Fundamentals of Combustion for Propulsion
Dr. S Varunkumar
Department of Mechanical Engineering
Indian Institute of Technology, Madras

Lecture - 12
Solid propellant combustion

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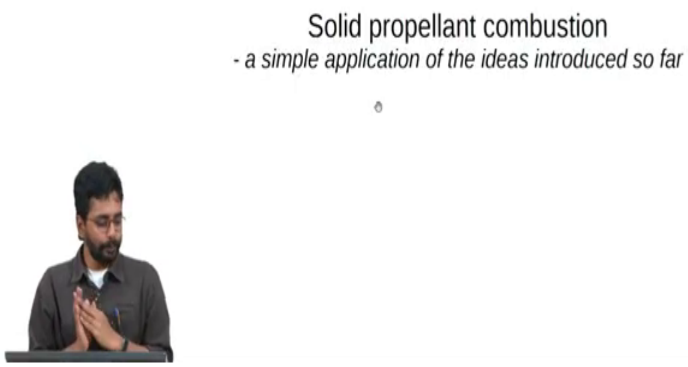

Let us continue. A natural progression from what we discussed in the previous lecture is to look at Compulsive Solid Propellant Combustion which I used as an example to explain the importance of length scales of fuel and oxidizer supply and its connection to lateral diffusion and pressure. So, we will discuss focus on solid propellant combustion starting from this lecture onwards. This is intended as a simple application of the ideas introduced so far, but before we go to the full description of the flame structure over a compulsive solid propellant, let us do a quick overview of the basics yeah.

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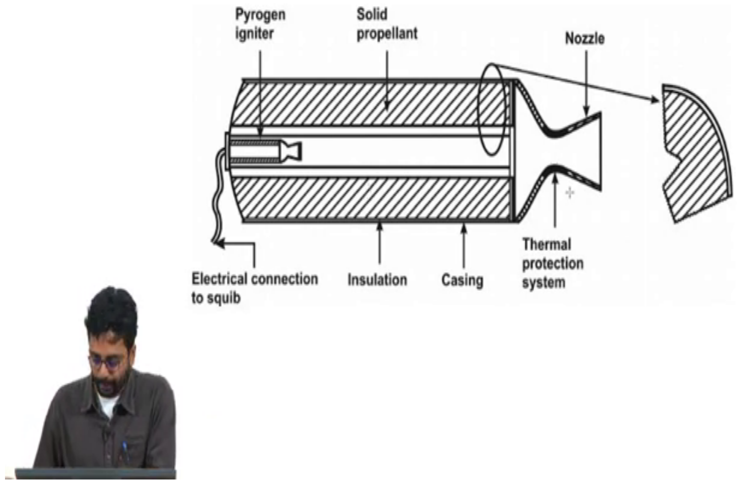

So, this is how a solid rocket motor looks. A metal casing into which a propellant is filled, and the propellant cross section can have variety of shapes, remember that the burning happens along the local normal. And therefore, several different shapes are conceived to have a certain burning area variation with time ok. If you have a circular cross section as the propellant burns, the area will increase with time; and if you have a block which burns cylindrical block which burns from one end, the area remains constant with time.

It is also possible to construct grains where the burning happens from in a cylindrical grain, the burning happens from inside to outside, but you start with special you know surface shapes, for example, a star grain or a cylindrical grain attached to a Finocyl grain as it is called, this can have complex burning surface evolution with time. For example, a star grain can be designed, so that the burning area remains constant with time ok.

So, the cross section can have variety of shapes. And depending on the size of the rocket motor you have igniters of special kinds, it is usually a pyrogen igniter connected to a electrical connection to the squib ok. This is the converging part of the nozzle, the throat and the diverging part. There is an insulation between the casing and the solid propellant. In case of long propellant grains there can also be the long propellant grains are made by putting together segments of grains, and there will be there can be insulations between or inhibitors between different sections of the grains also ok.

