

Fundamentals of Combustion for Propulsion
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Lecture - 32
Summary - solid rocket propulsion

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The slide contains the following elements:

- SRM Diagram:** A cross-sectional diagram of a Solid Rocket Motor (SRM) showing the grain, nozzle, and various internal components. A handwritten red arrow points to it with the label "SRM".
- Equilibrium State of SRM:** A red-bordered box containing the following text:

$$\frac{dm}{dt} = \frac{d(\rho V)}{dt} = V \frac{d\rho}{dt} + \rho \frac{dV}{dt} = V \frac{d\rho}{dt} + \rho_i t A_0 = \dot{m}_{in} - \dot{m}_{out}$$

$$\eta = 1 + 0.02 \ln(g^{*4} - g_0^{*4}) \ln(g - g_0), \quad (12)$$
 where $g = g_0 (Re_0/1000)^{-0.125}$ and $g_0 = 35.0$.
 A handwritten red note next to it says "Equilibrium state of SRM".
- NPTEL Logo:** The National Programme on Technology Enhanced Learning logo.
- Summary of basics of solid propulsion systems:** A central heading in red text.
- Graph:** A plot showing pressure (P) and thrust (F) versus time (t). The pressure curve is a damped oscillation, and the thrust curve is a step function. A handwritten red arrow points to it with the label "DC shift".
- Composite Solid Propellant:** A diagram showing the internal structure of a composite solid propellant grain, which is a porous material with internal voids. A handwritten red note next to it says "Composite solid propellant".
- Equation for Burn Time:** A red-bordered box containing the following equation:

$$\frac{T_{eff} - 1250}{T_{end} - 1250} = \frac{1 - e^{-Z}}{Z}; \quad Z = \frac{d_{AP}}{d_0}$$
- Presenter:** A video of a man in a black shirt, identified as Prof. S. Varunkumar, speaking and gesturing with his hands.

That, completes the summary of the fundamental ideas. We will move on to a summary of three different Propulsion systems that were discussed in the course the Solid Propulsion system, the liquid propellant propulsion system and air breathing propulsion systems, with use in which we in the analysis of which we used the ideas the fundamental ideas that were discussed and summarized till now. Let us move on to a summary of basics of solid propulsion systems.

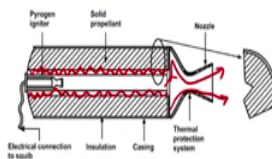
Here again I will try to use as much of pictures as I can, what I have shown here is a basic layout of a Solid Rocket motor these equations as you may recall or what are used to calculate the equilibrium state ok, of the solid rocket motor or the equilibrium pressure of the solid rocket motor. Whether that equilibrium is a stable equilibrium or an unstable equilibrium is a much more difficult problem to solve which is also something that we discussed in some detail, and one particular aspect of an unstable equilibrium is the D C shift ok, this is another thing that we discussed in the course.

And an other aspect of solid propellant combustion which we discussed in some detail is predicting the burn rate of composite solid propellants ok. Because what is required for this I will explain in the next slide in some detail, what I want to emphasis here is the key idea which was used for the prediction of the composite solid propellant burnt rate is the idea of lateral diffusion distance.

This d_0 that appears here is exactly the lateral diffusion distance and the ratio of the AP particle size to the lateral diffusion distance is a measure that was used to decide what is the extent to which reaction rate controls the deflagration rate of a solid rocket propellant ok.

