Engineering Mechanics
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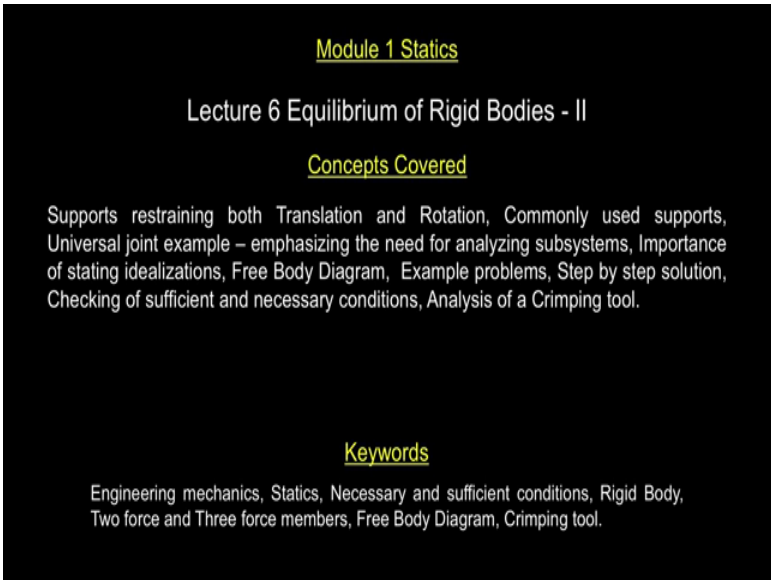
Module - 01
Statics
Lecture - 06
Equilibrium of Rigid Bodies - II



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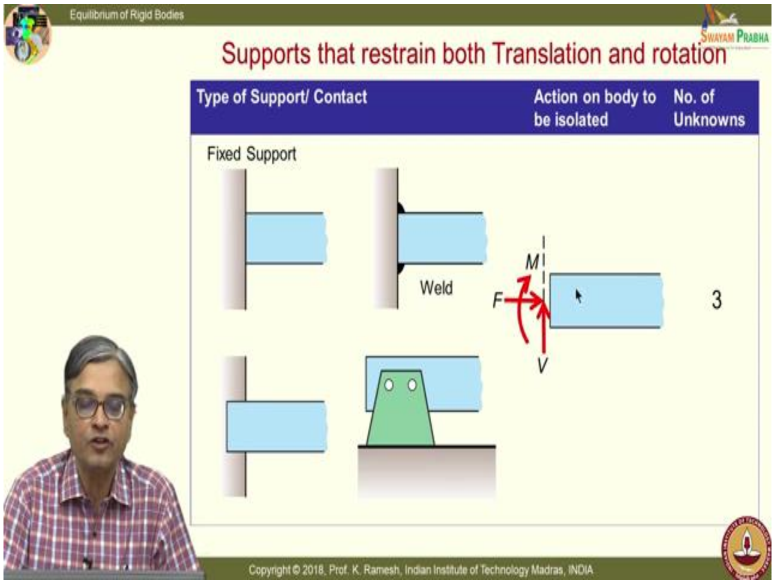
Let us continue our discussion on Equilibrium of Rigid Bodies.



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In the last class we had looked at supports that restrain both translation and rotation. We saw the fixed support in various forms. You could weld it or you could put it inside the wall and then do it. In the simplest case you can even put two pins and then make it as a fixed joint. And when I isolate the body I would in general have a force whose direction is not known hence, I have put a horizontal component and also a vertical component.

And I am also using this to introduce various terminologies when I have a force tangential to the surface you call that as a shear force. And we usually use the symbol *V*. I could also put this as R_x and R_y; it does not make any difference; my interest is to bring in various terminologies that we normally use in this course. So, I have an unknown force and an unknown moment. So, I have three unknowns if I idealize the support to be a fixed support.

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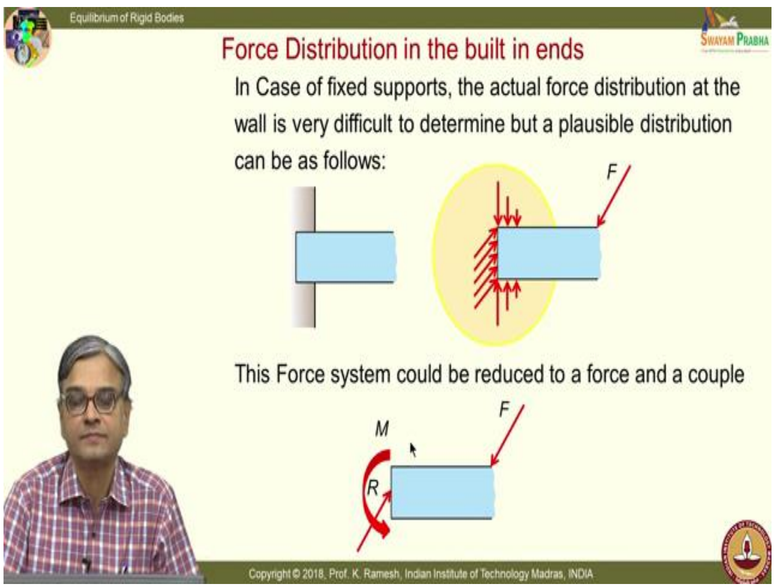

And one of the questions that come in anybody's mind is what happens inside the support. So, one possible way is I could have some such distribution under the action of an external force. I would have a distribution something like this which when I take the resultant, I could simplify it as a force in an arbitrary direction which balances the external force and you have a resisting moment.

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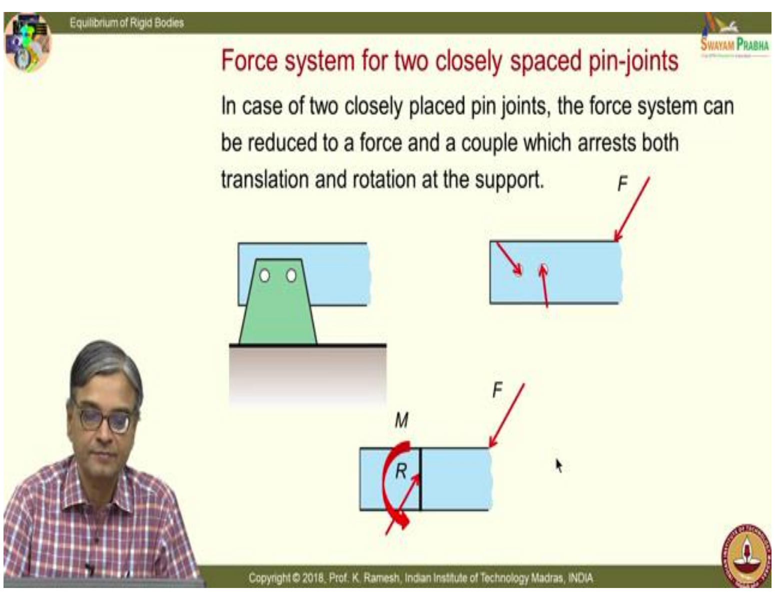
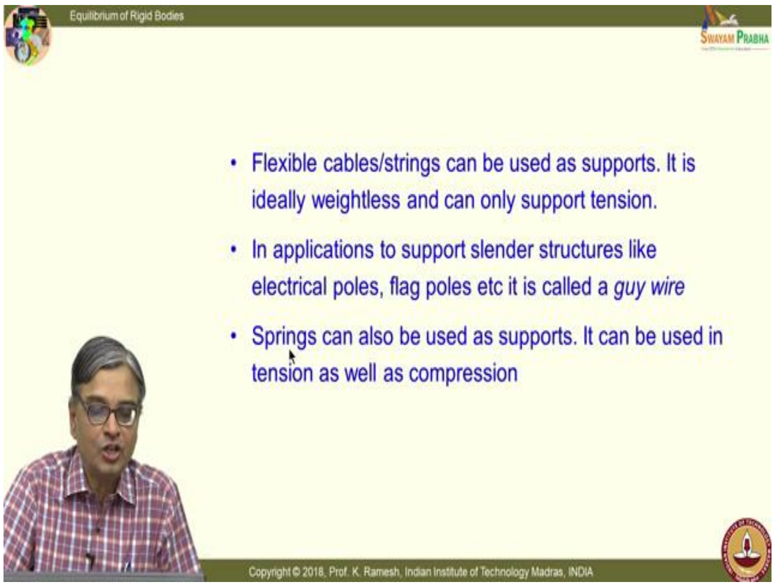


And we have seen physically in the previous class when I put 2 pins it also behaves like a fixed support. I have these two pins and a pin in general will transmit a force and the force interaction could be something like this. And the resultant would again balance the external force. See it is very difficult to build a truly fixed end. When a problem is posed to you take it as a fixed end you do not have to think.

