Lecture -22

Theory of mechanisms

Coupler curves -111, symmetrical coupler curves

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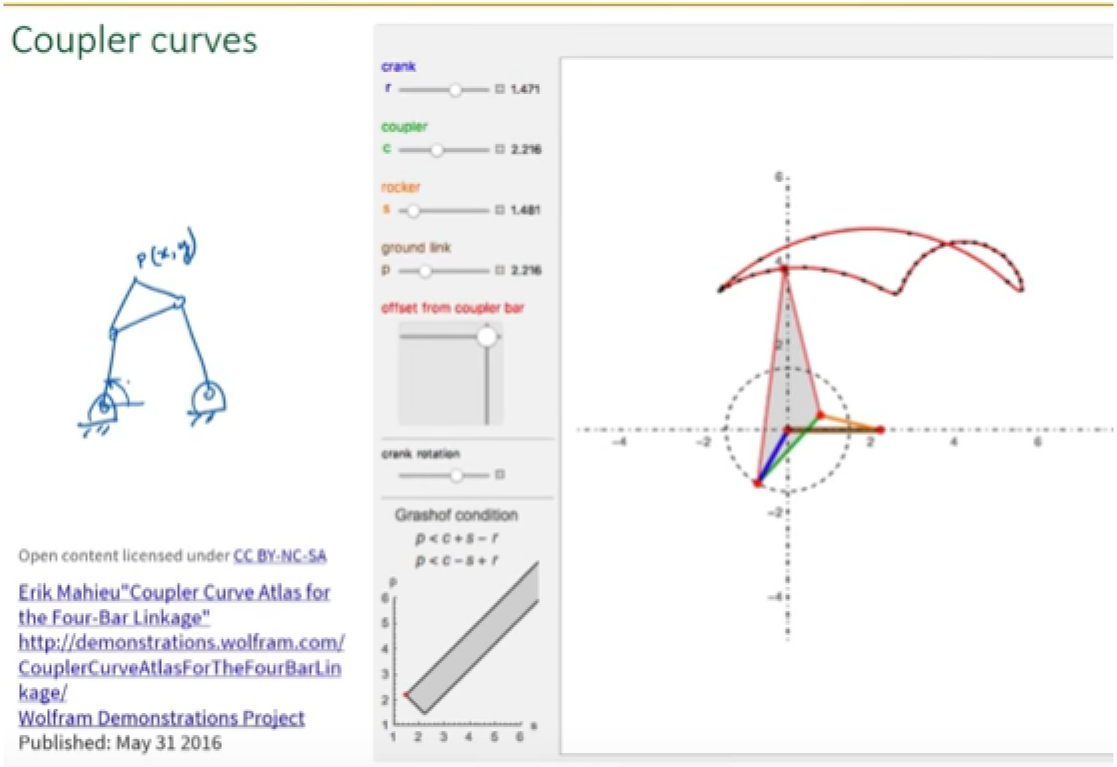

We when been, looking at the coupler curves. So, if you look at, the four bar and you try to write the equation ,of the coordinates of a coupler point ,you will find that, it is a sixth order equation, and that's the reason you can so, if I write the you know, I can get the coordinates x and y ,if I eliminate. so, write the equation of the curve, okay. Of the locus of point B, by eliminating say, the input angle, from the equation in terms of the link lengths, if I write that will get a sixth order curve, which kind of explains, why you get such complex curves?

