Engineering Mechanics
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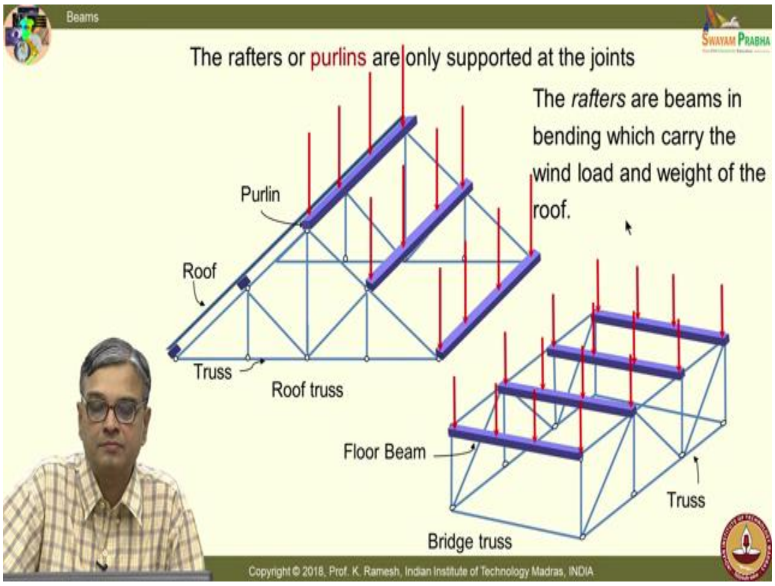
Module – 01
Statics
Lecture – 10
Beams – I



Let us move on to our next chapter on Beams.

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You know we have already seen this animation we will we have seen it in the context of trusses, now we will refocus our attention on the beams. First striking feature what you get he is the cross-sectional dimensions are much smaller than the length of the member, and they are also called as rafters and you should notice that they are supported only at the joints supported at the ends.

They are the simplest beams that you can analyze and what is the kind of load that, this experiences essentially transverse load. You have the beams used in a roof truss; they are also used in a bridge truss. I have water known as floor beams, they are again supported only at the ends, they are very simple to analyze there are also beams that are supported in between we will see a classification of them in little by later.

Here again the loading is transverse to the member. What you have to notices a beam supports transverse load; the cross-sectional dimensions are much smaller compare to the length.

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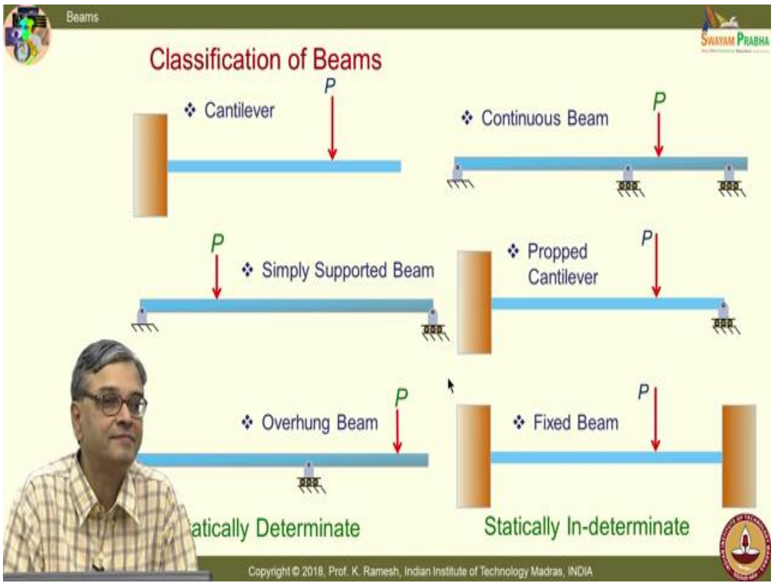
And you have a various classification of the beams right now we should be able to identity, what is the support that I have shown on this. You can identify that this as a fix



support and I have a slender member like this, I have an end load and this has a particular name this is called a cantilever.

You have many examples of cantilever that you come across in your surroundings. One simple example is you a sun shade. Many of the sun shades in the building are essentially cantilever, and look at the branch of a tree; branch of a tree is again a cantilever and there is also something very interesting in the cases of a branch. How does the cross section of the branch vary along its length; very intelligent. God has understood engineer mechanics much better than you and I learn.

You have that paper so, that where you need higher cross section to withstand the bending moment you have a higher cross section, it is not like a constant cross section that I have shown here. So, nature is very intelligent and you call this beam as simply supported. You should be able to identify what is the kind of end support I have used, please note that this is only a symbolism you should know how to interpret the symbols what is the striking difference between this support and the support shown here. The difference is in the rollers; there are many ways books represent these supports.

So, I have tried to show different forms of them so, that you can interpret the symbolism whenever you come across a problem for any book. Such a beam is very simple to analyze and you also call that as simply supported and I have a slight difference between the beam shown here and the beam shown below. What is done here is the support on the right is moved inside and you have the beam precluding out of the support. Have you come cross the any such practical example in your life, in your surroundings? Because I have always been emphasizing that you are trying to analyze systems around you. That is the purpose of this course.

How many of your swim swimmers? No, we are all very studious people, we never go near sports, that is very unfortunate and you know if you have gone their swimming pool, if you find the person is stepping on to these spring board to die. The support may not be this far away this may be closer to this. So, you have a practical example where diverse when they jump into a swimming pool you have a over hung beam either a over hung beam or it could be a cantilever depending on the construction that they have made in that place and this is called an overhung beam.

