Fundamentals of Combustion for Propulsion
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Lecture – 14
Instability in solid rockets

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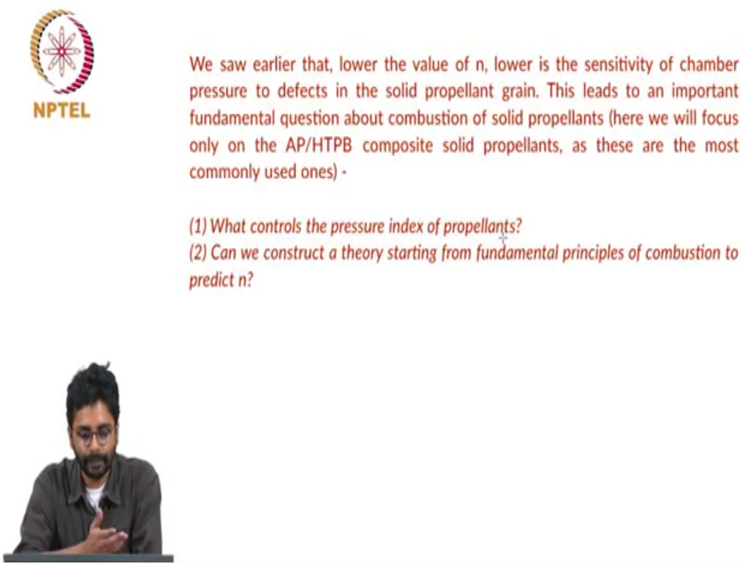

Let us continue from where we left off, where we left off was we saw earlier that lower the value of the run rate pressure index which we refer to as n which is the n and a p raised to n; the lower it is the lower is the sensitivity of chamber pressure to defects in the solid propellant grain. And, this is the most important consideration in developing solid rocket motors and this conclusion leads to an important fundamental question about combustion of solid propellants.

Here we will focus only on AP HTPB compulsion propellants such as these are the most commonly used ones. The question the principle question is what one principle question is what controls the pressure index of the propellants and the following question. And, the follow up question we construct the theory starting from fundamental principles of combustion to predict n.

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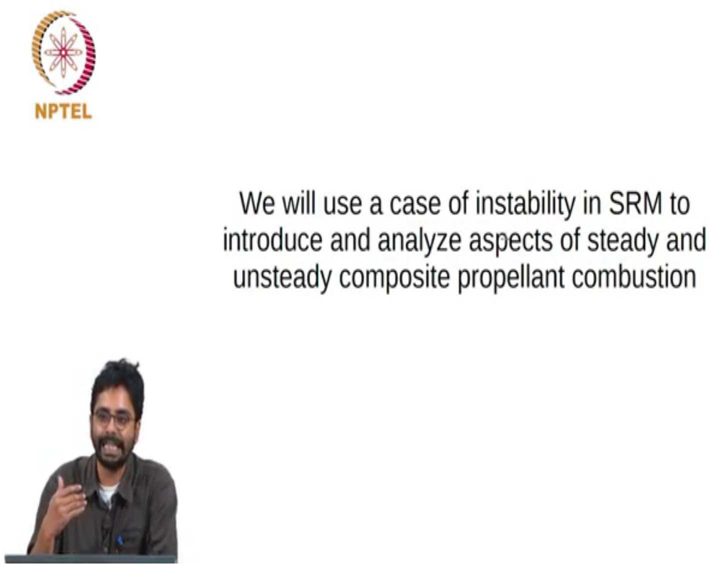

Instead of going into the details of the theory right away, I felt that the context will be you know will have a better context to look at this question with if we have some background on instability which is more serious problem when it comes to solid rocket motors. I think I will set the stage for discussion of detail discussion of the model using instabilities so, that the need for such a model becomes clear.