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Cusps

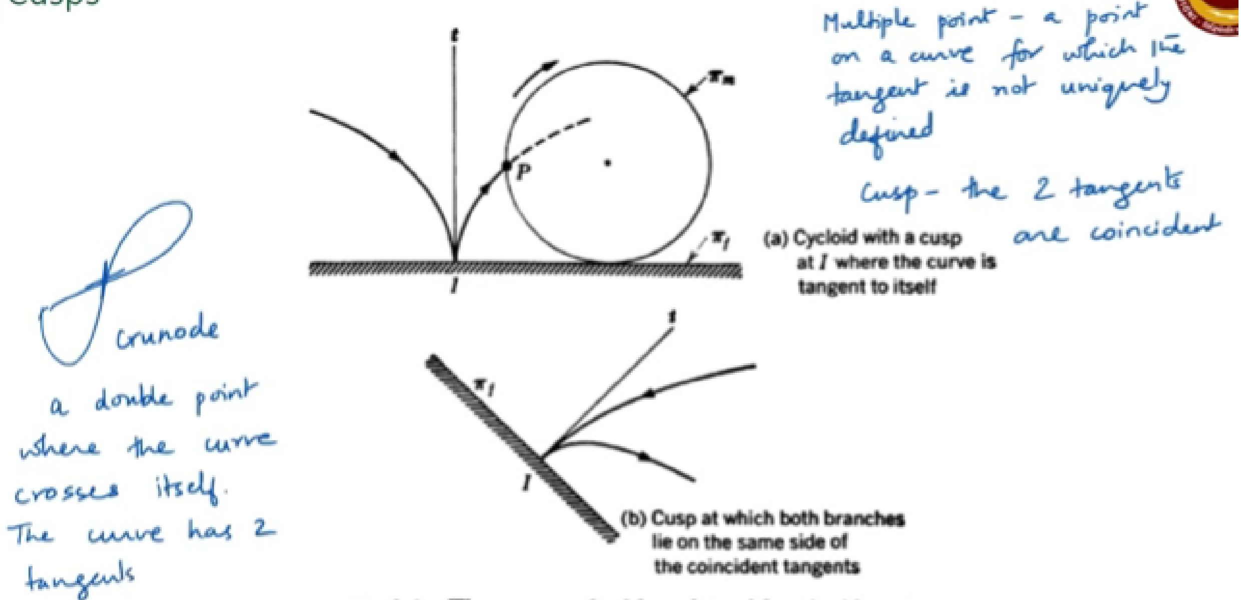

FIGURE 6-6 The cusp, a double point with coincident tangents.

So, last class we looked at, we were looking at, the concept of the cusp right. So, you have multiple points in the coupler curve, where you know multiple point, is a point where the tangent is not uniquely defined, okay?

So, a multiple point here is a point, on a curve for which the tangent is not uniquely defined. So, in the case of the cusp, for instance if you look at this as the curve, then you have this tangent, for this curve as well as, for this curves the curves at coincident, in the case of that in the cusp of a cusp the two tangents are coincident, okay .so, these are a cusp and a crew node, the crew node is that figure-eight, that we saw. So, if you say if the coupler curve has a point like this. Then again the tangent is not uniquely defined. Because, at this particular point you could have a tangent, like this or a tangent like this, okay. So, these are called double points, cusp and Cru node, in the case of the cusp, the two tangents are coincident, whereas in the case of the crow node, which is also double point. So, at which the curve crosses itself. So, crew node is a double point, where the curve crosses itself, and there are two tangents, at this point. So, the curve has two tangents, at this double point.

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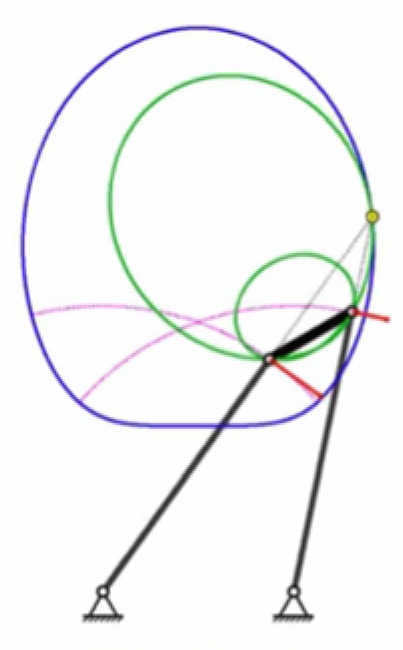
Fixed and Moving Centrodes



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so, the cusp is formed we looked at, the concept of fixed and moving cancroids in the last class, and you saw that the motion of the 4-bar, can be or the motion of the coupler, relative to the fixed link, can be replicated you know, by the same motion as the moving central, role rolling on the fixed central, and a cusp is formed so, this is another I will show you this. So, this is another case, the green one is the moving central, moving on the rolling, on the blue fixed central. Okay. So, if you choose points on the moving center. Okay. From that it follows that as you saw, with the rolling disk right, where it contacts the fixed central, where the point on the moving central, contacts the fixed central, you have a cusp, okay. So ,that is so in this case, for instance you have so this is my in position one, this is my instant Center, see one okay. The coupler point that I have chosen. Okay? Is contacting the fixed central. Because, it is the instant Center at that point, the moving centroid, when it contacts the fix central, that point is the instant Center, right. So, here for position one, you see that, that is the instant Center and you see that, there is a cusp in the path of the coupler point, okay. These will hit at different other at other points, okay. Because, at any instant there is only one Contact, between the moving centroid, and the fixed central, these are other possibilities so, so points D,E,F, also lie on the moving centroid but, they are going to contact ,the fixed centroid at other rotations of the crack. Okay, and here it probably indicates so, you have the numbers indicate, when a particular point contacts the fixed centroid, or when that is the instant Center, okay. So, add the instant that is the in **if**, if e is the instant Center, at some rotation of the crank, then there is a cusp, that's formed in the coupler curve of E.