So, there are different names given to these beams based on the way supports are located and how the supports are made. I have a fix support here, I have a hinged support here or a pinned connection, I have a roller support here, I have in this beam I have a pinned connection here, I have roller support shown in the rest of the beam. And you have to give a name to this, what is the kind of name the people have coined; I have many supports and people call this as a continuous beam. See for every type of beam we come cross you can always find the practical structure around you. Only then you lean engineering.

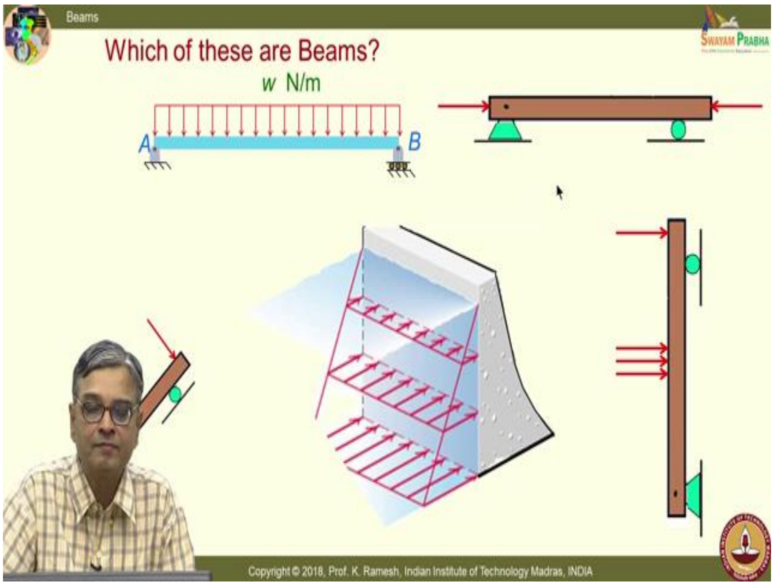
Can you identify a practical structure which everybody has to use that convenience when you go home, when you travel by a train please also note the rails how the rails are supported and you have this support at periodic intervals; you have not three you have thousands of supports, and not only that people also go and qualify those supports as beam on a elastic foundation. So, people can go very close to reality when you go for higher level courses.

In this level of the course we say everything is rigid. We say the supports are the again rigid in the sense they do not sink whereas, in a practical rail, the floor is not rigid you also accommodate the elastic behaviour of the soil beneath you. I have another example here what is the difference between a cantilever and what I see here? The end is not free, but the end is supported. Can you tell me a practical example that you normally come across; you have a big bungalow and you have a big portico. Many of the big porticos they will have a support at the end.

And I have another type of beam, I have shown this as a fixed support I also have another fix support you call this as a fixed beam. See I have the luxury of putting the beam as the nice rectangular piece you can simply write it as a line. If you can closely look at the way I have listed the beams, do you think that you can learn something more from the way they are classified from whatever you have leant earlier.

See in this course we assume everything as rigid and I use equilibrium equations, and this is a planar situation I can have only three equations written; so, I can determine only three unknowns. So, one of the basic exercises that you will do for all problems is whether the problem is solvable from equations of statics and you call it by the name statically determinate. I have given you the clue. Can you apply your mind and find out how the beams are listed in this how they can be classified? What do you have in this segment and what do you have in this segment? Even if you take one example on the left, one example on the right you can find out the classification.

When I have a; let us take a simply supported beam how many unknowns I have here? I have two and I have one, I can write three equations I can solve for all of them. On the other hand, when I go here how many unknowns I have; I will not be able to solve it from equations of statics, I have to bring in the deformation behaviour. So, that I can write one more equation I need as many numbers of unknowns, I should have as many numbers of equations; so, the way the beams are listed here. This set can be classified as statically determinate and this side can be classified as statically indeterminate.



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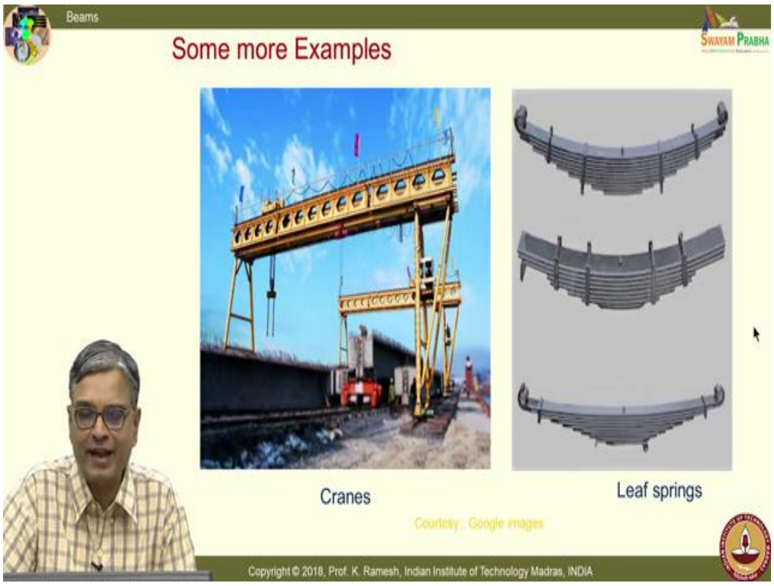
And I have shown a variety of members here and you have to tell me which of these are beams. In the previous slide we saw everything as horizontal members, I have a horizontal member like this, but I have a vertical member here, can you call this as a beam? Yeah, 50 percent of you say you can call it as a beam and 50 percent of you say you cannot call it as a beam.