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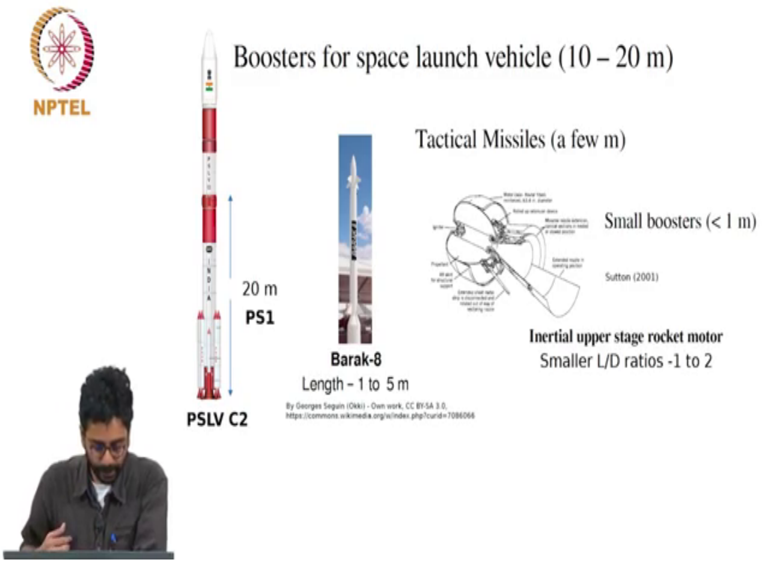

Fine; so, solid propellant rockets of variety of sizes are used for variety of applications; three broad categories: very long about 20 meters long rocket motors used as boosters in space launch vehicles and also tactical I am sorry and also ballistic missiles. And, the second category is tactical missiles where the length of the rocket is of few meters longs ok, boosters are about 10 to 20 meters long.

So, our ballistic missile like the Agni missile, but if you look at tactical missiles they are sharped, they are about the few meters long ok. And, the third class are small boosters which are used for inertial upper stage rockets ok, these are the three classes. The first class is 10 to 20 meters long, the second one is about 1 to 5 meter long, the third one less than 1 meter with an L by D ratio of 1 to 2 ok.

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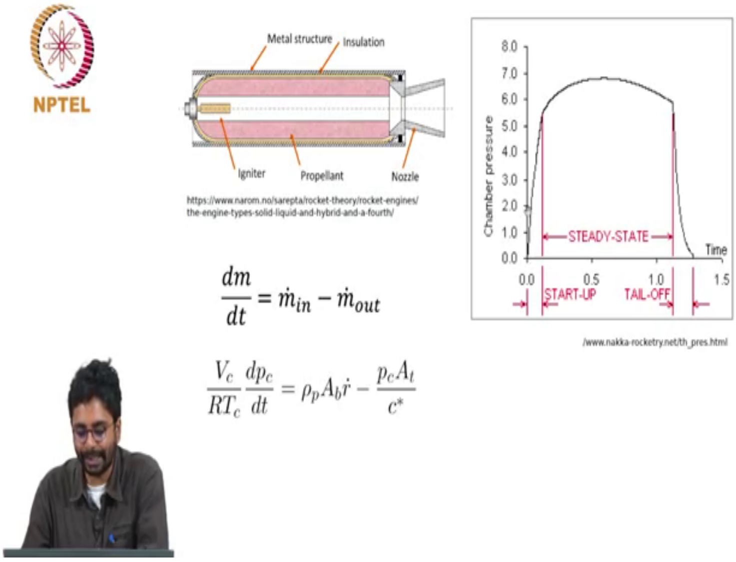

This we have seen already solid rocket motor has general structure like this and the pressure inside the rocket as a function of time can be calculated from a simple mass balance; we derived this equation in the morning. And, when you calculate the equilibrium pressure of course, you can include you can extend such an such an approach to get the initial ignition transient as well as the fall of after the propellant is burnt out and your account for some transient terms. So, you will get a curve like this, this is the predicted pressure time behavior of a rocket ok. So, there is a startup, there is a steady state and a tail off ok.

