

Fundamentals of Combustion for Propulsion
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Lecture – 15
Analysis of p-t traces – Part II

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Of course not all tactical missile rockets are not
unstable!



Well, of course not all tactical missiles rockets are unstable, and therefore, we need to understand why some are if the explanation I provided is universally correct, then all tactical missiles must be unstable, but they are not some are and some are not. So, we need to understand why some are unstable and some are not ok.

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So the question is, why are some stable and
some aren't?



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Inputs from earlier literature are generic (non-specific) and conflicting. So we decided to take a fresh look.



I have post that question, but we found that inputs from earlier literature on this topic is a vast amount of literature are non-specific and sometimes conflicting also. So, we decided to take a fresh look. If you look at literature there was more confusion than clarity because suggestions of the following class were made some suggestions were to increase ap particle size, some suggestions were to degrees ap particle size it was not clear what does the fundamental principle that is the basis for these suggestions and that is what we set out to find out ok.

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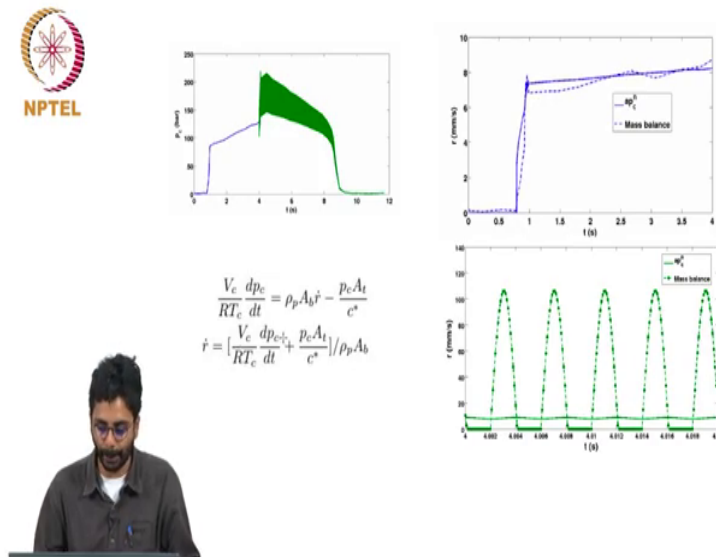


Q1. What can we learn from the static firing p-t traces?



So, we started from the basics. So, the question is what can we learn from the static firing pressure time traces ok?

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We go back to the most basic equation that we know which is that the rate of change of mass inside the rocket chamber is equal to the mass in minus the mass out. What it means for this kind of a pressure time trace is something that we wanted to calculate. And what must be the burn rate of the propellant to sustain such wild oscillations inside the rocket chamber.

So, we use this equation, we know pressure as a function of time, and we can calculate from here the burn rate of the propellant ok. This equation is rearranged to get the burn rate of the propellant, because we know pressure as a function of time ok.

As you can see for the steady part of the operation which goes from 0 seconds to 4 seconds, the burn rate that is estimated from the equation which is the dotted line shown here, closely matches with the expectation from ap rise to n ok. But for the part starting from four seconds onwards, where the instability is certain and d c shift has happened, what the equation shows

the dotted lines is that the for this is very important I due to pay attention to it remember that this motor is 2 meters long and the fundamental mode is 250 Hertz, it was found here that the dominant frequency is 250 Hertz as expected.