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$$\frac{dm}{dt} = \frac{d(\rho_c V)}{dt} = V\frac{d\rho_c}{dt} + \rho_c\frac{dV}{dt} = V\frac{d\rho_c}{dt} + \rho_c \dot{r} A_b = \dot{m}_{in} - \dot{m}_{out}$$

$$\frac{V}{RT_c}\frac{dp_c}{dt} = (\rho_p - \rho_c)\dot{r}A_b - \frac{p_c A_t}{c^*} = \rho_p \dot{r} A_b - \frac{p_c A_t}{c^*}$$

$$\frac{V}{RT_c}\frac{dp_c}{dt} = \frac{p_c A_t}{c^*}\left(\frac{\rho_p \dot{r} A_b}{\frac{p_c A_t}{c^*}} - 1\right)$$

$$\boxed{c^* = \sqrt{RT_c/\mathscr{M}}\,/\,\Gamma(\gamma)}$$

$$\frac{L^*}{\Gamma^2 c^*}\frac{1}{p_c}\frac{dp_c}{dt} = \left(\frac{\rho_p \dot{r} A_b}{\frac{p_c A_t}{c^*}} - 1\right)$$

The simplest the question first question that we can ask about the behavior of a solid rocket motor is how the pressure inside, once ignited, how the pressure inside the rocket changes with time ok. This can be calculated from a simple mass balance ok. So, once the rocket the propellant is ignited, the solid propellant gets converted into hot gases and that is the hot gas generation rate which happens which is controlled by the combustion rate of the propellant.

And this will increase the quantity of gas inside the chamber, and therefore the pressure inside the chamber and because of this the flow will start going out through the nozzle of the rocket.

So, we have burning solid propellant issuing hot gases into the chamber into the port as it is called. And this will cause the hot gases to flow through the nozzle and the nozzle because of its dimensions can accept only a certain dimension and the thermodynamic state of the gases that are coming out can accept only a certain maximum flow condition under which the nozzle becomes choked.

So, for a given upstream pressure and the thermodynamic state of the hot gases that are coming out which by the way can be calculated from equilibrium calculations that we discussed yesterday. Will, can actually admit only a certain maximum flow which under which conditions the nozzle becomes choked. And generation of more gases inside will simply increase a pressure inside the rocket, and the nozzle will continue to remain choked. Of course mass flow rate through the nozzle will increase because the pressure inside is changing, but the Mach number at the throat will remain 1 ok. So, the pressure inside the port is controlled by the rate of generation of hot gases because of the combustion of the solid propellant.

And the rate of outflow of hot gases through the nozzle ok. This controls the pressure inside the, this balance controls the pressure inside the rocket. And this we can simply calculate by a mass balance equation which is the rate of change of mass in the port which is also equal to the mass inside the port is the density in the combustion chamber or the density in the port multiplied by the volume of the port. So, the rate of change of mass inside the port is equal to the mass that comes into the port because of propellant combustion, the mass that goes out because of flow out flow through the nozzle.

What I have done here in between is just I have expanded this equation d by d t of rho c V is written as V d rho c by d t plus rho c d V by d t using a chain rule ok. And V d rho c by d t will remains as it is, but the density inside the chamber multiplied by the rate of change of volume inside the chamber. See the volume you start with a certain volume inside the port, and as the propellant burns that volume will increase right. So, that increase in volume is nothing but the volume created by the conversion of the solid propellant into gases which is nothing but the rate of burning of the propellant multiplied by the burning area of the propellant that is the increase in volume.

Imagine a cylinder of radius r 1 at time instant t 1, let say a time instant t 2 the radius increase from r 1 to r 2 because of the regression of the propellant normal to the surface or along the in the radial direction, the increase in volume is simply the area which is the circumferential area multiplied by r 2 minus r 1. So, the rate of increase perhaps this is something that I can just write it down just to make sure.