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Cusps

If a coupler point lies on the moving centrode of the coupler,
– a cusp will form at the pt. where point and the IC are coincident
in other words, a cusp will form when a point on the moving centrode contacts the fixed centrode

(a) Four-bar linkage $O_A A B O_B$ (in position 1) and fixed centrode π_f

(b) Plane of coupler showing the moving centrode π_m

Cusps provide for reversal of motion without impact or shock.

so ,if a coupler point lies, on the moving centroid, then one a cusp will form at, the point where, point and the instant Center or coincident, in other words when, the point contacts the fixed central. a cusp will form when, a point on the moving central, contacts the fixed central, okay. The case of the crew load**,** those are all set up those are coupler curves, for these points. So, F is a point on the coupler. Okay? So, f will trace this curve, significance of cusp is ,the point the coupler point, comes in a particular direction, it is velocity goes to zero, and it reverses direction without any shock. So, if you want to do a pick-and-place operation, Okay. You want to play, it will you locate it at the cusp .okay? So, it will place it and smoothly, retract, okay. so ,there is no shock due to the velocity ,the change in the direction of the velocity .yeah,it depends so yeah, like for instance you could have this kind of a curve, or you could have one some other point, having a cusp up here yeah ,so this, this would be so, it could pick, one place and drop it at some other place ,for instance this would be. Okay? Or it could just be a pick operation. so ,something is there it picks it, it approaches it, picks it up takes it or just a place operation drops it, okay .so, it's very even for that film strip right, it pulls it down then, neatly the retracts, okay. it's just able to neatly retract change, the direction and come out of the hole, in the film strip example, that we looked at so ,that's the advantage of the cusp reversal, you get cusps, provide for reversal of motion, without impact or shock. so ,they are very useful in mechanisms for that reason, again for that you know, you would have to actually look at the whole curve ,in order to be able to if that is, the objective of your design ,then that is something where, you look at the nature of the coupler curve, itself rather than just precision points. Okay? Here we are, looking at and you can use the atlas to find curves that have a cusp, and find couple of points, that will have a cusp at a particular location. Hmm, why not? it will just contact the fixed centroid, at a different if you have like this one right, here yeah so, if you look at let's see, I don't know, if I have you can have a coupler curve, that's shaped like this for instance right. So, this is a cusp, this is also almost a cusp ,these are, these are also you know it's so, you could have so, this is probably not a very good example but, there are some where it's sharp on both sides, you have a cusp on both sides. So, it is possible so, here on this one does not I am trying to say, if this one that I can show you?

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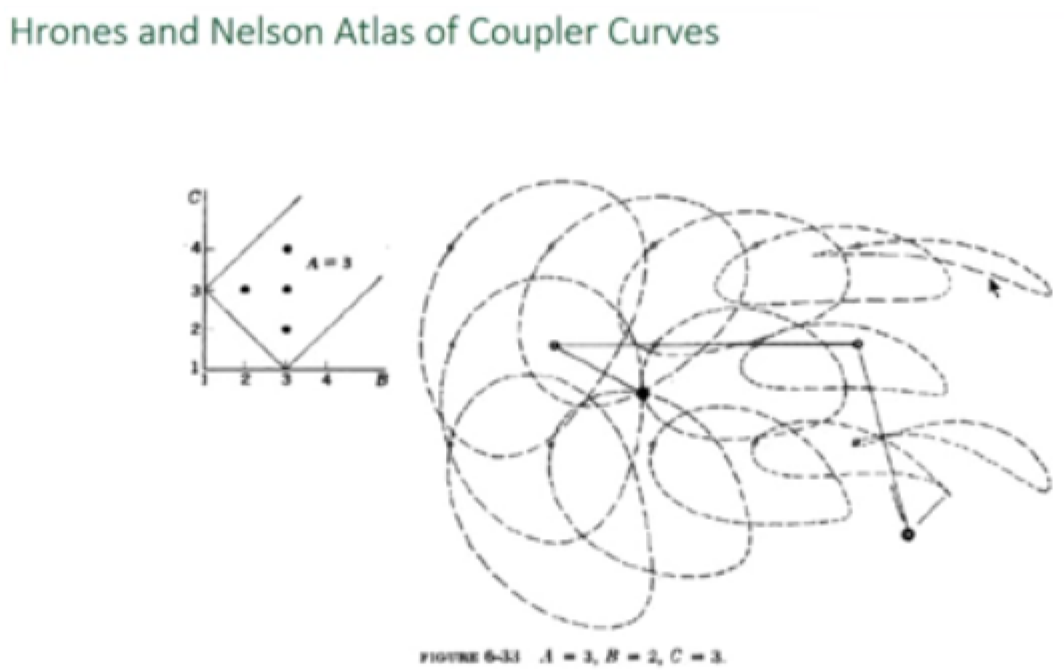

