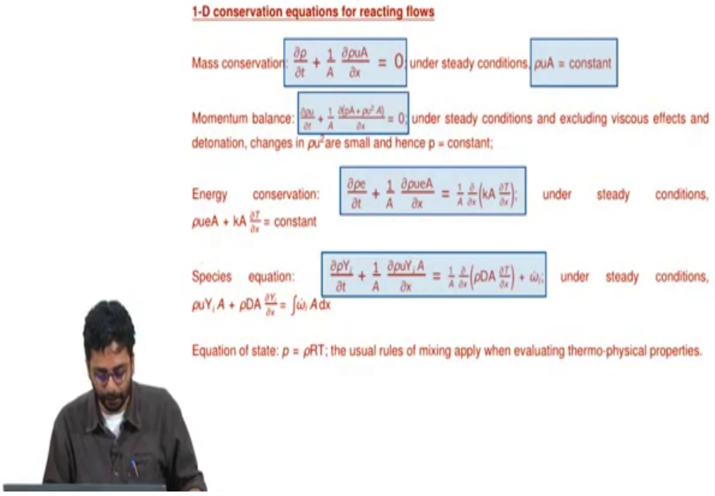
Fundamentals of Combustion for Propulsion
Dr. S Varunkumar
Department of Mechanical Engineering
Indian Institute of Technology, Madras

Lecture – 11
Integral Analysis of flame

(Refer Slide Time: 00:16)



1-D conservation equations for reacting flows

Mass conservation: $\boxed{\frac{\partial \rho}{\partial t} + \frac{1}{A}\frac{\partial \rho u A}{\partial x} = 0}$; under steady conditions, $\boxed{\rho u A = \text{constant}}$

Momentum balance: $\boxed{\frac{\partial \rho u}{\partial t} + \frac{1}{A}\frac{\partial (pA + \rho u^2 A)}{\partial x} = 0}$; under steady conditions and excluding viscous effects and detonation, changes in ρu^2 are small and hence p = constant;

Energy conservation: $\boxed{\frac{\partial \rho e}{\partial t} + \frac{1}{A}\frac{\partial \rho u e A}{\partial x} = \frac{1}{A}\frac{\partial}{\partial x}\left(kA\frac{\partial T}{\partial x}\right)}$; under steady conditions, $\rho u e A + kA\frac{\partial T}{\partial x} = \text{constant}$

Species equation: $\boxed{\frac{\partial \rho Y_i}{\partial t} + \frac{1}{A}\frac{\partial \rho u Y_i A}{\partial x} = \frac{1}{A}\frac{\partial}{\partial x}\left(\rho D A\frac{\partial T}{\partial x}\right) + \dot{\omega}_i}$; under steady conditions, $\rho u Y_i A + \rho D A\frac{\partial Y_i}{\partial x} = \int \dot{\omega}_i A\,dx$

Equation of state: $p = \rho R T$; the usual rules of mixing apply when evaluating thermo-physical properties.

Well that completes the simple form of the conservation equations.

(Refer Slide Time: 00:20)

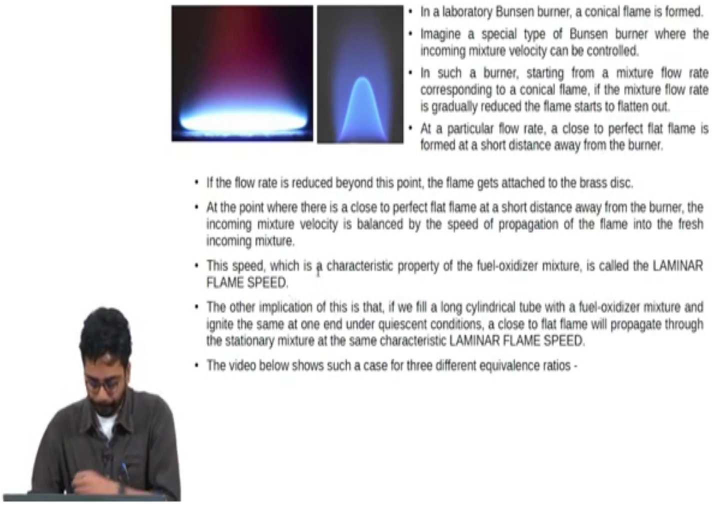
Integral analysis of flames

Let us see how we can use it for a simple integral analysis; some of this was discussed yesterday, but let us go over it one more time, ok. We will look at premixed flames first, ok.

(Refer Slide Time: 00:28)

Premixed flames

(Refer Slide Time: 00:34)



- In a laboratory Bunsen burner, a conical flame is formed.
- Imagine a special type of Bunsen burner where the incoming mixture velocity can be controlled.
- In such a burner, starting from a mixture flow rate corresponding to a conical flame, if the mixture flow rate is gradually reduced the flame starts to flatten out.
- At a particular flow rate, a close to perfect flat flame is formed at a short distance away from the burner.
- If the flow rate is reduced beyond this point, the flame gets attached to the brass disc.
- At the point where there is a close to perfect flat flame at a short distance away from the burner, the incoming mixture velocity is balanced by the speed of propagation of the flame into the fresh incoming mixture.
- This speed, which is a characteristic property of the fuel-oxidizer mixture, is called the LAMINAR FLAME SPEED.
- The other implication of this is that, if we fill a long cylindrical tube with a fuel-oxidizer mixture and ignite the same at one end under quiescent conditions, a close to flat flame will propagate through the stationary mixture at the same characteristic LAMINAR FLAME SPEED.
- The video below shows such a case for three different equivalence ratios -

This is the flame that we used for the discussion of equilibrium yesterday; I have added one more picture let me explain what the other picture is all about, ok. So, commonly when you set up or you when ignite a Bunsen burner in a laboratory, a conical flame is formed of this type, excuse me. Now imagine a special type of Bunsen burner where the incoming mixture velocity can be controlled, you do not have in a usual one that is available in the lab, you do not have that control; but let us say a built once a special one in which you can control the incoming velocities.

In such a burner, starting from a mixture flow rate corresponding to a conical flame; that is the flame here, if the mixture flow rate is gradually reduced the flame will start to flatten out, ok. So, you start from this condition, you keep reducing the mixture velocity; the flame will slowly become a flat flame, ok. So, at a particular flow rate, a close to perfect flat flame is formed at a short distance away from the burner; like the one I have shown on the left.

Now, if you continue to reduce the flow rate, if the flow rate is reduced beyond this point; the flame gets attached to a brass disc that is at the inlet. If there is no brass disc at the inlet, the flame will flash back, ok. What does happened is that in the conical flame, the incoming mixture velocity is higher than the flame speed ok; therefore, a conical flame was formed in such a fashion that, the mixture velocity normal to the flame is equal to the flame velocity.

That is what that is the reason why you have a conical flame here and when you start, when we had started reducing the incoming mixture velocity; there comes a point where the incoming stream velocity is perfectly matched by the flat flame perfectly matched by the laminar flame speed and therefore, you have a flat flame.

If you reduce it any further, the flame will start moving into the unburnt mixture and that is what we call as flashback. And in the reverse direction, starting from here where you have a conical flame, you keep increasing the velocity of the incoming mixture. What would happen is that, the angle of the cone; the flame will become longer, the angle of the cone will become smaller, because now the angle that is required to make sure that the component of the velocity is matched by the flame velocity is smaller and smaller.

And there will come a point, where the flame cannot actually hold on to the burner anymore, because incoming steam velocity so high that; there is no point anchoring point available for it, so that it can match the flame velocity, it will simply blow off, ok. So, this is the phenomena of steady flame, flat flame; steady flat flame and steady conical flame, flashback and blow off, ok. It is simply determined by the local relationship between flame speed and the mixture velocity.