Because we have seen in the previous slide all beams are horizontal. But I asked you to notice one important fact when we looked at the roof truss and the bridge truss which also had rafters and floor beams, the loading was transverse to the beam. See in English, if you go say what is a beam, I can consider that as a light beam or I can also say somebody with a beaming smile; the word has multiple meanings.

So, in the context of engineering mechanics a member that supports transverse load is called the beam. The member can be vertical like this, but still the load is applied transverse to this. The member could be inclined like this, but the load is transverse to this. One of the common confusion students have is whenever they look at a member which is horizontal, they do not think, they jump on to an immediate conclusion that it is a beam.

I have clearly shown that this is supporting only an axial load; you do not call that from engineer mechanics point of view as a beam. And you know recently Kerala is in state and all dams are overflowing they are sending out water; can you extend your knowledge of whatever you have discussed as a beam to what you see in a dam. I can see the dam cross section like this and I would have a water column applying a force here and this you know very well the pressure increases as you go down, you have a basically a triangular loading acting on the dam.

So, this is one practical example where you have a distributed loading acting. And I said when I do an analysis of any civil engineering construction the self weight is so important cannot be ignored. How do I model that? That is what is shown here, I have a beam, I have a uniformly distributed load; this is nothing but accommodating the weight of the beam as a load acting on the beam. See the books in order to train you to handle distributed load gives different types of distributed loading to get a mathematical practice.



But if you look at from a practical stand point, you will come across either uniformly distributed. In many cases representing self weight of the beam or you will have a loading which is triangular indicating the load that could come from a liquid column. So, we should be comfortable in handling uniformly distributed load and triangle loading, you cannot escape out of it they are very very common and you should be comfortable in handling this.

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And you have many other examples; you have a crane this is acting like a beam. And you should also this surprise to learn that there are springs which are used in automobiles; if you see a huge truck, if you look at the real wheel, it is supported on a structure like this. See, normally you have seen a spring which has coiled like this that is a normal string that you come across, this also behaves like a spring, but you have to analyze the members subjected to bending they are called leave springs they are practical examples.

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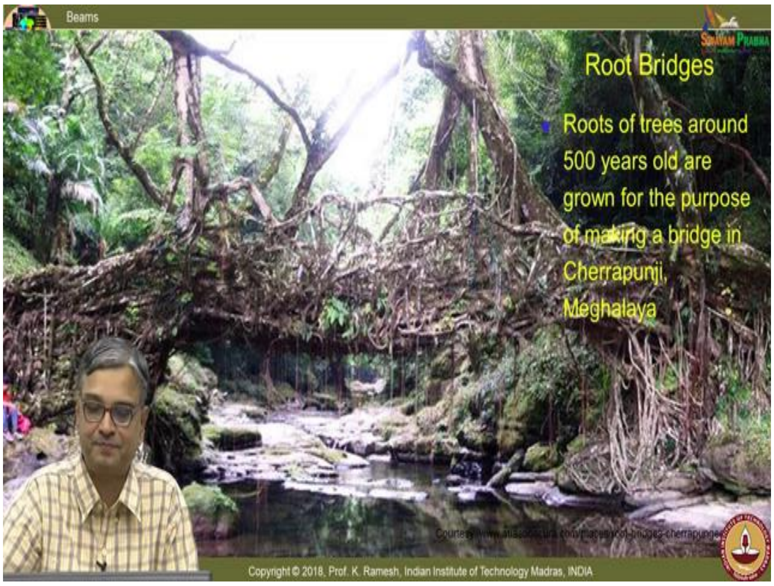

Then we move on to some fun facts we have seen that Romans where good in building aqueduct and we recently visited this place it is called a Mathoor hanging trough. This is the longest trough bridge in Asia at the height of 115 feet and I have this side view of the bridge, you could see very tall columns supporting the bridge and essentially an aqueduct. See this is taken at a time and there are no floods in Kerala; so, there is no water in the bridge that we had seen where that gives you an idea where you are at about 115 feet from the floor and this is the beam of 115 feet long.

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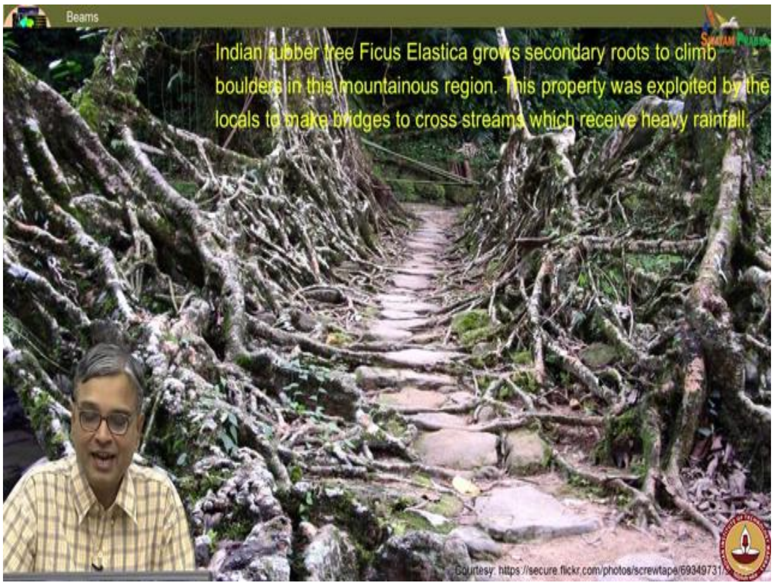
And there is also another interesting bridge, this bridge is made of a root. Roots of trees around 500 years old are grown for the purpose of making a bridge in Cherrapunji, Meghalaya; which is known for the heaviest rain fall in India. And the locals were very clever they had made this and you can understand some solid mechanics point of view, engineering mechanics point of view whenever I have a transverse loading their structure behaves like a beam.