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Basic layout




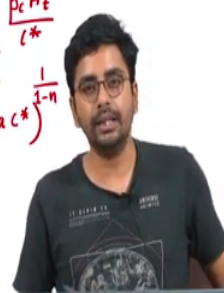
$$\frac{dm}{dt} = \frac{d(\rho V)}{dt} = V \frac{d\rho}{dt} + \rho_c \frac{dL}{dt} = V \frac{d\rho}{dt} + \rho_c r A_b = \dot{m}_{in} - \dot{m}_{out}$$

$$\frac{dm}{dt} = \rho_p \dot{r} A_b - \frac{p_c A_t}{c^*}$$

$\dot{r} = \dot{r}_0 + \dot{r}_{erosive}$ (Thermodynamic)

$\dot{r}_0 = a p_c^n$ $\rho_p \dot{r} A_b = \frac{p_c A_t}{c^*}$

Hagen-D

$$\rho_p a p_c^n A_b = \frac{p_c A_t}{c^*} \Rightarrow p_c = \left(\frac{A_b}{A_t} \frac{1}{\rho_p a c^*} \right)^{\frac{1}{1-n}}$$



Let us go into some details this is a basic layout of a solid rocket ok, once ignited using a pyrogen igniter in this case, the propellant starts burning normal to itself along the local normal and the solid propellant will get converted into hot gases at very high temperatures typically about 3000 Kelvins. Those high pressure high temperature gases expand through the nozzle producing thrust this is the basic operation of a solid rocket motor.

And the pressure inside the rocket motor is governed by a simple balance of how much mass is generated by the conversion of the solid propellant to the gaseous propellant and how much mass goes out ok. So, the rate of change of mass which I have expanded here which is well ok, the rate of change of mass is equal to the rate at which the solid propellant gasifies, which is $\rho_p \dot{r} A_b$ minus the outflow through the nozzle. $p_c A_t$ by c^* , we call that the c^* is a thermodynamic property of the propellant, this comes from thermodynamics.

And \dot{r} which is empirically determined usually has two components one is called the intrinsic burn rate of the propellant and other is the erosive burning component. The other is the erosive burning component. This \dot{r}_0 , is usually power law of, power law function of pressure and in the simple case where let us for a moment ignore the erosive burning component the equilibrium pressure of a rocket will be simply governed by the balance of the incoming mass with the outgoing mass.

C^* and $\rho P \dot{r}$ goes as $p^{1/n} A_b$ is $p^{1/n} A_t$ by C^* and from here we can simply get the equilibrium pressure to be A_b by $A_t \rho p^{1/n} C^*$, raised to power $1/(1-n)$. I think in the equation that I wrote in the lecture I made a small mistake the $a C^*$ appeared in the denominator in this equation, but this is the correct expression for the equilibrium pressure ok.

And the ignoring the erosive burning component this is this will be the equilibrium pressure in inside the rocket chamber and as the burning surface evolves the burning area will change ok, and the pressure inside the chamber will also change. \dot{r}_0 is typically obtained from experiments. But, it takes a lot of trial and errors given for a given machine there is a particular requirement of thrust which translates into a particular requirement of pressure or is to how the equilibrium pressure should change with time, and that determines what should be the burn rate of the propellant as a function of pressure.

And it is not a simple task to actually design a propellant such that it gives the required burn rate pressure variation. And therefore; creating a model with which a designer can design propellants without having to go through a trial and error process is a challenging task and the heterogeneous quasi 1 D model solves this problem. The heterogeneous quasi 1 D model results in a in a theoretical frame work with which \dot{r}_0 can be calculated as a function of pressure without having to do a lot of trials ok.

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$$\eta = 1 + 0.023(g^{0.8} - g_{th}^{0.8}) \mathcal{H}(g - g_{th}), \quad (12)$$

where $g = g_0 (Re_0/1000)^{-0.125}$ and $g_{th} = 35.0$.

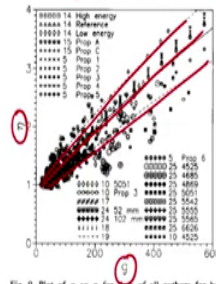


Fig. 9. Plot of η vs g for data of all authors for both double-base and composite propellants.

$$\eta = \frac{\dot{r}}{\dot{r}_0}$$



With this background let us quickly go to the erosive burning the idea of erosive burning, this was discussed in some detail in the as a part of the course. The key idea is that when expressed ok, eta which is the burn rate because including the effect of erosive burning divided by the intrinsic burn rate ok. When expressed as expressed in non-dimensional form as a function of the non dimensional port flux, port mass flux and the Reynolds number irrespective of the type of the propellant irrespective of whether a particular composition is composite or double base ok.