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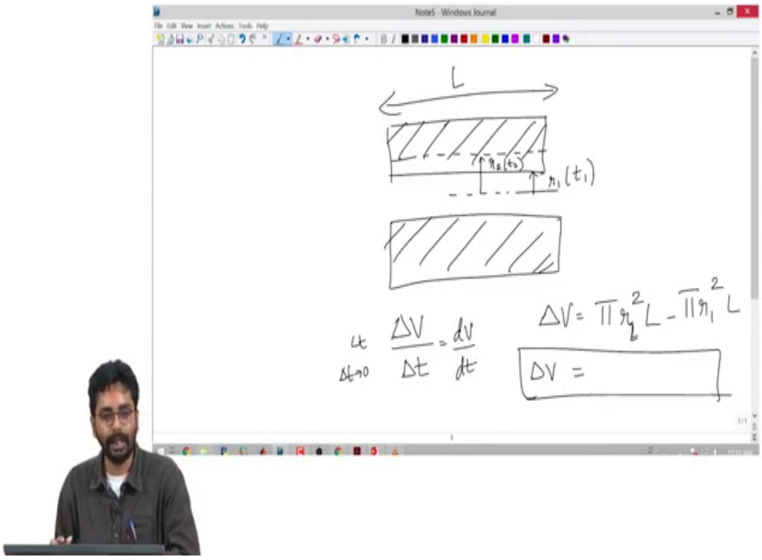

So, you have the starting let us say the starting diameter is this. So, you have start with the radius r 1 ok, let say this is the solid propellant, it is the cylindrical grain yeah, this is the radius at time t 1, let say a time t 2 the solid propellant burns and the radius increases to r 2 at time t 2 ok. So, now the port volume to begin with this, port volume to begin with was corresponding to a radius of r 1, now the port volume at time t 2 corresponds to a radius of a r 2. So, the change in volume is the volume at time t 2 which is 2 pi let us say the length is L 2 pi L r, I am sorry this is this is the volume not the surface area, is pi r 1 squared time is L is the volume of the cylinder at time t 2 is r 2 squared L minus pi r 1 squared L ok. So, this is. So, this is a delta V ok.

From the delta V you can calculate delta V by delta t as delta t goes to 0 you will get dV by dt ok that you can show is nothing but the density of the propellant times regression rate of the propellant multiplied by the area burning area ok. So, this is equal to, so the rate of change of mass inside the port is equal to the mass generation rate minus the mass outflow rate from mass conservation. So, now, we have different terms, we have the density can be written as pressure divided by R T c by the pressure in the chamber divided by p r T c. The T c is the equilibrium temperature which does not change. So, it is constant it does not change with time it is only a function of the composition of the propellant.

So, V by R T c dpc dt is equal to I am just rearranging the equation, these terms go the right. I must explained the rate of generation of gases will be equal to the rate of burning of the propellant multiplied by the density of the propellant multiplied by the area which we just saw the increase in volume that volume would have become gases at a lower density and that is this term rho p r dot A b, this is the rate of conversion of the solid propellant into gaseous propellant. The mass that is flowing out is the flow rate is the mass flow rate through a chocked nozzle ok, where the gases are starting from a stagnation condition at a pressure equal to the chamber pressure ok.

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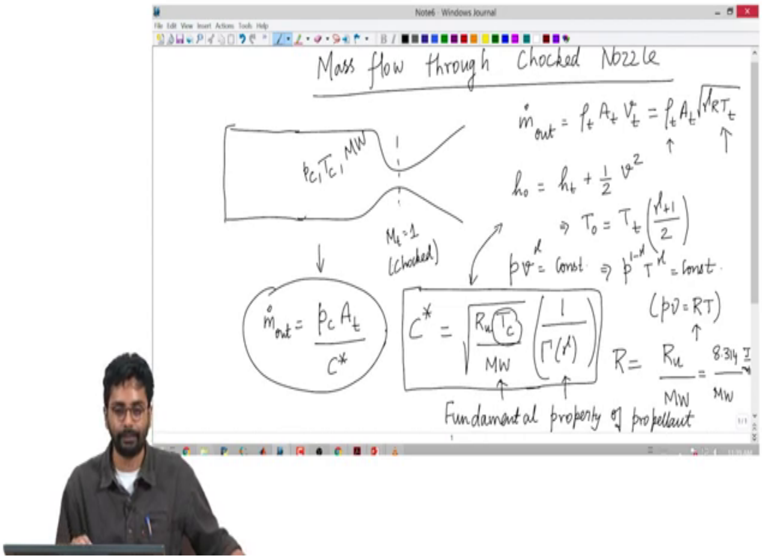

So, this let us go through a chocked nozzle we have something like this we have pressure pc here and the chamber temperature is T c. The conditions at the throat are such that the Mach number at the throat is 1 that is a chocked condition ok. The question that we are interested in is one is the flow rate that goes through a nozzle that is chocked with an upstream pressure of p c, temperature T c and the molecular weight of the gas is flowing through which is m ok. Let me call it m w, so that we do not confuse it with the Mach number ok. So, the mass flow rate through the nozzle m dot out is a density at the throat multiplied by the area of the throat multiplied by the velocity at the throat ok.