so, here for example this could be so, in terms of a perfect cusp because, it means your two instant centers will, coincide it happens only at one. so, you will have a so, pure cusp you could probably have only at one location, in the coupler curve yeah. Okay? It's a good question, yeah .so, I yeah you can because, you know a sharp that, that is your cusp right, where the point comes and there is reversal in direction. So, that's where you know, this may be close but, may not so it really depends on so, here also for instance now there it just enters right in the case of the film strip, you only have yeah, you are only using the lower portion, as the cusp. So, no yea. You are only using one. So, a pure cusp in terms of the definition yea,h it there should be only one, for a couple of curve it is intersecting here so, that means it is a contacting It, had two different instances so, it is possible then say, if you if you look at this one here, you have the point so, you have say, a coupler point on the move. See, the question is? Whether the same point will contact, the fix central multiple times, which may not happen because, you could have you be able to have multiple points, on the center which will generate cusps. Right but, is the same point, going to contact the fixed central, it will start at a point and come, come back to that yeah. So, is the same point will not because, it is yeah .so, the same point so, you can only have one cusp in the year but you can have something close to that but, probably not in the strictest definition of the cusp, you can only have one cusp in the curves, coupler curves. Okay? Because, if you pick one point on the moving central, it's going to contact the fixed centroid at, only one column is it possible for it to contact, it that's I will just double check, that just in case I will ,okay. All right, Point on the moving centered contacting the fixed centroid, at multiple places. Point yeah, that's what's happening here no? Yeah, yeah, yeah correct, Bennett yeah, nobody's asked me this question before. Okay? Yeah. So, here it is that same point, if you look at that the green curve, the red that shows the velocity .so, you can see here that it yeah. again goes to zero, and so it so, for the same point, I guess it is I will holla, holla I will have to look at That, you will have to look at that, though That is a good question. Okay? So, in this one it appears that you can have, it doesn't have to move, it has to roll at that point no, yeah. So, is that the point on the moving central? See, if you look here it is, it’s if you look at this area here, you can actually see some overlap in the two greens, they are not identical ,they are not one on top of the other, this is a point on the rocker, yes. And it is, yeah. That is, that is the magenta. So, that's basically a special point on the coupler, right because, it's yeah. It only moves along the circular arc, yeah. that point actually reverses, direction because at, at the limit ,limit positions so if you consider that as a point on the coupler, that's reversing direction, all the rockers, will have you know but ,we don't really consider that as a because, it's a it's the degenerate case, you only get a circular arck, right. So, the circular arc is basically like something with a double cusp, you know kind of collapsed that's, that's essentially what the circular arc is so, maybe that's the only so, true coupler point. Okay? So, something that is not on the input or the output link, on the crank or the follower, will probably have only one cusp. Okay? Okay. But, I yeah. I need to do some reading on that to see if but ,these two points yeah because you do have reversal of motion, even with the crank-rocker if you look at it, you have it goes to velocity goes to zero, then it reverses. So, that's what this? in this case the coincident points are but, we do not really treat those two pivots as ,couple of points because, you are not going to get anything more interesting, than a circular arc. Okay?

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Crunodes

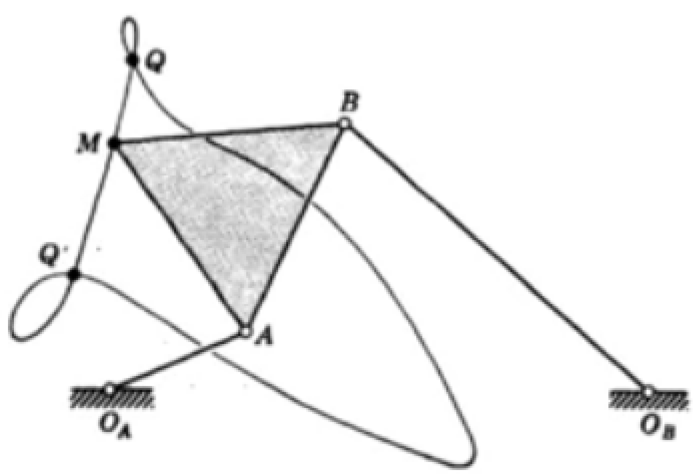

FIGURE 6-9 Coupler curve with double points Q and Q'.

So, a crew node to look at crew notes. So, these are, like your figure eight, double points where, it comes to that point and then follows a different path. so, it you know it reaches that point twice but, follows different paths each time. Okay? So, let's look at where we can locate a crew node, on the coupler curve.