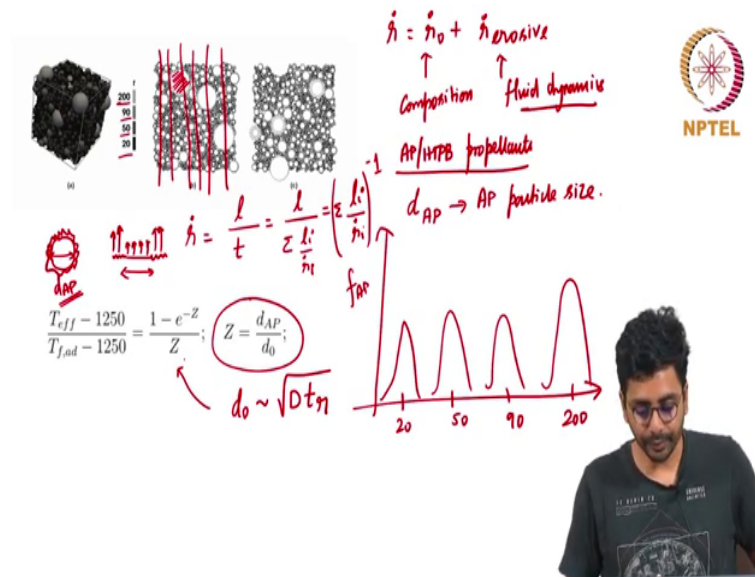
All data when expressed as eta versus g which is the non dimensional port mass flux follows more or less falls within a very narrow band that is what is shown here, this is data from several sources for different kinds of propellants using different kinds of ingredients and irrespective of all these variations, when expressed as eta versus g all data falls within a very

narrow band and this equation is the is a good fit for this behaviour ok. For some reason I am not able to draw the box. So, this equation is a good fit for the for this variation.

What is the conclusion the conclusion is very simple erosive burning is a fluid dynamic effect ok, because the only variables that appear on the right hand side of the function are the port mass flux or the non-dimensional port mass flux and the Reynolds number. There is no variable that is the function of the composition of the propellant or the burn rate of the propellant ok. Therefore, while the intrinsic burn rate which is \dot{r}_0 is a strong function of the composition and many other factors and dependent on the particle size of AP and ingredients very specific ingredients that are used for making propellants.

Erosive burning is independent of all those effects, all those composition effects are absorbed into \dot{r}_0 , erosive burning is only a fluid dynamic effect and that is the main conclusion from this theory ok. This is called the universal erosive burning model and the conclusion is that erosive burning is the fluid dynamic phenomena ok.

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So, now we have seen recall that \dot{r} as \dot{r}_0 plus $\dot{r}_{erosive}$, we saw that $\dot{r}_{erosive}$ is a fluid dynamic effect only. \dot{r}_0 is composition dependent and in the case of AP HTPB propellants, in the case of AP HTPB propellants it is a strong function of the size of AP or the size distribution of AP, AP particle size distribution d_{AP} is AP particle size.

And in a typical propellant, a typical propellant can contain particles of in a typical composite solid propellant there can be AP of different mean sizes for example, one case that is shown here has 4 mean sizes 200 90 50 and 20 20 50 90 and 200, not just that around each mean size there will be a distribution ok. Mass fraction of AP around each size there will be a distribution of particle sizes.

Therefore, a typical composite solid propellant having AP as an oxidizer, ammonium perchlorate as an oxidizer a cross section of it will look like this as I have shown here large AP

particles, surrounded by very small AP particles and here and there some medium sized AP particles ok. So, a cross section is highly heterogeneous containing particles of different sizes and the burn rate \dot{r}_0 is a strong function of this distribution.