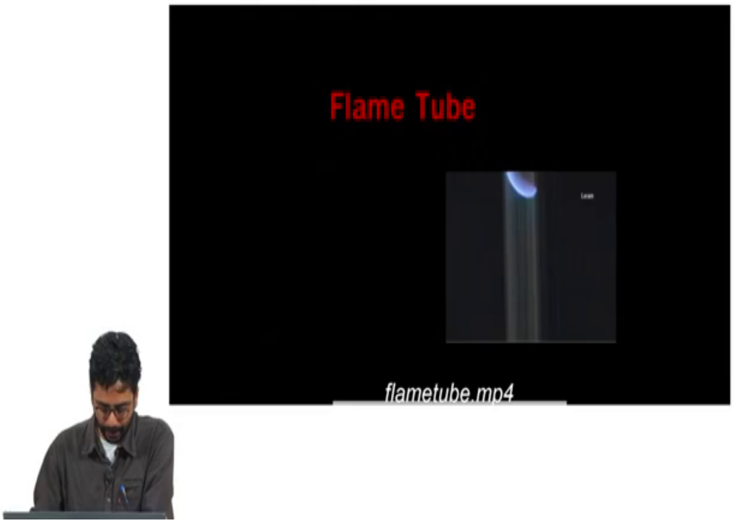
So, at the point, where there is close to perfect flat flame; like the one that is shown on the left at the short distance away from the burner, the incoming mixture velocity is exactly balanced by the speed of propagation of flame or the laminar flame speed into the fresh incoming mixture, ok. And this kind of arrangement has the following advantage that, the flame is not strained; we saw yesterday that straining the flame or imposing gradients along the flame can change the structure of a flame.

This flame is unstrained because it is a flat flame with velocity normal to it; there are no gradients of velocity along the flame, ok. So, this flame speed which is for a flat, which is that is one dimensional unstrained flame is a characteristic property of the fuel air mixture, the fuel oxidizer mixture; it is not dependent on the flow at all, it is a characteristic property of the mixture and is called the laminar flame speed, ok.

This also means that, the that I have a flat flame that is fresh mixture approaching the flat flame at exactly the flame speed, I can also do the following; I can create a mixture which is stationary and if I ignite, put an ignition source in it, what I will see is that, a flat laminar flame propagates into the unburnt mixture at exactly the flame speed. It is these two scenarios are just change in the coordinate system.

So, if you see it from the perspective, if you have a coordinate frame fixed to the flame; then flame is stationary and you have mixture approaching the flame at the flame speed. The other situation is when you have the coordinate frame fixed with respect to the unburnt mixture; then you have a flat flame propagating at the flame speed going into the mixture, ok. I will show you a video of that arrangement in a moment, ok.

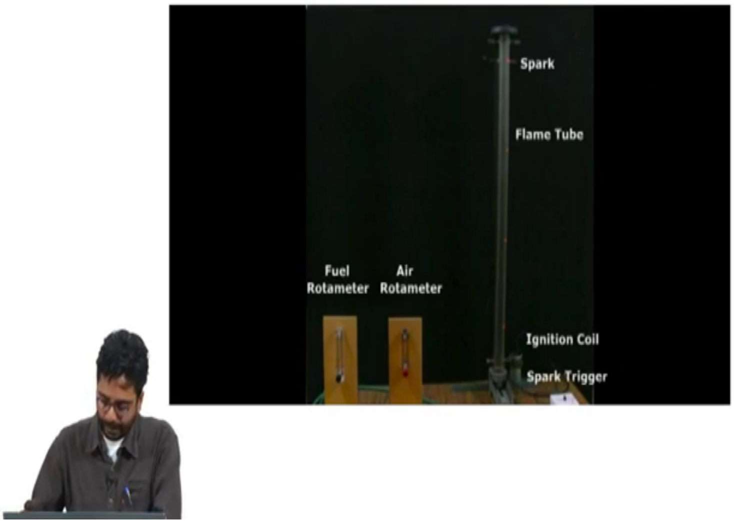
(Refer Slide Time: 05:31)


Flame Tube

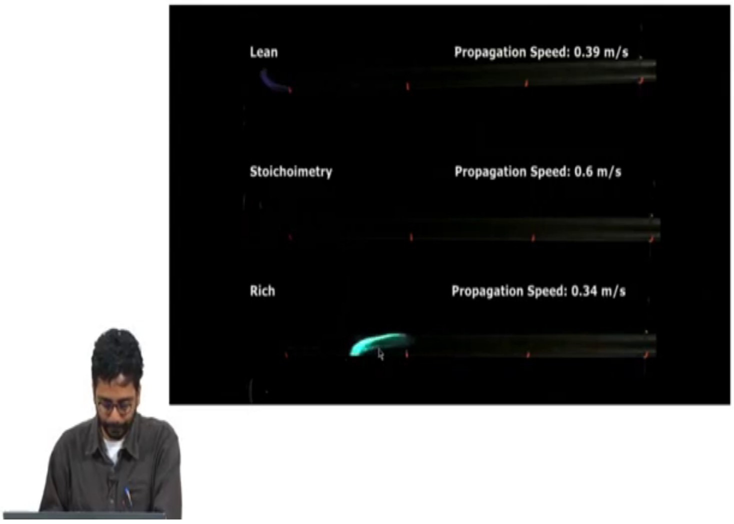
flametube.mp4

So, this is the arrangement where the unburnt gases are stationary and flame propagates through it.

(Refer Slide Time: 05:35)



(Refer Slide Time: 05:40)

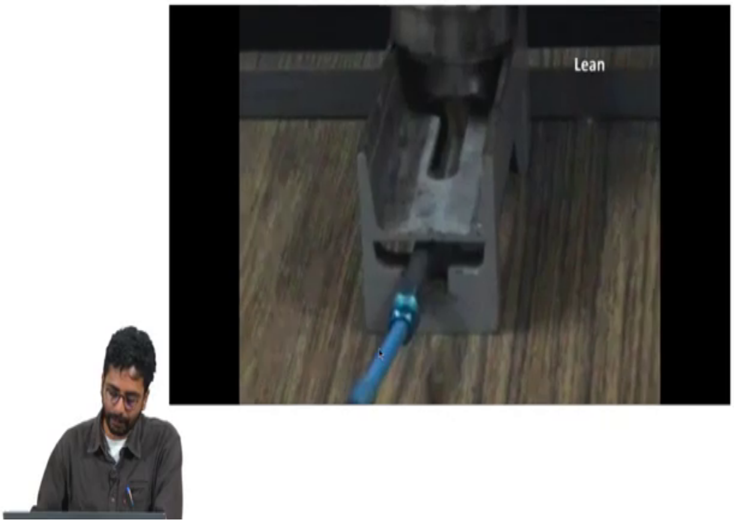


Lean Propagation Speed: 0.39 m/s

Stoichoimetry Propagation Speed: 0.6 m/s

Rich Propagation Speed: 0.34 m/s

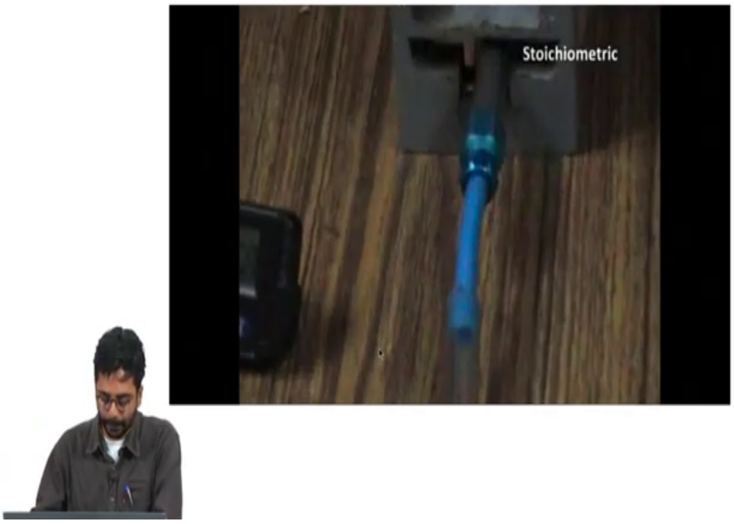
(Refer Slide Time: 05:44)



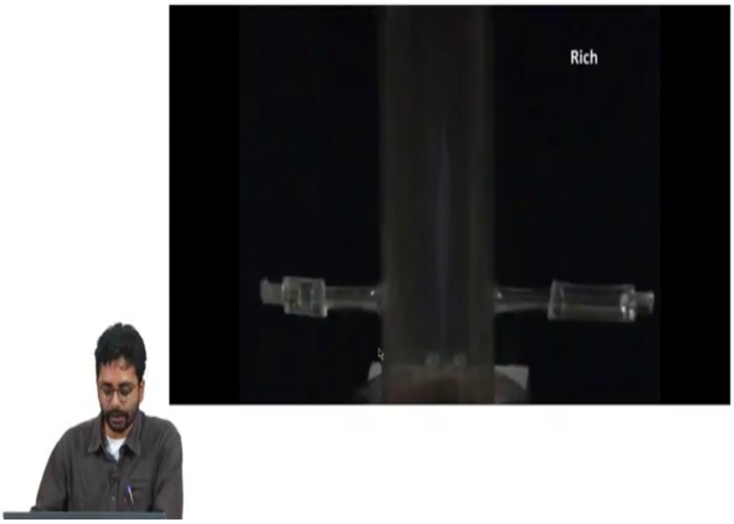
(Refer Slide Time: 05:48)

So, this is a tube in which the this is a tube in which the unburnt mixture is filled, it is ignited at the top.