But as an engineer if you have to go and idealize a physical system there is always a challenge to see whether it could be idealized as a true fixed support or a pin joint or something in between. Usually it is in between a pin joint and a fixed support. So, that is where I said you analyze the problem once with a pin joint you get one solution. You analyze the problem again with a fixed support you will have another solution. They form lower and upper bound solution. So, there is always a challenge when you want to idealize a physical system.



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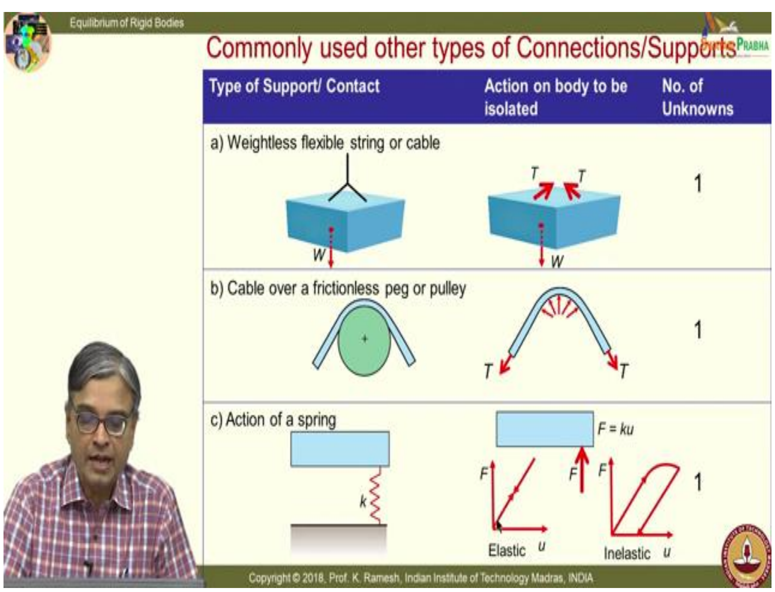

Next we move on to other types of supports we have already seen flexible cables. I told you long time by that it can use as a guy wire. We saw in the previous class it is also used as a railing in the case of a bridge to reduce the weight. Even though we talk about rigid body mechanics here, we would like to bring in springs because many appliances use springs for their functionality; right from your cycle carrier you have a spring used there.

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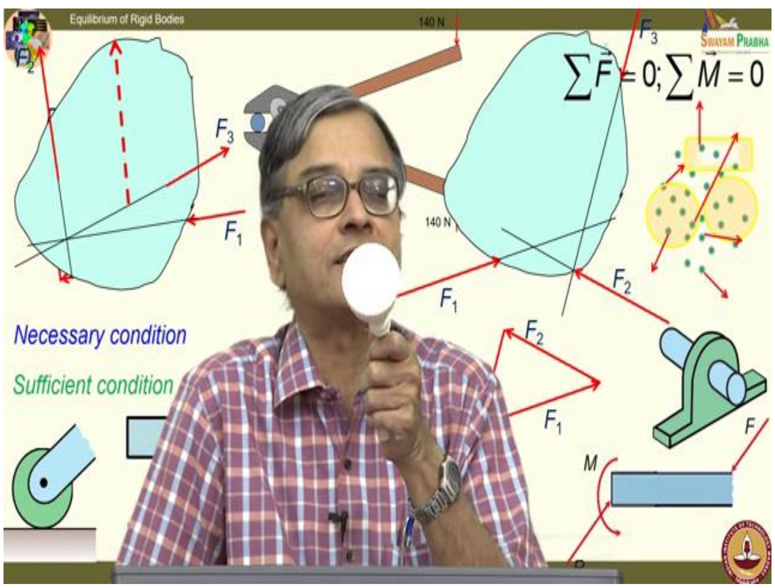
And if you take a simple jotter pen you have a spring there it is deformable; macroscopic ok. And you have seen cranes lifting weights when they lift weight when I have a cable it will support only a tensile force all along its axis. So, the direction is known only the magnitude is not known. So, I have only one unknown.

I also have a cable over a frictionless pulley. That is to be underlined. Later on, when we discuss friction if I have friction on the pulley how does it help us in creating many devices for useful applications. When I have this as frictionless the cable will support only an axial tension. I will have only one unknown. Then I have a body supported by a spring and spring is normally denoted by it is stiffness k and the spring could be a linear spring or a non-linear spring.

I have an elastic loading and elastic unloading in one case it could have an inelastic unloading in another case. And I have only one unknown in the direction shown here and F is given as $F=ku$. So, these are other class of supports which you normally come across. The whole focus here is to solve a variety of day to day problems not all problems that you come across. You can solve a variety of them no doubt.



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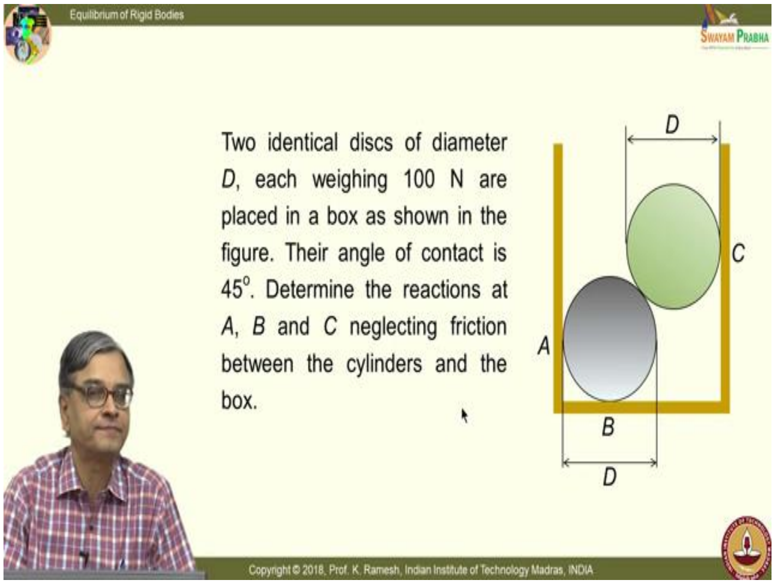
See so far, we have seen two dimensional supports now let us see a three-dimensional support here. Now this is a simple shower cap what is there in your bathroom. I can turn it like this, it allows me rotation in this axis, it allows me rotation in this axis. And it also allows me to rotate along the axis.

So, I have rotation permitted in on all three directions. Do you have that kind of a joint in your physical body? We have a joint many of you are able to recognize that you have a nice shoulder god has given, you have a ball and socket and all the cricketers use it so well to bowl and then get the wickets. And how do I idealize my elbow; how do I idealize my elbow? Pin joint. So, now you have the hang of how do we model a physical system.