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And what is the tree is, is actually an Indian rubber tree Ficus Elastica grows secondary roots to climb boulders in this mountainous region. This property was exploited by the locals to make bridges to cross steams which receive heavy rainfall. It is a local answer to a human need and mind you these bridges stay for 200 years whereas man made bridges life span is 75 to 100 years.

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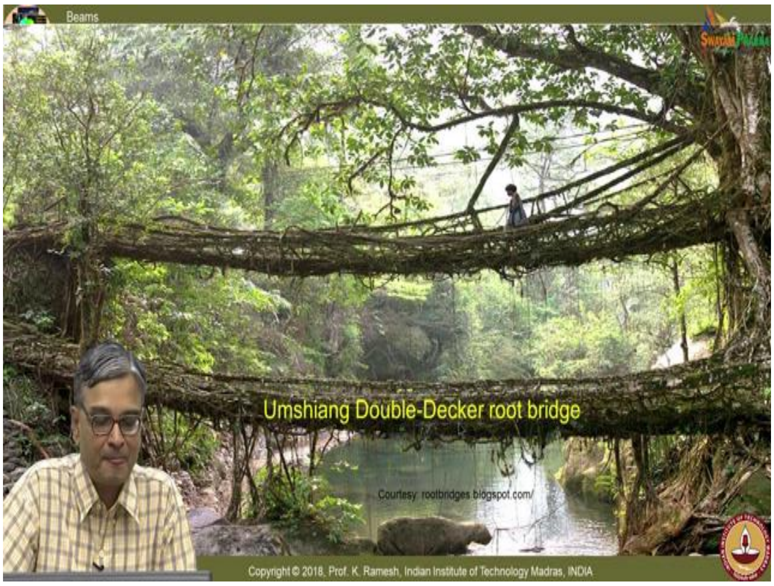
The difference is here only humans can walk that is how the bridge is constructed and I was also very surprised they had multilayer bridges. So, this is the bridge in Umshiang and it is a double decker root bridge. So, when there is heavy flow of water, they will use the top bridge and the other one in the bottom bridge.



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And there is also another very interesting bridge here, this is at Dwarka. This is the

Dwarka temple you have this Gomti river. This is a suspension bridge; what is interesting is, very similar to the roots that you have seen, you have array of cables they are needed to support this hanging bridge.

We read beam separately, we have seen the roof as well as the bridge a combination of truss and floor beams. And here you have a combination of a beam and cables and we will also see a combination of trusses, floor beams and cables all of these are needed to support transverse loading. See this bridge are short bridge may be about the 500 meters on Gomti river, supported by strings as well as your bridge you have the pier.

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And one of the very famous bridge which people would like to go and see when they want to tour the world, this is a golden gate bridge at San Francisco. And what is interesting to note here is, this is a very long bridge of 3 kilometers across 3 kilometers and from a distance can you perceive this as a slender member, this also gives you some ideas about idealizations. If I want to find out the natural frequency of this the bridge, I could consider this as the slender beam and get first level my results will be far away from the final result, but may be acceptable as a starting figure for you to aid your thinking.

Suppose I take a closer picture of it, I do see that what I see as the slender member is actually a truss; I have this truss which you have seen and this is suspended by cables. And, one of the records show they have used 1 lakh 30000 kilometers of wire to support this bridge main boggling number fine, and it is a massive structure you will feel it only when you travel on this.

So, you have the feeling of travelling on the bridge you can see these cables supporting this thus a massive peer which is supporting this suspension, and what I want you to notice is when we see here the board. The board would say there is a maximum speed limit of 45 miles per hour. See the idea here is when you design bridges you will also have to load them carefully.

See after the bridge was established in the 1960s they had a 50th year celebration and they had invited people around to come and visit the bridge without their anticipation about a lack of people lender on the bridge, bridge was not designed for the rate it is a suspension bridge. So, you develop some structural problem of the mid span of the bridge. So, they have now decided for the 75th year they are not going to invite public because loading is very important, fine and this is on the Pacific Ocean that is separating the San Francisco bay and this and they have designed the bridge to withstand wind speeds of 160 kilometers per hour.

See the scientific community learnt a lesson from Tacoma Narrows Bridge that collapsed because of wind forces and it appears it can accommodate a swing of about 8 meters.

Imagine, if there is going to be a swing like this you should not get on to the bridge and then walk. See, now Kerala is in spate all the bridges where you have; they all have over flying water beneath on tendency for human beings to go and watch and it is having a heavy current of water beneath. And imagine thousands of people stand on the bridge you will also have to go with the water. So, do not try out all of this is the stunts.