(Refer Slide Time: 05:50)



(Refer Slide Time: 05:52)

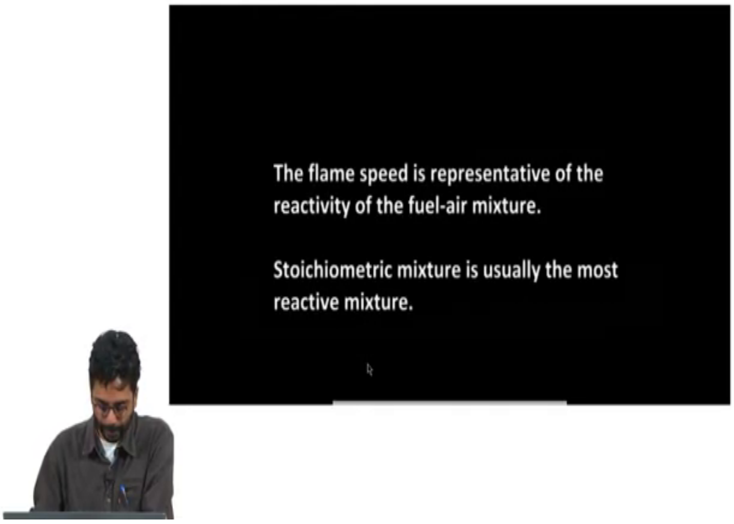


(Refer Slide Time: 05:55)

At slower speed

And you see a video of lean and close to stoichiometric and a rich mixture; flame propagating in these three mixtures. This will show you the same in slow motion; of course, there are there is formation of boundary layer and other aspects which distort the flame.

(Refer Slide Time: 06:09)



The flame speed is representative of the reactivity of the fuel-air mixture.

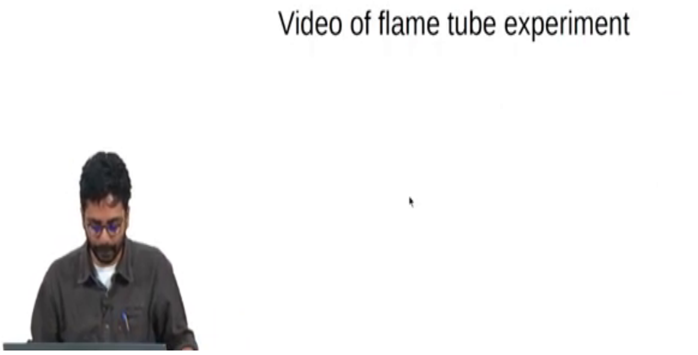
Stoichiometric mixture is usually the most reactive mixture.

This is for LPG; let me play it one more time. This is a nice close to flat flame, this is stoichiometric flame which is nearly flat and a rich conditions there are several instabilities that make the flame distorted. Let me not go into the details; maybe I will pause here, this is the good frame to look at all the three flames. So, this at this moment the lean flame is more or less flat, this is gone from being flat to spread out a little bit because of the velocity gradients in the tube. And the rich flame which is a weak flame is propagating slower.

Of course, other point I wanted to make is that, this is a bright and strong flame close to stoichiometry with the flame speed that is the maximum of three cases. The lean and the rich cases both propagate at the velocity that is lower than the stoichiometric speed, and flames also look visually weak; the heat release in these flames is are much lower than what it is in stoichiometric conditions, ok. These are broad features of laminar premixed flames, ok.

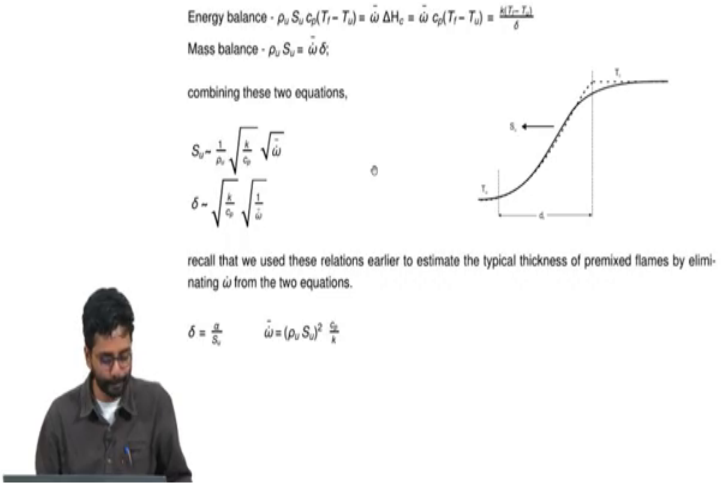
So, we say that this is more reactive than the other two cases and that is what we want to formally express using simplified conservation equations, ok.

(Refer Slide Time: 07:53)



Video of flame tube experiment

(Refer Slide Time: 7:54)



Energy balance - $\rho_u S_u c_p(T_f - T_u) = \bar{\dot{\omega}}\, \Delta H_c = \bar{\dot{\omega}}\, c_p(T_f - T_u) = \frac{k(T_f - T_u)}{\delta}$

Mass balance - $\rho_u S_u = \bar{\dot{\omega}}\, \delta$;

combining these two equations,

$$S_u \sim \frac{1}{\rho_u}\sqrt{\frac{k}{c_p}}\,\sqrt{\bar{\dot{\omega}}}$$

$$\delta \sim \sqrt{\frac{k}{c_p}}\,\sqrt{\frac{1}{\bar{\dot{\omega}}}}$$

recall that we used these relations earlier to estimate the typical thickness of premixed flames by eliminating $\dot{\omega}$ from the two equations.

$$\delta = \frac{\alpha}{S_u} \qquad \bar{\dot{\omega}} = (\rho_u S_u)^2\, \frac{c_p}{k}$$

Let me go back to; I have shown you the video of the flame tube experiment, this we saw yesterday. So, you can start from the steady one dimensional energy equation and if you integrate it from negative infinite to positive infinite; the gradients in these two extremes are 0. So, all the gradients with respect to x, derivatives with respect to x will drop out, you will be left only two terms; one is the heat release because of chemical reaction, which we just saw omega dot delta H c ok, for a one step irreversible reaction.

This is the heat that is released in the flame zone, where the temperature increases because of the chemical reaction. Remember that for a steady flame, the heat that is released in reaction zone ok; the same rate at which, the rate at which it is released in the reaction zone and the same rate heat must be removed out of the reaction zone. If not, the temperature in reaction zone will continuously increase, you see the point.

So, for a steady temperature profile, the rate of heat generation must be matched by the rate of removal of heat from the reaction zone. So, all the heat that is released is used to rise the temperature of the incoming mixture from initial temperature to the adiabatic flame temperature. And the heat must be removed out of reaction zone at the same rate because of conduction; you have high temperature here, low temperature here, there is a gradient in temperature. So, heat transfer will happen from the high temperature zone to the low temperature zone.