The throat is chocked, therefore the Mach number at the throat is 1, therefore, the velocity at the throat is equal to the sound speed and corresponding to the local condition. So, this is density at the throat area of the throat square root of gamma r temperature at the throat ok. So, this is the mass flow rate through the nozzle. Now, what we do is that all the throat parameters, this is the density at the throat and the temperature at the throat are related to the stagnation conditions upstream through the isentropic relations ok.

The isentropic relation is that the total enthalpy at every point is the same this is one part following from energy conservation which also from here you can show that the total temperature is temperature at the throat multiplied by gamma plus 1 by 2 ok. And this is the adiabatic part, the isentropic part is pv raise to gamma equals constant which also implies p over 1 minus gamma temperature raise to gamma is constant using the ideal gas equation. And using V, for both velocity, I should use u for specific volume ok. One important thing to remember is that this R is not the universal gas constant this is the specific gas constants. So, this R is a universal gas constant divided by the molecular weight which is equal to 0.8314 joules for mole Kelvin divided by the molecular weight ok.

You can now plug all of these relations into this equation. You can show that m dot out is equal to p c pressure in the chamber multiplied by the area of at the throat divided by a quantity called c star called a characteristic velocity where c star you can show if you do the algebraic manipulation which I am skipping, you can show that this is R the universal gas constant multiplied by the temperature in the chamber square root divided by molecular weight of the gases entering the nozzle divided by a function that is function of gamma ok, because of all these gammas appearing in the isentropic relationship you will have a complicated looking function of gamma. But you can show that c star is equal to square root of R u T c by molecular weight, I am explicitly writing this specific gas constant as the universal gas constant divided by the molecular weight multiplied by a function of gamma.

Function of gamma it does not change a whole lot, gamma changes from 1.1 to 1.674 monatomic gases. So, this is this does not change the whole lot. So, c star is a strong function of two things, one is the equilibrium temperature of the propellant combination. So, this is one use for the equilibrium composition or equilibrium calculation where you can get T c ok. And the situation is such that that enough time is usually available in the combustion chamber for the reactions to go to equilibrium and reach the equilibrium temperature.

The other most important factor is the molecular weight ok. So, you, so one of the reasons why, well the reason why in the hydrogen oxygen system that the c star peaks at hydrogen rich conditions is because the molecular weight of hydrogen is 2 grams per mole. And therefore, the value of c star even though the temperatures are lower than what it is under the stoichiometric conditions the value of c star is much higher because there is lot of hydrogen available ok. So, the c star peaks under rich conditions even though the maximum temperature is equals to this stoichiometry ok.

So, the so c star is a fundamental thermodynamic property of the propellant composition ok. And it is related to the flow outflow through the nozzle by this relationship ok, through this relationship. c star is a fundamental property of the propellant, the propellant combination ok. Please go through the algebra. I have not done the entire algebra, make sure that you get this equation by starting from here ok. So, therefore, the mass coming in is dependent on the rate of burning of the propellant as expected, it is rho pr dot into burning surface area this is because the propellant burns normal along the direction of the local normal.

And the m dot out we just saw is pc A t by c star where c star is a fundamental property of the propellant combination which is equal to square root R T c, here R is indeed the universal gas constant the subscript u is missing, but this is the universal gas constant which is 8.314 joules per mole Kelvin, because the molecular weight explicitly appears in the denominator ok. So, c star will increase with increase in T c or the equilibrium temperature, but always keep in mind that molecular weight changes must be carefully accounted for especially when you are dealing with species like hydrogen which have a very low molecular weight compare to the oxidizer.

Now, this equation is now rearranged we have V by R T c dpc dt is equal to rho p minus rho c r dot A b minus the outflow which is pc A t by c star propellant densities especially when you are looking at solid propellants the propellant densities are much higher than the gas densities, therefore, the second term which is the density of the gases is much smaller than this and can be neglected. So, the right hand side we are left with just two terms which is rho p r dot A b which is the rate of conversion of solid propellant into gaseous hot gases minus pc A t by c star ok. you can you can scale this equation the way it is done is we pull out pc A t by c star from the right hand side, so that the terms within the brackets are non-dimensional.