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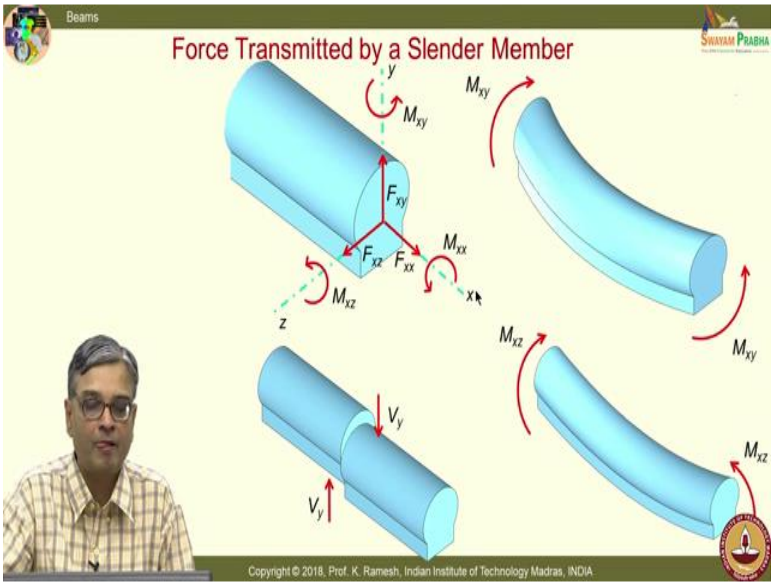
And I would also like to show you another interesting application where the technology is going you all feel that glass is brittle and this is a 70-foot cantilever bridge at Grand Canyon. Say a scientific development is so good that we have mastered how to make bridges out of glass, they are very strong; you can temper them and you use some kind of a coating and fibers and so on and so forth you have this.



And what you will have to look at is you see the canyon beneath you very clearly and they have also said that it has a good section. So, then you can easily walk, you can also see people working on top of it. Of course, this is not meant for your car to travel on this; human being can walk over it, to that extent you know this is the achieved and you find that glass is sued. Glass is also attractive because there is a green material. If glass is broken, I can melt it, use it for some other purpose.

So, that is how all civil engineering constructions are now heading towards too. You go to any mall you will find railings are replaced by glass unless you understand glass from mechanics point of view you cannot employee them there.

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Let us now get onto our understanding of a slender member. You know I have shown small portion of the member imagine that this member is very long, the cross-sectional dimensions are much smaller compared to its lengths and I am also taken a member which has a plane of symmetry.

I have also shown the axis, I have the x axis along the access of the member y and z axis. Suppose I pass an imaginary plane through this slender member, what are the forces that their slender member can transmit? We have to understand that and there is also a symbolism that is used in higher level studies they use the symbolism like this. I could have a force acting on the surface of the member.

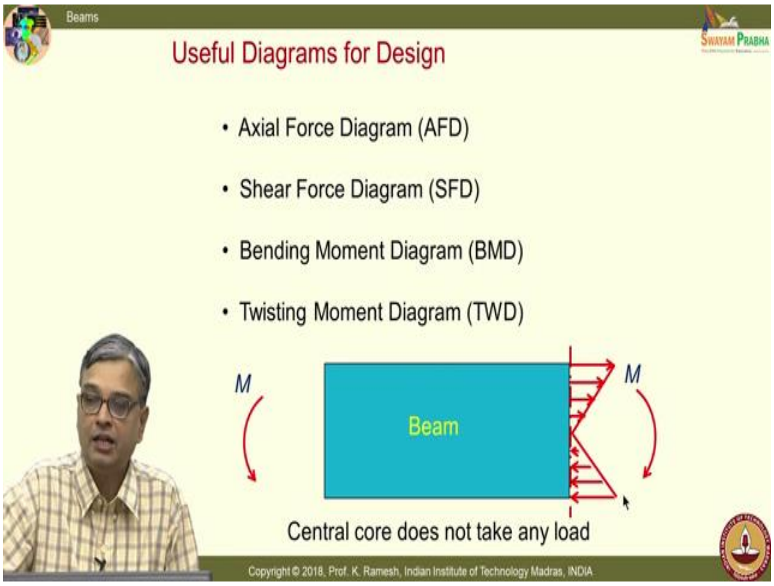
This is denoted by two subscripts; first subscripts denote the plane on which it is acting. So, you can very well see that the surface is actually the x-plane, plane is indicated by the outward normal and the force is in the direction of y. So, that is why this put as F_{xy}, this is the notation that you will come across in your second level of course, but in this course, this is simply represented as capital V with a subscript y indicating in the y direction there are multiple symbols used in text books.

My interest is to expose you to these symbolisms, you have to know that for you to interpret when you come across, I would like you to make a need sketch and put these forces. I could also have a moment like this when I have a moment like this, the member can bend in this plane I have a slender member this moment will bend it. See only in this course you make a distinction between what is the moment that causes bending of the member, what is the moment that causes twisting.

I think in your earlier learning you have always associated moment as a twisting moment, you have never even bothered about whether a moment can produce bending, if I have the moment acting on the axis the member will twist. So, I could have a moment like this, this is denoted as M_{xz}. So, z is the direction this is the positive direction that I have here, this will cause bending in xy plane. I have this member; I have bending in this plane or bending in this plane ok, when I apply the bending moment anticlockwise it will bend like this, when I apply the bending moment clockwise it will bend like this.

I could also have a bending moment denoted as M_{xy}. So, when I have this member it was initially bending in this plane, now I can also bend like this or like this in the horizontal plane. Then finally, I can also have a force in the x-direction and the twisting moment. The moment M_{xx} is also a moment has the same units as Newton meters, because it does the action of twisting the member you qualify that as a twisting moment; you also call this as a torque. I think in your study of a physics, you have worried about torque and you have always associated moment to a torque; you are never associated the moment to bending like this or bending like this.