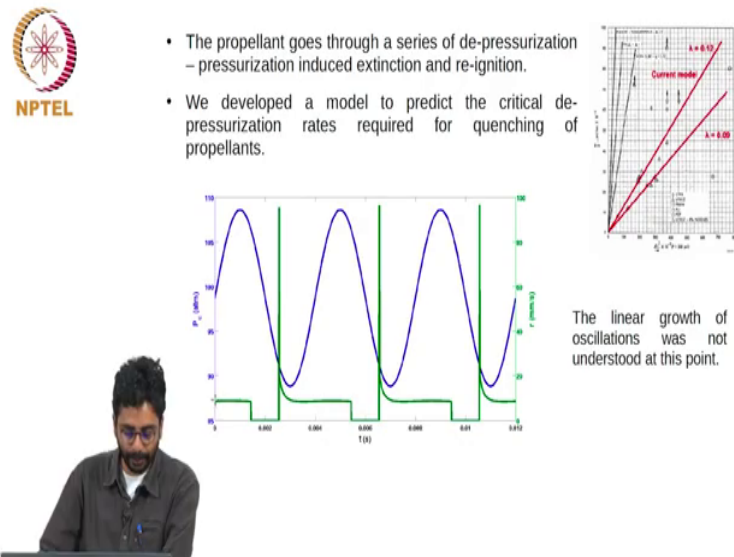
So, one cycle of 250 Hertz is 4 milliseconds ok, 1000 divided by 250 milliseconds is 4 milliseconds. So, one immediate thing that you can see in the plot at the bottom is that there is a cyclic variation of one rate with a period of 4 milliseconds. It goes from 4 to 4.002, from 4.002 to 4.004; this is certain kind of behavior, so that is 250 Hertz. The next important thing to notice is that for this half of the cycle 4 to 4.002 seconds, the propellant burn rate estimated from this equation is 0.

In fact, it will you will get a negative value for the burn rate because the $p \cdot c \cdot d \cdot t$ is negative, but of course, there is no such thing as negative burn rate of the propellant, the only inference from the negative burn rate is that the propellant to stop burning ok. So, for this section from 4 to 4.002, 2 milliseconds, one-half of the cycle the propellant is not burning.

And when it starts burning at 4.002 seconds or 2 milliseconds later, the burn rate goes up to as high as or the burn rate must go up to as high as 100 millimeters per second if this kind of a pressure time curve can be a realized. It is not to say that this is how the propellant burns. What is being said is that unless something similar or equivalent of this is happening inside the rocket motor, it would be impossible the energy avail the energy required to sustain such intense oscillations will not be available ok.

So, the propellant must be going through very going through cycles of very high burn rates, and almost no burn rate for such high amplitude fluctuations to be sustained ok.

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So, the conclusion from that exercise is that the propellant goes through a series of depressurization pressurization induced extinction and re-ignition. Let me explain what is meant by that. Around the DC shift point ok, the mean pressure at the start of the DC shift the mean pressure is about a 100 atmospheres the amplitude of oscillations is about 10 percent, so about 10 atmospheres.

So, when a propellant is subjected to oscillations of this kind during this part of the cycle when the pressure increases from 100 atmospheres to 110, it is subjected to pressurization. And when it goes from 110 to 90, it is subjected to depressurization. The rate of pressurization is 0 at the beginning, it increases and then reaches 0, increases and then comes back to 0 at the peak ok.

This is, imagine this is a $\sin \omega t$ curve, and you take the derivative of this with respect to time. You can see that the slope is 0 here, the slope is also 0 here. At the peak points the slopes are 0, that means, the pressurization and the depressurization rates at the peak values are 0.

Therefore, as you go from low pressure to high pressure, it goes from 0, it goes through a positive $\frac{dp}{dt}$, it goes back to 0. Similarly, during the depressurization cycle, it will start from 0, $\frac{dp}{dt}$ is 0 at the start, it will go to a certain negative value and go back to 0 again, increase back to 0 ok. So, during the depressurization cycle, the propellant is experiencing the following thing.


The begin with that is no change in pressure, it is starting from the maximum value, the depressurization rate slowly goes up it becomes more and more negative, reaches a maximum negative value, and then increases back to 0. And now let us look at what the burn rate of the propellant behaves like when subjected to pressurization and depressurization of this kind. During pressurization, of course, there is no problem the propellant burn rate increases in response to the increase in pressure, and reaches some maximum value at the maximum pressure.

Once the pressure starts decreasing, propellant burn rate it response to that also starts decreasing, but there is a critical depressurization rate ok. Remember that the depressurization rate to begin with the 0, and then it becomes more and more negative as you go along this curve. When it crosses a certain threshold value, the flames over the propellant cannot be sustained the flames get blown off and the propellant crunches. So, the burn rate drops to 0.

And it remains so till the depressurization rate remains less than the critical or more than the critical depressurization rate. And once it comes out of that critical phase, it ignites again and the burn rate goes up to as high as 100 millimeters per second ok, and the cycle repeats ok, ok. I before I proceed any further I just want to make sure that you have understood this point ok.

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Handwritten notes and diagrams illustrating heat conduction concepts.