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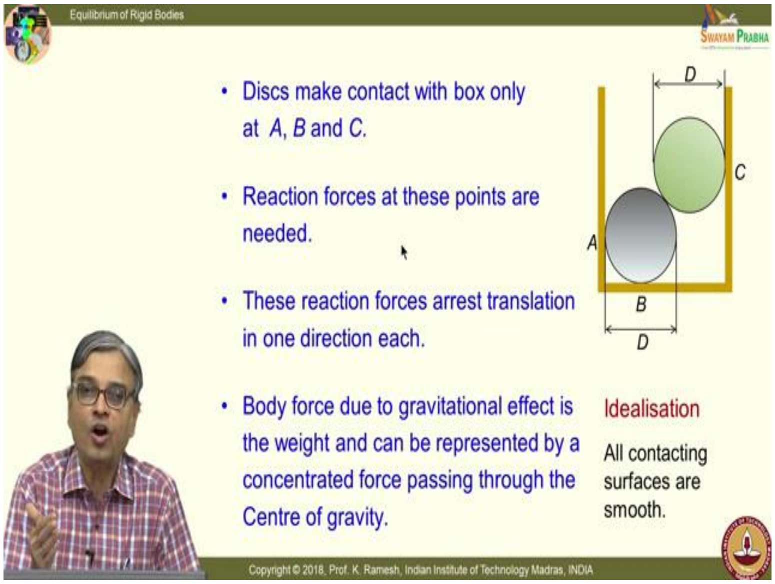
Now, let us move on to solving simple problems just to understand the procedure and the concept. So, I have two identical discs of diameter *D*, each weighing 100 Newtons are placed in a box as shown in the figure. Their angle of contact is 45°, you have to determine the reactions at *A*, *B* and *C*. Neglecting friction between the cylinders and the box just to make your life simple we have removed friction.



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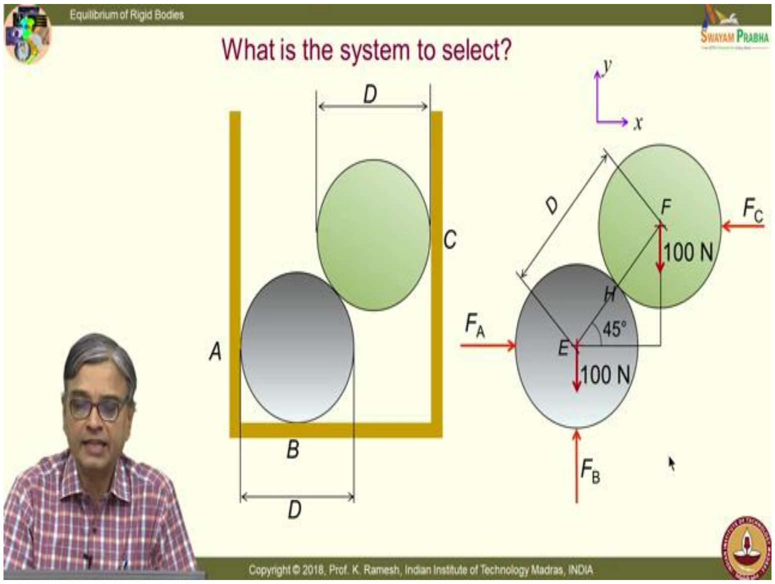
My focus here is to get a good habit on how to draw free body diagrams and what is the kind of solution procedure I would like you to have. First of all, we have to recognize that discs make contact with box only at *A*, *B* and *C* that is very clear from the problem statement. And I would like you to say very clearly the first step is; what are the idealizations that we have used to solve this problem. Some may be given in the problem statement itself, some you might want to idealize to solve the problem.

So, I definitely want a statement when you solve a problem what are the idealizations that you have used it is a must. In this case it is given in the problem statement all contacting surfaces are smooth; that means, there is no friction. And we will have to get the reaction forces, that is the problem statement and it is very clear from the geometry I have a disc in contact with the flat surface or a vertical surface.



We have already seen the idealized supports; you can borrow knowledge from that or you can also look afresh what is the kind of interaction and you can recognize that it arrests translation in one direction. The problem statement also gives what are the weights of this disc that act like a body force and where does the body force act; it acts through the centroid of the discs.

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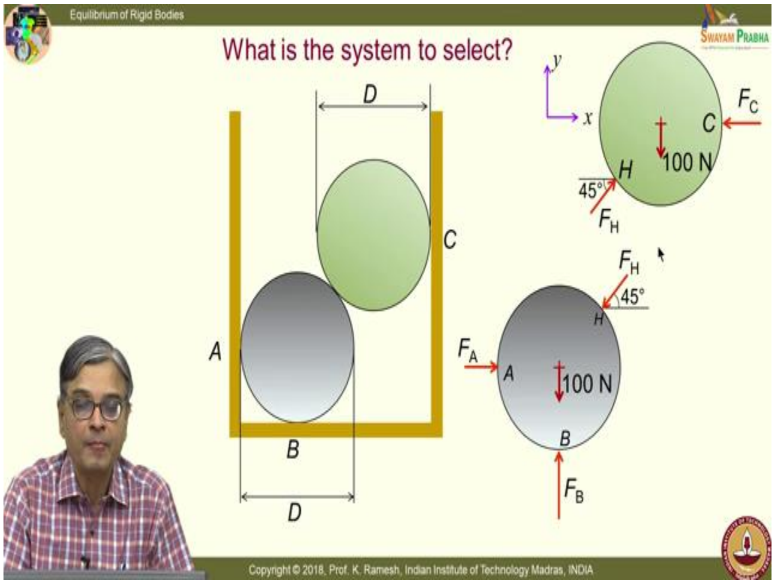
Now, the next question is what is a system to select? I could have two discs as a system, I could also take each disc separately and solve the problem. The next step what I want you to do is I would like you to put the coordinate axis. Then I am putting the body force which is acting on the centroid of these discs.

Then we will have to put the unknown forces at points *A*, *B* and *C*. If I do not know the direction, I can start with any direction. My mathematics will help me to correct the direction later. But here since I have the physics of the problem very clear I could put the force at *A* as acting in this direction. I could put force F_B acting in this direction and I could put force F_C acting in this direction which is very clear from the problem itself.

So, the next step is applying the conditions $\sum \vec{F} = 0; \sum \vec{M} = 0$ and determine the 3 unknowns. I have 3 equations and I can find out 3 unknowns. Let us solve the mathematics. The problem statement also gives that they are oriented at angle 45° that is also labeled. And the distance between *E* and *F* is also given from the geometry simply as the distance *D*. It is fairly, a simple problem my interest is to specify what all things I would like in a problem solving. You have to have idealizations, you have to clearly mark the coordinate axis, you have to correctly label these forces then put the equations.



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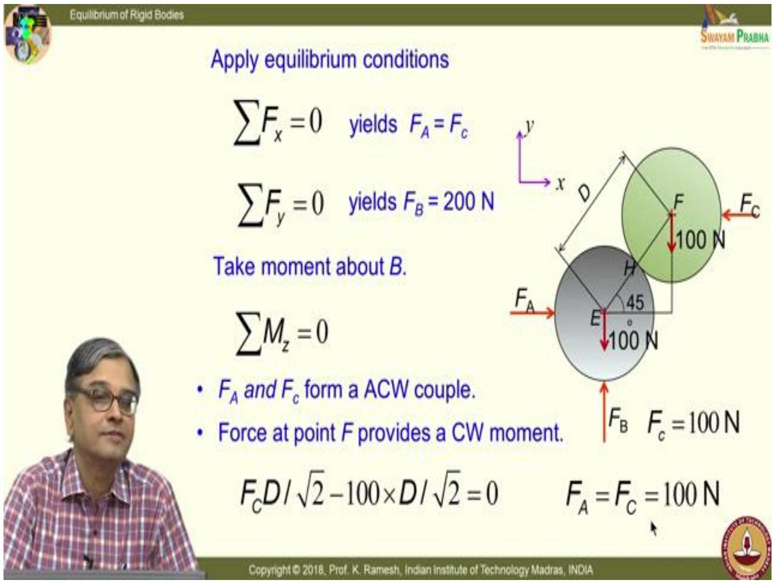
So, the other way is I could also solve by taking individual discs. When I take the individual disc, I will have to put the interaction of the other disc acting on this. Now what I have done is from the physics of the problem this disc is going to push this out. So, I have put the direction in one way, these two discs are in contact. So, when I draw the free body diagram for the other disc, I have no choice; at least for the force interaction acting at point *H*. I will have to use Newton's third law and put equal and opposite, do not start thinking afresh and then put another direction then you mess it up. I will have to put the interaction at this point only equal and opposite.