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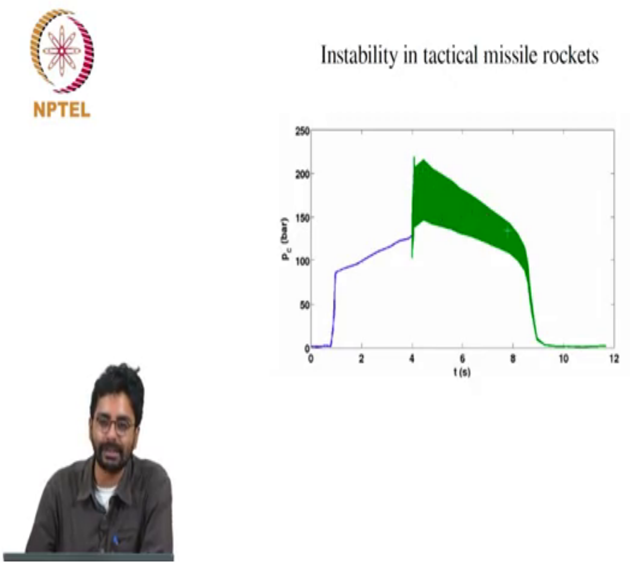

But that is not what always what you get reality, sometimes predicted burnt rate the predicted pressure time rate is much very closely with reality, sometimes it does not. We saw one such example yesterday which is also repeated here that the rocket starts off as expected follows the expected curve. But, at some point along the line it the oscillations develop and there is a DC shift, shift in the mean and then large amplitude oscillations around the increased mean value ok.

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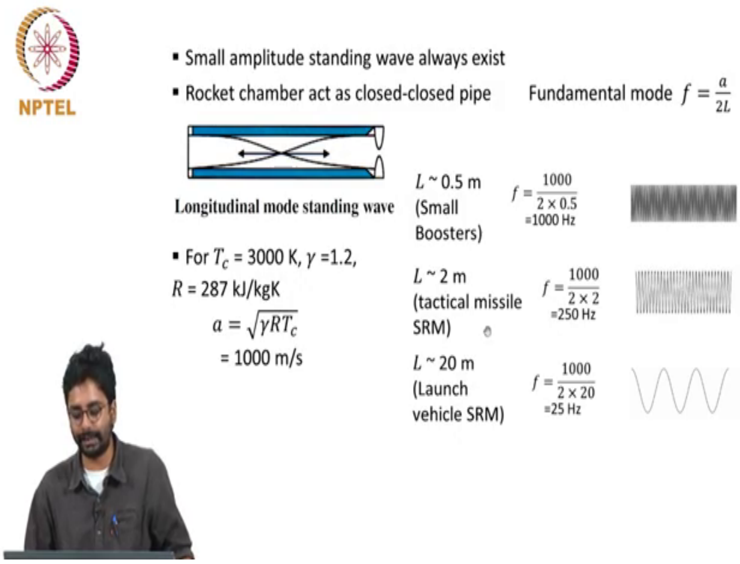

Let us look at it in some details ok, the one important thing to note is that fluctuations always exist in system where there is a turbulent flow ok. So, small amplitude standing waves always exist in a chamber through which flow is happening and rocket chamber act as a closed, closed pipe that is because of course, the head end is close to the rigid wall and the throat is close to choked. I mean the throat is closed acts as a closed end because the flow is choked in that ok, definition of a closed end is that the velocity at that point should not fluctuate.

So, at the throat the velocity is always a speed of sound and therefore, it acts as a closed end ok. So, in such closed, closed pipe we know from the basic solution of linear (Refer Time: 05:06) acoustic equation that standing modes right and left travelling waves super impose and form standing modes. And, these standing modes have very specific wave length and frequency ok. The simplest way to see it is you can take right traveling wave to be cosine k x minus omega t, take a left travelling wave to be cosine of k x plus omega t.

If you add these two function you can show that the resulting function will satisfy the boundary conditions of closed ends, only for very specific discreet values of k which is the wave number. And, since the speed of the sound is fixed; that means, there will be specific discreet frequencies only specific discreet frequencies that will be allowed in allowed as standing wave solutions ok. So, the first such standing mode will have a frequency of the sound speed divided by twice the length ok. I suggest that you to try doing the simple case just add cos k x minus omega t with cos k x plus omega t. You will get a function that looks like cosine k x multiplied by cosine omega t or sin k x multiplied by sin omega t.