So, at this rate of heat removal must be equal to the rate of heat release because of combustion; otherwise the temperature profile will not be steady ok. And now this combined with the fact that the incoming mixture must be consumed in the reaction zone, because we know that the state to the right of the flame is closed to equilibrium state; therefore, the net chemical reaction rates is 0 here. So, all the reactions happen in this zone and the reaction rate is such that all the reactants are consumed, ok.

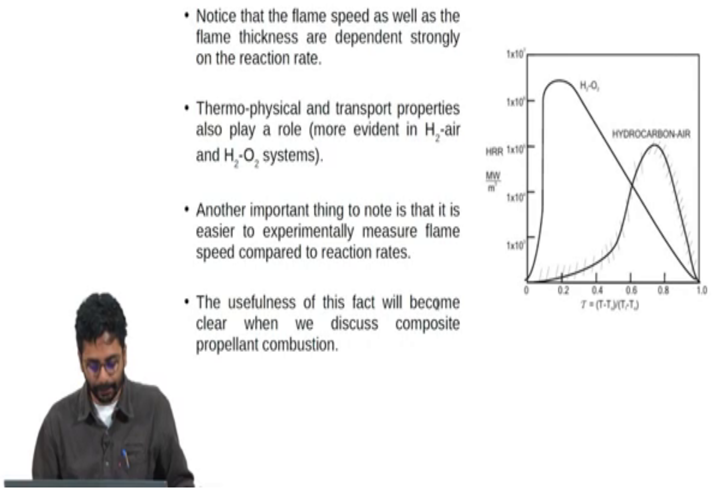
If it is stoichiometric both fuel and oxidizer are consumed; if it is lean, all the fuel is consumed; if it is rich all the oxidizer is consumed. So, the smallest quantity whichever is in smaller, deficient reactant will be consumed completely. Now you can combine these equations and you can show. See you have two equations and you have two variables; one is the flame speed, the other is the thickness of the flame, ok. Now you can combine these two equations to get a relationship of between the flame speed and reaction rate; the thickness and the reaction rate, ok.

This is exactly what was introduced yesterday; the flame speed goes as the square root of the overall reaction rate ok, goes square root k by c p square root omega dot. And the thickness of the flame goes as 1 by square root of omega dot, ok. From here you can also show the other thing that, S u delta by alpha which is the flame Reynolds number is 1, ok. We can just multiply these two and divide by alpha; you can show that it equal to 1. So, the point that I want to emphasize again is that, the flame speed is a measure of reactivity of the mixture. So, higher the flame speed, higher is the reactivity; the higher is the reactivity, thinner is the flame ok.

So, now recall that we used these relations earlier to estimate typical flame thickness of premixed flames by eliminating reaction rate term. You can do that and show that delta is alpha by S u; this also means that, S u delta by alpha is 1, that is the flame Reynolds number is 1. And can eliminate delta from these two equations; you will get omega dot equals rho u S u the whole square c p by k; remember that this is the form for the reaction rate that we used yesterday to get an estimate for delta, ok. This is also the equation that I use to emphasize that, it is much it is easier to measure flame speed than reaction rate.

In fact, the question of what it means to measure reaction rate itself is discussed earlier. But it is much easier to measure flame speed which is at which has a direct relationship to the reaction rate; and the reactivity or the relative reactivity of mixtures can be estimated simply from this relationship. And this relationship follows from a simple energy and mass balance for the flame zone, ok.

(Refer Slide Time: 12:28)



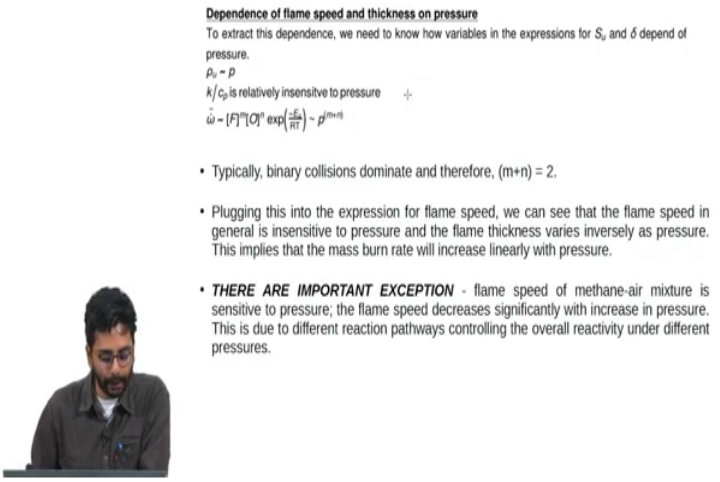
- Notice that the flame speed as well as the flame thickness are dependent strongly on the reaction rate.
- Thermo-physical and transport properties also play a role (more evident in H_2-air and H_2-O_2 systems).
- Another important thing to note is that it is easier to experimentally measure flame speed compared to reaction rates.
- The usefulness of this fact will become clear when we discuss composite propellant combustion.

Notice that the flame speed as well as the flame thickness are dependent strongly on reaction rates; this is a reaction controlled phenomena. Thermo physical and transport properties also play a role, it is not that diffusion is absent; but there is a reaction diffusion balance, and reaction effects are significant.

And the effect of flame speed or thermo physical and transport properties on flame speed are more evident in cases where is there is large disparity in the transport properties. For example, if you have hydrogen which diffuses much faster than the oxidizer, and other fuels like methane and LPG; you will see significant effects of transport properties, ok. That is what is reflected here; in terms of heat release as a function of non dimensional temperature between a typical hydrocarbon air case and hydrogen oxygen case.

Another important thing to note which I already mentioned is that, it is easier to experimentally measure flame speed compared to reaction rates, ok. The usefulness of this fact will become clear when we discuss composite solid propellants, ok.

(Refer Slide Time: 13:35)



Dependence of flame speed and thickness on pressure

To extract this dependence, we need to know how variables in the expressions for S_u and δ depend of pressure.

$\rho_u \sim p$

k/c_p is relatively insensitve to pressure

$\dot{\bar{\omega}} \sim [F]^m[O]^n \exp\left(\frac{-E_a}{RT}\right) \sim p^{(m+n)}$

- Typically, binary collisions dominate and therefore, (m+n) = 2.
- Plugging this into the expression for flame speed, we can see that the flame speed in general is insensitive to pressure and the flame thickness varies inversely as pressure. This implies that the mass burn rate will increase linearly with pressure.
- ***THERE ARE IMPORTANT EXCEPTION*** - flame speed of methane-air mixture is sensitive to pressure; the flame speed decreases significantly with increase in pressure. This is due to different reaction pathways controlling the overall reactivity under different pressures.

Quick discussion on the dependence of flame speed and thickness on pressure, ok. We know that we will use this expression for flame speed; S u goes 1 over rho u and square root k by c p and square root omega dot bar. And the density is directly proportional to pressure, k by c p is relatively insensitive to pressure, and the reaction rate has the following dependence on pressure; it is it comes through the dependence of concentration on pressure.

It is a concentration of the fuel rise to some exponent, concentration of oxidizer rise to some exponent; this is the erroneous term which we saw yesterday, exponential minus E a by R T. Therefore, omega dot has of course, a very strong dependence on temperature; it is an exponential dependence on temperature which we already know. And the pressure dependence comes through the concentration dependence on pressure, which will become pressure rise to m plus n, ok.

In typical conditions of pressure and temperature; binary collisions dominates and therefore, m plus n is equal to 2, ok. So, if you plug this into the expression for flame speed; we can see that flame speed is in general is insensitive to pressure, and the flame thickness varies inversely as pressure. Flame speed goes as p rise to close to 0 and the thickness goes as 1 over pressure rise to n by 2, ok. But I want to emphasize that there are important exceptions to this; flame speed of methane-air mixture is sensitive to pressure, because different reaction pathways are dominant at different pressures and therefore, m plus n is not, the effective m plus n is not always 2.