Diagram 1: A surface with a wavy line representing a boundary, with a vertical arrow labeled q_c pointing downwards.

Diagram 2: A cross-section of a material with a temperature profile. The top surface is at $T_s + \Delta T_s$ and the bottom surface is at $T_s = 300 \text{ K}$. The temperature profile is shown as a curve starting from $T_s + \Delta T_s$ at $x=0$ and decreasing to $T_s = 300 \text{ K}$ as $x \rightarrow \infty$. The temperature at $x=0$ is also labeled $T_s = 800 - 100 \text{ K}$.


Equation 1: $\delta \sim \sqrt{\alpha t_n}$

Equation 2: $T_{\text{conduction}} = \frac{\alpha}{\beta^2} \left(\frac{\text{m}^2 \text{s}^{-2}}{\text{s}^2 \text{m}^2} \right)$

Equation 3: $T_{\text{conduction}} \sim \frac{0.1}{50} (\text{s}) = 2 \text{ ms}$

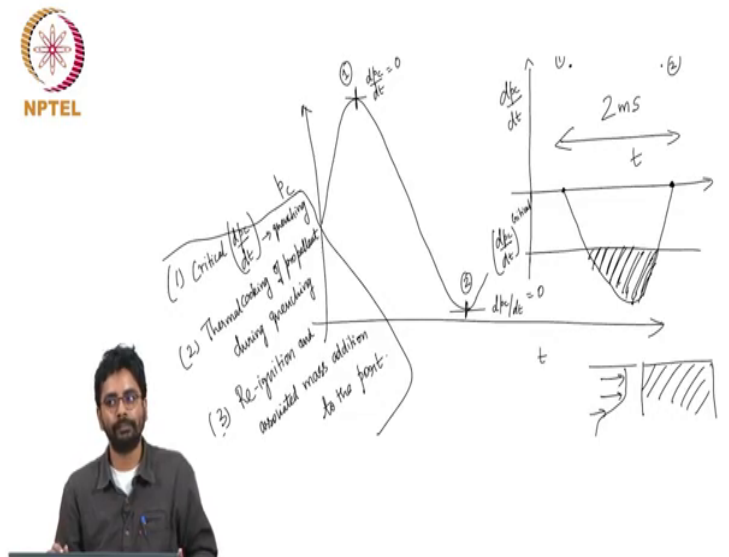
Equation 4: $\alpha = 0.1 \text{ mm}^2/\text{s}$; $\beta = 7 \text{ mm/s}$

Equation 5: $f_{\text{conduction}} = \frac{1}{2 \text{ ms}} = 500 \text{ Hz}$



I, it could be easier to do it here I guess.

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You have something like this, pressure versus time. The slope here is 0, 0, the slope here is also 0. So, the value of $\frac{dp}{dt}$ starting from this point that this point will look like this if I plotted $\frac{dp}{dt}$ as a function of time, so I will start let us say this is point number 1, this is point number 2 ok, let us say this is point number 1, let us say this is point number 2, in both these places the $\frac{dp}{dt}$ value is 0 ok.

What happens is that between these two points the $\frac{dp}{dt}$ becomes negative, because the pressure is decreasing with time $\frac{dp}{dt}$ is negative ok. So, it $\frac{dp}{dt}$ is negative reaches a maximum negative value here, and then it goes back to 0. And this is time. And when propellants are subjected to depressurization, remember that when pressure increases the flame stand off distance decreases, you have seen this earlier you have a certain ensemble of premixed and non-premixed flames over the surface of the propellant.

So, when the pressure goes up, the overall flame structure will go closer to the surface of the propellant, and the pressure goes down the flame will go away. But when subjected to very fast oscillations in pressure and very high amplitude oscillations in pressure, when it goes back the pressure may go down so fast that the profiles of temperature may not be able to adjust and the flame can get more or less blown off.

So, there is a critical depressurization rate which is here let us say dp/dt critical. If the depressurization rate goes below that in this zone, there may not be a gas phase a sustained gas phase combustion on the surface of a propellant. And therefore, the heat that is received by the surface of the propellant because of the gas phase combustion becomes unavailable, and therefore, the propellant stops burning.

So, in this phase, so it will start from here go into a phase of more burning and come back ok. Something interesting happens when it comes out ok. In the hashed zone which I am showing here, there is no heat flux that is coming from the surface, there is no heat flux that is coming from the gas phase to the surface ok. But remember that the propellant itself has the temperature profile which looks like this ok.

