

Fundamentals of Combustion for Propulsion
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Lecture - 19
Instability in solid rockets – II

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Back to unsteady conditions...



I mention that I just want to give you a recap we went started with an analysis based on the mass balance equation for a realistic pressure time trace for an unstable tactical missile rocket motor. From there we moved to the question of the connection if any between low index and stability, I made a statement that low index does not necessarily mean that the rocket will be stable ok.

And then we went to the question of predicting the index, and therefore, I discussed in some detail the steady state burning behavior of propellants ok. Now, I would like to get back to the

question of stability its connection to index especially in cases where the index is lower than 0.3 ok.

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Q3. Now can we calculate the frequency response of composite propellants? Yes. Using the equations below.


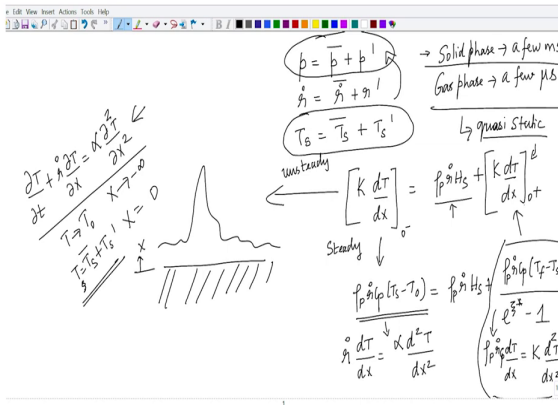
$$|R_{p,i}| = \frac{2+h_s/(1-h_s)(0.6\bar{p})/\bar{H}_s(1/g(B_{eff})) + z_r[e_g + (1/g(B_{eff}))\bar{T}_{eff}/(\bar{T}_{eff}-\bar{T}_s)]}{2+(\theta_{fs,eff}(1-h_s) + A_c \cos(\phi_c) - e_s)/(g(B_{eff})(1-h_s)e_s)}$$

$$\phi_{p,i} = \cos^{-1} \left(\frac{\langle r'_i, p' \rangle}{\sqrt{\langle r'_i, r'_i \rangle \langle p', p' \rangle}} \right) \quad R_p = \frac{\dot{r}'/\bar{r}}{p'/\bar{p}} = \bar{r} \sum_i \frac{l_i}{\bar{r}_i} R_{p,i}$$



Took very about this equation, this we have a set of equations that describe the steady state behavior of the model ok.

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The whiteboard contains the following handwritten notes:

- Top left: $\frac{\partial T}{\partial t} + \rho \frac{\partial T}{\partial x} = \alpha \frac{\partial^2 T}{\partial x^2}$ with $x \rightarrow -\infty$ and $x = 0$.
- Top right: $\rho = \bar{\rho} + \rho'$, $\dot{q}_1 = \dot{q}_1 + \dot{q}_1'$, $T_s = T_s + T_s'$. Notes: "Solid phase \rightarrow a few ms", "Gas phase \rightarrow a few μ s", "quasi-steady".
- Center: A diagram of a solid phase (hatched) with a temperature profile $T(x)$ showing a peak at the surface.
- Left of center: $T \rightarrow T_0$, $T = T_s + T_s'$, $x = 0$.
- Right of center: "unsteady" with the equation $\left[K \frac{dT}{dx} \right]_0 = \rho_p \dot{q}_1 H_s + \left[K \frac{dT}{dx} \right]_0$. Below it, "Steady" with $\rho_p \dot{q}_1 (T_s - T_0) = \rho_p \dot{q}_1 H_s + \left[K \frac{dT}{dx} \right]_0$.
- Bottom right: $\rho_p \frac{dT}{dx} = \alpha \frac{d^2 T}{dx^2}$ and $\int \rho_p \dot{q}_1 dt = K \frac{dT}{dx}$.