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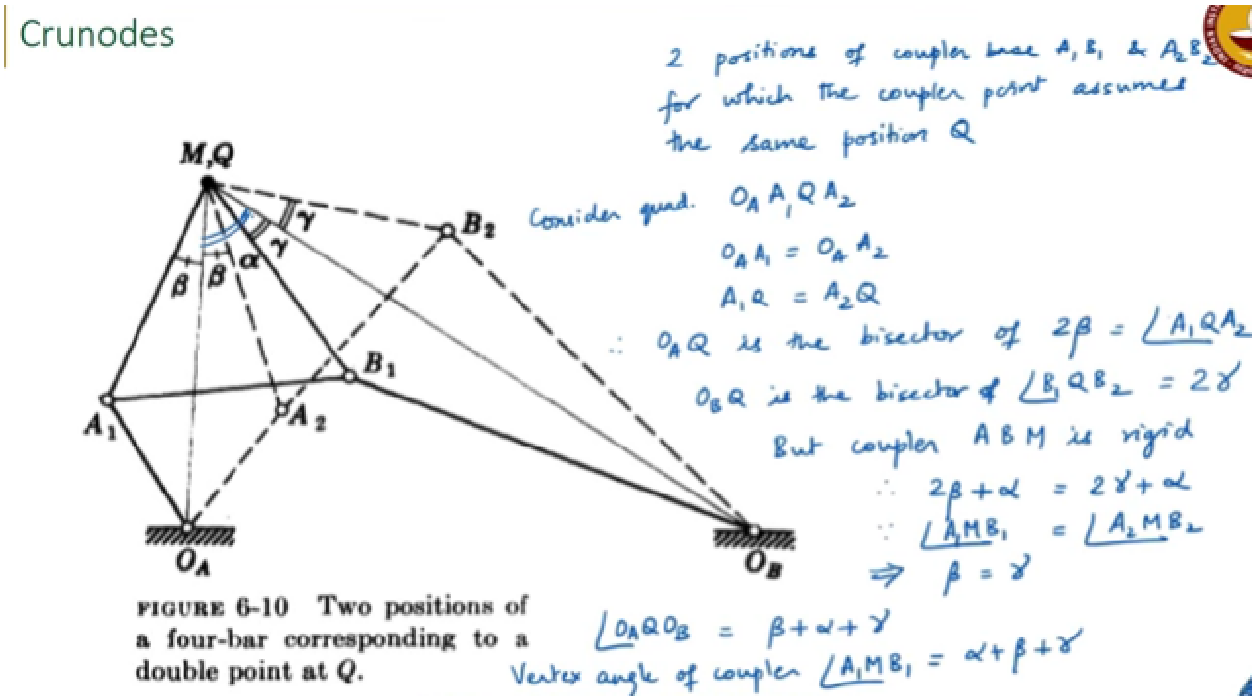

So, suppose these are, two positions I have, two positions of the crank here OA, a1 and OA,A2 and let's say, the coupler point M, is at this Q both types .okay? So, that means it's reaching Q with two different input angles essentially so, two different configurations of the linkage, give rise to the same location for that coupler point. Okay? So, these are two positions of coupler base a 1, B 1 and a 2, B 2 for which the coupler point, assumes the same position q. okay? So, now consider the quadrilateral OA , AQ ,A2, OA, A1,QA2, quadrilateral OA, A1,QA2 all right. here so, here you know OA,A1 equals OA,a2 then A1 Q which is the A1 Q equals A2Q and side OA, ms Common OA, MS is common. Rate of you Therefore, this is the OAQ is the bisector of that angle to beat them. Okay? So, similarly if I look at obq, that's the bisector of angle b 1, q, b2 .okay. Which I call to gamma. Okay? But, my coupler a1, b1, m, A,B,M is rigid. Okay? Therefore, what does it? I can just call It A, B,M therefore, this angle to beta plus alpha equals to gamma plus alpha. This is the common; this is to beta .so, if I look at, a1 because angle a1, M, B 1 equal toA2, M, b2. Therefore, beta equals gamma angle. If I look at this if I look at angle OA,q, OB. Okay? that angle equals beta plus, alpha plus, gamma yes. Okay? And if I look at the vertex angle of the coupler because, beta equal to gamma. That is angle a one, m,b1, I can also write that as alpha plus, beta plus, gamma because, beta and gamma are equal. I just showed that beta and gamma are equal therefore, that is also equal to this thing. So, this implies if I consider. so, that means this angle OA, m,OB this angle is the same as a1, M,B1 or A2,M,B2 whatever, ok. The coupler angle, vertex angle, at the point M.

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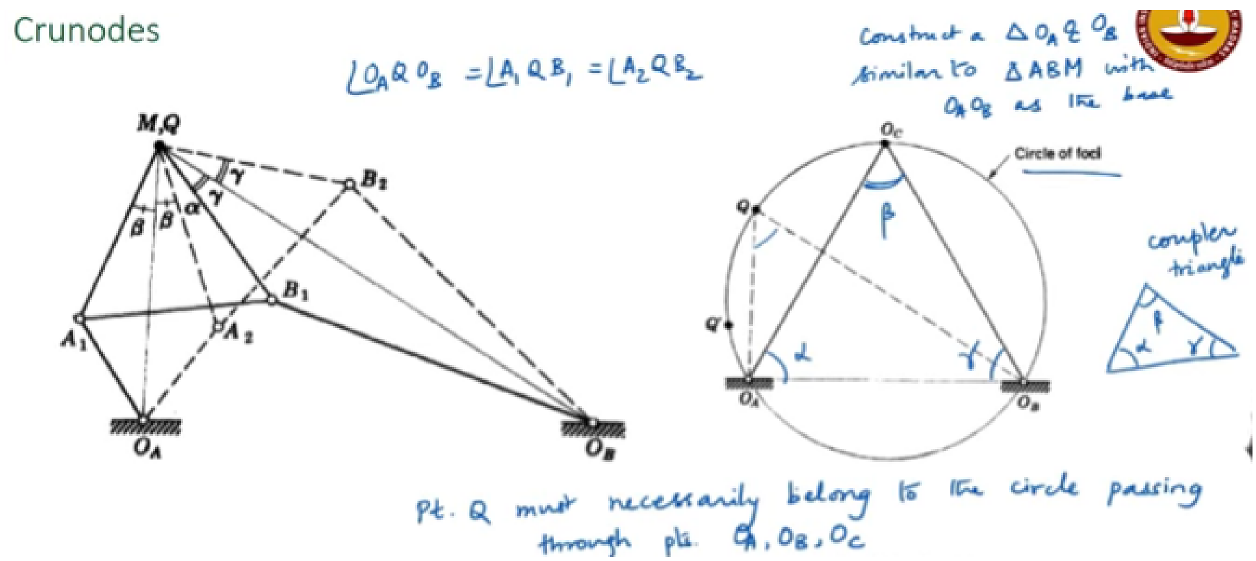