If I choose it differently modify this differently. In simple problems you can directly put the interaction there is no difficulty. Only in complex problems where you are unable to visualize;



you can still take one direction where mathematics will finally, help you to do that provided you took care of this interconnected bodies carefully. You should not make a mistake there.

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Now, I would solve the problem by taking this as a system and we have already learnt certain concepts earlier. We will exploit all those concepts in problem solving. So, when I put

$$\sum F_x = 0 \quad \text{yields } F_A = F_c$$

$$\sum F_y = 0 \quad \text{yields } F_B = 200 \text{ N}$$

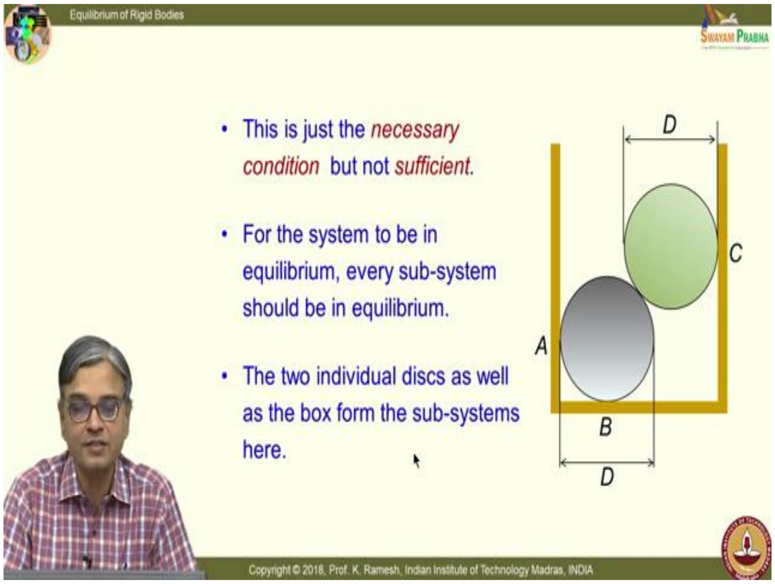
And they are parallel then I can easily say when I want to do further simplification, I can bring in my concept of couple force system and make my life simple I can solve the problem quickly. I do not have to find out what is that moment created by this force separately and this force separately. I have used sigma $F_X = 0$, then I use sigma $F_Y = 0$. I can easily find out what is F_B that turns out to be 200 N. Then I will have to calculate these forces I take moment about B.

So, that I do not have to worry about these two forces they do not cause a moment. I will have a moment caused by this force this will be a clockwise moment and these two forces form a couple force system that forms an anticlockwise moment. So, I write my moment equation in a very simple form

$$\sum M_z = 0$$

$$F_C D/\sqrt{2} - 100 \times D/\sqrt{2} = 0$$

This gives me $F_A = F_C = 100 \text{ N}$



Have we solved the problem? We have not solved the problem yet. I have always been saying you should verify whether these conditions are sufficient; we have not verified whether these conditions are sufficient. How do I verify it?

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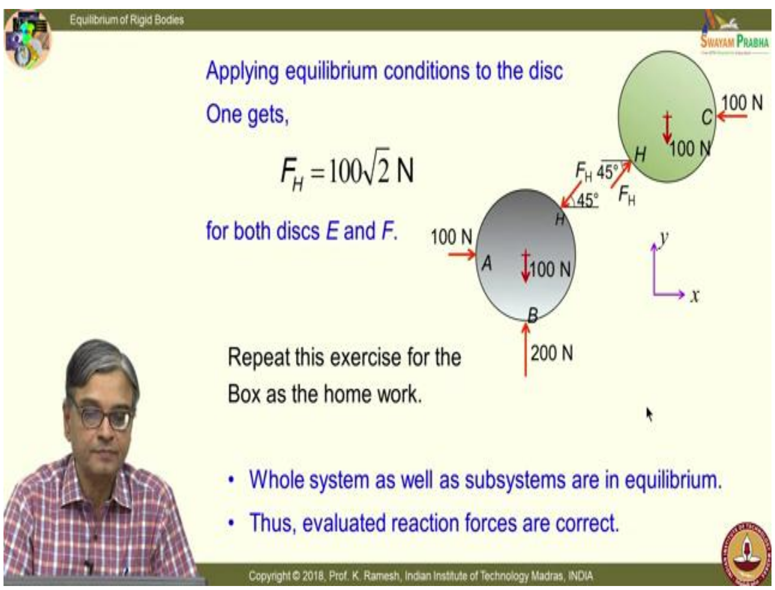

I have to verify it take a sub-system and then see whether these conditions are satisfied. I am not going to solve the problem. I will only be going to take a subsystem and take individual discs as well as the box from the sub-systems. I could take individual discs or box. I will do it for individual disc. I am only going to verify the solution.

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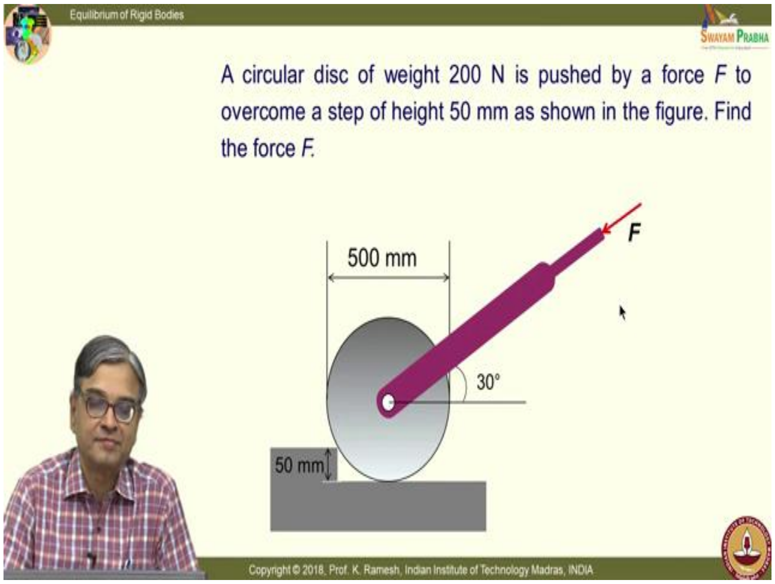
So, I am going to find out what is the value of F_H from one of this. It is very simple arithmetic. I get

$$F_H = 100\sqrt{2} \text{ N}$$

I get the same result by solving these two and I find that these two discs are in equilibrium. System as a whole is in equilibrium and sub-systems what we have selected are also in equilibrium and I would leave verifying the box as in equilibrium as a homework.



Only when you do all of this you can confidently say solution is completely determined; your idea is you should always look for necessary and sufficient conditions. In every case where I draw the free body diagram you need to have the axis drawn clearly.

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Now, let us move on to another problem which is fairly simple, but you have to think how do you solve it? The problem is like this I have a roller; I have a handle attached to it. When I am pushing this roller, I have come across a step to make our life simple these are given as nice values, you have this as step of 50 mm.