All that is done is that a simple perturbation analysis, where the pressure is taken to be mean pressure plus perturbation corresponding perturbation in \dot{r} and other variables ok. In the steady state equation that we have, do you remember is $k \frac{dT}{dx} \big|_0$ minus is equal to $\rho_p \dot{q}_1 H_s$ plus $k \frac{dT}{dx} \big|_0$ plus; this is the basic equation both for steady as well as unsteady regression.

For the steady case, we took this to be steady case this was $\rho_p \dot{q}_1 c_p T_s - T_0$ is equal to $\rho_p \dot{q}_1 H_s$ plus the gas phase heat flux term which we found is $\rho_p \dot{q}_1 c_p T_{\text{flame}} - T_s$ divided by exponential $\eta^* - 1$. This is the steady state flux balance, which we have already discussed. When it goes to unsteady, remember that this particular term was obtained as a solution to the steady conduction equation ok.

And this was obtained, sorry, there is no I am using an alpha on the right hand side, $\rho P r$ dot $d T / d X$ is the gas phase assuming a thin flame, this was what was done. This term came from here this term came from here. Now, when the conditions are not steady, we cannot use the steady conduction equation. The slope must be obtained from solving the unsteady conduction equation, which in this case is simply ok.

So, this flux term the heat flux that goes into the solid phase must be calculated, as a function of time, because the surface temperature will now fluctuate with time. Why is the surface temperature fluctuating with time? Because the propellant itself is subjected to the fluctuation in pressure ok. So, this equation must be solved with the following boundary conditions T goes to T_0 as x goes to minus infinite, but T equals T_s plus T_s dash, sorry T_s is the mean surface temperature plus the fluctuating component at X equals 0 ok.

So, therefore, this term will fluctuate with time, this term will have value that is fixed by the flux balance. What about the gas phase? Remember that the response time for the solid phase, solid phase responds with a time scale of a few milli seconds ok. On the other hand, gas phase response is much faster, probably a few or few 10s of microsecond. And therefore, remember that the we are interested in pressure fluctuations with a frequency of about a few 100Hertz, and therefore, a time scale of a few milliseconds ok.

So, in a few milliseconds when the pressure changes from the peak value to the lowest value when it oscillates the gas phase has sufficient time to adjust to the changes in pressure, and therefore, the gas phase can be considered quasi-static. Gas phase is quasi-static, that means, we do not need an unsteady version of this term we can use exactly the same equation that we have been using and assume that it instantaneously adjust changes in temperature and pressure.

Student: Coordinate (Refer Time: 06:01).

No. So, this is the propellant, you have the flame structure here something like that. So, this is X coordinate, and the coordinate is assumed to be attached to the regressing surface. Yes.

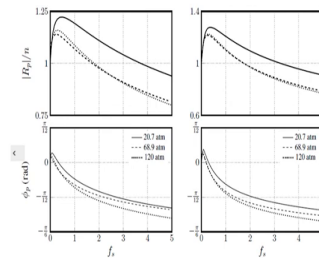
Student: (Refer Time: 06:22).

Yeah. So, I mention that in the surface heat flux balance equation, we have these three terms. Under unsteady conditions, we need to account for the change in the heat flux that goes into the condense phase with time ok. Therefore, this term is obtained by solving the unsteady conduction equation. One might think that we need to do the same thing for the gas phase also, but it is not required, because the pressure changes in the gas phase are happening at a time scale of a few milliseconds, but the gas phase can adjust to changes in pressure within a few microseconds or a few 10s of microsecond. And therefore, the gas phase can always be assumed to be at a steady condition. Corresponding to the local changes in pressure and temperature it instantaneously adjusts to the steady profile ok. Answer your question?

When this perturbation analysis is done, please do not worry about the equation. All I want you to recognize is that this is the response function of a particle of size d_i . We had an equation for the burn rate; this is the corresponding equation for the fluctuation in the burnt rate for the same particle size. The fluctuation and burn rate is in general a complex quantity; because it can have a certain magnitude and it can have phase difference with pressure. And this is the magnitude and this is the phase.