And the propellant takes about for the temperature profile inside a propellant to change when the boundary condition is changed takes about 2 milliseconds ok. So, in this is happening only in a fraction of the 2 milliseconds, this whole duration is 2 milliseconds ok. This is only a fraction of the 2 millisecond duration. And therefore, in that small short amount of time, the temperature profile would not have changed significantly inside the propellant ok.

So, when it comes out of the depressurization or when it comes out of the critical zone, the propellant is still hot there is a thermal layer of a few 10s of microns thick which is cooked ok, which is suddenly exposed to a gas phase flame when the propellant comes out of the critical zone.

When this happens, the implications are that this cooked layer of propellant will simply get thrown into the gas phase, adding a lot of mass in a very short duration of time. The net effect

of it is equivalent to the propellant burn rate going up as high as 100 millimeters per second. Is it better now, is it clear now ok.

So, the crypt the important ideas are, one is critical depressurization rate leading to quenching, then thermal cooking of the propellant during quenching re-ignition, re-ignition, and associated mass addition to the port ok, these are the key ideas ok, ok. And I just want to tell this one more time that the typical time that it takes for the temperature profile to adjust is about 2 milliseconds, this depressurization with critical zone is only a fraction of the 2 milliseconds.

And therefore, the solid phase will not would not have had enough time to adjust to the new conditions in the gas phase ok. Question, is it clear now, should we move on?. So, now, this plot I hope looks clear. So, when the propellant actually is undergoing steady combustion ok, subject to depressurization, it quenches and goes to 0 burn rate at a critical dpc dt , and it remains at quenched conditions still the critical depressurization rate condition prevails.

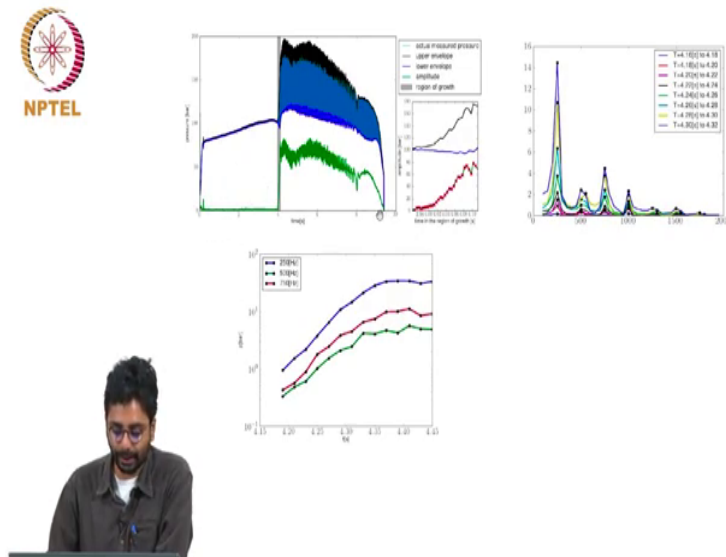
And when it comes out of it the burn rate goes to shoots up to very large values of about 100 millimeters per second ok. So, we have done two things, one is that we have back calculated what kind of propellant combustion behavior is required to sustain these high pressure amplitudes. And in addition to that, we have also shown that such a behavior is indeed possible when the propellant is subjected to high amplitude fluctuations in pressure ok.

Of course as a an additional outcome from this work cause that we calculated we developed a model to calculate the de critical depressurization rates, and it matched with the literature data better than the models that were available from earlier studies ok. But the more important conclusion is what you see here in this plot ok. So, this gave us some idea of the propellant behavior when it is undergoing DC shift.

But the more important question was the actual growth of oscillations in the initial phase which eventually becomes or its eventually transitions into DC shift. So, the linear growth of oscillations were was not understood at this point. Of course, the DC shift is something that

happens after the oscillations have grown to a certain amplitude to cost quenching and re-ignition, but the main question is why the oscillations grow in the first place.

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So, a set of analysis was done by a Dr. Aravind Ayyar at that time at he was at IIC – Institute of Science, Bangalore, the data was made available to do the analysis. So, what was done is to do a Window Fourier transform of the data. And look at amplitude evolution for different modes ok, that is the first fundamental mode is 250 Hertz, the first harmonica as 500 Hertz and so on. It was found that the first three modes were dominant or significant. And the data was used to extract the amplitude as a function of time.