And the position of the handle at that time was about 30 degrees. You have a circular disc of weight 200 N. You are asked to find out what is the force required to climb over the step that is the problem. The key point here is how do you go and idealize the interactions? If I idealize my interaction and understand the problem statement clearly, I need just one equation to solve this problem.

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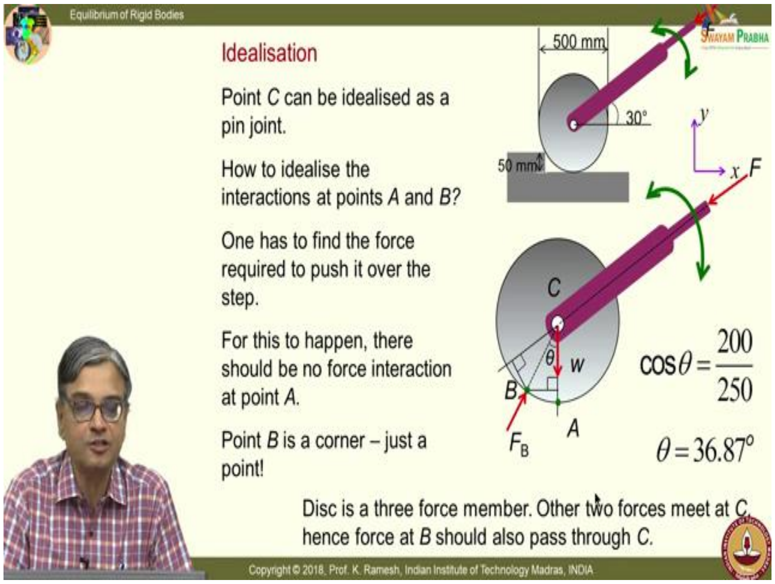
Let us get on to the free body diagram, I have a problem statement like this. The first step is I need to idealize and you know you have a handle; the handle can be rotated. It allows rotation in this plane. So how can I idealize the joint here? I can idealize the joint as a pin joint, but before I do that, I should also label my key points on the diagram so that it is easy for us to document whatever that we want to do.



The handle can be moved like this I have put an axis of reference. The disc will have interaction here it will have interaction at this corner. So, they are important points let me label them as point *A* and point *B*, so I label this as point *C*. From the problem statement and the understanding of the physics behind the handle and the wheel I could idealize this joint as a pin joint. Point *C* can be idealized as a pin joint.

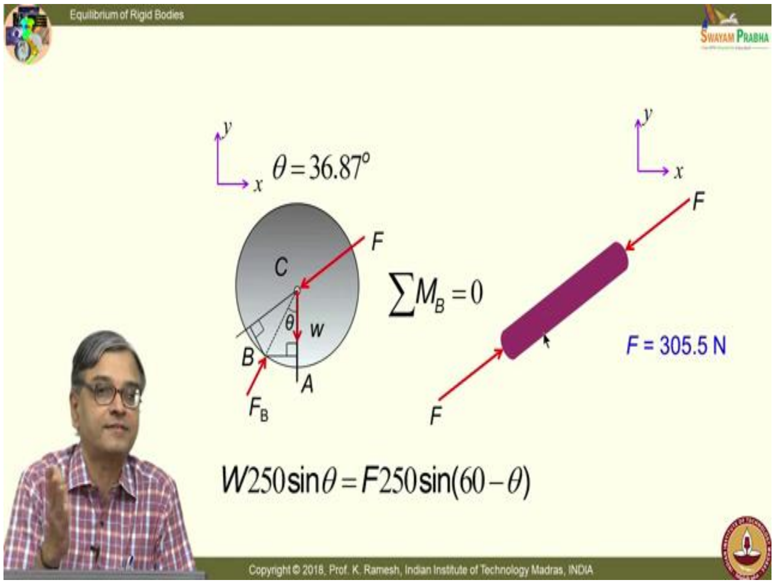
Then the next question is how to idealize interactions at points *A* and *B*? If you idealize them correctly you would find the answers for the problem statement. What the problem statement is? I need to find a force so that I roll over the step. If the wheel has to roll over the step what is the kind of condition you anticipate at that point in time? What would happen at point *A* and what would happen at point *B*? What is it that you should impose?

Conversation: Student: (Refer Time: 22:36).

No contact at *A*; that is very good then you got the physics of the problem that is very important. You should not blindly say that I have a roller and support replace it by a vertical force. You should look at what is the problem statement and then put in that problem statement in your problem; then only you are solving the problem that is asked for; is the idea clear? Because I have to push the roller so that it climbs over the step, if it has to climb at that point in time it has to lift from the floor. So, the first stage is there should be no contact at *A*; we are not looking at a situation where it is lifted when it is about to lift it has to lose contact at *A* and that is the key aspect in this problem.

There should be no force interaction at point *A*, point *B* is a corner it just a point fine. You can recognize solving this problem in many different ways. I have shown you the weight *W* acting for the centroid and the man is pushing this roller and this is the force. They meet at this point, the third force has to pass through this point because it is a three-force member this is one way of looking at it, I can also find out the direction of force at *B* from another perspective. I have this as a corner and this as a smooth surface that is another idealization that we can say that all surfaces are smooth which is not listed here. I think we should also list that.

So, I can put a tangent to the circle and it has to be perpendicular to that. So, this direction becomes the direction along the radius of this disc. So, you can find out the direction of force at *B* either by taking it as a three-force member or taking it as a smooth contact with a corner. Put the tangent at the interface and put a force perpendicular to that either way you can look at this. And I will have to find out what is the angle theta that comes from the simple geometry which you can easily figure it out and I can write this as I have put a perpendicular line. So, from the radius I can find out this value and this is the perpendicular distance that I have for me to find out the moment of this force.



So, from the geometry given I can write this

$$\cos\theta = \frac{200}{250}$$

because this height you know as 50 mm as the height of the step. So, now I know what is the angle theta; $\theta = 36.87^{o}$

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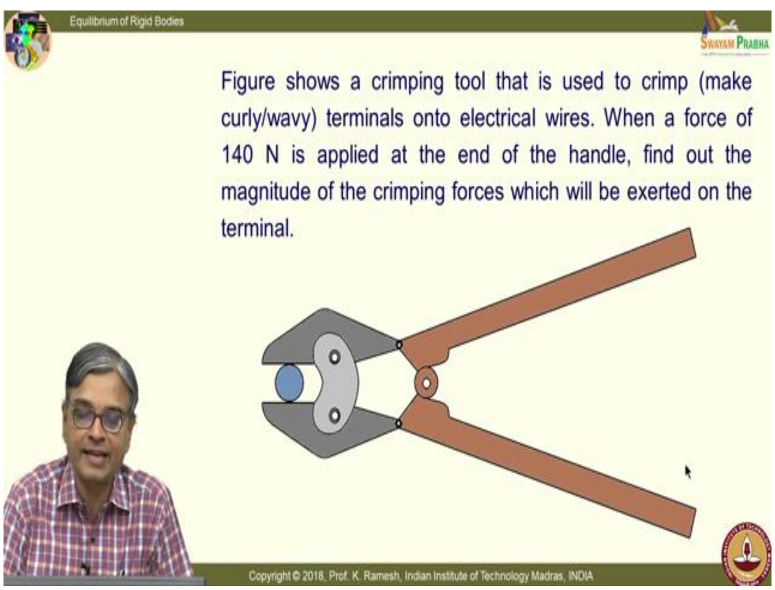
If I have the geometry of the problem, now I told you my focus is to find out F, my focus is not to find out what is the force happening at point *B*. My focus is only that and we have seen I have to use equations $\sum \vec{F} = 0; \sum \vec{M} = 0$, of the three equations which one would you choose to get the force at *F* so that I just made one calculation; is the question clear? One can use $F_X = 0$, $F_Y = 0$ and keep doing the problem.