And, for this combination of this product function to satisfy the boundary conditions the k value must have a very specific form and from that you can extract the frequencies that are allowed. So, the fund so, called fundamental mode is the more that has the wave length that is the twice the length of the chamber ok. And, the frequency that is sound speed divided by wave length which is twice the wave length of the chamber ok. It will have a structure as shown here, it is a standing wave; that means, that that the special variation of it is fixed and it oscillates in a temporal way.

The wave that is shown here is like this whereas, pressure above the mean on the left hand side in the 0 to L by 2 and pressure below the mean from L by 2 to L. And, overtime this will fluctuate like this and the pressure at the center will be fixed, it will not fluctuate ok. So, in one half of the cycle the pressure on the left hand side is above the mean, the pressure on the right hand side is below the mean and the other half of the cycle the pressure on the left, the left is lower the pressure on the right is higher sure.

Student: The standing wave mode is considered as throat wave.

The node is not the throat; the node is at the center of the chamber.

Student: Center of the chamber.

Yeah. In this case the functional form is such that if you the functional form that satisfy the boundary condition, the value of the pressure ok; the value of the special function at L by 2 will be 0 ok. If I remember right in this case this will be cos 0 at the end cos pi at this end and cos pi by 2 at the center, cos pi by 2 is 0 ok. Any other questions? Yeah. So, let us look at estimate frequencies for typical the three classes of rocket motors that we saw in the previous slide.

The chamber temperature is typically about 3000 Kelvin's, the ratio of specific heat is 1.2, R which is the specific gas constant is 8.314 divided by the molecular weight of the gases is about 300, it must be 287 Joules per kilograms Kelvin, not kilo Joules ok. And therefore, for these conditions the speed of sound will be about 1000 meters per second ok. So, a is about 1000 meters per second. So, now, we know the speed of sound, we can calculate what the frequencies of the fundamental modes will be for different lengths of this rockets ok.

So, let us look at small boosters, length is about half a meter therefore, frequency would be about 1000 Hertz, the fundamental mode, a tactical missile will have a length of about 2 meters. Therefore, the frequency is about 250 Hertz and for a launch vehicle booster frequency will be about 20 Hertz because, the length is 20 meters ok; 20-25 Hertz; so, this is high frequency, intermediate frequency and low frequency. So, the take away is that small boosters have longitudinal modes which are very high frequencies, tactical missiles have frequency of about 250 Hertz and launch vehicles have a frequency of 25 Hertz ok.

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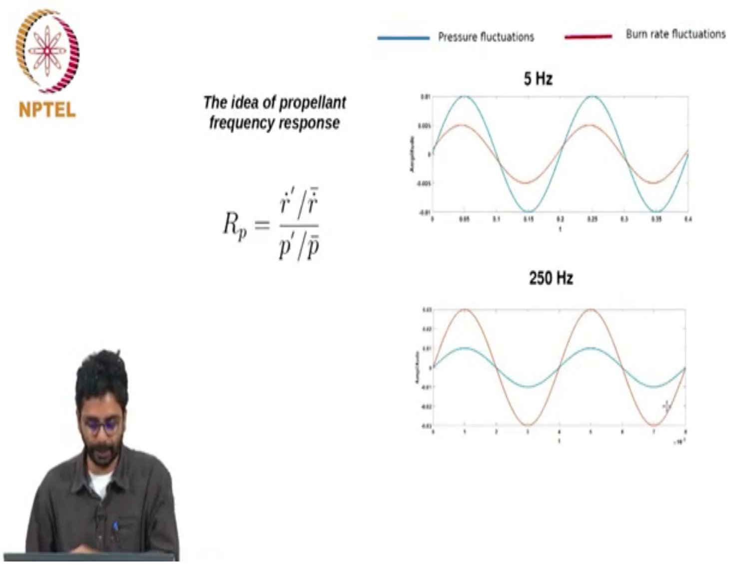

So what? So, this actually is at the is becomes most important consideration in when it comes together these rockets can exhibit instability. What is shown here is two plots showing fluctuations in burn rate of a propellant when subjected to pressure fluctuations ok. Remember that there are always pressure fluctuations that exist inside a chamber, a rocket motor and the solid propellant rocket, the solid propellant that is being used inside this rockets, the burn rate of this solid propellant depend on the static pressure that they experience.