And when you rearrange the equation, you will get an equation of this kind where you have L star which is nothing but V divided by c star divided by it is V divided by c star, and you have 1 by pc dpc dt ok, there is a time factor that you need to watch out for ok. I am sorry let me correct myself L star is simply V divided by A t, that is L star we have a gamma square and c star appearing from the simplification of the terms on the right, 1 by pc dpc dt, and the right hand side is non-dimensional ok.

The reason why I have highlighted this in blue color is L star over c star has dimensions of time ok. In fact, the typical time that is available for the gases to fill the port is about this time. And if you make an estimate of this quantity for a typical rocket, you will find that it is few tens of milli seconds ok. And this is typically the time that is available for filling of the port, and therefore the this is also the time that is approximately available for the gases to go equilibrium which is generally much higher than the chemical reaction rate times. Therefore, assuming that the gases are at equilibrium is a very good approximation ok. So, with this equation right hand side is a non-dimensional we will see how this equation behaves for different conditions ok. This will be the left hand side; this is the right hand side.

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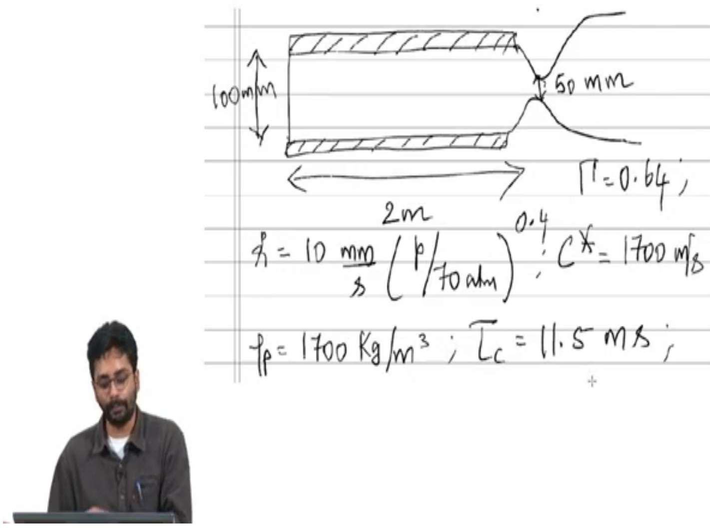

We will take a typical example. Solid rocket motor with a port diameter of a throat 100 millimeters, the throat diameter of 50 millimeters, the burn rate remember goes as a p raise to n. So, the way the burn rate expression is written here means the following, it is written as 10 millimeters per second multiplied by it has written as 10 millimeters per second multiplied by p by 70 raise to 0.4 this means that the propellant has a burn rate of 10 mm per second at 70 atmospheres, because if you substitute p equals 70 burn rate will become 10 millimeters per second. And the rather pressures it changes as 10 multiplied by p by 70 raise to 0.4, the n value is 0.4, the A value is 10 mm per second at 70 atmospheres.

The c star of a typical compulsive solid propellant will be in rage of 1700 meters per second. Propellant density will also be about 1700 kilograms per meter cube. Tau c is this quantity L star divided by gamma squared c star which is 11.5 milliseconds which is plenty of time available for the reactants to go to equilibrium ok. And therefore, the c star value which is calculated from the equilibrium result is accurate ok.

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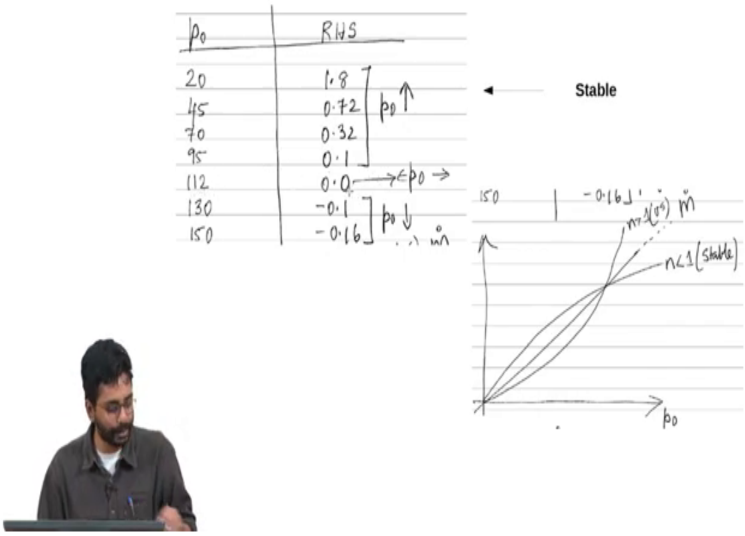