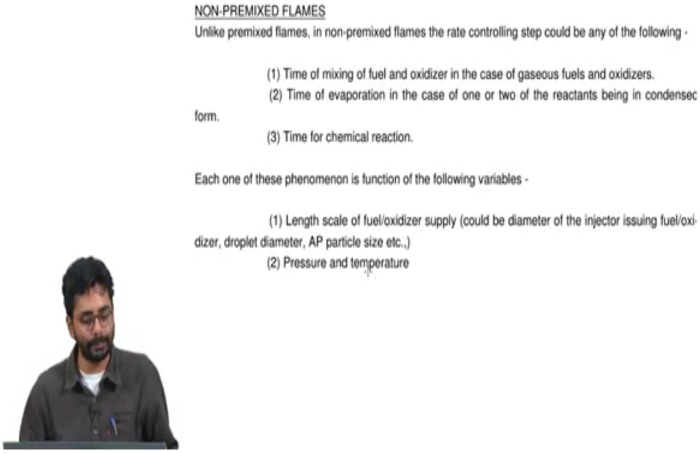
The flame speed actually decreases significantly with increase in pressure; this is due to different reaction pathways controlling the overall reactivity under different pressures, ok. So, we take away is the general conclusion is the flame speed is insensitive to pressure; there are exceptions, and the flame thickness will decrease with increase in pressure, ok.

(Refer Slide Time: 15:54)

Non-premixed flames

That completes the discussion on premixed flames. If there are any questions, now is the right time on premixed flames, ok. If there are no questions, let us move on to non premixed flames.

(Refer Slide Time: 16:09)



NON-PREMIXED FLAMES

Unlike premixed flames, in non-premixed flames the rate controlling step could be any of the following -

(1) Time of mixing of fuel and oxidizer in the case of gaseous fuels and oxidizers.
(2) Time of evaporation in the case of one or two of the reactants being in condensec form.
(3) Time for chemical reaction.

Each one of these phenomenon is function of the following variables -

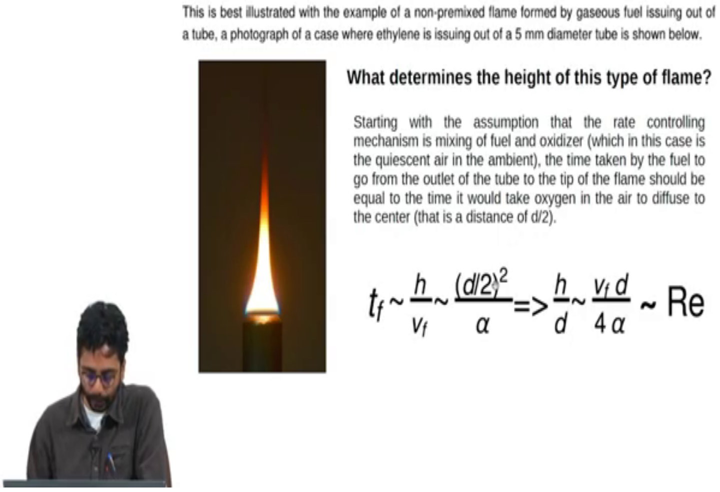
(1) Length scale of fuel/oxidizer supply (could be diameter of the injector issuing fuel/oxidizer, droplet diameter, AP particle size etc.,)
(2) Pressure and temperature

So, unlike the premixed flames, in non premixed flames the rate controlling step could be any of the following; in premixed flame remember the reaction rate is the rate controlling feature, in non premixed flame it could be any of the following or the combination of these. Time of mixing of fuel and oxidizer in the case of gaseous fuels and oxidizer; remember that the fuel and oxidizer must mix before the reaction can happen. And the time for mixing can be much larger than the time it takes for the reactions to happen; therefore, mixing can be rate limiting.

In the example that we saw earlier of ethanol droplet burning; time of evaporation is rate limiting, ok. And in under some extreme conditions, it is possible that the mixing times are made much smaller; I will give you one example of where this happens, the time for chemical reaction can also become rate limiting, ok. So, in premixed flames, the behavior is more or less universal; the dynamics is controlled by reaction rate under all conditions, but in non premixed flames it can be any of these three, ok.

So, each one of these phenomena is a function of the following variables; the length scale of fuel oxidizer supply, which could be for example, one of the examples that I showed was combustion of matrix of AP and HTPB. It could be the diameter of the injector issuing fuel or oxidizer; the droplet diameter or the AP ammonium perchlorate particle size and other variables include pressure and temperature. I will demonstrate what I mean by the statement with a simple example, ok.

(Refer Slide Time: 17:49)



This is best illustrated with the example of a non-premixed flame formed by gaseous fuel issuing out of a tube, a photograph of a case where ethylene is issuing out of a 5 mm diameter tube is shown below.

What determines the height of this type of flame?

Starting with the assumption that the rate controlling mechanism is mixing of fuel and oxidizer (which in this case is the quiescent air in the ambient), the time taken by the fuel to go from the outlet of the tube to the tip of the flame should be equal to the time it would take oxygen in the air to diffuse to the center (that is a distance of d/2).

$$t_f \sim \frac{h}{v_f} \sim \frac{(d/2)^2}{\alpha} \Rightarrow \frac{h}{d} \sim \frac{v_f\, d}{4\,\alpha} \sim \mathrm{Re}$$

This all everything that I mentioned in the previous slide is the best illustrated with the example of a non premixed flame formed by a gaseous fuel issuing out of a tube. This is a similar arrangement to the premixed flame that we saw earlier except that, where we had stiochiometric close to stiochiometric mixture of methane and air coming out of the tube. Here what we have is pure ethylene coming out of the tube, ok. There is only fuel coming out of the tube, it burns by taking oxidizer from the surroundings.

So, in the photograph that is shown below, ethylene is issuing out of a 5 millimeter diameter tube, ok. And forming a non-premixed or in this case a diffusion a pure diffusion flame. One for the first question that we could ask is; what determines the height of this type of flame, or this can be the functional the principle dependents of this can be extracted from a simple analysis.

So, starting with the assumption, that the rate controlling mechanism is mixing of fuel and oxidizer, which in this case is the quiescent air in the ambient. The time taken by the fuel to go from the outlet of the tube to the tip of the flame should be equal to the time it would take oxygen and the air to diffuse to the centre.

Let me explain what I mean by that, see the fuel comes out the tube here, and almost all the fuel is consumed at this point right, all the fuel is consumed at this point; that means, the oxygen that is required, stiochiometric amount of oxygen that is required to burn off all this fuel, must have entered a volume that is enclosing this flame within this height. Remember that diffusion flame, it is always stoichiometric; where ever reaction happens, it is always stiochiometric. So, for all the fuel to be consumed, yes.

Student: But will it be visible at some of the stiochiometric.

Yes.

Student: But coming to the real flame where it recreates active happening.

Yeah.

Student: That flame may be, may not be exactly what we are in discussing like during the fire, the length may be even smaller.

The length may be even smaller is the comment that you are making.

Student: May be within the visual spectrum that we are seeing.

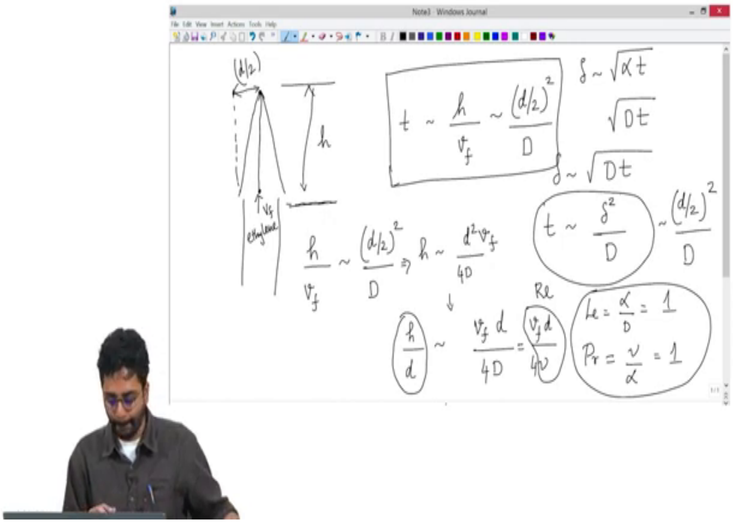
That is.

Student: You go for commercial diagnosis o h not refund all; then the length may be the shorter where we exactly match the map the flame.