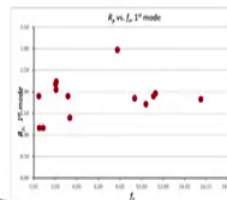
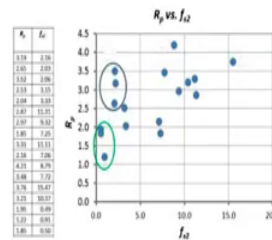
What you see here is the Fourier transform for different windows, and what you see here is the amplitudes extracted from the Fourier transform as a function of time plotted on a semi log axis. Plotted on a semi log axis, and it shows a linear growth ok, a linear curve in the semi

log axis is as an exponential growth of the amplitudes. And therefore, this is nothing but an exponential of αt from the slope of this line we can get the growth rates for these three frequencies.

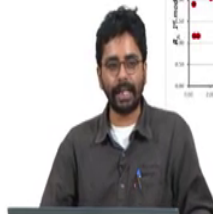
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The final results



- This was very surprising from a theoretical point of view.
- The rule of thumb that as long as you keep the propellant index small (<0.3), there won't be any instability, is incorrect – the index of the propellants used in this rocket is 0.25.



So, from the growth rates, we can calculate the response function of the propellants, the response function was calculated for a variety of combinations, and these are the results from the analysis you can see that the response function was somewhere between 1 and four and a half. And response function for different modes decomposed were also plotted. It is also from about point from 1 to about 3, three and a half.

And this was a variety of data was available from a different conditions and this was very surprising from a theoretical point of view from the following reason. The rule of thumb was that was being followed was that as long as you keep the propellant index small that is less

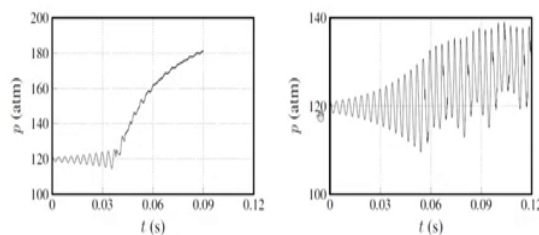
than 0.3, there would not be any instability ok. The logic is something like this. Remember that the response function becomes equal to the index at 0 frequency ok.

So, the logic was that at the response function at other frequencies cannot be an order of magnitude higher than the index, and therefore, lower the index lower will be the response function was the logic that was followed, and it is not correct, because the propellants that were used in these rockets had very low index 0.3 or less than 0.3. And they were unstable and the response functions calculated from the data analysis showed that the response function is about 10 times as large as the index itself ok.

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That a response as high as 3 is required for linear growth and accounting for de-pressurization -- pressurization induced extinction – re-ignition is the cause of DC shift is confirmed using CFD calculations.



Vishal Wadhai and Varunkumar S (2017), 11th ASPACC; Vishal Wadhai (2018) MS Thesis.



Calculations, CFD calculations were done, and this was a surprising result because it was all earlier literature all theoretical models, the result was that the index, and the response function must be of the same order that it is one order more for sub racing. And therefore,

calculations were done to check what value of response function is required to trigger linear growth.

So, this was confirmed by CFD calculations that unless you have a response functions as high as 3, there would not be linear growth of oscillations that is point number 1. You can see linear growth here linear growth here as well. And this is the first part of the prediction. The second part of the prediction is that unless you account for depressurization-pressurization induced extinction - re-ignition DC shift cannot be explained ok.

So, both these aspects that the response function greater than three and ignition quenching the re-ignition cycles were put into the CFD model couple to a flow solver, the DC shift and linear growth could not have been explained.

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Q2. What is the basis for the idea that low index leads to stability? Why is it not correct?

To answer this question, we needed to know what controls the index of the propellant. To our surprise, there was no satisfactory theory for predicting the steady deflagration of composite solid propellants, especially the ones with low pressure index ($n < 0.3$). So, we set out to develop one.



So, that is the discussion on the first question as to what we can learnt from the pressure time traces. What we have learnt is that the ignition re quenching – re-ignition has a major role in DC shift and linear growth can be caused by a response functions as high as 3. The second question is what is the basis for the idea that low index leads to stability and why is it not correct ok.