And the response function, we saw the definition for response function already; it is the relative fluctuation in \dot{r} to the fluctuation in pressure. And the response function of the propellant can be related to the response function of individual particles is exactly the same you just analogous to what we did in the steady state. Let us the burnt rate of the propellant is related to the burnt rate of a individual particles and the statistical particle path. The response function of the propellant is related to the response function of individual particles in the statistical particle path. The line average fraction appears again here.

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SD-III-18 from Miller (1982)

- Though interesting and very useful for practitioners, these results are not relevant for instability for two reasons.

- R1 – the minimum index is 0.4 – lateral diffusion alone cannot explain low index of 0.25.

- R2 – $R_p/n \sim 1$ – these propellants are very well behaved in the context of 'instability'.



Now, response functions can be calculated for a variety of compositions or the same by extending the code ok. What I want to emphasize is that though very interesting and very useful, these results are not very relevant for instability for two reasons. The reason one set the maximum index that we could predict with this approach is only 0.4, but remember that propellant compositions that are used have index as low as 0.2, 0.25. We still do not know how that is achieved ok. So, just accounting for lateral diffusion brings down the index from 0.8 to 0.4, not below that.

Reason 2, and it is the most important, the response function divided by the index or the response function scaled by the index is of the order of 1. Remember that the response function becomes equal to the index at 0 frequency. So, R_p/n will be 1 at 0 frequency, but more importantly the maximum value of the response function is only about 1.25 times T_m , n is about 0.4, therefore, the response function is 0.4 times 1.25, which is nowhere close to

what is required to trigger linear growth of oscillations which eventually leads to DC shift ok. Therefore, the only description I can think of for these propellants are they are very well behaved as far as instability is concerned ok.

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Q4. What causes instability if the 'regular' compositions are well behaved?



Strontium carbonate and the associated of binder melt.



So, question remains what causes instability if the regular compositions are well-behaved? It looks like the only thing that we have not accounted for so far in the model is are the effects of the inhibitors. One particular inhibitor that I want to mention here is strontium carbonate and the associated binder melt. And of course, it could be other inhibitors like Oxamate also.

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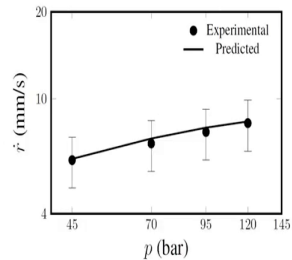


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$$f_{II} = 2.1 f_{SC} (0.2 + B_{ds})^{-2}$$

$$B_{ds} = \frac{T_d - T_s}{T_s - T_0 - H_s/c_p}$$

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- Only by this way, index as low as 0.25 could be explained.
- The physical mechanism is shielding of active AP surface by binder melt – while this occurs in all propellants (due to heterogeneous fuel distribution), it is exacerbated by SrCO_3 .
- What it does to the frequency response?

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What do they do? The theory or the hypothesis is that that strontium carbonate which is a inhibitor added to the binder inhibits the decomposition of the binder for through the endothermic decomposition, the strontium carbonate itself goes through ok. Remember that the enthalpy or the heat flux required for the decomposition of the binder comes from the gas phase heat flux, and a significant fraction of it comes from the flame between AP decomposition products and the binder because that flame is what is sitting at the interface between the AP and binder ok. And this fraction, this component of the flux decreases with increase in pressure because lateral diffusion decreases with increase in pressure.

And therefore, even in normal regular composition, the heat flux that comes for the decomposition of the binder, the fraction comes down with increase in pressure, and this is made worse by addition of addition of inhibitors like strontium carbonate which can make this problem worse ok. And what happens, the binder can melt, but it will not decompose and

therefore, there is accumulation of binder at the surface which can laterally move and cover AP surfaces ok. And therefore, active AP surfaces are blocked, thereby bringing down the burnt rate as well as the index ok. So, this effect is accounted for through what is called the liquid layer effect, which is simply the functional form is taken as what is shown here.