So, what is shown in this table here is remember that the RHS is rho p r dot A b divided by p c A t by c star minus 1, and the left hand side is this quantity. What I have done is for various values of chamber pressure, I have calculated the RHS values ok. So, at what this means is that at 20 atmospheres, the right hand side is a positive number. So, the left hand side is dpc dt, and the right hand side is a positive number, that means, if you start from 20 atmospheres dpc dt is positive and the pressure will increase ok. So, the pressure will increase till a point where the right hand side becomes 0, because dpc dt equals zero means the pressure is constant, it cannot change once it reaches that point.

So, if you start from 20 atmospheres, the right hand side is positive, the pressure will increase, the magnitude of the right hand side term will slowly decrease as a pressure increases, and there will come a particular pressure at which the right hand side is 0 that is the equilibrium pressure at which the rocket will settle down ok, that in this case, is 112 atmospheres.

Instead if you start it from value that is higher than 112 atmosphere, let say you start from 150 atmospheres, obviously, as expected the right hand side is negative, so the pressure will decrease till this point when pressure becomes 112 and then stop there ok. So, this 112 atmosphere is called the equilibrium pressure ok. So, if you build a rocket and you want to calculate the equilibrium pressure, the equilibrium pressure is the pressure at which the left hand side is 0, that means these terms the mass generation rate and the mass outflow rate are exactly matched ok.

In reality it has to be done in a quasi-static fashion, because remember that at different instances of time the burning area will also change and that is how it is done in using ballistic codes when developing solid rockets ok. But the reason why I brought this up is for a different reason and a fundamental reason, this behavior this equilibrium state where dpc dt is 0 can be stable or it can be unstable.

What I mean by that is look at the graph that I have shown on the right bottom, it is a plot of chamber pressure versus m dot in and m dot out. Remember that m dot out goes as pc A t by c star, therefore for a fixed throat and the fixed propellant composition the exit mass flow rate will linearly increase with pressure and that is what I have shown with the line in the centre. It is a straight line which is m dot out ok. And the other two curves are the mass generation rate which is because of the combustion of the propellant ok. As you can see the combustion of the propellant or the mass generation rate because of the combustion of the propellant goes as rho pr dot A b and r dot itself goes as a p raise to n.

So, when n is less than 1, that means, the pressure the burn rate pressure index of the propellant is less than 1, the mass generation rate will change like this for the n less than 1 case. You can see two curves one marked n less than 1, one marked n greater than 1. The n less than 1 curve goes above the straight line till some pressure and then crosses the straight line at a certain pressure. The point where it crosses over is the equilibrium point that is where the right hand side is 0 ok, and that is equilibrium pressure. So, if you drop a vertical from that point, you will get 112 atmosphere is for the case which I described in the previous slide. Remember that the index of this propellant was 0.4 therefore, n was less than 1. And therefore, this curve and the straight line are representative of what is happening in this rocket. And now if you are at the equilibrium point for this case, let us say there is some perturbation that brings down the pressure I think I better explained this in a in some detail ok.