It is possible that the flame length in this case is probably somewhere here ok; there is another phenomena that is happening which I have not mentioned, there is also some smoke that is out going out soot that is going out. But for the purpose of the discussion, we can assume that the bright yellow cone where it terminates is where the flame stops, ok. What you see downstream of that is probably some hot carbon particles that are leaving and emitting some radiation, ok.

Even with that the all the oxidizer that is required for consuming the fuel must enter the volume surrounding the flame within that height, ok. So, the time that is available for the fuel to go from the tip, from the inlet to the tip is the height of the flame divided by the velocity of the fuel. And within that time, oxidizer should go from the periphery to the centre of the flame by diffusion. Remember with the diffusion time scale and length scale are related by delta goes square root d t r or alpha t r which we saw earlier.

(Refer Slide Time: 21:31)



$$\boxed{t \sim \frac{h}{v_f} \sim \frac{(d/2)^2}{D}}$$

$$\delta \sim \sqrt{\alpha t}$$

$$\sqrt{Dt}$$

$$\delta \sim \sqrt{Dt}$$

$$t \sim \frac{\delta^2}{D} \sim \frac{(d/2)^2}{D}$$

$$\frac{h}{v_f} \sim \frac{(d/2)^2}{D} \Rightarrow h \sim \frac{d^2 v_f}{4D}$$

$$\frac{h}{d} \sim \frac{v_f d}{4D} = \frac{\overset{Re}{v_f d}}{4\nu}$$

$$Le = \frac{\alpha}{D} = 1$$

$$Pr = \frac{\nu}{\alpha} = 1$$

So, maybe I will just do the algebra here. What I am trying to say is that, you have a jet is showing ethylene; the fuel is coming out of the jet and forming a flame a non-premixed flame, which has a height of h. The point that I am making is the time, let us say the fuel is entering with a velocity of v f; the a good measure of the time it takes for the fuel to go from the inlet to the tip ok, the time it takes for the fuel to go from the inlet to the tip is the height divided by the velocity. This is a good measure of the time.

Within this time, the oxidizer should go from here to here; the oxidizer must diffuse from here to here, which is a distance of d by 2, ok. Unless this happens, the flame will not terminate here; there will be extra fuel left which has to get more oxidizer. Or the if this had a happened little earlier; there will be more oxygen than what is required for burning off all the fuel. So, the flame would have terminated a little earlier, ok. So, this is the limiting condition.

Remember that the diffusion length and time scale are connected by this kind of an expression for thermal diffusion.

For mass diffusion, the thermal diffusion coefficient is replaced by the mass diffusion co efficient. So, delta goes as mass diffusivity multiplied by time; if we assume that lowest number is 1 alpha is equal to D, let us not worry about that. So, delta goes as square root D t; therefore, time goes as delta squared, which is a diffusion length scale divided by the diffusion co efficient. Here the diffusion length scale is the distance that the oxidizer must diffuse is d by 2; therefore, this time will go as d by 2 squared over the diffusion coefficient, ok.

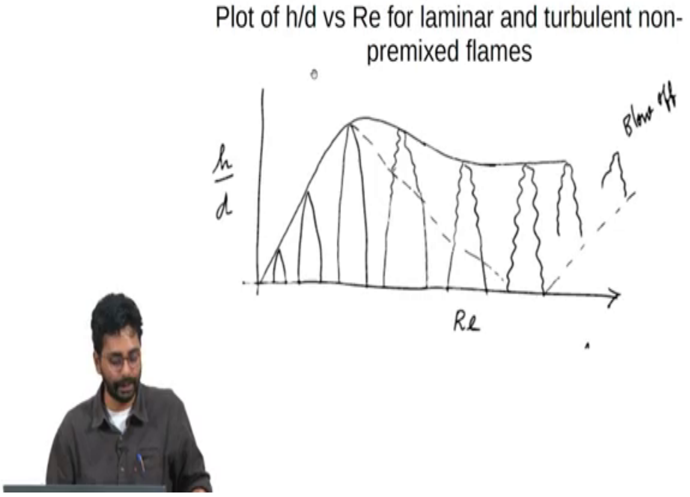
So, this time; the time it takes for the fuel to go from the inlet to the tip should be matched by the time it takes for the oxidizer to go from the edge of the burner to the centre. So, this is what controls the height of the flame, ok. So, h by the velocity of the fuel goes as d by 2 squared by D ok; you can now rearrange this, this implies that h goes as d squared by 4 times D multiplied by the velocity of the fuel, ok.

From here can show that, h divided by d; the length or the height of the flame measured in terms of the diameter of the burner goes as v f multiplied by d which is a diameter divided by 4 times the diffusion co efficient. Let us assume that lowest number which is alpha by D is 1; let us also assume that Prandtl number which is mu by alpha is approximately 1, therefore, alpha is equal to D is equal to mu.

So, this is nothing, but v f d (Refer Ti me: 25:00) by 4 times mu and you can recognize that this is nothing, but the Reynolds number; the Reynolds number of the jet that is issuing from the inlet, ok. So, the non dimensional height or the height of the flame measured in as multiples of diameter of the burner is proportional to the Reynolds number of the jet that is issuing out of it. Of course, the these assumptions are not restrictive, you can assume that Prandtl number is not 1, you will get an extra term, ok. Here in terms of Prandtl number or Schmidt number, ok.

This is what is summarized here in the last line. So, the time for the fuel to go is h by v f this is the diffusion time, which implies that the non dimensional height goes as Reynolds number or Reynolds number divided by 4, I am leaving out the factor of 4 there, ok.

(Refer Slide Time: 26:00)



Plot of h/d vs Re for laminar and turbulent non-premixed flames

This is also, this is not, this is a case where simple analysis of this type reveals some interesting dynamics. In fact, the experimental results show indeed that, when you plot h by d as a function of Reynolds number the following thing happens; as long as you are in the laminar regime, as expected from the scaling analysis the h by d increases linearly with Reynolds number till some point, ok. And the Reynolds number crosses a certain threshold, you see effects of turbulence, ok.

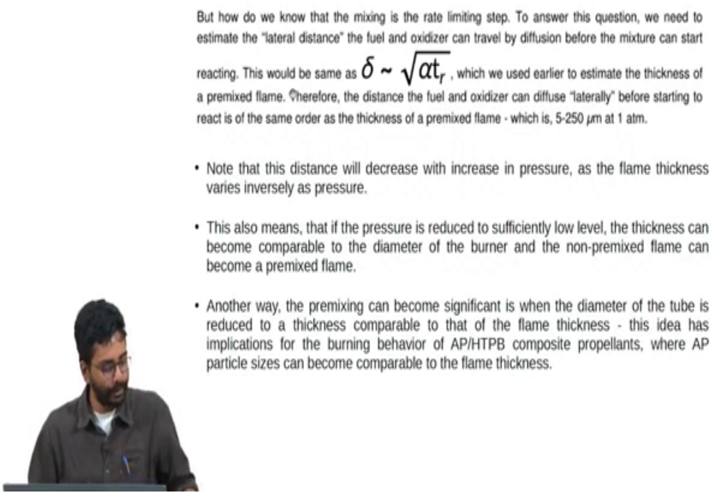
So, what happens is the base of the flame remains laminar, the tip starts becoming turbulent; and as you increase the Reynolds number, the height remains more or less the same in fact, it comes down a little bit and then remains constant. But what changes as you increase Reynolds number beyond this point is; more and more of the flame will become turbulent, ok. Here look at this case, here all the flame is laminar; when you went here, the tip became turbulent and rest of the flame was laminar.

As you increase the Reynolds number, the fraction of the height of the flame that will be turbulent will increase, ok. In fact, this is a simple demonstration of the fact that turbulence increases the consumption rate of the fuel, ok. The height remains constant; as you increase Reynolds number, you are increasing the amount of fuel that is going to the flame, ok. And therefore, within the same volume; look at this case and this case, Reynolds number on the right is larger than the Reynolds number on the left.

