this  $1/\mu$  is here know the same  $\mu$  which is here and the  $c_p (T_f - T_s)$  divided by  $U$  which is here and you multiply by  $\tau_w$  ok. I can rearrange it in the next step and you will discover finally,  $k$  by  $c_p \mu$  which is essentially  $\mu c_p$  by  $k$  is essentially prime number 1 by prime number  $c_p (T_f - T_s)$ .

And I will introduce what is called  $c_f$  by  $c_{f,naught}$  to  $c_{f,naught}^2$  into  $\rho U$  and the  $c_{f,naught}$  is introduced to account for the blowing effect which I spoke to you a few minutes ago  $c_f$  by  $c_{f,naught}$ . From this if you do the algebra you will get  $1/\text{prime number}$  and you divided multiplied by  $\Delta h_s$  you will get essentially transfer number which is what is put together,  $\rho U$  which is present continuously in all this in this discussion  $\rho U$  here you see here same  $\rho U$ . And so, it goes into the simple expression of  $c_{f,naught} \log$  of  $1 + B$  into  $G$  power 0.8.

This result is not very different from what Lenoir and Robillard said he only added the non erosive burn rate along with that. So, the physics is about the same and there are many issues related to  $B$ , I do not think we need to discuss it here, but I have a similar dependence on Reynolds number or mass flux this is something that we need to keep in mind that is all I want to tell you on response to your question. And, I should you wish to raise any questions on this or anything else any other question you are most welcome we will debate them now.