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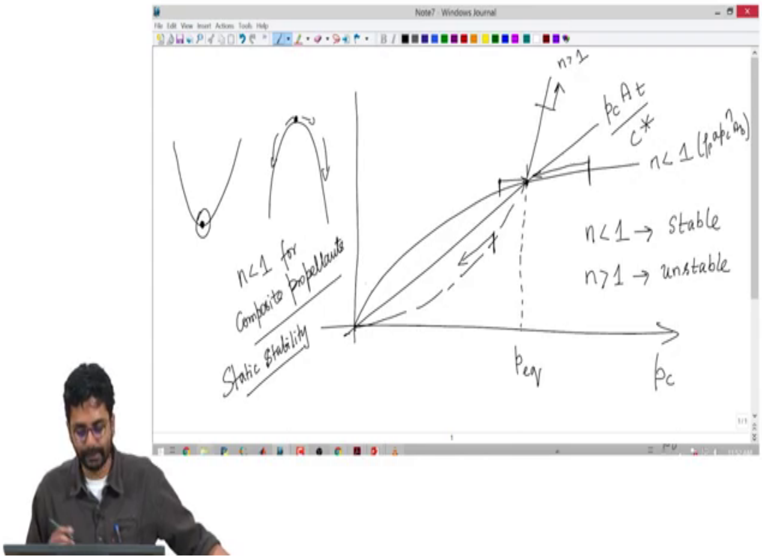

What I am talking about is this is the chamber pressure this is p c A t by c star when n is less than 1, the mass burn rate of the propellant behaves like this. This is n less than 1 ok. This is the equilibrium point this is the equilibrium point ok. So, if you start from a pressure that is less than the equilibrium pressure, the mass generation rate is greater than the mass outflow, therefore the right hand side is positive the pressure will increase and come to the equilibrium value. This starts from here, you will go to the equilibrium point ok.

Instead if you start from a higher pressure, the mass generation rate is smaller than the mass out flow rate therefore, the right hand side is negative this will automatically push you to the equilibrium point ok. So, any tiny perturbation that causes the chamber pressure to either go down or go up is not is not very serious, because you will get automatically the rocket will automatically go back to the point of equilibrium because that is how the relationship between the inflow and the outflow are ok. This is the case when n is less than 1.

But consider the case where n is greater than 1 ok, let us say this n is greater than 1 ok. Here also the equilibrium point is here, but if you start from a point that is at a pressure lower than the equilibrium pressure, the outflow rate is greater than the mass generation rate, that means, the right hand side is negative ok. So, it will actually if you start here it will pull you it will push you towards the origin ok.

And if you start here on the at a pressure that is above the equilibrium pressure, the mass generation rate is more than the mass outflow rate, that means, the right hand side is positive and the pressure will continue to increase ok. So, by some delicate balance if you started at the equilibrium points any tiny disturbance will either put push the rocket towards zero pressure or the pressure will blow up, and the rocket will explode ok. So, n less than 1, the equilibrium point is stable, and n greater than 1 the equilibrium point is unstable.

What we mean by that you can imagine something like this. The stable equilibrium is like this the rocket is sitting at the state that is here. You push it a little to the left a little to the right, it will come back to the equilibrium point, but n greater than 1 is as if the rocket operating point is sitting on top of a hill, you push it a little to the left, you push it a little to the right it will run away from the equilibrium point ok. But as it turns out for all the compulsive solid propellant index is usually less than 1, and therefore, they are statically stable. So, they are statically stable so static stability is always ensured with compulsive solid propellants ok. This is pc A t by c star and this rho pa pc raise to n A b. So, n less than 1, the curve goes like this; n greater than 1, goes like this. Is that clear? So, that is what I was trying to explain.

So, the right for n less than 1, you have at pressures lower than the equilibrium pressures right hand side is positive, you are push to the equilibrium point; for pressure is greater than the equilibrium pressure the right hand side is negative, again you are push to the equilibrium point if n is greater than one this order will be flipped. So, you will be pushed away from the equilibrium point, therefore the equilibrium point for n greater than 1 is unstable.

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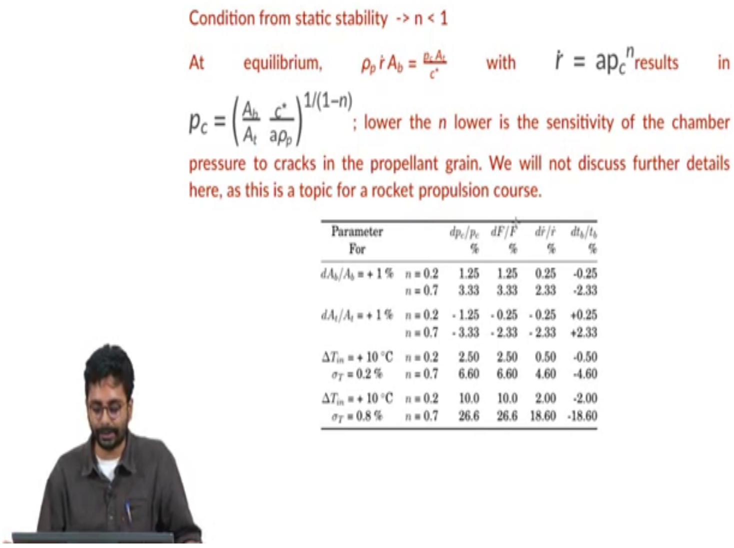