You are issuing more fuel and the extra fuel less consumed within the same reaction volume and therefore, the reaction rate or the consumption rate of the fuel is enhanced by turbulence ok, not by reaction rates, simply because of enhanced mixing because of turbulence, ok. So, at some point you would see that the entire flame would have become turbulent; if you increase the Reynolds number beyond that point, the flame lifts off and if you increase it further it blows off.

Because once it lifts off ok; it is lifting off because it is not able to anchor to the burner, because locally the transport rate of the fuel, the rate of transport of fuel to the flame is not matched by the reaction rate. And if you increase it any further, it blows off; because you are issuing fuel at the rate that is more than what the flame can consume, it brings down the temperature and blows off the flame, ok. So, of course in the laminar regime, this linear relationship is accurate, ok.

(Refer Slide Time: 28:36)



But how do we know that the mixing is the rate limiting step. To answer this question, we need to estimate the "lateral distance" the fuel and oxidizer can travel by diffusion before the mixture can start reacting. This would be same as $\delta \sim \sqrt{\alpha t_r}$, which we used earlier to estimate the thickness of a premixed flame. Therefore, the distance the fuel and oxidizer can diffuse "laterally" before starting to react is of the same order as the thickness of a premixed flame - which is, 5-250 μm at 1 atm.

- Note that this distance will decrease with increase in pressure, as the flame thickness varies inversely as pressure.
- This also means, that if the pressure is reduced to sufficiently low level, the thickness can become comparable to the diameter of the burner and the non-premixed flame can become a premixed flame.
- Another way, the premixing can become significant is when the diameter of the tube is reduced to a thickness comparable to that of the flame thickness - this idea has implications for the burning behavior of AP/HTPB composite propellants, where AP particle sizes can become comparable to the flame thickness.

So, that is an example of diffusion of a non premixed flame which was a ideal diffusion flame, where the height of the flame was simply controlled only by the diffusion phenomena and the reaction rates are much faster, ok. The question I am posing is how do we know that mixing is rate limiting step; of course, for that case we know, because we assume that mixing is a rate limiting step, we derived a conclusion that h by d should linearly vary with Reynolds number.

If that is the case and the experimental results indeed show that h by d goes as Reynolds number in the laminar regime. So, there in that case we know that mixing is rate limiting step. But we can pose a general question; can we come up with a way of finding out, for a given condition is the flame going to be controlled only by mixing or will other chemical reaction effects come into picture. So, I am posing the general question here.

And to answer this question we go back to the same idea that we have been using so far; we need to estimate the lateral distance, the fuel and oxidizer can travel by diffusion before the mixture can start reacting, ok. What I am trying to convey here is that, you are issuing fuel from here; the oxidizer is diffusing from outside. And let us say you just send fuel and do not ignite; what will happen is that, the fuel and oxidizer will mix will diffuse into each other and mix, and when you ignite, a diffusion flame is formed.

The question that I am posing is that, is the reaction rate always much much higher than the diffusion rate, ok. The way to determine that, is by estimating how much the fuel can diffuse out, or the oxidizer can diffuse in; you have a jet that is coming out. The fuel will diffuse out, because the concentration of the fuel is high at the centre and low at the periphery; the oxidizer diffuse in, because the concentration of oxidizer is high at the outside and low on the inside, ok.

So, the question that we are posing is how much distance the fuel can move out and the oxidizer can move in before a flame gets established; because once a flame gets established and reaction rate in the flame is significantly higher than the mixing rate, all the fuel that comes to the flame will be consumed. All the I am sorry all the fuel that comes to the flame will be consumed, and all the oxidizer that comes to the flame will be; because diffusion flames stabilize, or diffusion flames establish itself at stoichiometric value, ok.

So, the question is how much lateral distance the fuel and oxidizer can travel by diffusion before the mixture can start reacting. This would be same as square root d t r or square root alpha t r ok; that is the distance that the fuel and oxidizer can move laterally before they start burning ok, because it takes t r seconds for them to react ok. And once reaction starts, there is no further movement of the fuel; because the fuel is all consumed, the oxidizer is all consumed. So, the time that is available for it to move laterally and mix is only the time it takes for the reaction to happen ok, this is the same estimate.

Student: Sir, where one pi is a thermal diffusion coefficient.

Yes of course, I am making a simplification that lowest number is 1, so all the transport coefficients are.

Student: Sir we have been talking about the diffusion of course, specious in different directions.

It should be, technically it should be d t r; of course, I am assuming for simplicity that lowest number is 1, therefore, all the transport coefficient mass, heat and momentum.

Student: Thermal through diffusion code.

They are equal. So, this is the same relation which we used earlier to estimate the thickness of the premixed flame; therefore, the distance of fuel and oxidizer can diffuse laterally before starting to react, this of the same order as the thickness of the premixed flame, ok. Because t r again will be will be calculated from the reaction rate of the premixed flame and therefore, this estimate will be same as thickness that we calculated at calculated earlier when doing thermodynamic equilibrium calculations, which we saw is between 5 and 250 microns at 1 atmosphere, ok.

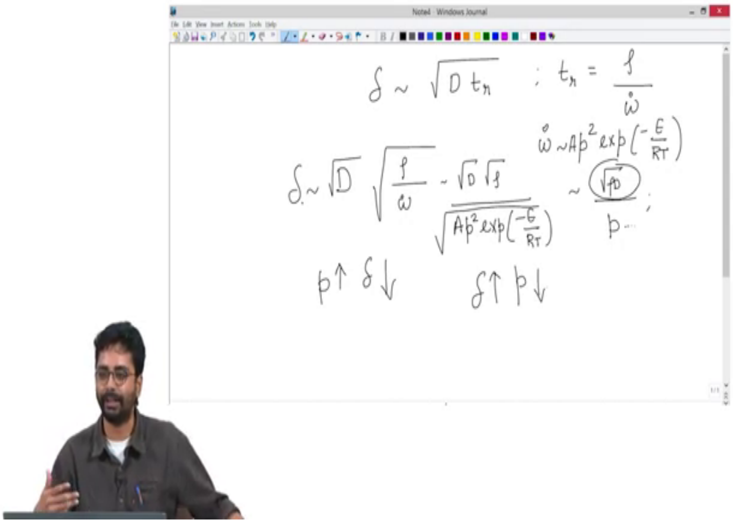
So, the point is that, in this case also there is a certain amount of there is a certain thickness over which the fuel and the oxidizer are in mixed state; only that distance is somewhere between 5 and 250 micron or 0.05 to 0.25 micron 0.25 millimeters. And that distance compared to the 5 millimeters of the burner is much smaller, ok. And that is the reason it is correct to assume that the reaction zone thickness is very small; the thin flame approximation is valid, because the distance over which the fuel and oxidizer can coexist is as small as 5 microns or a maximum of 250 microns and that compared to 5 millimeters is very small.

But remember that this distance will decrease with increase in pressure, because the reaction rates will increase with increase in pressure and therefore, the time for reaction will decrease with increase in pressure. And therefore, the delta will decrease with pressure and also because as the flame thickness is inversely goes as pressure or the thickness of the flame will come down with pressure.

This also means that, instead of increasing the pressure, I can decrease the pressure. So, if we start with the diffusion flame and if the pressure is reduced to sufficiently low level ok; the thickness, the diffusion layer thickness can become comparable to the diameter of the burner, ok. So, the point that I am making is you can start with the same 5 mm ethylene, 5 mm tube issuing ethylene; instead of 1 atmosphere you keep decreasing the pressure.

You can if you go to sufficiently low pressure, the same diffusion flame can actually become a premixed flame, ok. Simply because the mixing times are approximately the same as you change the pressure; but you are reducing reaction rate and reducing reaction time by reducing pressure, ok. So, I will summarize that in this point; this also means that, if the pressure is reduced to sufficiently low level, the thickness can become comparable to the diameter of the burner and the non premixed flame can become a premixed flame, ok.