So, OA,q,OB, equals a1, Q, B1, equals a 2 ,Q, B2 angles, right. That's what I? Now, if I draw a circle. Okay? So, I construct a triangle, such that this angle. Okay? So, if I construct, if I have a circle where this is the, angle subtended at the circumference, then I can pick any point there and it will have the same angle, right. This called OA, OB will subtend the same angle at any point on the circumference of this circle, that means I can pick my point Q anywhere, on this circle. So, this circle if constructed triangle, OA, OC, OB which is similar to the dub the point triangle A, B, M with OA,OB as the base. Then, if I take any point Q, that angle is going to be the same as, this angle subtended here. Okay? So, Q must belong to the circle,Q must lie on the circle passing through OA, OB and OC, the point Q that I choose, the point Q where the crew node happens, will lie on the circle that is formed by OA, OB and OC. this, this circle is, called the circle of four cycle, it has some other significance .but, will not. So, Point Q ,must necessarily belong to the circle passing through points ,OA, OB and OC. So, where the coupler curve intersects, the circle of force I you will have a double point .okay? a double point exists at the intersection of the coupler curve, with the circle of force,a circle of singular force and it is actually called circle of singular force and the circle of singular force, I is essentially the circle passing through something that has the same so, if I have a coupler if this is my coupler, it's similar to it's a triangle that is similar to, the coupler triangle, you need a point M. okay? And if you want it so, where that point M, may have a double point is so, you have the point M. okay? You construct the circle of Versailles with OA,OB,S base and similar to a 1 ,M, b1, okay. So, if the coupler curve, of that point M, intersects this circle you will have a crew node at that point. Okay? So, here M and Q are coinciding at this you know 2.so, Q is a point on the plane. Okay? M is a point on the coupler. So, you choose a coupler triangle, the point M will have a crew node, if its couple curve intersects this, circle of singular foresight.

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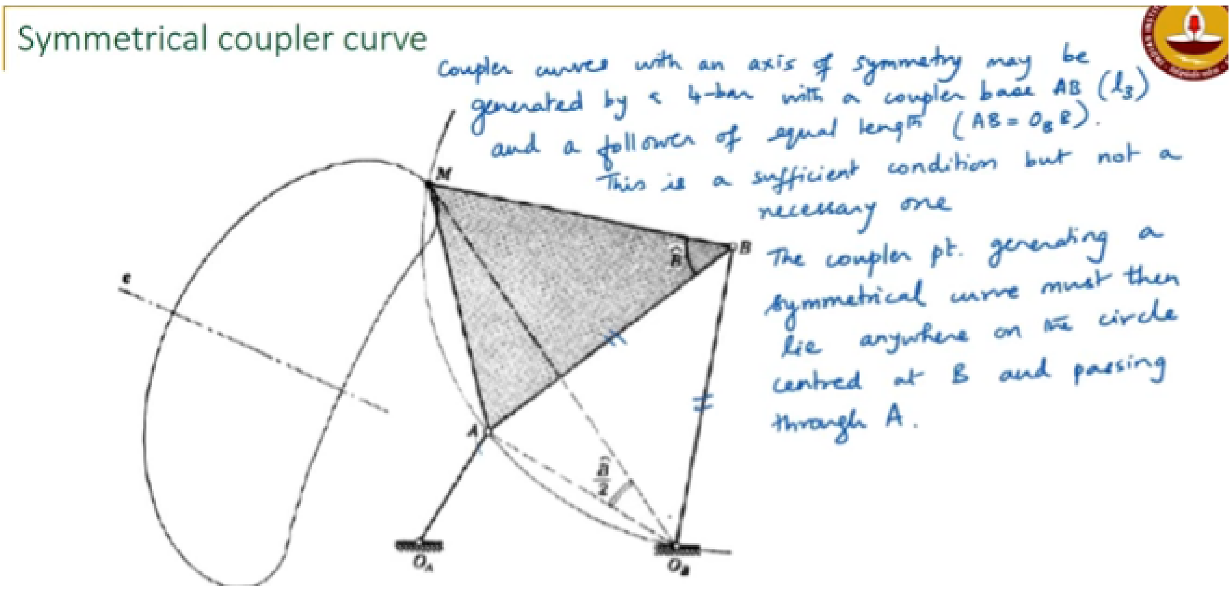