And, how do we calculate at what rate this propellant will burn when it has particles of different sizes. So, the simple idea that was introduced is that the burn rate is nothing, but the average burn rate of a large number of random lines drawn through a cross section like this ok. So, the burnt rate of a propellant, is simply the length divided by the time the average time that it would take for such a statistical particle path to burn, and that time is the sum of the time of particles of different sizes.

So, the time for the line to burn the average line to burn is the sum of the times for individual particles which is the length of the individual particle divided by the burn rate of the individual particle and therefore, the burn rate of the propellant is $\frac{l_i}{\dot{r}_i}$ sum inverse. We can arbitrarily take l to be unit length ok. So, this is the simple idea that is used to calculate the burn rate of a propellant containing AP particle sizes different AP particle sizes.

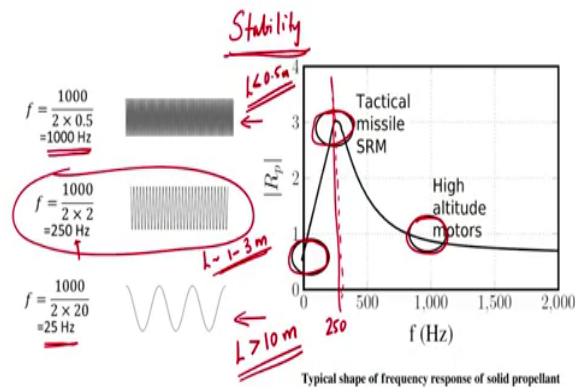
And now the question comes what controls the burn rate of a particle of size d_{AP} ok. And this is where the idea of lateral diffusion comes into picture. So, we have a particle of size d_{AP} covered by fuel and therefore, when you look at it in the cross section there is fuel AP and fuel, oxidizer is issuing from the centre fuel vapours are coming off from the periphery ok.

So, the question is what controls the structure of the flame and therefore, the heat flux that goes to the surface which in turn controls the regression rate, the idea is very simple. The flame structure will be controlled by mixing if d_{AP} is much larger than the diffusion distance ok, and the behaviour will be close to premix if d_{AP} is much smaller than the diffusion distance. And, that simple idea is what was used to define this non dimensional variable which is nothing, but the ratio of the AP particle size to the lateral diffusion distance.

If you go back and check the equation the equation for \dot{d}_0 is exactly this reaction controlled diffusion distance ok. This simple idea was encoded into this to calculate the burn rate of

different particles that constitute the statistical particle path, and from that the burn rate of the propellant can be calculated ok. This is the crux of the heterogeneous quasi one d model ok.

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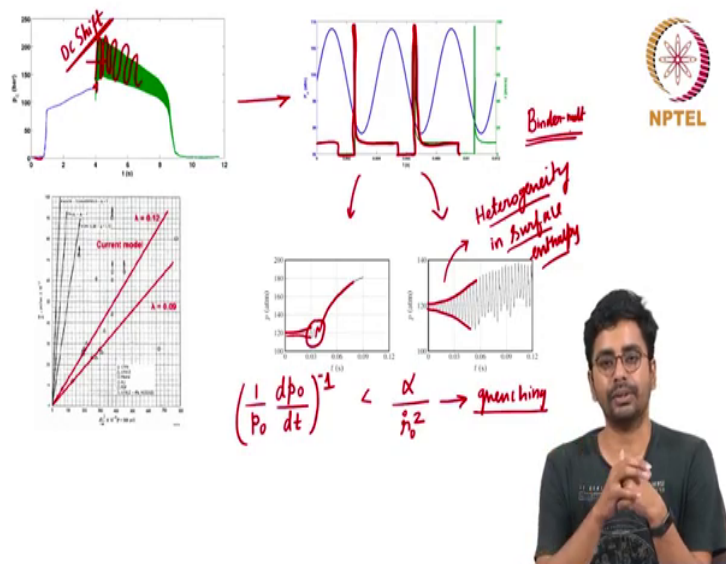
Moving onto the problem of stability ok. So, far we have concerned ourselves with calculating the equilibrium state of the rocket, but we have not ask the question whether that equilibrium is stable or unstable. As it turns out the equilibrium pressure and the equilibrium pressure variation with time that we calculate using the classical ideas ok.