To answer this question, we needed to know what controls the index of the propellant, and we were surprised that there was no satisfactory theory for predicting the steady deflagration of composite solid propellants, specifically the ones with low pressure index ok.

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Heterogeneous quasi 1-D model for AP/HTPB
composite solid propellants

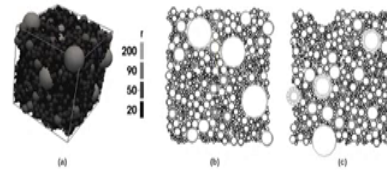


You set out to develop a model, and the result is the heterogeneous quasi 1-D model for AP HTPB de-composite solid propellants.

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- In solid rocket propellants, fuel and oxidizer are mixed together.
- Energetic combinations of exotic chemicals are used as fuels (NC, PU, HTPB etc.) and oxidizers (NG, KP, AP etc.).
- Of these Ammonium perchlorate (AP) and HTPB have emerged as the widely used combination – due to ease and safety in processing, ballistic flexibility etc.
- AP/HTPB based mixtures are called 'composite/heterogeneous propellants' as opposed to 'double-base/homogeneous propellants' (NC-NG for example).



Heterogeneous packing of AP particles

contd...



So, as we all know in solid rocket propellants, fuel and oxidizer are mixed together. Energetic combinations of chemicals of exotic nature are used nitrocellulose, polyurethane, HTPB, etcetera, and oxidizers include nitroglycerine potassium perchlorate and ammonium perchlorate. Of these combinations ammonium perchlorate, and a HTPB have emerged as the widely used combination, due to several reasons including ease and safety and processing and ballistic flexibility etcetera.

And mixtures of AP HTPB are called composite or heterogeneous propellants as opposed to double-base homogeneous propellants mixtures of nitro cellulose and nitroglycerin for example. The reason for calling it a heterogeneous propellant or a composite propellant is that a typical propellant cross section or a typical propellant element of AP HTPB class will look like this.

It will, if you look at the cross section, you will see particles of different sizes. Here you see four particle size classed 200 microns, 90 microns, 50 and 20 micron embedded in a HTPB matrix ok. The reason why it is called hydrogenous is because there is if you look at the cross section, there is a wide variation in the distribution of oxidizer and fuel. Around large AP particles, and in zones where there are large AP particles, you will have lot more oxidizer than fuel.

And in zones that contain fine AP particles you will have more fuel than oxidizer ok. So, the O by F ratio is highly heterogeneous and hence these propellants were called heterogeneous propellants ok.

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Propellant specifications

- Motor operating pressures range from 10-150 atm depending on applications.
- For a given application the operating pressure range is usually narrow (excluding the ignition and burn out transitions).
- In addition to having high energy (high specific impulse),
 - Propellants must burn at a certain rate at a given pressure.
 - And the burn rate must have a certain specified dependence on pressure and initial temperature (T_0).

$$\dot{r} = \dot{r}_{20} \left(\frac{p}{20} \right)^n \quad \sigma_p = \frac{1}{\dot{r}} \left[\frac{\partial \dot{r}}{\partial T_0} \right]_p$$

contd...



Propellant specifications, motor operating pressures range from 10 to 150 atmospheres depending on applications. And for a given application the operating pressure range is usually

narrow excluding ignition and burn out transitions ok. And in addition to having high energy, so that the performance is the maximum possible, propellants are expected to burn at a certain rate at a given pressure demanded by the design ok. And in defense applications sensitivity to initial temperature is also an important factor, because the systems are expected to work at very high temperature is also very low temperatures ok.

These two requirements are can be seen here. The first is the well known power law which connects the burn rate and the pressure. So, n is the pressure index ok, and \dot{r}_{20} is the burn rate of the propellant at 20 atmospheres. And σ_p is a quantity that is a measure of the sensitivity of the propellant to initial temperatures. It is a fractional change in burn rate with change in initial temperature at constant pressure ok.