In higher studies you will have two subscripts one of the first subscript denotes the plane and the second subscript denotes the direction. So, a slender member; in fact, can support three forces; it can transmit three forces as was three moments. If I have to design a slender member, I need to know what are the forces acting on it and I would like to find out the critical location so, that I can design the cross section appropriately. So, I essentially need to know the variation of these forces not at one section how does it vary along the length of the member.



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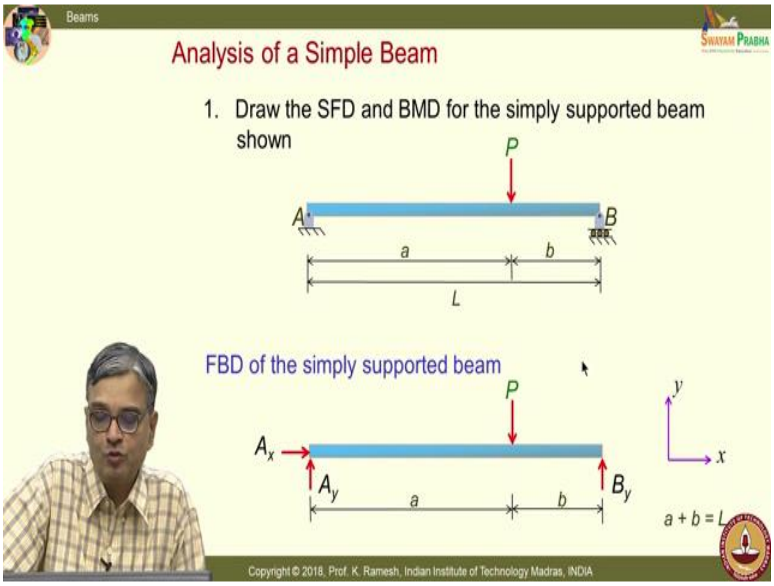
That is what you get from diagrams like axial force diagram we had seen F_{xx} is acting along the axis of the member. You may want to see how does it change from section to section along the length of the beam, we call that as axial force diagram. I could also have a diagram which indicates the variation of shear force, it could be F_{xy} or F_{xz} either of the two. Then I would also be interested in knowing the variation of the bending moment along the length of the member, I call such a diagram as bending moment diagram. And, I may also be interested how does the twisting moment varies from cross section to cross section.

So, the idea of this chapter is to learn how to plot of the four two of the diagrams; shear force diagram and bending moment diagram. And in this what is important I have a beam like this. Suppose the beam is subjected to a bending moment like this, you could see I am applying a clockwise bending moment fine. And, how do you find what happens the top fiber this is obviously, not rejected, it is a very flexible member; I am not a superman, but still I can show the deflection so easily here with small forces this is bending like this.

Can you see something happening to the top five or physically and something happen in to the bottom fiber physically, what happens to the top fiber; it is stretched and the bottom fiber is compressed. This is what I had shown in your earlier discussion when we are comparing a truss and the beam; in the case of a beam their forces, the stresses developed vary in a triangular fashion. One half of it experiences tension another half of it experiences compression.

I have shown the bending moment like this clockwise suppose I have the bending moment anticlockwise reverse of this would happened. So, from your design point of view more than the sign of the bending moment you are worried about the magnitude of the bending moment that is very important. The magnitude is very important for your design purpose that is what you are really looking at. Nevertheless, when I want to draw the diagram along the length of the beam, I should have both the magnitude and direction of that beam bending moment properly denoted that requires a systematic training; we will see the nitty gritty details now.



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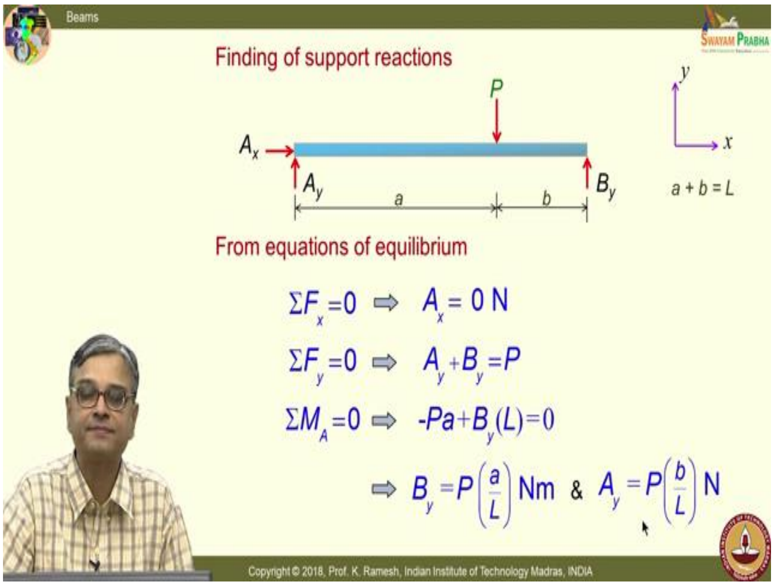
Let me take a simple beam I have a beam supported on a pin joint and a roller support. Here also given the dimensions, you could replace this bar as a simple line for your nodes. And, one of the first steps in analyzing this problem is to determine the reactions, it is also given the distances a and b are such a plus b equal to the total length L of the beam. Let us write the free body diagram of the simply supported beam, you should put the coordinate axis and replace the supports by the forces which we have already learnt; how do I replace a pin joint, I do not know the direction of the resistance. So, I denote this by a horizontal component and a vertical component.