But one can also look at the moment equation and choose an appropriate point about which you to take the moment, so you make your life simple. So, do not jump on solving a problem think about it for a minute. What are the ways that you can use the equations effectively; I have given enough clues can you tell me what is the point about which I had to take a moment?

Student: *B*.

B; very good, so I write only one equation $M_B = 0$; I get we have already looked at what is this distance which we can easily do. So,

$W 250 \sin\theta = F 250 \sin(60 - \theta)$, it gives me



$F = 305.5 \text{ N}$

Because we are given what is the value of *W* and this is a two-force member, I have a force acting on this like this. So, this member is in equilibrium, the problem is not completely solved you should also see sub-system.

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It is a good practice to understand the concepts. Next we move on to solving a practical problem.

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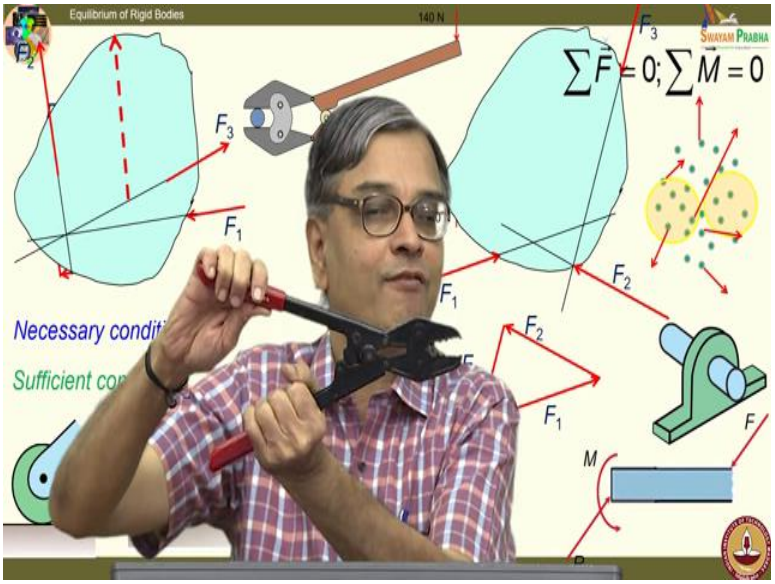

See we have seen this cutter in one of the classes when I said that you have interconnector rigid bodies is the crimping tool. Please do not put your finger in this we have this circulated to the class. And before we solve the problem, I should understand what is the way this gadget is made; only when I know how it is made, I can do the idealization. I have good animations on my screen also let us see that. And what is the problem statement this shows the crimping tool; I have simplified it I have put this as straight lines rather than curves that is given there.

The rest of the dimensions are very similar to what I have shown as the actual crimping tool. The idea is when a force of 140 N is applied at the ends. Find out the magnitude of the crimping force at this point this is what I have to find out.

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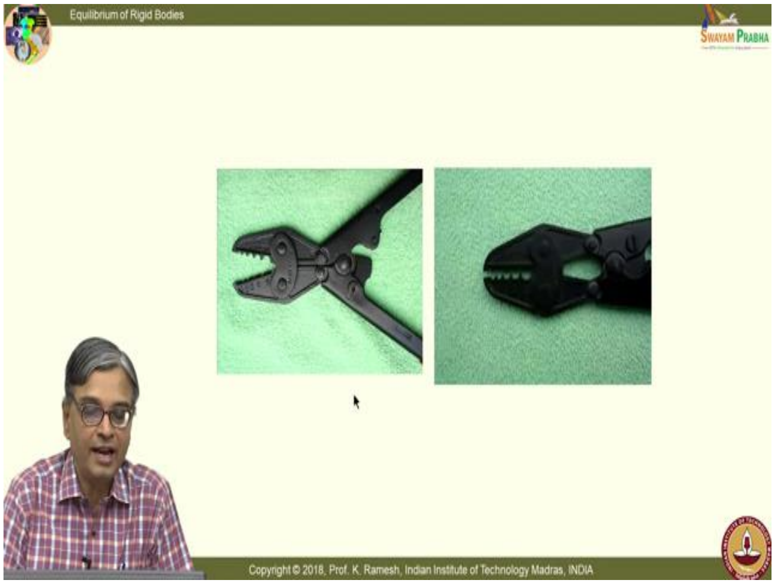
And let us understand the crimping tool; this is the crimping tool in an enlarged picture.



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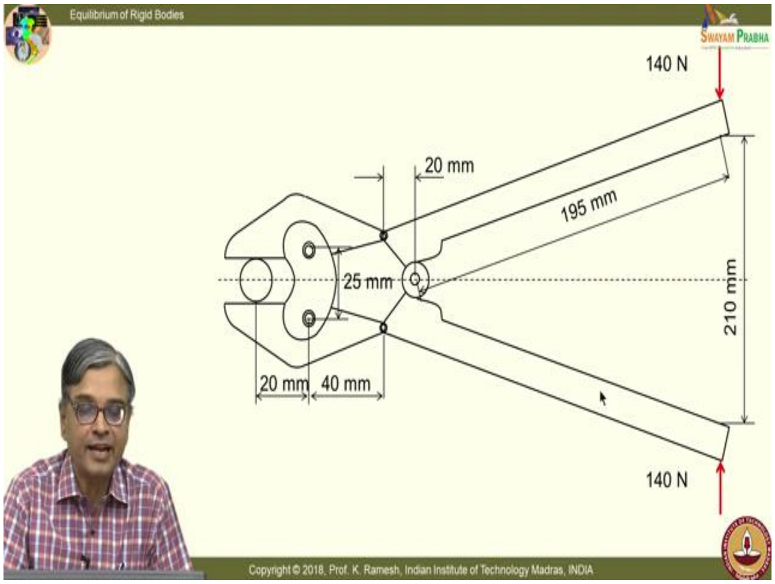
And I would also move this handle and understand what happens here; what happens here; what happens here and what happens here. You have to appreciate what is the degree of freedom those joints provide you. You can recognize that they allow rotation in the plane; when it allows rotation in that plane how do you idealize those joints. I can idealize them as pin joints; a pin joint allows rotation; you cannot get this unless you see the actual gadget verified for yourself whether it behaves like a pin joint.

Even if it is tied; you put a drop of oil you will have the situation becomes drastically different. I am not quite sure how many of you maintain your cycle. You do not care about any one of your belongings; you want somebody else to do that; that is a luxury you have in India and you will see periodically when you put even a drop of oil in any one of the joints you will find a dramatic improvement in the joint behaviors.



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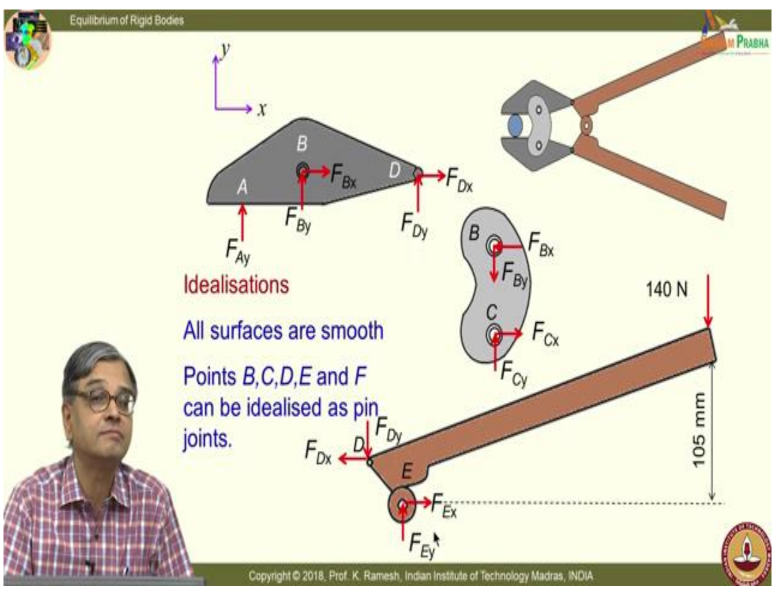
So, the idea is I have joints here and these joints allow rotation in that plane and you are justified in idealizing these joints as pin joints. You can make a sketch of this diagram it gives the geometric details what is the distance between these two joints it is given as 25 mm. And I have the wire put here in the cross section it is seen as a circle, this is need to be pressed by this tool.