And, when the static pressure inside fluctuate the burn rate will also fluctuate. What is shown here? There are two cases where there is fluctuation in pressure of 1 percent amplitude. The blue curve is a pressure line; it has an amplitude of 0.01, that is 1 percent of the mean and a frequency of 5 Hertz, a very low frequency of the kind that is found in launch vehicles ok.

When subjected to very low frequency oscillation you can see that the burn rate also fluctuates at about the same frequency.

But, the amplitude of fluctuation of the burn rate is about half percent, but the same propellant when subject to one percent amplitude fluctuation at 250 Hertz; the blue line still is 1 percent ok. In both the plots the blue line is the maximum value is 0.01 that is 1 percent, here also it is 0.01 it is 1 percent. At when the same propellant is subject to oscillation say 250 Hertz, the burn rate can fluctuate have a fluctuation up to 3 percent ok.

This the amplitudes are the same, just the frequencies are different. This fluctuation in burn rate, a measure of the fluctuation of burn rate is what is called the response function of the propellant which is defined as simply the percentage fluctuation in burn rate to percent fluctuation in pressure. So, for the 5 Hertz case the response function will be half, it is about 0.005 divided by 0.01 ok, not half it will be yeah it will be about half correct. And, for the 250 Hertz it will be 0.03 divided by 0.013 ok.

So, the response function at 5 Hertz is half, the response function at 250 Hertz is 3 ok, just keep this in mind. And, one more example and one more thing it is not shown here is if the same calculation is done at let say 2000 Hertz at 1000 Hertz of relevance to small boosters, you will find that the response function comes down again. So, it is lower at lower frequencies reaches a peak at about 250 Hertz and then drops back to lower values at higher frequencies. Why is this?

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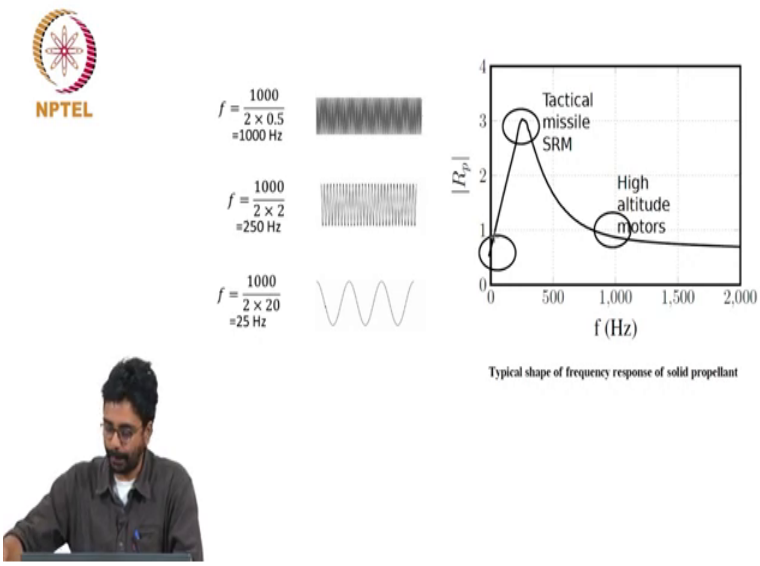

The reason for this is the following, before we I will just summarize this slide. So, at low frequencies the response function is very close to the index of the propellant which we know is a number that is less than 1 and most of the cases it is less than half. And, at about 250 Hertz it has a response function of about 3 and then it drops again to at very high frequency. This is very typical response curve which is very similar to that of the response of the simple harmonic oscillator. So, at low frequencies it just follows the forcing then there is resonance and then it comes back to no response case ok. As to what is so special about 250 Hertz, I will just explain that quickly.