Are stable for very short rockets l less than 0.5 metres, and for very long rockets l greater than 10 meters. And as it turns out, this is because the fundamental longitudinal mode frequency for a very long rocket motor is very low its only 25 Hertz and for a very short rocket is very high about 1000 Hertz. It is only intermediate sized rockets which are a few metres long l to 3

metres or something like that, the frequency is about 250 Hertz and the composite solid propellant responds to perturbations which are around 250 Hertz.

If you have very short very short motors your frequencies are very high you are far to the right your response function is lower and therefore, there is no instability uh. If you go to the left which is very long motors the fundamental frequencies are very low again the response function is very small and therefore, there is no instability. But tactical solid rocket motors are sort of in the instability switch part where the frequency and the response the peak response and the frequency matches. So, what are the consequences of this?

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The consequences that the instability will cause, a growth of small perturbations like this and what is unique to solid rocket motors is what is called D C shift is the shift in the mean ok. Is

the shift in the mean, is the shift in the mean pressure and very large amplitude oscillations around this mean, this is the characteristic of solid rocket motors.

And we saw that during the DC shift the propellant are subjected to very high depressurization and cyclic depressurization and pressurization phenomena, that causes cyclic extinction and dramatic reignition where the burn rates can go to 10 times the mean value before it comes to the mean position, quenches again because of high depressurization rates remains constant ignites dramatically and comes back ok.

This, that this is the origin of DC shift is conclusively shown using c f d calculations ok. Where linear growth or exponential growth of oscillations when a pressure when a critical amplitude is crossed, where the depressurization rates have crossed the critical value the propellant undergoes extinction reignition cycles leading to DC shift ok.

And a simple scaling for the critical depressurization rate is that the timescale for depressurization $1/p_0$ you start from a pressure of p_0 ok. This the inverse of this is the timescale for pressurize depressurization, if this becomes comparable or smaller than the conduction timescale which is α/r_0^2 . Then the propellant will extinguish this will lead to quenching. If the propellant does not have enough time to adjust to the changes in the gas phase temperature profile it will quench ok.

And in the case of cyclic depressurization pressurization, there is dramatic reignition because when the propellant quenches the solid phase still remains at a higher temperature it remains in the cooped state. And, when pressurization happens establishment of the gas phase flame will simply gasify this entire mass of cooped propellant in one shot leading to very high burn rates, and that is the origin of the DC shift in solid rocket motors ok.

So, that completes the discussion of uh. Well one point that I missed is what causes increase in the amplitude of the? What causes the growth in the amplitude of small perturbations called the linear instability? This is because of the heterogeneity in surface enthalpy ok. To put it in simple terms it is the binder melt fluctuation binder melt fluctuation you have AP surface

surrounded by binder and instability is usually associated with the use of burn rate inhibiting additives and these inhibitors.

Because they are added to the binder they inhibit the gasification of the binder leading to overflow of the melted binder over AP, causing the burn rate to go down. This is what happens when under steady pressure conditions, but when the pressure oscillates the same binder melt can actually fluctuate leading to a very large fluctuation in the burn rate.

Which when coupled to the pressure fluctuations or the acoustic fluctuations in the rocket motor can lead to growth of oscillations, and when the oscillation amplitude crosses the critical value this leads to D C shift ok. That completes, that is the complete picture of instability in solid rocket motor.