So now, let's look at the synthesis so, you can generate coupler curves that are symmetrical about an axis. Okay? And there's a fairly simple construction that can give you a symmetrical coupler curve. Okay? So, then again you know if you want to generate something that has, two cusps we have to say, that again it's an interesting come back to that. Okay? So, let's look at how we would generate, how we would design a linkage, to generate a symmetrical coupler curve. Okay? So, the condition for that is a coupler curve, with an axis of symmetry, maybe generated by a four bar with a couplet bass, AB there is your L3 in your truck. And a follower of equal length so, here I am saying AB equals OB b. okay? This is, a sufficient condition but, not a necessary one. So, there may be other linkages that can generate a symmetrical coupler curve, this is one way to generate a symmetrical coupler curve. Okay? It's not necessary, that this condition has to be met in order to generate a symmetrical coupler curve. Okay? Then, the coupler point generating a, symmetrical curve, must then lie anywhere on the circle, centered at b and passing through A. what this say, is if this equal to this. Okay? then if I create M, such that it lies on a circle that is centered at B, and passes through OB and a ,then that all those points M will generate symmetrical couple of curves. So, let's look at why that is the case .so, if you look here if you, look at this coupler triangle A,B,M. okay? This angle B cap, okay. if this, is the angle B cap, then A, M so, this is the angle that a,m ,the cord a,m subtends at the center of the circle b.okay? That means A, M subtends the angle be kept by two at OB right. Okay? Okay. So, we have established that, this angle is be kept by two. Okay? So, now let's look at so, in the previous case if you see here b,o,b equals B a therefore, the circle passes through OB also right. Okay? BA equal to this, equal to b, M. So those three are equal.

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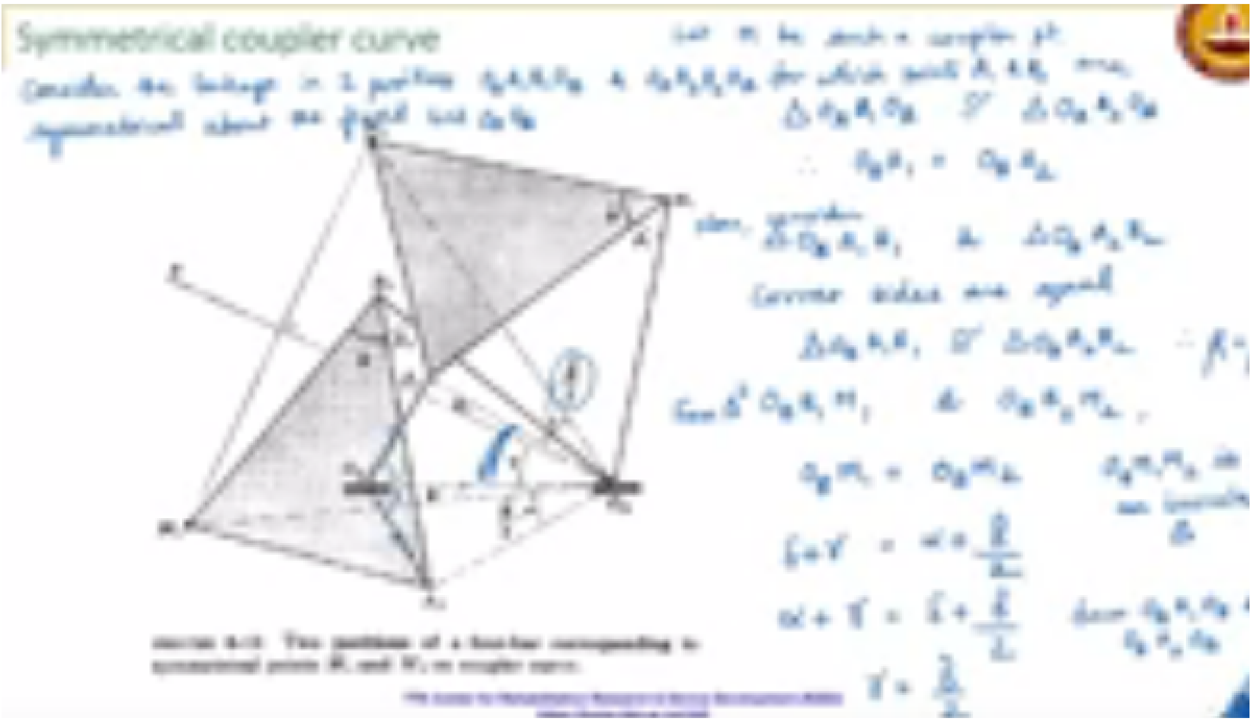