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- Propellant development so far has occurred by trial and error, by playing around with,
 - Predominantly the AP particle size distribution
 - Burn rate modifiers (catalysts and inhibitors)
- Simple theories (BDP and other extensions like HYPEM etc.) did not evolve to the stage of being used as design tools.
- In the past decade CFD-based simulation tools have been developed (Buckmaster, Jackson, Gross, Beckstead and group)
 - projected as the last resort solution for the propellant design problem.
 - Very time consuming and cannot be extended to accommodate effects of additives like for example, SrCO_3 , oxamide, nitramines etc., which creates complex interface interactions (a major cause of combustion instability).
 - In addition to the common additives mentioned above more and more candidates are tried and adopted frequently.

$$\dot{r} = \dot{r}_{20} \left(\frac{p}{20} \right)^n$$



Propellant development, so far has occurred mostly by trial and error. And the variables that are available for the developers to get the required burn rate behavior is predominantly the AP particle size distribution and burn rate modifiers in the form of catalysts which enhance the burn rate of the propellant or inhibitors that decreased the burn rate and also the index of the propellant. There have been several attempts in the past to develop a comprehensive theory to predict the burn rate and the index of the propellants.

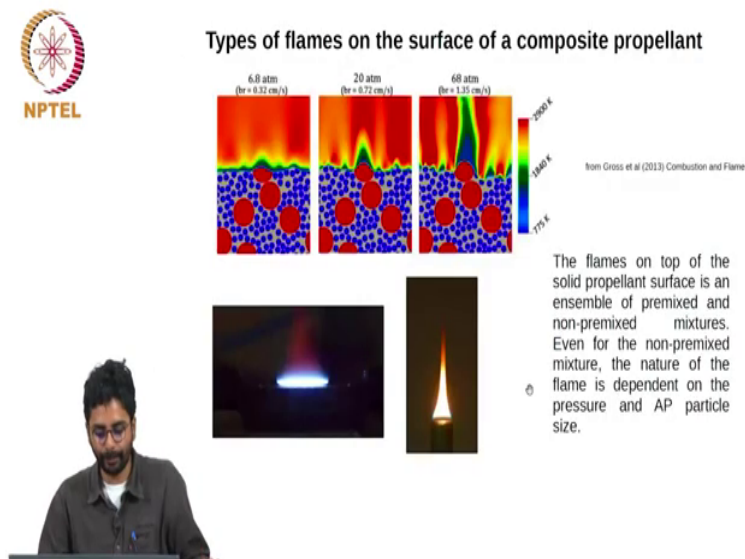
The Beckstead Derr and Price model is the most famous of this group or one of the first models that were proposed. And there were several extensions which used BDP as the BDP model as a basis one is the high energy petite ensemble model, and there were several other variants also. But none of these theories evolved to the stage of being used as design tools.

And what has happened in the last 10-20 years especially in the US is CFD based simulation tools have been developed mostly by group led by Buckmaster, Jackson, Grosse, Beckstead and others. It is projected as the last resorts solution for the propellant design problem, but it is very time consuming and cannot be extended to accommodate the effects of additives like strontium carbonate, oxamide nitramines etcetera.

These create, these additives I already told you that the surface behavior is quite complex. Parts of the surface that have exposed AP will be exothermic parts of the surface that have exposed fuel will be endothermic, excuse me. And in addition to these two substances, several other additives are added. For example, strontium carbonate and oximide will make the surface locally endothermic. Adding nitromines can make the surface locally exothermic ok.

So, these additives can make the heterogeneity in terms of exothermicity or endothermicity of the surface, it will make it more extreme than what it is with AP and HTPB. And to account for the effects of these additives and the list of these additives never increasing there are always new compounds that are suggested tried and so on. It becomes very difficult to do it with the CFD framework ok.

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This we have already briefly discussed of the types of flames on the surface of the composite propellant. The best description I could think of is that the flames over the surface of the propellant can be considered to be an ensemble of premixed and non-premixed flames. Even for the non-premixed mixture, the nature of the flame is dependent on the pressure and the AP particles sized this is something that we discussed in the morning. It is non-premixed, but not essentially diffusion control. There could be significant pre mixing depending on the particle size and pressure.

So, I have shown two pictures here the one on the left is the flat premixed flame that we saw in the morning. The flames over AP surfaces are more like flat premixed flames. And the flames around AP surfaces, so surrounded by binder matrix could be close to this flame ok or

somewhere in between the flat flame and this flame depending on the extent of premixing and pressure and size of the AP particle ok.