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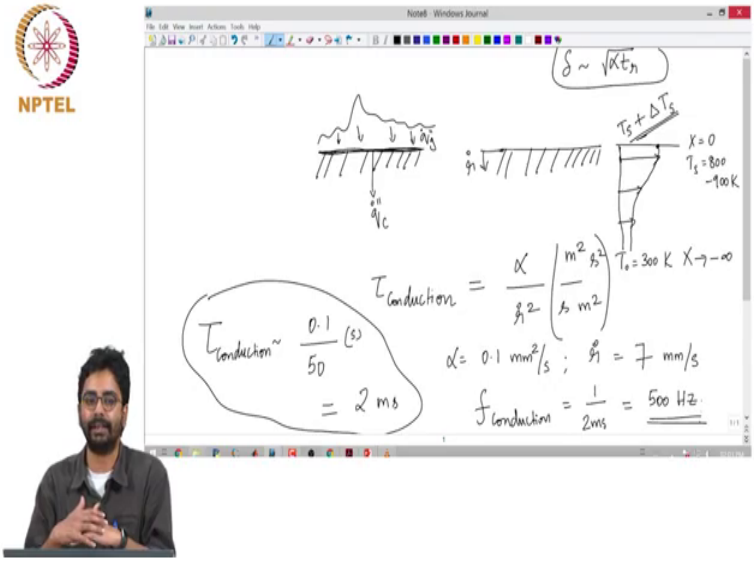

So, you have a typical propellant this that is burning, it has a let say some complex flame structure over it which is transferring heat flux to the surface. This is the heat flux it is coming from the gas phase; part of this heat will go into the condensed phase. Heating of the propellant from room temperature to the surface temperature and there is some heat release that is happening at the surface of the propellant. It could be heat release because, of exothermic decomposition of certain components. It could also be heat absorption because, of endothermic decomposition some components ok.

So, the condition the surface is generally a complex mixture of spots which undergo exothermic decomposition and other spots that undergo endothermic decomposition ok. If you look at the temperature profile inside the solid part of the propellant, this is the surface let say it is at X equals to 0, if you go far away from the propellant the temperature is room temperature let say 300 Kelvin’s. And, at the surface the temperature is T s which is usually about 800 to 900 Kelvin’s ok.

So, the temperature is high here and then it is low here, it usually looks like this the profile ok. This is high temperature, this is low temperature ok. The propellant burns at burn rate of r dot; that means, the surface regresses at a rate r dot ok. So, the time scale for conduction, the time scale for conduction is alpha divided by r dot square ok. You can check that it is dimensionally correct that gives you another number which is seconds, let me also make sure this is meter square per second, meter square second square so, this has dimensions of seconds ok. So, typical value of alpha is both 0.1 mm square per second and typical value for r dot is 5.

Student: 7.

7 ok; 7 millimeters per second ok. So, the time scale for conduction is 0.1 weighted by 7 square that is 50 seconds. This will be equal to 20 milliseconds, I am sorry this will be equal to 2 milliseconds. What this means is that when the let say you start under steady condition the surface temperature is let say T s and let say you change T s to T s plus delta T s. So, suddenly change from T s to T s to delta T s, the typical time that it would take for the temperature profile to adjust to the new surface temperature would be about 2 milliseconds that is what the alpha over r bar square means ok.

It can be the time scale can be direct more rigorously, you will see how to do it later, but right now I just want you to remember that the time scale is alpha over r bar square. Of course, it can be related to be kind of relation kind of expression that we have been using between delta and alpha t r. This two can be connected, but this is usually the length and time scale relationship in diffusive phenomena governed by diffusion. But let us not worry about that, what it means is it when you change the temperature suddenly from a steady value to a new value; the typical time that it would take for this temperature profile to adjust the new value is about 2 milliseconds ok.

So, the frequency associated with conduction time scale as you can see is 1 by 2 milliseconds which is about which is 500 Hertz. So, one of the reasons that the propellant responds to fluctuations in pressure, only in the frequency range of a few 100 Hertz is because the condense phase can provide the coupling which causes the fluctuation in burn rate only in that frequency range. When the changes in this temperature are too slow the response is causes static; that means, it follows the changes in the pressure. And, when the pressure fluctuations are of very high frequency kind, the condense phase simply does not respond ok.

Because, it is being forced at the frequency that is much higher than the frequency at which it can change or the time scale that it can change ok. So, the reason why the frequency response of a propellant or one of the reasons why the frequency response of the propellant is low at low frequencies, it reaches a peak at intermediate frequencies and then drops off is simply because the condense phase can respond to changes in or fluctuations in pressure only in the frequency range of few 100 Hertz. It is associated with the time scale for conduction in the condense phase ok.