(Refer Slide Time: 35:21)



$$\delta \sim \sqrt{D t_r} \; ; \; t_r = \frac{\rho}{\dot{\omega}}$$

$$\dot{\omega} \sim A p^2 \exp\left(-\frac{E}{RT}\right)$$

$$\delta \sim \sqrt{D} \sqrt{\frac{\rho}{\omega}} \sim \frac{\sqrt{D}\sqrt{\rho}}{\sqrt{A p^2 \exp\left(\frac{-E}{RT}\right)}} \sim \frac{\boxed{\sqrt{\rho D}}}{p \cdots} ;$$

$$p \uparrow \; \delta \downarrow \qquad \delta \uparrow \; p \downarrow$$

We are just so, we do not delta goes as square root D t r; remember that t r we saw in the earlier lecture is rho divided by omega dot ok. And omega dot goes as pressure squared exponential minus E by R T multiplied by a pre exponential factor ok. So, that delta will go as D; instead of t r, I will substitute A P squared, I am sorry t r is square root D square root of rho divided omega dot ok, this goes as square root D square root rho divided by square root of reaction rate which is A psquared exponential minus E by R T square, ok. So, D goes as square root rho D by p, ok.

With some factors which I have not written there; rho D is insensitive to pressure, ok. So, the thickness as you decrease pressure; as you increase pressure thickness will decrease; pressure increases, delta will decrease. This we already know, from the premixed flame analysis the thickness of the premixed flame will decrease with increase in pressure; but it also means that, the delta will increase if the pressure decrease.

So, if I decrease pressure to a level where delta becomes let us say 3 mm or 2 and a half, 3 mm; that means, the fuel in a 5 mm tube, if delta comes to 2 and a half mm, the fuel and oxidizer can move laterally by 2 and a half mm which covers the entire diameter of the tube. So, the entire mixture will become premixed before the reaction begins and that means, you will have a premixed flame. In fact, this would be a very nice experiment to do and take pictures of.

Student: Sir, the in case of the temperature comes from exponential terms wherein.

Yeah.

Student: As far the thickness is concerned.

Yeah.

Student: And in the amount we keep the temperature to a very high level.

Correct.

Student: Also we have the same rate.

Yes, if you increase the ambient temperature, the reaction rate will increase. So, the thickness will decrease. So, you have to reduce the temperature to have the effect; but it is not as significant as a pressure effect, because if you decrease the temperature the whole lot, it may not even ignite.

Student: So, for the flame is spread pressure buried sir; I assume that the boundary conditions for governing pressure completes.

Ok.

Student: And having the Doppler travelling in more faster, and the same conditions like the diffusion coefficient and the thermal diffusion rates are almost like the same number is valid for you know for Doppler diffusion.

You need to look at the vaporization time also. So, here there are only two timescales; one is a time for mixing and the time for the reaction. When you have a droplet it is it will be likely that the evaporation is much slower than any of the other phenomena; it may still remain rate controlling. Mixing and chemical reactions will be much faster than the vaporization process.

So, this is one way that diffusion, a purely diffusion flame can become premixed; there is also another way which the same equation suggests, I fix the diameter reduce the pressure to increase the diffusion length, but I can also do the other thing. I can cause premixing, an other way to cause premixing is; when the diameter of the tube is reduced to a thickness comparable to that of the flame thickness, ok.

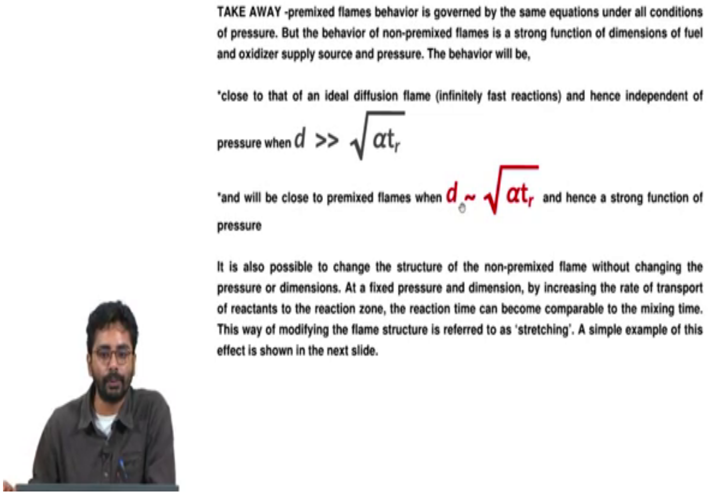
Remember that the delta at 1 atmosphere is can be anywhere between 5 microns and 250 microns. Now, if I will keep reducing the diameter of the tube where the size of the tube becomes 250 microns; then there will be significant premixing. You may think that this is unrealistic; but we saw an example of AP HTPB combustion, where there is an AP particle which is issuing a jet of oxidizer and the size of the AP particle can be 200 microns.

In fact, the typical AP sizes that are used in composite solid propellant making, can vary from as small as 1 micron to as large as about 500 micron. So, in a given propellant, you may have zones where you have AP particles as small as a few microns, ok. And we may be tempted to assume that, because it is giving out oxidizer surrounded by fuel that the dynamics is controlled by non premixed diffusion flames; but it may not be the case.

Because the diffusion distances are comparable to the size of the AP particle that is issuing the oxidizer, significant premixing can happen because of lateral diffusion before the flame ignites. So, the effects of premixing cannot be ignored. In fact, the classical thinking of multiple flames that form between AP surrounded by binder is that you have an AP mono propellant flame surrounded by what is called a primary diffusion flame.

But it may not be the correct description, because is the possibility of significant premixing exists; the right terminology is probably to only call it as a primary flame and not a primary diffusion flame. It can be a primary diffusion flame under some circumstances or some conditions of particle size in pressure; at some other conditions of particle size in pressure, it could be a primary premixed flame also.

(Refer Slide Time: 41:09)



TAKE AWAY -premixed flames behavior is governed by the same equations under all conditions of pressure. But the behavior of non-premixed flames is a strong function of dimensions of fuel and oxidizer supply source and pressure. The behavior will be,

*close to that of an ideal diffusion flame (infinitely fast reactions) and hence independent of pressure when $d >> \sqrt{\alpha t_r}$

*and will be close to premixed flames when $d \sim \sqrt{\alpha t_r}$ and hence a strong function of pressure

It is also possible to change the structure of the non-premixed flame without changing the pressure or dimensions. At a fixed pressure and dimension, by increasing the rate of transport of reactants to the reaction zone, the reaction time can become comparable to the mixing time. This way of modifying the flame structure is referred to as 'stretching'. A simple example of this effect is shown in the next slide.

So, we take away is that premixed flame behavior is governed by same conditions under all conditions of pressure; reaction, rate, controls everything is already thoroughly mixed. So, there is no question of mixing time, everything is controlled by reaction time. It is not the case for non premixed flames; because there are varieties of phenomena at play. And the two important phenomena are; one is the time for mixing and the other is the time for reaction. And in cases where there are droplets, the time for evaporation is also significant factor or will be the rate limiting step, ok.

So, the behavior of non premixed flames is a strong function of dimensions of fuel and oxidizer supply source. When I say that, I would like you to imagine a 10 micron diameter AP particle issuing oxidizer, ok. These are you know as real as it gets; it is not 10 micron fuel sources are not there, it is there in sitting inside in compulsive solid propellants.

Therefore, the behavior will be close to that of an ideal diffusion flame; what I mean by an ideal, I am referring to the ethylene flame the picture of which I showed you, where the assumption of infinitely fast reactions and hence independent the dynamics being independent of pressure is valid, because the dimension of the fuel issuing tube is much larger than the lateral diffusion distance, ok.

But the dynamics or the behavior will become close to a premixed flame, when the size of the fuel issuing tube becomes comparable to be lateral diffusion distance and hence a strong function of pressure. Remember that now the extent of diffusion is dependent on square root alpha t r which goes as 1 by pressure and therefore, a non-premixed situation, the behavior burning behavior becomes sensitive to pressure, ok.

So, it is also ok, it is also possible to change the structure of the non premixed flame without changing the pressure or dimensions. I let me just quickly, no I think this is going to take sometime; I think this is the right time to take a break. So, I will stop with this these two points, I think these two points are the important take away; d much much greater than square root alpha t r, dynamics are is controlled by mixing and not by chemical reactions. But when d becomes comparable to square root alpha t r, dynamics become controlled by reaction rates and hence a strong function of pressure.