So, let M be such a coupler point, and we want to show here, that let's take the crank OA to lie in symmetrical positions about the frame. Okay? OA, OB is here, let's say for position 1, m1 is where? The coupler point is, okay. And let's say that for this position OA, A 2 when it is reflected, about thee for that crank position m2 is the point. You, want to show that C is basically the perpendicular bisector of m1, m2 because, then that will establish, it and that it makes a constant angle to the frame, if you can show that then it implies that, it is a line of symmetry ,it does not change. Okay? So, for all positions of the crank, on either side of the fixed link, you will find a corresponding you, you will find that M is reflected about this line of symmetry. Okay? So, you get a symmetrical coupler curve, okay. I have the linkage in these two positions OA, a 1, B 1, OB and OA, a 2, B 2 ,OB. ok ?these are the two positions of the linkage. Now, I want to show that, okay. Let's consider this triangle, may first show that A, M, A1 equal to OB,2, okay. O, B, M1 equal M. okay? Let me consider triangle O, B, B in this case o if, okay. If I consider these two triangles OA, A1, OB and OA,A 2, OB I am taking this, symmetrical about OA, OB this is, the common side OA,A1 equal to OA. So, these two triangles are congruent. Okay? Therefore, OB, A1 equals OB, A2. Okay? So, OB, a1 equals OB, A2. now ,let me take triangles OB,A1, M 1 is it and OB,A2, M2 that will give me, I want to show a O,B,M1 equal to a horseshoe attick OB, A1, B 1. Okay? OB, a 1, B 1 and OB, A2, B2.ok? Oh, ok. First I considered, ok .let me first do it systematically, I look at triangle OA,A1,AB is congruent to triangle OA,A2,OB therefore ,I have OB, a1 equal to OB,A2.so, these are two positions a1 ,a2 are symmetrical. Okay? I am considering the linkage because, that's important .consider the linkage in two positions OA, a1,b1, OB and OA, a2,b2, OB for which points a1 and a2 are symmetrical about, the fixed link OA,OB. now ,if I look at triangle OB, a 1, B 1, OB, a 1, B 1 and triangle OB,A2, B 2. Okay? Now, a 1, B 1 equals a 2, B 2 it's the same coupler base right? Then OB, b 1 equals o B, B 2 follower. Okay? And I have shown that OB, a 2 equals OB, a 1 because, the corresponding sides are equal. So, by SSS I have triangle OB, a 1, B 1 is congruent to triangle OB, A2, B2.okay? Now, consider the triangle OB, b1, M1 and OB, b1 sorry, will be B2, M2. OB, B1, M1 and OB, B2, m2 yes. Yeah, yeah, yeah. Oh B this is, this alleged body and you're rotating it, rotating it about this.

So, that means this we already know is this angle therefore, beta 1 equals beta 2 from these two rights. Because, again the three sides are equal, beta 1 equals, beta 2. Beta 1 equals, beta 2 I say from this itself OA, A1 where is it, OB a sorry OB, a1, B1 I have shown these two are equal right? OB, A1, B1, OB, A2, B 2 those two are equal therefore, corresponding sides of congruent triangles therefore, beta 1 equal to beta 2. Ok? So, using that because this is B plus beta 2, this is B plus beta 1 I show that, from these two triangles, from congruence of these two triangles, I have OB, m1 equal to OB, m 2. Okay?

Now, let's look at these angles. Okay? I have this line C. okay? Which makes some angle gamma, with respect to the frame? Okay? So, I have if I look at this Triangle, I have shown that OB, m 2, B 2 is congruent to OB ,m 1 ,B 1 right? So, I have this angle, Delta plus gamma this. Okay? this is Delta, this is gamma this, is equal to alpha plus B cap by 2, Because, this is now an isosceles triangle OB, m1, m2 okay. Ob, m1, m2 is an isosceles triangle therefore, these two angles it bisects that. Okay? So, I have Delta plus gamma equals, alpha plus B cap by two, then from the other one I have from because, these two are equal, I already showed that these two angles are equal, I have B cap by 2 plus this one, alpha plus gamma this angle. Okay? Equals, Delta plus B cab by 2.okay? From these two triangles, this is from OA, a 1, OB and OA, a2,OB. Okay?

If I add these two equations, I get rid of alpha and Delta therefore, I get 2 gamma equals B cap or gamma equals B cap **by** 2. B cap is a constant angle it's not an angle that's varying with time because, B cap basically locates the point M, with respect to the coupler base. Okay? since this is the rigid body, this is this angle is constant B cap is a constant angle therefore, the angle that this perpendicular bisector makes with the frame, does not change, which implies that M 1 and M 2 will always be equidistant, from this so, this becomes the line of symmetry C becomes the line of symmetry for this coupler point M 1. Ok? So, the construction of so, to find a coupler point with say a certain line of symmetry. Okay? Say, I want a line of symmetry, at a certain angle to the base; I would take the follower length to be equal to the coupler base, ok. Draw a circle that passes through, that centered at B and passes through, OB and a1. Okay? And then choose that angle, has twice the angle so, I would locate the point m1 on the circle such that it makes twice the angle, as the angle of symmetry as the line of symmetry. So, here this is b cap by 2 that is what I have shown, ok. So, this is one way of constructing a symmetrical coupler curve.