This is at a distance of 20 mm from this joint and the distance between these two points is given as 40 mm. And on the handle, you have one pin joint here, you have another pin joint here and this is at a distance of 20 mm. And then the length of handle is given as 195 mm and for this configuration the distance between the handles at this end is given as 210 mm and the force that is applied is given as 140 N.

So, you have the geometry of the system available. We have also looked at physically what is the way the crimping tool behaves. We have made ourselves clear and ensured that joints shown here, shown here, and shown here. All of them allow rotation in that plane. They are reasonably smooth. So, they could be idealized as pin joints; that recognition will come only when you see a physical system and that is the reason why I keep telling you; you learn engineering by looking at systems around you. You must ask a question how it is made, why it is made that way; both these questions are very important. And I also take a conscious effort to show physical systems either in the form of an animation or a photograph and in this class, we have a luxury of solving a practical problem.

Let us also label these points so that we could easily identify how to discuss the force interaction at these points. I have labeled it in one way I put this as the *A, B, C, D, E, F* like this. So, you follow an ordered sequence do not put the label randomly, because I can easily recognize this triangle member as *ABD* and this member as *BC*; it is easy for discussion. So, label them carefully, follow some kind of an ordered sequence.



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And this shows what are the elements in the system. I have one link like this, I have another link like this, I have the third link like this and we have also labeled them. Let us put those labels here; I have this as *A*, *B*, *D* this is between *B* and *C* and this is *D* and *E*.

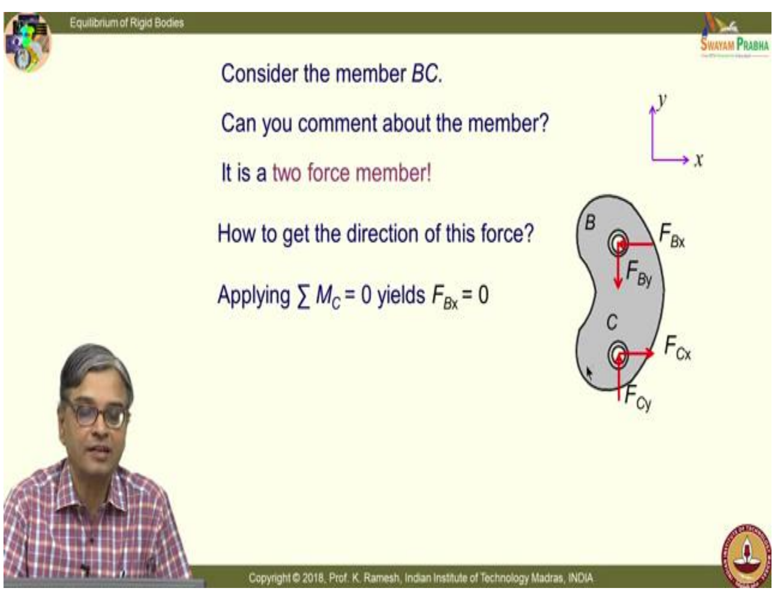
And we have discussed enough that the joints at *B*, *D*, *C*, *E* behave like pin joints. All surfaces are smooth; it goes without saying. In many of the problems we would solve without friction it makes our life simple. As I told you if you have a cycle which has not smooth running just put a drop of oil, it will improve the frictional conditions.

So, we have looked at the joints *B*, *C*, *D*, *E* and *F* behave like pin joints. I have a circular object here on a flat surface. So, this interaction is nothing but what you have seen as a roller support; I can replace this interaction by a single vertical force. And I put this along the positive direction of the *Y* axis, it also satisfies for the physics of the problem.

How do I put the force interaction at point *B D* and so on? Points *B* and *D* behave like pin joints. So, I will have two unknowns I do not know what direction it is; I have 2 unknowns. So, I will put this as F_{By} and F_{Bx}. Because the problem is little involved, I just put these unknown forces in the positive direction of the coordinate system. Now I can also replace the force at point *B* on this link.

If I assumed it in this fashion, I have to necessarily write it in the opposite direction here this is one place students can make a mistake do not close your eyes be alert. When the body is interconnected you have a freedom to choose for one of the connections to start with. Once you have assumed what happens in this link you have no choice on how to put these forces this is very important and I can put the interaction at point *D*; I have F_{Dy}, F_{Dx} at point *D*.

Similarly, I have F_{Cx} and F_{Cy} at point *C*. Since the handle is connected at *D* whatever the forces, I have put on this link has to be reversed on the handle at point *D*. And similarly, I have used it for the other joint. So, what you need to look at is at this stage I seem to have too many unknowns, there is definitely symmetry in the problem. I end up looking at 1 plus 2; 3 plus 2; 5 plus 2; 7 unknowns.



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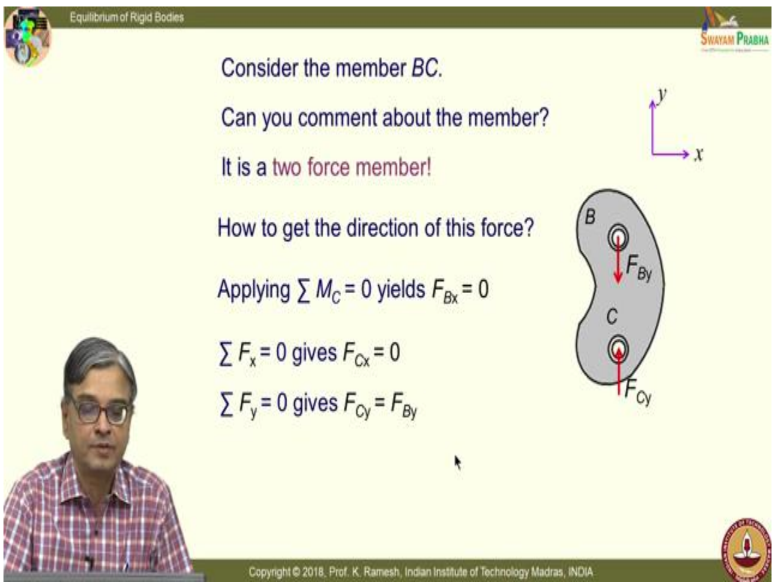
And how many equations I have? I can write three equations each for each of the links. At this stage the problem looks as if it is not easy to comprehend, but it is not so. We have already looked at certain recipes looking at this as a two-force member, three force members so on and so forth. We will go back and see whether this could be simplified.