See earlier we have put this as a reaction and put it as R_{Ax}, R_{Ay} and so on, it will be two cumbersome to write like this I have labeled this as A so, you can also simply write it as A_x and A_y. And how do I replace the support at the; it is a roller support so; I have only one reaction B_y.



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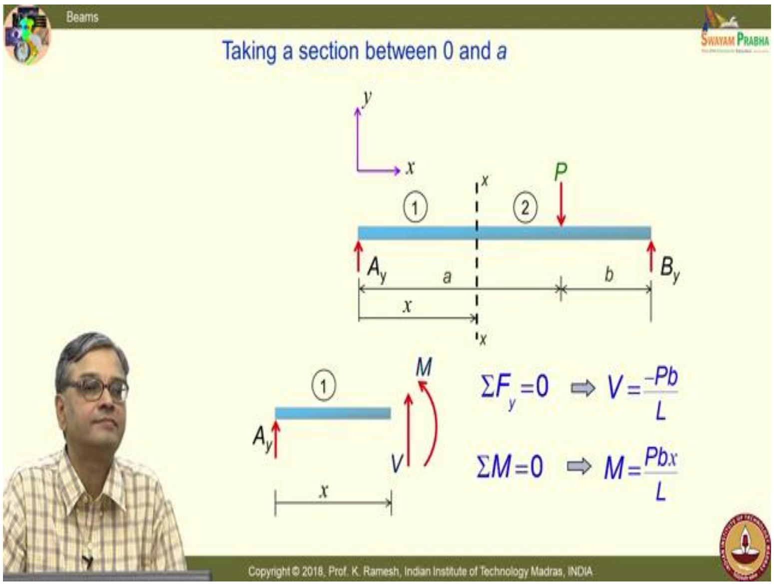
So, I can use the equations of statics

$$\Sigma F_x = 0 \Rightarrow A_x = 0 \text{ N}$$
$$\Sigma F_y = 0 \Rightarrow A_y + B_y = P$$

and I can take moment about any point I take it about point A. When I take it about point A I have the force P and I have the reaction B_y. And what is the way force P gives the bending moment, it is clockwise; I would like you to visualize that it gives the clockwise bending moment and the reaction B_y gives the anticlockwise bending moment when I look from point A. So, I get this as minus P a plus B y into L equal to 0, I get this reaction at B as

$$\Sigma M_A = 0 \Rightarrow -Pa + B_y(L) = 0$$



$$B_y = P\left(\frac{a}{L}\right) \text{N}$$

$$A_y = P\left(\frac{b}{L}\right) \text{N}$$

In fact, once you are experienced you should be able to determine this reaction just by inspection. When you are learning it in the initial stages, you will write $\sum F_x = 0$, $\sum F_y = 0$, $\sum M = 0$; after solving 10 problems, you should be able to write them by inspection because this is very simple ok.

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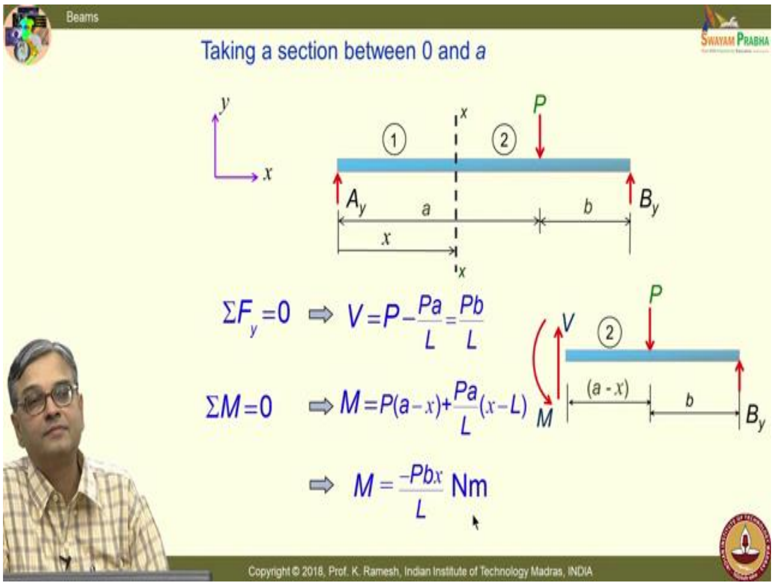
I have to find out what is the bending moment. So, I taken arbitrary section through beam between 0 and a, I have taken the origin here. So, between 0 and a I will take a section; so, that will tell me what happens in this section. If I take another cut between a and a + b, I would be able to find out what is the general variation in the section.

Now, let me make an imaginary cut at a distance x from the point *A*. I call this as section 1 and section 2 and I draw the free body diagram. When I say the free body diagram, I should put what force the slender member is transmitting. We have already seen in general it can transmit three forces and three moments. Here the problem is simple; so, it will be essentially transmitting a shear force and a bending moment.

And in your earlier discussion we have said this is a unknown I have the reference axis like this, I could represent the unknowns as positive quantities to start with I can put the shear force *V* like this and we have followed a convention anticlockwise moment is positive and clockwise moment is negative. I do not know what is the magnitude of this, what are the directions. To start with I put this as in the positive direction. My mathematics will tell me whether the assumption is right or wrong when I analyze the free body, but I just follow the principle fine.