So, launch vehicle booster is fall here, tactile missiles are in a backs spot where the propellant can actually respond very intensely to pressure fluctuations. High altitude motors do not also respond to fluctuations in pressure because, the response function is small. I just want to mention one thing, emphasize one thing the response function is defined in such a fashion that as frequency approaches 0, the response function magnitude will be equal to the burn rate pressure index of the propellant fine; let us move on.

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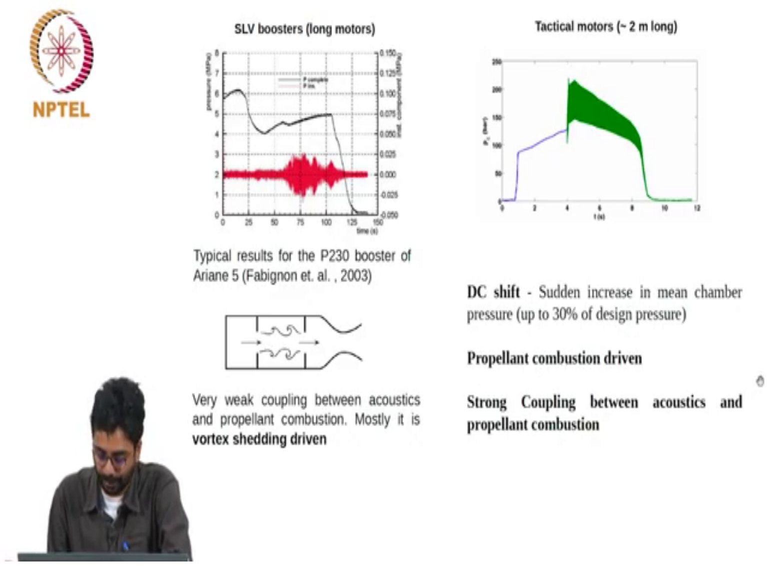

So, it is not that there are no serious pressure fluctuations in large launch vehicle class boosters. For example, this is data from Ariane 5 of P230 booster ok, where it operates at 2 mega Pascal's 20 bar. It operates fine till some point and then some oscillations are formed ok. This is the absolute pressure; this is the differential pressure ok. But, what I want you to notice is that there is no DC shift ok, in this rocket the fluctuation is around the mean.

So, when the fluctuations are around the mean and the fluctuation frequencies are also small because, of the long the motor itself being long leads to weak coupling between acoustics and propellant combustion. The we know that when forced at low frequencies, the propellant will simply passively respond to the fluctuations in pressure and that is what is happening. The pressure fluctuation itself are maintained by vortex shed vortex shed from the projecting inhibitors between different segments of the motor. The main difference between this class is instability and what is observed in tactical missiles as you can see is the DC shift ok.

Here its not just there are large amplitude oscillations, it is also that the mean itself has increased by about 25 to 40 percent and then there are large amplitude fluctuations around the new mean ok. So, DC shift the definition is the sudden increase in mean chamber pressure up to 30 percent of the design pressure. It to sustains such high amplitude oscillations with a shift in mean, the energy has to come from propellant combustion. So, it is propellant combustion driven and therefore, it is strongly coupled with acoustics and propellant combustion yeah.

Student: We use with the note that the red color stuff, the actual longitude is just about 2 to 5.

Yeah, correct.

Student: So, its small (Refer Time: 22:42).

That is correct yeah. So, since the oscillations in the booster is driven by a fluid flow phenomena of vortex shedding and the propellant is passively responding, the energy that is there in the oscillations is very small. So, the percentage fluctuation compared to mean is very small. But, if you see here, the mean is about 175 bar, the amplitude is about 25 bar ok.

So, the amount of energy that is required to sustain such very high amplitude oscillations has to come from propellant combustion ok. These are the key differences between instability in space vehicle boosters and tactical rockets. And, this is the key difference is these are long and these are intermediate sizes, where the frequencies are such that oscillations can be sustained by propellant combustion.