This is an extension of the blocking effect in hybrid rockets. To the extreme case where the surface is blocked by the melt itself ok. And only by this way index as low as 0.25 could be explained ok. Here I have shown prediction for one such composition and the index of this propellant is about 0.25 and index as low as this can be explained only by additionally accounting for the shielding of the heat flux reaching the surface because of the binder melt that is covering the surface ok.

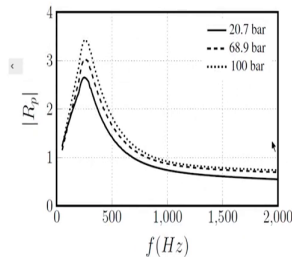
The physical mechanism is shielding of active AP surface by binder melt, while this occurs in all propellants, for example, small particles which are fuel rich can have binder melt even in regular compositions due to the heterogeneous fuel distribution, but it is made worse by strontium carbonate. What does it mean to the frequency response?

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$$R_{p,il} = \frac{2 + \frac{h_d}{1-h_d} \frac{c_p \bar{p}}{g(B_{eff})(H_s + f_{SC} H_d)} + z_r \left[e_g + \frac{1}{g(B_{eff})} \frac{\bar{T}_{eff}}{\bar{T}_s - \bar{T}_s} \right] + f_{il,amp} \frac{\bar{f}_{il}}{f_{init}}}{2 + \frac{\theta_{fs,eff}(1-h_d) + A_c \sin(2\pi f_s \tau + \phi_c) - c_s h_d - (1-h_d) e_s}{g(B_{eff})(1-h_d) e_s}}$$



• Fluctuations in binder-melt caused by acoustic pressure fluctuation is hypothesized to lead to high response values.

• How do we know that it is the case? We don't know for sure.




• But removing SrCO_3 from the composition significantly improved the stability margins of the rocket – not just statistical stability, but the dynamic stability (determined by pulse tests).

• So I guess we can't be so far off from truth!

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So, because of the addition of the liquid layer an additional term appears in the liquid layer an additional term appears in the response function equation; because of the liquid layer effect, and this fluctuations in blinder melt caused by acoustic pressure fluctuation is hypothesized to lead to high response values. Only by accounting for this affect response function as high as 3 can be explained. How do we know that? This is the case well cannot know for sure, but removing strontium carbonate from the composition significantly improves the stability margins of the rocket not just statistical stability, but also dynamic stability, that means, that that you can pulse the rocket create a pressure disturbance and see that it decays. The pulse decays with time instead of growing ok. So, I guess this cannot be far from the truth.

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Take away

- The current framework is a significant improvement over the earlier ones (usually BDP based models).
- More importantly, this framework can yield results much faster than CFD calculations, especially for response function calculation and can accommodate a variety of ingredients (while CFD calculations so far have dealt only with AP/HTPB + aluminum).
- We have found more willingness among practitioners in developmental agencies (space and defense) to accept the theoretical basis of the model compared to academics (experimentalists in particular).
- Right now we are working with two agencies, VSSC and HEMRL, to integrate the software based on HeQu1-D into their propellant development process.
- Progress has been slow but steady. We hope to soon see wider acceptance of the model for developmental as well as research purposes.
- Why it's important? Because we have an opportunity to lead, instead of follow, aided by insights from a theory.



So, to summarize the current frame work is a significant improvement over the earlier ones, usually BDP base models – Beckstead Derr and Price models which did not evolve to a stage where practitioners can use it for designing propellants. More importantly this frame work can yield results much faster than CFD calculations, especially for response function calculation and can accommodate a variety of ingredients which is difficult with CFD calculations. Well, I, well, let me go through. So, we have found more willingness among practitioners and developmental agencies space as well as defense to accept the theoretical basis of the model compared to academics mostly experimentalists.

Right now we are working with VSSC and HEMRL, to integrate the software into their propellant development process, progress has been slow, but steady, but we hope to soon see wider acceptance of the model for development as well as the research. Why it is important?

We think it is important because we have an opportunity to lead instead of follow aided by insights from a theory ok. I just stop with this.

Thank you.