So, when I do this, I get

$$\Sigma F_y = 0 \Rightarrow V = \frac{-Pb}{L}$$

$$\Sigma M = 0 \Rightarrow M = \frac{Pbx}{L}$$

Is there any difficulty at the stage? There is no difficulty at all. Now, let me solve the problem not by taking the section 1, let me solve the problem by taking section 2.

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I have the section 2, I can start afresh when I start afresh if I do not know that the direction assumes it in the positive direction and then proceed with it. So, let me put the shear force acting in the vertical direction like this and your bending moment is anticlockwise. Let me solve this free body diagram,

$$\Sigma F_y = 0 \Rightarrow V = P - \frac{Pa}{L} = \frac{Pb}{L}$$

$$\Sigma M = 0 \Rightarrow M = P(a-x) + \frac{Pa}{L}(x-L)$$

$$\Rightarrow M = \frac{-Pbx}{L} \text{ Nm}$$

individual free body diagrams are systemically analyzed absolutely no problem. Have we missed anything in a free body diagram, if you do not know anything you assume it in any direction your mathematics will give me the final direction ok? You will not say that whenever I solve the problem, I solve from left to right. We would solve a problem by taking a section out of the two sections which is simpler to solve we will take it and then determine the unknowns.

Ultimately, I want to find out what happens at section xx that is my interest. If my interest is only that it is simpler to handle the section 1 and get the answers, section 2 is not good for finding out for this section at a distance x. On the other hand, if I have to find out what is the variation in this segment of the beam, I can make a cut here, then, this one will be simpler to analyze then what you have it here.

So, I want to set a stage we may have to solve a given problem starting from left to right or right to left which one is simpler in handling mathematics. We had one convention if you do not know the unknown forces put them as positive and then find out the values is not sufficient; do you see the need for developing a sign convention. If you look at the answers that you have got in this and the answers that you have got earlier.

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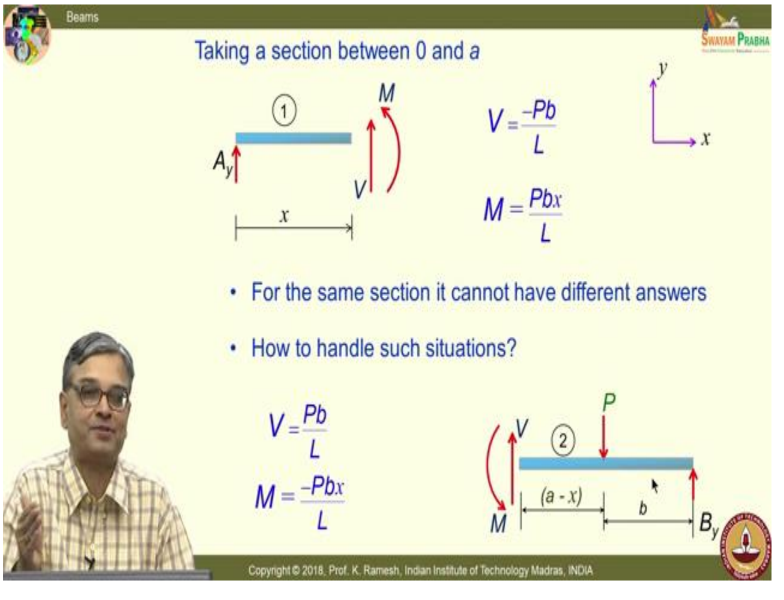
I have got the answer for this. When I summarize I get this as



$$V = \frac{-Pb}{L}$$
$$M = \frac{Pbx}{L}$$

when I analyze the segment 2 I get

$$V = \frac{Pb}{L}$$
$$M = \frac{-Pbx}{L}$$

When I finally, go and translate this on this, when I change them both the results will give me the same answer, but the question is when I want to plot a bending moment diagram or a shear force diagram how will I write this as positive or negative. Individually what I have solved in this free body diagram is correct; individually what I have solved this free body diagram is correct, but collectively we have missed a very important point.

You know I would like to spend sufficient time on the sign convention; the idea is each book follows a different sign convention. See after learning beams underneath you should be able solve problem from any book and verify your answer is correct or not with his sign convention because people rush through the sign convention. You should not rush through a sign convention you should get your fundamentals clear.

I want you to go back and ponder today: what are the ways that we could rationalize our steps. So, in this class we have looked at what is a beam, what are the different classifications of the beams, we have looked at cantilever simply supported, overhung, continuous beam, propped cantilever and also a fixed-fixed beam and we have also classified them as statically determinate and statically indeterminate.

Then we looked at several examples of what is a beam in things around you and I said of the distributed loading you should be comfortable in handling, uniformly distributed load and triangular loading. Because, this type of loadings you come across in many of the practical applications, you cannot miss them.

Then we have also looked at beam is essentially a slender member and a slender member can transmit in general three forces and three moments and in this chapter, we are confining our attention to the slender member behaving like a beam. So, we take a simpler beam for analysis the loads are in the same plane as the beam is drawn, you essentially have only a shear force and a bending moment.

And we have looked at then difficulty in an associating the sign for a given bending moment or shear force when we solve the problem from left to right or right to left, you need to have a sign convention. We have not done any mistake individually, but collectively when I want to solve a complicated problem with several loadings, I may have multiple sections, I should have a convention for me to analyze from left to right or from right to left. So that, I could identify a strategy how to solve the problem in the easiest manner possible with less mathematical computation yet, I get a result that can be interpreted without any doubt.

Thank you.