Now I take a member *BC*, I have force interaction at point B, force interaction at point *C* like this. And let me put the equilibrium conditions. Before I do that, I recognize that this is a two-force member. In a two-force member the forces have to be collinear equal and opposite; that is how it can remain in equilibrium. I apply the conditions



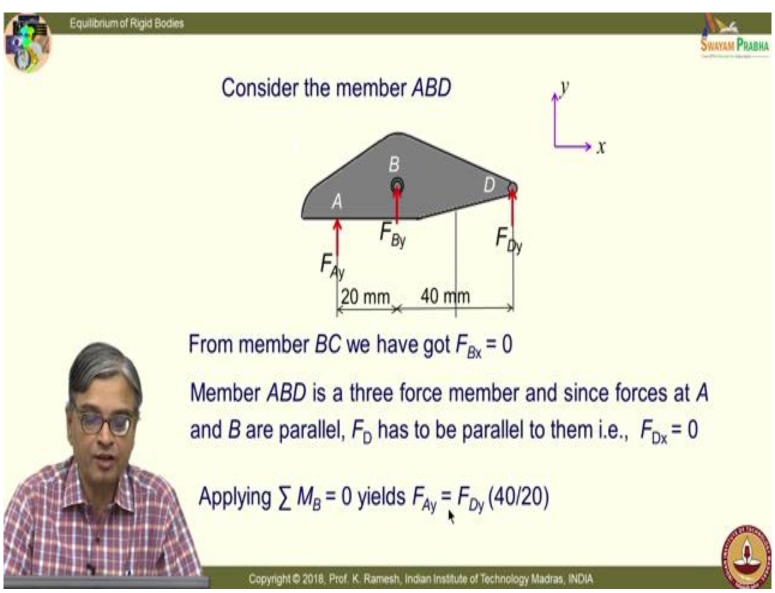
$$\sum M_C = 0 \text{ yields } F_{Bx} = 0$$

I take moment about this point; moment equilibrium should be satisfied. So, I get this

$F_{Bx} = 0$. So, this automatically tells me what should happen

$$\sum F_x = 0 \text{ gives } F_{Cx} = 0$$

$$\sum F_y = 0 \text{ gives } F_{Cy} = F_{By}$$



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So, I am now left with only the forces F_{By} and F_{Cy}; it is a two-force member. The forces are collinear equal and opposite only then this link is under equilibrium. Now I translate this back to the other member *A*, *B*, *D*.

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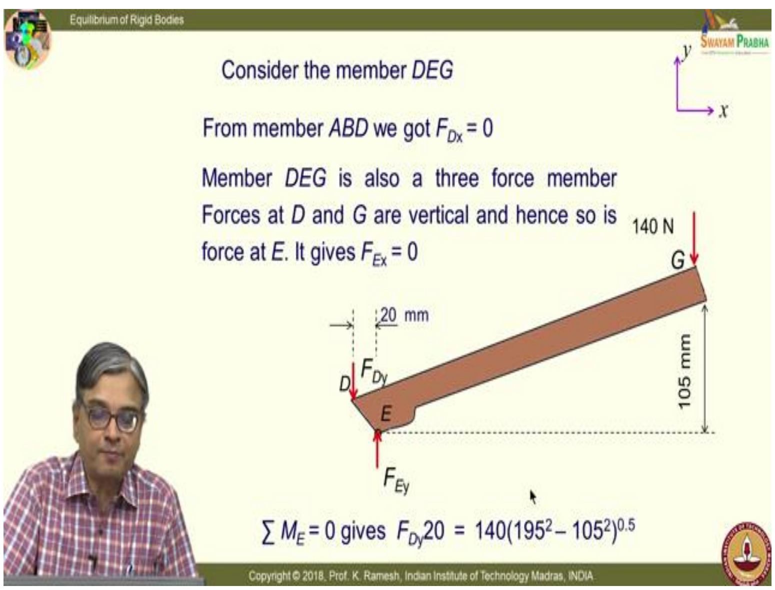
So, I get this as $F_{Cy} = F_{By}$. When I go to this link *ABD*, I had a force interaction at A shown like this. We have understood what happens at joint *B* by looking at member *BC*. What is it that we have learnt? F_{Bx} is 0. So, I get this as only this force. I do not know what is happening at point *D*. Can you comment by looking at this free body diagram what should be the force interaction at *D*; will I have both horizontal and vertical components?

You have to give me a reason; without a reason you cannot justify that. I have to get the key terminology. See I have neglected the weight of this member I am not considered the weight at all. It has force interaction at points A, B and D. So, what do you recognize; this as a three-force member. When it is a three-force member. under what condition can it remain in equilibrium; two forces are parallel to each other.

Obviously, the third force has to be parallel to that, but it should be at a particular proportion; that proportion we will do mathematics and find it out. But you get a very important clue when this is a three-force member; the third force has to be parallel to the other two forces they meet at infinity. So, this gives me a very interesting result $F_{Dx} = 0$, applying $M_B = 0$ yields

$$F_{Ay} = F_{Dy}\,(40/20)$$



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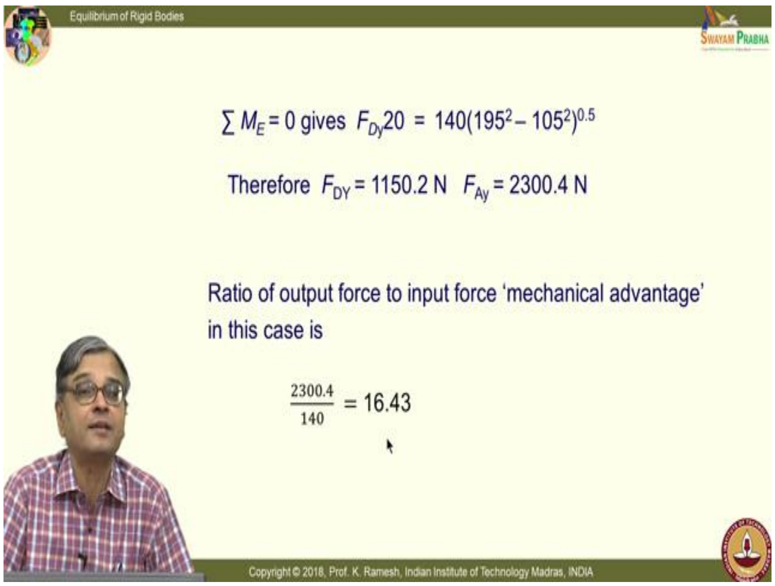
Now, consider the member *DEG*, here again I have this force is vertical, this force is vertical and this interaction has to be parallel to that. So, I will have this also as a three-force member, so I will have $F_{Ex} = 0$, I will have only this. So, I have the free body diagram like this, I know what is 140 N applied here. So, I can find out what is the value of F_{Dy} from this. This acts like a lever whatever Archimedes invented the lever system; we exploit it and here it has a multiplying effect I have two such leavers put in a simple tool it has a very high magnification.

So, I have this moment about E gives me F_Dy as the value like this.



$$\sum M_E = 0$$

$$F_{Dy}20 = 140(195^2 - 105^2)^{0.5}$$

$$F_{DY} = 1150.2\text{ N} \quad F_{Ay} = 2300.4\text{ N}$$

Once I know F_{Dy} , I can go back to the previous link and find out what is the force acting at the leaves of the crimping tool; that was the question.

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So, I have this $\frac{2300.4}{140} = 16.43$.

Ratio of output force input force which is known as mechanical advantage. See I am consciously trying to bring you certain engineering terminologies through these examples. This is a course that you do in your engineering degree. So, you should learn some of those terminologies by solving these problems.

And what I have this as magnification is of the order of 16.43. It is very high and if I appropriately take this dimension, I can have magnification of the order of 100 easily by having a hand tool. And this is used repeatedly by electrical engineers for clipping their electrical terminals.

So, in this class we have taken three example problems starting from a very simple problem which you all would have solved at some point in time earlier. The focus was to emphasize you should write down idealizations as a first step in solving the problem. Draw a neat free body diagram put all the forces appropriately and definitely put the reference axis. I have shown a right-handed coordinate system. Please put the appropriate coordinate system when you do the problem solving.

And I have also emphasized that when I have a problem do not get satisfied that you have taken a complete system and whatever the results that you have got; go back and verify the solution is indeed correct by verifying the sub assembly equilibrium only then the conditions become sufficient. Then we moved on to another problem where you have to determine what is the condition that you should specify at point A, when you have to push the roller. Finally, we solved a very practical problem that also gives you a sense of joy in solving a practical problem so that what you learn in the class is directly applicable.

Thank you.