Parameter For		dp_c/p_c %	dF/F %	$d\dot{r}/\dot{r}$ %	dt_b/t_b %
$dA_b/A_b = +1$ %	$n = 0.2$	1.25	1.25	0.25	-0.25
	$n = 0.7$	3.33	3.33	2.33	-2.33
$dA_t/A_t = +1$ %	$n = 0.2$	-1.25	-0.25	-0.25	+0.25
	$n = 0.7$	-3.33	-2.33	-2.33	+2.33
$\Delta T_{in} = +10$ °C	$n = 0.2$	2.50	2.50	0.50	-0.50
$\sigma_T = 0.2$ %	$n = 0.7$	6.60	6.60	4.60	-4.60
$\Delta T_{in} = +10$ °C	$n = 0.2$	10.0	10.0	2.00	-2.00
$\sigma_T = 0.8$ %	$n = 0.7$	26.6	26.6	18.60	-18.60

So, the condition for static stability is that the index must be less than 1. And at equilibrium where the left hand side is 0, dpc dt is 0, the mass generation rate is balanced by the mass production rate we should say the other way around, the left hand side is a mass generation rate the right hand side is a mass outflow, and with r dot equals A p raise to n this will give you an expression of this kind. The equilibrium pressure is dependent on the ratio of the burning surface area to the throat area multiplied by c star divided by A rho p raise to 1 by 1 minus n.

In practice it is generally preferred to have as low a value for n as a possible there are several reasons for that. But the most important reason as you can see from the table is for a n value of 0.2 ok, if there is a some process that causes a change or a abrupt change in the area of the burning surface ok, there could be cracks in the propellant which can create a perturbation in the burning surface area, this will lead to a change in pressure which is 1.25 times the differential change is 1.25, and the corresponding change in percentage change in thrust is a factor of 1.25 ok.

On the other hand, if you have an n value of 0.7, this can lead to a large change it can lead to a change in pressure that is about 3 times of what you get at n equals 0.2, and the thrust change is corresponding 3 percent. The other factor is if there is a small crack and the index is large, it will quickly become lead to a quick increase in the burning surface area because a crack can propagate because of further burning of the propellant. So, this is the most important limitation in seeking compositions with lower index. The other things are can be controlled in a in a much better way than the burning surface area of the propellant, anyway I will not go in to the details of this as a this is for a rocket propulsion course, we are not looking at the rocket propulsion per say ok.

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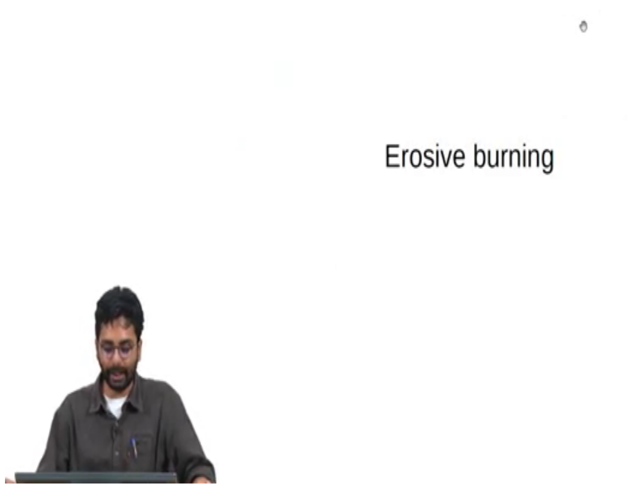

The next topic is something to do with r dot equals to a pc raise to n called erosive burning, and I will hand it over to Prof Mukunda for this.